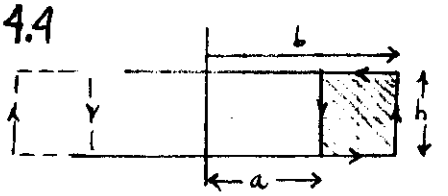
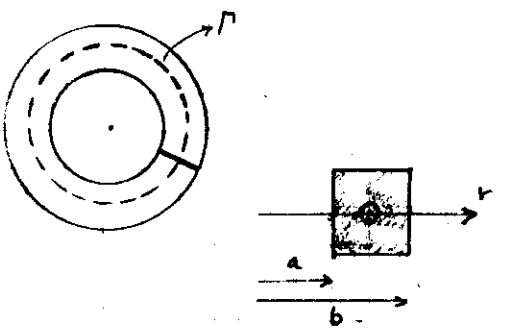
4.4



$$\int_\Gamma \vec{B}\cdot d\vec{\ell} = \mu I_c$$

$$B_\varphi 2\pi r = \mu N i$$

$$\vec{B} = \frac{\mu \cdot Ni}{2\pi r}\hat{\varphi}$$



$$\Phi = \int \vec{B}\cdot d\vec{S} = \frac{\mu\, Ni}{2\pi}\int_a^b \frac{1}{r}\, h\, dr =$$

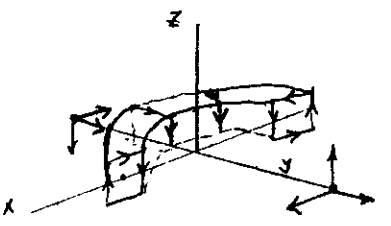
$$d\vec{S} = \hat{\varphi}\, dz\, dr$$

$$\Phi = \frac{\mu\, Nih}{2\pi}\cdot \ln(b/a)$$

$$L = \frac{N\Phi}{i} = \boxed{\frac{\mu\, N^2 h}{2\pi}\cdot \ln(b/a)}$$

La autoinductancia será

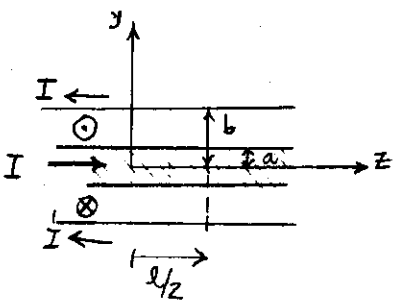
Ya supusimos $\vec{B} \neq \vec{B}(\varphi) \rightarrow \vec{B} = \vec{B}(r,z)$



Simetría de reflexión en xz $B_z = 0$, $B_r = 0 \rightarrow$

$$\vec{B} = B(r,z)\hat{\varphi}$$

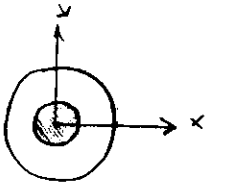
4.5



Fuera del conductor:

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\varphi}$$

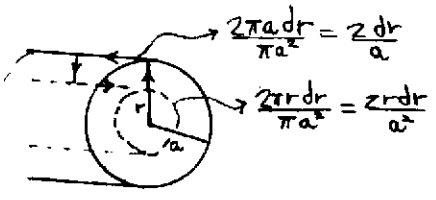
$$\Phi = \int \vec{B}\cdot d\vec{S} \qquad \hat{\varphi}\, dz\, dr$$



$$\Phi = \frac{\mu_0 I}{2\pi}\, l \ln\left(\frac{b}{a}\right) \rightarrow L_{fc} = \frac{\Phi}{lI} = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)$$

Dentro del conductor

$\vec{B} = \frac{\mu_0 I r}{2\pi a^2}\hat{\varphi}$ (del problema 4.1) Pero en anillos de radio 'dr' existe una corriente $\frac{2\pi r\,dr}{\pi a^2}$



$$d\phi = \int_r^a \frac{\mu_0 I r'}{2\pi a^2}\,dr' = \frac{\mu_0 I}{2\pi a^2}\left(\frac{a^2 - r^2}{2}\right)$$

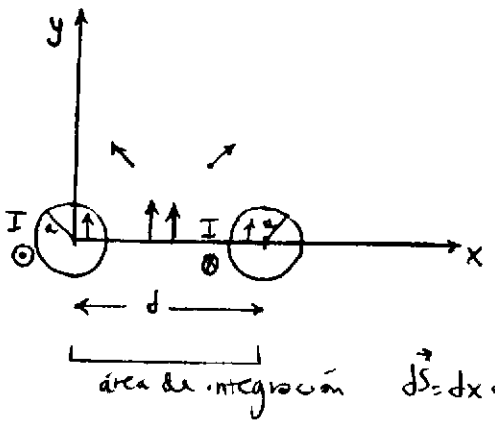
$$\Phi = \int_{-l/2}^{l/2}\int_0^a \frac{2r}{a^2}\,\frac{\mu_0 I}{4\pi a^2}(a^2 - r^2)\,dr\,dz = \frac{2\mu_0 I}{4\pi a^4}\left(\frac{a^4}{2} - \frac{a^4}{4}\right)$$

$$\Phi = \frac{\mu_0 I l}{8\pi}$$

$$L = \frac{\mu_0}{8\pi}$$

$$\boxed{L = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi}\ln\left(b/a\right)}$$

4.6



Fuera de los conductores

En $y=0$ es

$$\vec{B}_1 = \frac{\mu_0 I}{2\pi}\frac{1}{x}\hat{y}$$

$$\vec{B}_2 = -\frac{\mu_0 I}{2\pi}\frac{1}{(x-d)}\hat{y}$$

$x > a$, $x < (d-a)$

$$\Phi = \int(\vec{B}_1 + \vec{B}_2)\cdot d\vec{S}$$

$$= l\int_a^{d-a}\left(\frac{\mu_0 I}{2\pi}\frac{1}{x} + \frac{\mu_0 I}{2\pi}\frac{1}{(d-x)}\right)dx$$

$$\frac{l\mu_0 I}{2\pi}\left(\ln(x)\Big|_a^{d-a} + \ln(z)\Big|_a^{d-a}\right)$$

$\int_a^{d-a}\frac{dz}{z}$, $d-x \equiv z$

$$\Phi = l\frac{\mu_0 I}{\pi}\ln\left(\frac{d-a}{a}\right)$$

$$L = \frac{\Phi}{I l} = \frac{\mu_0}{\pi}\ln\left(\frac{d-a}{a}\right)$$

Dentro de los conductores

Por cada conductor $L_{dc} = \frac{\mu_0}{8\pi}$ → $L_{dc} = \frac{\mu_0}{4\pi}$

$$\boxed{L_{TOTAL} = \frac{\mu_0}{\pi}\left(\frac{1}{4} + \ln\left[\frac{d-a}{a}\right]\right)}$$