ONDAS ELECTROMAGNÉTICAS EN CONDUCTORES (buenos, malos) Y EN DIELÉCTRICOS

$$\boxed{\begin{array}{ll} \vec{\nabla}\cdot\vec{B} = 0 & \vec{\nabla}\times\vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \vec{\nabla}\cdot\vec{D} = 0 & \vec{\nabla}\times\vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t} \end{array}}$$

Supondremos fuentes lejanas (pero que hacen campo en un recinto). Queremos ver qué sucede en un conductor y en un dieléctrico.

Asumimos medios LIH $\Rightarrow$ $\boxed{\vec{J} = \sigma \vec{E}}$, σ constante (conductividad)

$\vec{B} = \mu \vec{H}$ $\quad$ $\vec{D} = \varepsilon \vec{E}$

$$\left(\vec{\nabla}\times\vec{E}\right) = \left(-\frac{\partial \vec{B}}{\partial t}\right) \rightarrow$$

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right) = -\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{B}\right) \overset{\vec{B}=\mu\vec{H}}{=} -\frac{\partial}{\partial t}\left(\mu\, \vec{\nabla}\times\vec{H}\right) = -\mu\frac{\partial}{\partial t}\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$$

$$\vec{\nabla}\underbrace{\left(\vec{\nabla}\cdot\vec{E}\right)}_{=0} - \nabla^2\vec{E} = -\mu \frac{\partial(\sigma\vec{E})}{\partial t} - \mu\varepsilon\frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{-\nabla^2\vec{E} + \mu\sigma\,\partial_t\vec{E} + \mu\varepsilon\,\partial_t^2\vec{E} = 0}$$

Podríamos hacer lo mismo para $\vec{B}$. Consideremos

(1) $\quad \vec{E} = \vec{E}_0\, e^{i(kx - \omega t)}$ $\quad$ onda plana en $\hat{x}$

$$\partial_t\vec{E} = \vec{E}_0 e^{i(kx-\omega t)}(-i\omega) = -i\omega\vec{E}$$
$$\partial_t^2\vec{E} = \vec{E}_0 e^{i(kx-\omega t)}(-i\omega)^2 = -\omega^2\vec{E}$$

$$-\nabla^2\vec{E} - i\mu\sigma\omega\vec{E} - \mu\varepsilon\omega^2\vec{E} = 0$$

$$\boxed{\nabla^2\vec{E} + \mu\varepsilon\omega^2\left(1 + \frac{i\sigma}{\varepsilon\omega}\right)\vec{E} = 0}$$

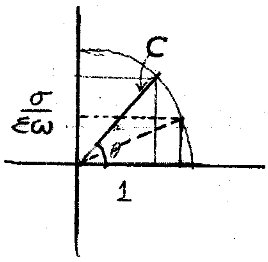
$$\nabla^2\vec{E} + \mathbb{K}^2\vec{E} = 0 \qquad \text{Helmholtz}$$

Dada (1) resultará:

$$(-k^2 + \mathbb{K}^2)\vec{E} = 0 \quad \rightarrow \quad k = \mathbb{K}$$

$$k = (\mu\varepsilon)^{1/2}\omega\left(1 + \frac{i\sigma}{\varepsilon\omega}\right)^{1/2}$$

Tenemos que hacer la raíz del complejo $\left(1 + \dfrac{i\sigma}{\varepsilon\omega}\right)$;



$$z = C . e^{i\theta} \qquad \sqrt{1+\frac{\sigma^2}{(\varepsilon\omega)^2}}\left(\cos\theta + i . \operatorname{sen}\theta\right)$$

$$\theta = \operatorname{atan}\left(\frac{\sigma/\varepsilon\omega}{1}\right)$$

$$z^{1/2} = C^{1/2} . e^{i\theta/2} \qquad z^{1/2} = \left(1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2\right)^{1/4} . \left[\cos\left(\frac{\theta}{2}\right) + i . \operatorname{sen}\left(\frac{\theta}{2}\right)\right]$$

$$\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\cos\theta}{2}} \qquad \operatorname{sen}\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\theta = \frac{1}{\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}}$$

$$k = (\mu\varepsilon)^{1/2}\omega\left[1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2\right]^{1/4}\left[\sqrt{\frac{1+1/\sqrt{1+H^2}}{2}} + i . \sqrt{\frac{1-1/\sqrt{1+H^2}}{2}}\right]$$

$$k = (\mu\varepsilon)^{1/2}\omega\,\frac{\cancel{(1+H^2)^{1/4}}}{\sqrt{2}}\left(\frac{\sqrt{(1+H^2)^{1/2}+1}}{\cancel{(1+H^2)^{1/4}}} + i\,\frac{\sqrt{(1+H^2)^{1/2}-1}}{\cancel{(1+H^2)^{1/4}}}\right) \qquad H \equiv \left(\frac{\sigma}{\varepsilon\omega}\right)$$

$$k = (\mu\varepsilon)^{1/2}\frac{\omega}{2^{1/2}}\left(\sqrt{\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}+1} + i . \sqrt{\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}-1}\right)$$

$$k = k_R + i . k_I \qquad \Rightarrow$$

$$\left.\begin{aligned} \vec{E} &= \vec{E}_0 . e^{i(k_R + ik_I)x - i\omega t} \\ \vec{E} &= \vec{E}_0 . e^{ik_R x} . \underbrace{e^{-k_I x}}_{\text{Factor de atenuación}} . e^{-i\omega t} \end{aligned}\right\}$$

- Caso Dieléctrico $(\sigma = 0)$

$$k = (\mu\varepsilon)^{1/2}\frac{\omega}{2^{1/2}}\left(\sqrt{2} + i\sqrt{0}\right) = (\mu\varepsilon)^{1/2}\omega$$

$$\vec{E} = \vec{E}_0 . e^{i(\mu\varepsilon)^{1/2}\omega x} . e^{-i\omega t}$$

$$c = \frac{\omega}{k} = \frac{\cancel{\omega}}{(\mu\varepsilon)^{1/2}\cancel{\omega}}$$

$$\boxed{c = \frac{1}{(\mu\varepsilon)^{1/2}}}$$

$$\nabla^2\vec{E} + \mu\varepsilon\omega^2\vec{E} = 0$$

- Caso buen conductor $\left(\frac{\sigma}{\varepsilon\omega} \gg 1\right)$

$$k = (\mu\varepsilon)^{1/2} \frac{\omega}{2^{1/2}} \left(\sqrt{\frac{\sigma}{\varepsilon\omega} + 1} + i \sqrt{\frac{\sigma}{\varepsilon\omega} - 1} \right)$$

$$k = (\mu\cancel{\varepsilon})^{1/2} \frac{\cancel{\omega}\,\omega^{1/2}}{2^{1/2}} \left(\frac{\sigma}{\cancel{\varepsilon\omega}}\right)^{1/2} (1+i) = \left(\frac{\mu\omega}{2}\right)^{1/2} \sigma^{1/2} (1+i)$$

$$\vec{E} = \vec{E}_0\, e^{i\sqrt{\frac{\mu\omega\sigma}{2}}\,x} \cdot \underbrace{e^{-\sqrt{\frac{\mu\omega\sigma}{2}}\,x}}_{\text{atenuación}}\, e^{-i\omega t}$$

$$\boxed{\nabla^2 \vec{E} + \mu\varepsilon\omega^2 \left(1 + \frac{i\sigma}{\varepsilon\omega}\right) \vec{E} = 0}$$

Propagación $(\mu\varepsilon\omega^2)$ Difusión $(\mu\sigma\omega)$

- Caso mal conductor $\left(\frac{\sigma}{\varepsilon\omega} \ll 1\right)$

$$k = (\mu\varepsilon)^{1/2} \frac{\omega}{2^{1/2}} \left(\sqrt{1 + \frac{1}{2}\left(\frac{\sigma}{\varepsilon\omega}\right)^2 + 1} + i \sqrt{\cancel{1} + \frac{1}{2}\left(\frac{\sigma}{\varepsilon\omega}\right)^2 - \cancel{1}} \right)$$

$$k = (\mu\varepsilon)^{1/2} \frac{\omega}{2^{1/2}} \left(\sqrt{2} \sqrt{1 + \frac{1}{2\sqrt{2}}\left(\frac{\sigma}{\varepsilon\omega}\right)^2} + i \frac{1}{\sqrt{2}}\left(\frac{\sigma}{\varepsilon\omega}\right) \right)$$

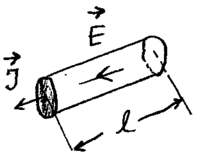
$$k = (\mu\varepsilon)^{1/2} \frac{\omega}{2^{1/2}} \left[\sqrt{2} \left(1 + \frac{1}{4\sqrt{2}}\left(\frac{\sigma}{\varepsilon\omega}\right)^2\right) + i \frac{1}{\sqrt{2}}\left(\frac{\sigma}{\varepsilon\omega}\right) \right]$$

$$\vec{E} = \vec{E}_0\, e^{i(\mu\varepsilon)^{1/2}\omega \left(1 + \frac{1}{4\sqrt{2}}\left(\frac{\sigma}{\varepsilon\omega}\right)^2\right) x}\, e^{-(\mu\varepsilon)^{1/2} \frac{\omega}{\sqrt{2}} \frac{1}{\sqrt{2}} \left(\frac{\sigma}{\varepsilon\omega}\right) x}\, e^{-i\omega t}$$

$$\vec{E} = \vec{E}_0\, \underbrace{e^{i(\mu\varepsilon)^{1/2}\omega x}\, e^{i\frac{1}{4\sqrt{2}}\left(\frac{\mu}{\varepsilon}\right)^{1/2}\frac{\sigma^2}{\omega}}} \cdot \underbrace{e^{-\frac{1}{2}\left(\frac{\mu}{\varepsilon}\right)^{1/2}\sigma x}}_{\text{atenuación}} \cdot e^{-i\omega t}$$

- Ley de Ohm microscópica

$$\vec{J} = \sigma \vec{E}$$



$$\int \vec{E} \cdot d\vec{l} = V$$

$$E = \frac{V}{l}$$

$$\int_{\Sigma} \vec{J} \cdot d\vec{S} = \int_{\Sigma} \sigma \vec{E} \cdot d\vec{S}$$

$$J.A = \sigma E.A = \sigma \frac{V}{l} A = \frac{V}{\frac{\rho . l}{A}} = \frac{V}{R}$$

con $\rho = \frac{1}{\sigma}$ resistancia (inversa de la conductancia)

$$\boxed{I = \frac{V}{R}}$$ Ley de Ohm

- Medios LIH

Lineales	$\vec{A} = \overleftrightarrow{\varkappa} \vec{B}$	$\varkappa$ es un coeficiente
isótropos	$\overleftrightarrow{\varkappa} = \varkappa \overleftrightarrow{\mathbb{I}}$	Mismo en toda dirección
homogéneos	$\varkappa \neq \varkappa(\vec{x})$	No depende de la posición