Serie 9: Modelo de Ising. Fenómenos Críticos

1.

Modelo de Ising

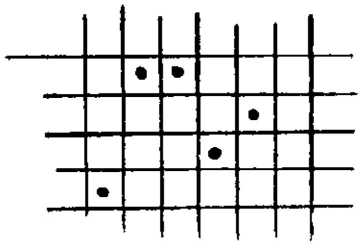
$$E = -\sum_{i=1}^{N} \mu H s_i - J \sum_{\langle i,j\rangle}^{N} s_i s_j \qquad s_i = \pm 1$$

sobre 1eros vecinos

, con campo externo nulo es:

$$E = -J \sum_{\langle i,j\rangle}^{N} s_i s_j = -J \sum_{i}^{N} s_i s_{i+1}$$

EL Modelo de lattice gas es un esquema de fluctuaciones de densidad y transiciones de fase en un sistema líquido-gas.
El espacio se divide en celdas que pueden estar



ocupadas $n_i = 1$ | vacías $n_i = 0$

Las atracciones entre partículas vecinas se tienen en cuenta diciendo que cuando se hallan en celdas contiguas su energía es $-\varepsilon$

$$E = -\varepsilon \sum_{\langle i,j\rangle}' n_i n_j$$

← un estado de energía en particular (N celdas)

N es el tamaño del sistema: No la cantidad de partículas

$$Z_C = \sum_{\{n_i\}}'' e^{\beta\varepsilon \sum'_{\langle i,j\rangle} n_i n_j} \quad \text{, con vínculo } \sum_i^N n_i = N_a$$

$$Z_{GC} = \sum_{N_a=0}^{\infty} z^{N_a} \sum_{\{n_i\}}'' e^{\beta\varepsilon \sum'_{\langle i,j\rangle} n_i n_j} = \sum_{\{n_i\}=0,1} e^{\beta\mu \sum_i^N n_i + \beta\varepsilon \sum'_{\langle i,j\rangle} n_i n_j}$$

Haciendo el cambio de variables:

$$n_i = \frac{s_i+1}{2} \qquad \begin{aligned} n_i = 0 &\Leftrightarrow s_i = -1 \\ n_i = 1 &\Leftrightarrow s_i = +1 \end{aligned}$$

$$\beta\mu \sum_i^N \frac{s_i+1}{2} + \beta\varepsilon \sum'_{\langle i,j\rangle} \left(\frac{s_i+1}{2}\right)\left(\frac{s_j+1}{2}\right) \Rightarrow$$

$$Z_{GC} = \sum_{\{s_i\}} e^{\beta\mu \sum_i^N \left(\frac{s_i+1}{2}\right) + \beta\varepsilon \sum'_{\langle i,j\rangle} \frac{1}{4}(s_i s_j + s_j + s_i + 1)}$$

$$Z_{GC} = \sum_{\{s_i\}} e^{\frac{\beta\mu}{2}\sum_i^N s_i + \frac{\beta\mu}{2}\sum_i^N 1 + \frac{\beta\varepsilon}{4}\sum'_{i,j} s_i s_j + \frac{\beta\varepsilon}{4}\sum'_{i,j} s_j + \frac{\beta\varepsilon}{4}\sum'_{i,j} s_i + \frac{\beta\varepsilon}{4}\sum'_{i,j} 1}$$

$$\left\{ \begin{array}{lll} \sum_i^N 1 = N & \sum'_{i,j} 1 = \sum_i^N \sum_j^{\gamma_i} 1 = \frac{\gamma}{2} N & \sum'_{i,j} s_i = \frac{\gamma}{2}\sum_i^N s_i \quad \text{\} smart sums} \\ & \sum'_{i,j} 1 = \gamma N & \sum'_{i,j} s_i = \gamma \sum_i^N s_i \quad \text{\} dumb sums} \end{array} \right.$$

smart {dumb}

$$Z_{GC} = \sum_{\{s_i\}} e^{\frac{\beta\mu}{2}\sum_i^N s_i + \frac{\beta\mu}{2} N + \frac{\beta\varepsilon}{4}\sum'_{i,j} s_i s_j + \frac{\beta\varepsilon}{4}\sum'_{i,j} s_j + \frac{\beta\varepsilon}{8}\gamma \sum_i^N s_i + \frac{\beta\varepsilon N\gamma}{8}}$$

$$\left\{ \frac{\beta\varepsilon}{4}\gamma \sum_i^N s_i + \frac{\beta\varepsilon}{4} N\gamma \right\}$$

$$Z_{GC}^{\text{Lattice}} = \underbrace{\left[\sum_{\{s_i\}} e^{\left(\frac{\beta\mu}{2} + \frac{\beta\varepsilon}{4}\gamma\right)\sum_i^N s_i + \frac{\beta\varepsilon}{4}\sum'_{\langle i,j\rangle} s_i s_j}\right]}_{Z_c^{\text{Ising}}} \left(e^{\frac{\beta\mu N}{2} + \frac{\beta\varepsilon N}{4}\gamma + \frac{\beta\varepsilon}{4}\sum'_{i,j} s_j}\right)$$

N es una constante (el tamaño de la grilla)

Hemos considerado las sumas Dumb (así obtuvimos el resultado presentado en la bibliografía)

Para Ising teníamos

$$E_\mu = -\sum_i^N m_0 s_i H - J\sum'_{\langle i,j\rangle} s_i s_j$$

$$Z_c^{\text{Ising}} = \sum_{\{s_i\}} e^{\beta m_0 H \sum_i^N s_i + \beta J \sum'_{\langle i,j\rangle} s_i s_j}$$

Podemos trazar una correspondencia como sigue:

	Lattice	Ising
	ε	$4J$
	μ	$2m_0H - J\gamma 2$
celda ocupada }	N_{at}	spin up
celda vacía }	$N - N_{at}$	spin down

pot. químico del lattice; energía del lattice

$$\cancel{\beta}\frac{\mu}{2} + \cancel{\beta}\frac{\varepsilon}{4}\gamma = \cancel{\beta} m_0 H$$

$$\frac{\mu}{2} = m_0 H - \frac{\varepsilon}{4}\gamma$$

$$\mu = 2m_0 H - \frac{\varepsilon}{2}\gamma$$

$$\mu = 2m_0 H - 2J\gamma$$

$$Z_{GC}^{\text{lattice}} = Z_c^{\text{Ising}} \underbrace{e^{\beta N\left(\mu/2 + \varepsilon\gamma/4 + \frac{\varepsilon}{4N}\sum'_{i,j} s_j\right)}}_{\text{Esto es una constante para el lattice porque N es fijo allí.}}$$

$$\beta PV = \ln Z_{GC}^{\text{lattice}} = \ln\left(Z_c^{\text{Ising}}\right) + \beta N\left(\frac{\mu}{2} + \frac{\varepsilon}{4}\gamma + \frac{\varepsilon}{4N}\sum'_{i,j} s_j\right)$$

$$\cancel{\beta} PV = -A\cancel{\beta} + \cancel{\beta} N\left(\frac{\mu}{2} + J\gamma + \frac{\varepsilon}{4N}\sum'_{i,j} s_j\right)$$

$$PN = -A + N\left(m_0 H \cancel{-\gamma J} \cancel{+ J\gamma} + \frac{\varepsilon}{4N}\sum'_{ij} s_j\right)$$

$$P = -\frac{A}{N} + m_0 H + \frac{\varepsilon}{4N}\sum'_{i,j} s_j$$

Habría que realizar la suma:

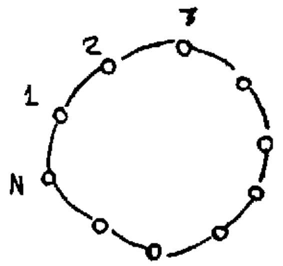
$$\frac{\varepsilon}{4N}\sum_i^N\sum_j^\gamma s_j = \frac{\varepsilon}{4N}\frac{N\gamma}{2} = \frac{J\gamma}{2} = \frac{1}{2}J\gamma$$

Lattice	Ising
P	$-\frac{A}{N} + m_0 H + \frac{1}{2}J\gamma$

2.

Ising en 1D

a)



con $s_i = \pm 1$

$s_1 = s_{N+1}$

Σ sobre 1eros vecinos

un estado de energía particular

$$E_\nu = -\sum_{i=1}^{N} \mu H s_i - J \sum_{i,j}{}' s_i s_j$$

$$- J \sum_{i}^{N} s_i s_{i+1}$$

$$Z_c = \sum_{s_i=\pm 1} e^{+\beta \mu H \sum_i^N s_i + \beta J \sum_i^N s_i s_{i+1}}$$

$$Z_c = \sum_{s_1, s_2, \dots s_N = \pm 1} e^{\sum_i^N \left(\underbrace{\beta \mu H}_{b} s_i + \underbrace{\beta J}_{K} s_i s_{i+1}\right)}$$

$$\boxed{Z_c(b,K) = \sum_{s_1} \sum_{s_2} \dots \sum_{s_N = \pm 1} e^{\sum_i^N \left(b . s_i + K s_i s_{i+1}\right)}}$$

b)

$$q = e^{\left(b\frac{(s+s')}{2} + K s s'\right)} \qquad ; s, s' = \pm 1$$

q contiene las 4 posibilidades para un spin individual y su interacción con dos vecino

$$q = \begin{array}{c|c} & \begin{array}{cc} +1 & -1 \end{array} \\ \begin{array}{c} +1 \\ -1 \end{array} & \begin{pmatrix} e^{b+K} & e^{0-K} \\ e^{0-K} & e^{-b+K} \end{pmatrix} \end{array} = \begin{pmatrix} e^{b+K} & e^{-K} \\ e^{-K} & e^{-b+K} \end{pmatrix}$$

Ahora considerando Z_c es

$$\sum_i^N b . s_i + K . s_i s_{i+1} =$$

$$\sum_i^N b . \left(\frac{s_i + s_{i+1}}{2}\right) + K s_i s_{i+1}$$

$$Z_c(b,K) = \sum_{s_1} \sum_{s_2} \dots \sum_{s_N = \pm 1} e^{\left(\sum_i \frac{b}{2} . [s_i + s_{i+1}] + K s_i s_{i+1}\right)} = \sum_{\{s_i\}} \prod_i^N e^{\frac{b}{2}(s_i + s_{i+1}) + K s_i s_{i+1}}$$

sumemos explícitamente s_1 y s_2 → correspondiente a considerar $i=1$ en $e^{\sum_i}$ ⇒

$$e^{\frac{b}{2}[1+1] + K.1.1} + e^{\frac{b}{2}[1-1] + K.1.(-1)} + e^{\frac{b}{2}[-1-1] + K(-1)(-1)} + e^{\frac{b}{2}[-1+1] + K(-1)(+1)}$$

Se puede pensar como matriz donde

$-J \uparrow\uparrow \quad \downarrow\downarrow \quad \uparrow\downarrow$

$s_i s_{i+1} = b_i$

$s_1 s_2 = b_1$

3. Ising en 1D con $\vec{H} = 0$

$$\varepsilon_\nu = -J \sum_{i,j}^{N} s_i s_j$$

Energía de un estado particular $\longrightarrow$ $\varepsilon_\nu = -J \sum_i^N s_i s_{i+1}$

con N muy grande vale esto ǂ

$$Z_c = \sum_{\substack{\{s_i\} \\ s_i = \pm 1}} e^{+\beta J \sum_i^N s_i s_{i+1}} \approx \sum_{\substack{\{b_i\} \\ b_i = \pm 1}} e^{\beta J \sum_i^N b_i}$$

cambio de variable sugerida $b_i = s_i s_{i+1}$

$$Z_c = \sum_{b_1} \sum_{b_2} \cdots \sum_{b_N} \prod_i^N e^{\beta J b_i}$$

$$Z_c = \prod_i^N \left(\sum_{b_i} e^{\beta J b_i} \right) = \prod_i^N \left(e^{\beta J} + e^{-\beta J} \right)$$

$$Z_c = \prod_i^N 2\cosh(\beta J) = [2\cosh(\beta J)]^N$$

$$\rightarrow \boxed{Z_c \triangleq (2\cosh(\beta J))^N}$$

ǂ con $s_i s_{i+1}$ se tiene

$$\begin{array}{ccccc} s_1 s_2 & s_2 s_3 & \cdots & s_{N-1} s_N & s_N s_{N+1} \\ b_1 & b_2 & \cdots & b_{N-1} & b_N \end{array} \qquad \text{donde } N+1 = 1$$

b_N aporta a la energía

$$\sum_{b_N = \pm 1} e^{\beta J b_N}$$

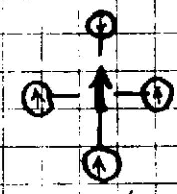
EL Libro de Resueltos de Chandler da una respuesta sobre este tema, pero no la entendí.

4.

$$E_\nu = -\sum_i^N m_0 s_i H - J \sum_{\langle i,j\rangle}' s_i s_j$$

$$\varepsilon_i = -m_0 s_i H - J s_i \sum_j' s_j$$

$$\varepsilon_i = -m_0 s_i \left(H + \frac{J}{m_0} \sum_j' s_j \right) \longrightarrow \text{en 2D sería}$$



$$\varepsilon_i = -m_0 s_i \underbrace{\left[H + \frac{J}{m_0} \gamma \langle s \rangle \right]}_{=H'}$$

La aproximación de campo medio pierde la información de las correlaciones locales reemplazándolo por correlaciones globales

Para 1 partícula será: $Z_c^{(1)} = \sum_i e^{\beta m_0 s_i [H + (J/m_0)\gamma\langle s\rangle]}$; $s_i = \pm 1$

$$Z_c^{(1)} = e^{\beta m_0 (H + \frac{J}{m_0}\gamma\langle s\rangle)} + e^{-\beta m_0 (H + \frac{J}{m_0}\gamma\langle s\rangle)}$$

$$Z_c^{(1)} = 2 . \cosh\left[\beta m_0 \left(H + \frac{J}{m_0}\gamma\langle s\rangle\right)\right]$$

Para N partículas: $Z_c^{(N)} = 2^N \cosh^N \left[\beta m_0 \left(H + \frac{J}{m_0}\gamma\langle s\rangle\right)\right]$

a) A campo nulo es $\vec{H} = 0 \rightarrow Z_c^{(N)} = 2^N . \cosh^N(\beta J \gamma \langle s \rangle)$

$$\langle s_i \rangle^{(1)} = \frac{\sum_i s_i . e^{\beta m_0 s_i H'}}{\sum_i e^{\beta m_0 s_i H'}} \qquad M = \sum_i^N m_0 s_i$$

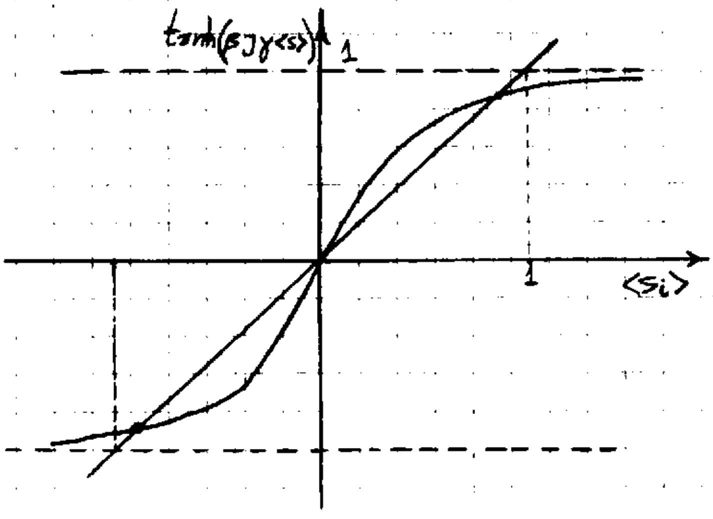
$$\langle s_i \rangle^{(1)} = \frac{e^{\beta m_0 H'} - e^{-\beta m_0 H'}}{e^{\beta m_0 H'} + e^{-\beta m_0 H'}} \qquad \langle M \rangle = \sum_i^N m_0 \langle s_i \rangle \quad \langle M \rangle = \langle s \rangle m_0 N$$

$$\langle m \rangle = m_0 \langle s_i \rangle$$

$$\langle s_i \rangle^{(1)} = \frac{\operatorname{senh}(\beta m_0 H')}{\cosh(\beta m_0 H')} = \tanh(\beta m_0 H')$$

$$\langle s_i \rangle^{(N)} = \tanh^N(\beta m_0 H')$$ con campo nulo será:

$$\langle s_i \rangle = \tanh(\beta J \gamma \langle s \rangle) \Rightarrow$$



Para que exista solución diferente a $\langle s \rangle = 0$ necesito que:

$$\beta J \gamma > 1 \rightarrow$$

$$\frac{J\gamma}{kT} > 1$$

$$\frac{J\gamma}{k} > T \Rightarrow$$

$$\boxed{T_c \equiv \frac{J\gamma}{k}} \Rightarrow \overset{\text{necesito}}{T < T_c}$$

$$\langle m\rangle = m_o\langle s_i\rangle = m_o \tanh(\beta J\gamma\langle s\rangle) \qquad \tanh(x)\approx x-\frac{x^3}{3}$$

$$\langle s\rangle \cong \beta J\gamma\langle s\rangle - \frac{1}{3}(\beta J\gamma)^3\langle s\rangle^3$$

sea $T\lesssim T_c \rightarrow 1\lesssim \frac{T_c}{T}$

$$\langle s\rangle(1-\beta J\gamma)\cong -\frac{1}{3}(\beta J\gamma)^3\langle s\rangle^3$$

$$\langle s\rangle = \tanh\left(\frac{T_c\langle s\rangle}{T}\right)$$

$$\frac{3(\beta J\gamma - 1)}{(\beta J\gamma)^3}\cong \langle s\rangle^2$$

$$\langle s\rangle \approxeq 3^{1/2}\left(\frac{J\gamma}{kT}-1\right)^{1/2}\cdot\frac{(kT)^3}{(J\gamma)^3}$$

$$\cong\sqrt{3}\left(\frac{T_c}{T}-1\right)^{1/2}\left(\frac{T}{T_c}\right)^3 = \sqrt{3}\,(T_c-T)^{1/2}\underbrace{\frac{T^2}{T_c^3}}_{\sim 1}$$

$$\langle s\rangle \propto (T_c-T)^{1/2}$$

$$\Rightarrow \boxed{\beta=\frac{1}{2}}$$

b)

$$\langle s\rangle \sim \beta m_o H' - \frac{1}{3}(\beta m_o H')^3$$

$$\langle s\rangle \sim \beta m_o H + J\gamma\langle s\rangle\beta - \frac{1}{3}[\quad]^3 \rightarrow \left(\frac{m_oH}{kT}+\frac{T_c}{T}\langle s\rangle\right)$$

$$\left(\frac{T_c}{T}\langle s\rangle\right)^3\underbrace{\left(1+\frac{m_oH}{kT_c}\right)^3}_{\sim 1+3\frac{m_oH}{kT_c}}$$

$$\langle s\rangle \sim \frac{m_o}{kT}H + \frac{T_c}{T}\langle s\rangle - \frac{1}{3}\left(\frac{T_c}{T}\langle s\rangle\right)^3 - \left(\frac{T_c}{T}\langle s\rangle\right)^3\frac{m_oH}{kT_c}$$

En $T=T_c$ es:

$$\cancel{\langle s\rangle} \sim \frac{m_oH}{kT_c} + \cancel{\langle s\rangle} - \frac{1}{3}\langle s\rangle^3 - \langle s\rangle^3\frac{m_oH}{kT_c}$$

$$\langle s\rangle^3\left(\frac{m_oH}{kT_c}+\frac{1}{3}\right)\sim\frac{m_oH}{kT_c}$$

$$\langle s\rangle^3 \sim \frac{m_oH}{kT_c}\cdot\frac{1}{\left(\frac{1}{3}+\frac{m_oH}{kT_c}\right)} = \frac{m_oH}{kT_c}3\left(1+\frac{3m_oH}{kT_c}\right)$$

$$\langle s\rangle^3\sim\frac{3m_oH}{kT_c} \rightarrow m_o\langle s\rangle = \langle m\rangle$$

$$\langle m\rangle(T_c,H)\propto H^{1/3}$$

$$\Rightarrow\boxed{\delta=3}$$

c)

$$\chi\equiv\lim_{H\to 0}\frac{\partial\langle M\rangle}{\partial H} = \lim_{H\to 0} N\frac{\partial\langle m\rangle}{\partial H} = \lim_{H\to 0} N.m_o\frac{\partial\langle s\rangle}{\partial H} \qquad T\lesssim T_c,\ H\to 0 \Rightarrow$$

$$\langle s\rangle\sim\beta Hm_o + \beta J\gamma\langle s\rangle.$$

$$\frac{\partial\langle s\rangle}{\partial H}\sim\beta m_o + \beta J\gamma\frac{\partial\langle s\rangle}{\partial H}$$

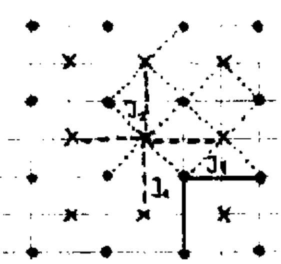
$$\frac{\partial\langle s\rangle}{\partial H}(1-\beta J\gamma)\sim\beta m_0 \Rightarrow \qquad \frac{\partial\langle s\rangle}{\partial H}\sim\frac{\beta m_0}{1-\frac{T_c}{T}}=\frac{\beta m_0 T}{(T-T_c)}$$

$$\chi \equiv \lim_{H\to 0} N m_0 \frac{\partial\langle s\rangle}{\partial H} = \lim_{H\to 0}\beta\frac{N m_0^2 T}{(T-T_c)}$$

$$\chi = \beta m_0^2\frac{N T}{(T-T_c)}$$

$$\Rightarrow \boxed{\gamma = 1}$$

5.



- • sitios B
- × sitios A
- → Red cuadrada 2D
  - → $\gamma = 4$

Considero un H

Recorre la red A y cuenta la interacción con vecinos en A — Recorre la red B y cuenta interacciones entre vecinos en B — Recorre una red y cuenta interacción con los vecinos de la otra. Como es un conteo doble te meto un 2

a) $$\mathcal{H}=\sum_i^{(A)}\mu_A s_i^A H \underbrace{}_{-\sum_i^{(B)}\mu_B s_i^B H} - J_1\sum_{\langle i,j\rangle}^{(A)} s_i^A s_j^A - J_1\sum_{\langle i,j\rangle}^{(B)} s_i^B s_j^B + \frac{J_2}{2}\sum_{\langle i,j\rangle}\left(s_i^A s_j^B + s_i^B s_j^A\right)$$

Para cada i, el índice j va hasta $\gamma/2$

b) En campo medio cada spin de la red A siente una interacción que depende del H imperante y del acople de sus vecinos en B y en A.

$$E^{(A)} = -\sum_i^{(A)}\mu_A s_i^A H - J_1\sum_{\langle i,j\rangle}^{(A)} s_i^A s_j^A + \frac{J_2}{2}\sum_{\langle i,j\rangle}^{(A)} s_i^A s_j^B$$

$$E^{(B)} = -\sum_i^{(B)}\mu_B s_i^B H - J_1\sum_{\langle i,j\rangle}^{(B)} s_i^B s_j^B + \frac{J_2}{2}\sum_{\langle i,j\rangle}^{(B)} s_i^B s_j^A$$

$$\Rightarrow \mathcal{H} = E^{(A)} + E^{(B)}$$

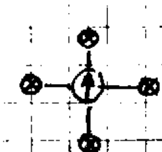
$$\varepsilon_i^{(A)} = -\mu_A s_i^A\left(H + \frac{J_1}{\mu_A}\sum_j^{(A)} s_j^A - \frac{J_2}{\mu_A 2}\sum_j^{(A)} s_j^B\right)$$

$$\varepsilon_i^{(A)} = -\mu_A s_i^A\left(H + \frac{J_1}{\mu_A}\gamma\langle s^A\rangle - \frac{J_2}{2\mu_A}\gamma\langle s^B\rangle\right)$$

Importante

Para campo medio (como es para uno única spin) es:

$$\sum_j s_i = \gamma\langle s_i\rangle$$



Sumo en todos los vecinos

Para el caso de la red B será:

$$\varepsilon_i^{(B)} = -\mu_B s_i^B\left(H + \frac{J_1}{\mu_B}\sum_j^{(B)} s_j^B - \frac{J_2}{2\mu_B}\sum_j^{(B)} s_j^A\right)$$

$$\varepsilon_i^{(B)} = -\mu_B s_i^B\left(H + \frac{J_1}{\mu_B}\gamma\langle s^B\rangle - \frac{J_2}{2\mu_B}\gamma\langle s^A\rangle\right)$$

$$\boxed{H^{eff}_{\text{sobre spin A}} \equiv H + \frac{J_1}{\mu_A}\gamma\langle s^A\rangle - \frac{J_2}{2\mu_A}\gamma\langle s^B\rangle}$$

$$\boxed{H^{eff}_{\text{sobre spin B}} \equiv H + \frac{J_1}{\mu_B}\gamma\langle s^B\rangle - \frac{J_2}{2\mu_B}\gamma\langle s^A\rangle}$$

* En el caso del spín de la red A vemos que el término $J_1\gamma\langle S^A\rangle/\mu_A$ suma al campo en el sentido de que «añade» a la tendencia a apuntar en dirección de H. El término $J_2\gamma\langle S^B\rangle/Z\mu_A$ va contra el campo y tiende a "desordenar". Este término es el responsable del acople. Para el spin de la red B la situación es ídem.

$$\mathcal{H} \doteq \sum_i -\mu_B S^B_i H^{eff}_B - \mu_A S^A_i H^{eff}_A$$

Para cada tipo de spín el campo efectivo que "siente" es el H imperante, la interacción con los vecinos de su misma red y la interacción con los vecinos de la otra red.

c)

Las ecuaciones para $\langle S^A_i\rangle$ y $\langle S^B_i\rangle$ surgirán de

$$\frac{\langle M\rangle}{N} = \langle m\rangle \longleftarrow \text{Magnetización media por partícula}$$

La aproximación de campo medio consta de pararse sobre un spín y sumar sobre sus γ vecinos que tendrán un S_j dado por el promedio de S_j en todo el dominio (será un $\langle S_i\rangle$)
De este punto de vista la Z que se derivará con este modelo puede factorizarse, con lo cual:

$$Z^{total} = \left(Z^{(1\,spín)}\right)^N$$

Para la magnetización tenemos:

$$M = \sum_i^{N_A} \mu_A S^A_i + \sum_i^{N_B} \mu_B S^B_i$$

$$\langle M\rangle = \sum \mu_A\langle S^A_i\rangle + \sum \mu_B\langle S^B_i\rangle = \sum \mu_A\langle S^A\rangle + \sum \mu_B\langle S^B\rangle$$

Supongamos $N_A = N_B$

$$\langle M\rangle = \mu_A\langle S^A\rangle \sum_i^N + \mu_B\langle S^B\rangle\sum_i^N = N\left(\mu_A\langle S^A\rangle + \mu_B\langle S^B\rangle\right)$$

$$\frac{\langle M\rangle}{N} = \langle m\rangle = \mu_A\langle S^A\rangle + \mu_B\langle S^B\rangle$$

$$Z_c^{(A)} = \sum_{i=+1,-1} e^{\beta\mu_A S^A_i (H^{eff})} \rightarrow \text{Función de partición para 1 spín de la red A}$$

$$\langle S^A\rangle = \frac{\sum S^A_i \, e^{\beta\mu_A S^A_i\left(H + \frac{J_1}{\mu_A}\gamma\langle S^A\rangle - \frac{J_2}{Z\mu_A}\gamma\langle S^B\rangle\right)}}{Z_{1c}}$$

$$\boxed{\langle S^A\rangle = \frac{\operatorname{senh}(\beta\mu_A H^{eff}_A)}{\cosh(\beta\mu_A H^{eff}_A)} = \tanh\left(\beta\mu_A H^{eff}\right)}$$

, donde H^{eff} tiene dentro $\langle S^A\rangle$ y $\langle S^B\rangle$ (ahí está el acople de las ecuaciones)

En modo idem llegamos a:

$$\boxed{\langle s^B \rangle = \tanh\left(\beta \mu_B H^{eff}_B\right)}$$

, donde H^{eff} tiene dentro $\langle s^B \rangle$ y $\langle s^A \rangle$

d)

$$\chi = \lim_{H \to 0} \frac{\partial \langle M \rangle}{\partial H} = \lim_{H \to 0} N \frac{\partial \langle m \rangle}{\partial H} = \lim_{H \to 0} N \mu \frac{\partial \langle s \rangle}{\partial H}$$

El yeite está, entonces en $\langle s \rangle$. En torno a T_c por debajo $(T \lesssim T_c)$ vale

$$\tanh(x) \sim x - \frac{1}{3}x^3$$

$$\langle s^A \rangle \sim \beta \mu_A H + \beta J_1 \gamma \langle s^A \rangle - \beta \frac{J_2}{z} \gamma \langle s^B \rangle$$

Derivando implícitamente esta ecuación es:

$$\frac{\partial \langle s^A \rangle}{\partial H} \sim \beta \mu_A + \beta J_1 \gamma \frac{\partial \langle s^A \rangle}{\partial H}$$

$$\frac{\partial \langle s^A_i \rangle}{\partial H} \sim \beta \mu_A \cdot \frac{1}{(1 - \beta J_1 \gamma)} = \frac{\beta \mu_A}{\left(1 - \frac{T_c}{T}\right)}$$

$$\frac{\partial \langle s^A_i \rangle}{\partial H} \sim \frac{\beta \mu_A T}{(T - T_c)}$$

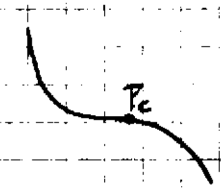
$$\chi = N \mu_A^2 \beta \frac{T}{(T - T_c)} \Rightarrow \boxed{\chi \sim \frac{1}{T - T_c}}$$

6.

$$(V - b)\left(P + \frac{a}{V^2}\right) = RT \qquad \text{Van der Waals}$$

$$P = \frac{RT}{V - b} - \frac{a}{V^2}$$

a) En el P_c tenemos inflexión de la curva $P = P(V) \rightarrow$



$$\frac{\partial P}{\partial V} = 0 \quad , \quad \frac{\partial^2 P}{\partial V^2} = 0 \quad \therefore$$

$$\frac{\partial P}{\partial V} = -\frac{RT}{(V - b)^2} + \frac{2a}{V^3} = 0 \qquad \frac{2a}{V^3} = \frac{RT}{(V - b)^2}$$

$$\frac{\partial^2 P}{\partial V^2} = +\frac{RT\,2}{(V - b)^3} - \frac{3 \cdot 2a}{V^4} = 0$$

$$-\frac{RT}{(V - b)^3} + \frac{3a}{V^4} = 0 \qquad \frac{3a}{V^4} = +\frac{RT}{(V - b)^3}$$

$$RT_c = \cancel{\frac{(V-b)^2 2a}{V^3}} = \frac{+3a(V-b)^2}{V^4}$$

$$+\frac{2}{3} = \frac{V-b}{V} \rightarrow +\frac{2}{3}V - V = -b$$

$$-\frac{1}{3}V = -b$$

$$\boxed{V_c = 3b}$$

$$P_c = \frac{R.T_c}{3b-b} - \frac{a}{9b^2}$$

$$P_c = \frac{R.T_c}{2b} - \frac{a}{9b^2}$$

$$R.T_c = \frac{(2b)^2 2a}{27b^3}$$

$$R.T_c = \frac{8a}{27b}$$

$$\boxed{T_c = \left(\frac{2}{3}\right)^3\left(\frac{a}{bR}\right)} \qquad \boxed{P_c = \frac{1}{27}\frac{a}{b^2}}$$

$$P_c = \frac{\cancel{8}^4 a}{27.\cancel{2}b^2} - \frac{a}{9b^2} = \frac{1}{27}\frac{a}{b^2}$$

$$\frac{2a}{27b^{\cancel{3}}} = \frac{RT_c}{4b^{\cancel{2}}}$$

$$P_c = \frac{5.a}{3\cancel{27}\frac{V_c^2}{\cancel{9}}} = \frac{5.a}{5V_c^2}$$

b)

$$(V-b)\left(P + \frac{a}{V^2}\right) = RT = \frac{8a}{27b}\frac{T}{T_c}$$

$$\left(V - \frac{V_c}{3}\right)\left(P + \frac{a}{V^2}\right) = \left(\frac{2}{3}\right)^3\frac{a}{b}\tilde{T}$$

$$V_c\left(\tilde{V} - \frac{1}{3}\right)\left(P + \frac{P_c 27 b^2}{V^2}\right) = \left(\frac{2}{3}\right)^3\frac{a}{b}\tilde{T}$$

$$\frac{V_c}{3}(3\tilde{V} - 1)\left(P + \frac{P_c 3 V_c^2}{V^2}\right) = \left(\frac{2}{3}\right)^3\frac{a}{b}\tilde{T}$$

$$(3\tilde{V}-1)\frac{V_c}{3}P_c\left(\tilde{P} + \frac{3}{\tilde{V}^2}\right) = \left(\frac{2}{3}\right)^3\frac{a}{b}\tilde{T}$$

$$(3\tilde{V}-1)\frac{\cancel{3b}\,\cancel{a}}{\cancel{3}\,\cancel{27}\,b^{\cancel{2}}}\left(\tilde{P} + \frac{3}{\tilde{V}^2}\right) = \left(\frac{2}{\cancel{3}}\right)^3\frac{\cancel{a}}{\cancel{b}}\tilde{T}$$

$$\boxed{(3\tilde{V}-1)\left(\tilde{P} + \frac{3}{\tilde{V}^2}\right) = 8\tilde{T}}$$

7. En el punto crítico $\tilde{V} = 1,\ \tilde{T} = 1,\ \tilde{P} = 1 \Rightarrow$

$$\tilde{P} = \frac{8\tilde{T}}{(3\tilde{V}-1)} - \frac{3}{\tilde{V}^2}$$

$$\tilde{P}(\tilde{V}=1) = \frac{8\tilde{T}}{2} - 3$$

$$\left.\frac{\partial\tilde{P}}{\partial\tilde{V}}\right|_{\tilde{V}=1} = -\frac{8\tilde{T}.3}{(3\tilde{V}-1)^2} + \frac{3.2}{\tilde{V}^3}$$

$$\frac{\partial^2\tilde{P}}{\partial\tilde{V}^2} = +\frac{2.8\tilde{T}.3.3}{(3\tilde{V}-1)^3} - \frac{3.3.2}{\tilde{V}^4}$$

$$\frac{\partial^3\tilde{P}}{\partial\tilde{V}^3} = -\frac{16.\tilde{T}.9\,3\,3}{(3\tilde{V}-1)^4} + \frac{4.18}{\tilde{V}^5}$$

$$\tilde{P} = (12\tilde{T}-3) + \left(-\frac{24\tilde{T}}{4}+6\right)(\tilde{V}-1) + \frac{1}{2}\left(\frac{144\tilde{T}}{8}-18\right)(\tilde{V}-1)^2 + \frac{1}{6}\left(-\frac{1296\tilde{T}}{16}+72\right)(\tilde{V}-1)^3$$

$$\tilde{P}-1 = 4\tilde{T}-4 + (-6\tilde{T}+6)(\tilde{V}-1) + \frac{1}{2}(18\tilde{T}-18)v^2 - \frac{1}{6}(81\tilde{T}-72)v^3$$

$$p = 4t - 6(\tilde{T}-1)v + 9(\tilde{T}-1)v^2 - \frac{1}{6}(81\tilde{T}-81+9)v^3$$

$$\boxed{p = 4t - 6tv + 9tv^2 - \frac{3}{2}v^3 - \frac{81}{6}tv^3}$$

b)

$$\varkappa_T = -\frac{1}{V}\frac{\partial V}{\partial P}\bigg|_T$$

Podemos poner : $p \approx 4t - 6tv$ (pues $v \sim 0$ por ser $\tilde{V}-1 \equiv v$)

$$v \approx \frac{4t}{6t} - \frac{p}{6t} = 2/3 - p/6t$$

$$\frac{\partial v}{\partial p} = -\frac{1}{6t} \rightarrow \boxed{\varkappa_T \propto t^{-1}}$$