Serie 1: Termodinámica

1.

a) $$S = \left(\frac{R^2}{v_0 \theta}\right)^{1/3} (NVU)^{1/3}$$

- S es función de variables extensivas
- S es aditiva, continua y diferenciable $S=S(U)$ creciente
- $\left.\frac{\partial U}{\partial S}\right|_{V,N} = 0$ en $S=0$

$$\left.\frac{\partial S}{\partial U}\right|_{V,N} = \left(\frac{R^2}{v_0 \theta}\right)^{1/3} (NV)^{1/3} \frac{1}{U^{2/3}}$$

$$\left.\frac{\partial S}{\partial U}\right|_{V,N} > 0 \quad \forall U \rightarrow \text{es creciente}$$

$$S(\lambda N, \lambda V, \lambda U) = \left(\frac{R^2}{v_0 \theta}\right) \lambda \, (NVU)^{1/3} \rightarrow \text{es aditiva, es continua}$$

$$U = \left(\frac{v_0 \theta}{R^2}\right) \frac{S^3}{NV} \rightarrow \frac{\partial U}{\partial S} = \frac{v_0 \theta}{R^2} \cdot \frac{2S^2}{NV} \rightarrow \left.\frac{\partial U}{\partial S}\right|_{S=0} = 0$$

Puede ser ec. fundamental

b) $$S = \left(\frac{R^2}{\theta^2}\right)^{1/3} \left(\frac{N.V}{U}\right)^{2/3}$$

$$\left.\frac{\partial S}{\partial U}\right|_{V,N} = \left(\frac{R^2}{\theta^2}\right)^{1/3} (N.V)^{2/3} \cdot -\frac{2}{3} \cdot \frac{1}{U^{5/3}} < 0 \rightarrow \text{No es creciente pues } \left(\frac{1}{U}\right)^{2/3} \text{ es decreciente}$$

No satisface postulado III

c) $$S = N.R \ln\left(\frac{U.V}{N^2.R\theta v_0}\right)$$

$$\frac{\partial S}{\partial U} = NR \frac{1}{U.V} . N^2 R \theta v_0 = \frac{N^3 R^2 \theta v_0}{U.V} > 0 \rightarrow \text{es creciente}$$

$$S(\lambda N, \lambda U, \lambda V) = \lambda \, N.R \ln\left(\frac{U\,V}{N^2 R \theta v_0}\right) \rightarrow \text{es aditiva}$$

$$U = e^{\frac{S}{NR}} \frac{N^2.R.\theta.v_0}{V} \rightarrow \frac{\partial U}{\partial S} = \frac{N^2 R \theta v_0}{V} . e^{\frac{S}{NR}} . \frac{1}{NR}$$

$$\left.\frac{\partial U}{\partial S}\right|_{S=0} = N \frac{\theta v_0}{V} \neq 0 \Rightarrow$$

No satisface postulado IV

d) $$S = \left(\frac{R}{\theta}\right)^{1/2} (NU)^{1/2} e^{-\frac{U.V}{NR\theta v_0}}$$

$$S(\lambda N, \lambda V, \lambda U) = \left(\frac{R}{\theta}\right)^{1/2} \lambda \, (NU)^{1/2} e^{-\left(\lambda \frac{UV}{NR\theta v_0}\right)} \rightarrow$$

No satisface postulado III (no es homogénea de orden 1)

$$\frac{\partial S}{\partial U} = \left(\frac{R}{\theta}\right)^{1/2} \left(N^{1/2} \frac{1}{2U^{1/2}} . e^{-\frac{UV}{NR\theta v_0}} - \left(\frac{V}{NR\theta v_0}\right)(NU)^{1/2} e^{-\frac{UV}{NR\theta v_0}} \right)$$

$$\frac{\partial S}{\partial U} = \left(\frac{R}{\theta}\right)^{1/2} e^{-\frac{UV}{NR\theta v_0}} \left[\frac{1}{2}\left(\frac{N}{U}\right)^{1/2} - \frac{V}{\theta v_0 R} (NU)^{1/2} \right]$$

$$\frac{1}{2.U^{1/2}} - \frac{V \cdot U^{1/2}}{\theta.v_0.R} > 0$$

$$\frac{\theta.v_0.R}{2} > U^2$$

Es creciente solo para

$$\sqrt{\frac{\theta.v_0.R}{2}} > U \Rightarrow$$ No es creciente en general

No satisface postulado III

e) $$U = \left(\frac{v_0.\theta}{R}\right)^{1/2} \frac{S^2}{V} . e^{\left(\frac{S}{NR}\right)}$$

$$\left.\frac{\partial U}{\partial S}\right|_{N,V} = \frac{v_0 \theta}{R} . \frac{1}{V} \left(2S. e^{\frac{S}{NR}} + S^2 . e^{\frac{S}{NR}} . \frac{1}{NR} \right)$$

$$\left.\frac{\partial U}{\partial S}\right|_{NV} = \frac{v_0.\theta}{R.V} . S.e^{\frac{S}{N.R}} \left(2 + \frac{S}{NR} \right) \Rightarrow \left.\frac{\partial U}{\partial S}\right|_{N,V} = 0 \quad \text{si} \quad S = 0$$

U es continua, diferenciable

$$\left.\frac{\partial S}{\partial U}\right|_{N,V} = \frac{1}{\left.\frac{\partial U}{\partial S}\right|_{N,V}} = \frac{R.V}{v_0.\theta} . \frac{e^{-\frac{S}{NR}}}{S} . \frac{1}{\left(2 + \frac{S}{NR}\right)} > 0 \rightarrow S \text{ es creciente}$$

S es continua, diferenciable, aditiva $\Rightarrow$ **Puede ser ec. fundamental**

3.

$$U = \left(\frac{v_0 \theta}{R^2}\right) \frac{S^3}{NV}$$

a) $$dU = \underbrace{\frac{\partial U}{\partial S}}_{T} dS + \underbrace{\frac{\partial U}{\partial V}}_{-P} dV + \underbrace{\frac{\partial U}{\partial N}}_{\mu} dN$$

Ecuaciones de estado son en función de S,V,N

$$\boxed{T = \frac{v_0 \theta}{R^2} \cdot \frac{3S^2}{NV}} \quad \boxed{P = +\frac{v_0 \theta}{R^2} \cdot \frac{S^3 2}{NV^2}} \quad \boxed{\mu = -\frac{v_0 \theta}{R^2} \cdot \frac{S^3 2}{VN^2}}$$

b)

$$\mu = -\frac{v_0 \theta}{R^2} \cdot \frac{2}{VN^2} \left(\frac{TR^2 NV}{v_0 \theta 3}\right)^{3/2} \qquad S^2 = \frac{TR^2 NV}{v_0 \theta 3}$$

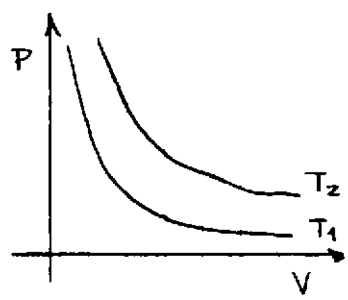
$$\mu = -2 \left(\frac{R^2 V}{v_0 \theta N}\right)^{1/2} \left(\frac{T}{3}\right)^{3/2} \qquad S^3 = \left(\frac{TR^2 NV}{v_0 \theta 3}\right)^{3/2}$$

$$\boxed{\mu = -2R \cdot \sqrt{\frac{V}{v_0 \theta N}} \cdot \left(\frac{T}{3}\right)^{3/2}}$$

c)

$$P = \frac{v_0 \theta}{R^2} \frac{2}{NV^2} \left(\frac{TR^2 NV}{v_0 \theta 3}\right)^{3/2}$$

$$P = 2 \cdot \left(\frac{R^2 N}{v_0 \theta V}\right) \left(\frac{T}{3}\right)^{3/2} \rightarrow P = 2R \sqrt{\frac{N}{v_0 \theta}} \left(\frac{T}{3}\right)^{3/2} \left(\frac{1}{V}\right)^{1/2}$$



$P \sim K \cdot \frac{1}{V^{1/2}}$, K constante

con $T_2 > T_1$

◂ Es un comportamiento tipo gas ideal

4.

$$f = f(x_1, \dots, x_n) \quad \text{con} \quad df = \sum_{i=1}^{N} U_i \, dx_i \quad \text{con} \quad U_i = \left(\frac{\partial f}{\partial x_i}\right)_{x_j}$$

1. defino $g \equiv f - \sum_{i=r+1}^{N} u_i x_i$

$$dg = df - \sum_{i=r+1}^{N} u_i dx_i - \sum_{i=r+1}^{N} x_i \, du_i$$

$$dg = \sum_{i=1}^{r} U_i dx_i + \sum_{i=r+1}^{N} U_i dx_i - \sum_{i=r+1}^{N} U_i dx_i - \sum_{i=r+1}^{N} x_i dU_i$$

$$dg = \sum_{1}^{r} U_i dx_i - \sum_{r+1}^{N} x_i \, dU_i \Rightarrow$$

$$\Rightarrow \boxed{g = g(x_1, x_2, \dots, x_r, U_{r+1}, U_{r+2}, \dots, U_N)}$$

2. $dE = T.dS - p.dV + \sum_i u_i dN_i$ $\qquad E = E(S,V,N)$

diferencial de energía interna

$dE = T.\frac{\partial S}{\partial V}\Big|_T dV + T.\frac{\partial S}{\partial T}\Big|_V dT - p.dV + \sum_i u_i dN_i$ $\qquad dS = A.dV + B.dT$

$$dE = \underbrace{\left(T.\frac{\partial S}{\partial V}\Big|_T - p\right)}_{\frac{\partial E}{\partial V}\big|_{T,N}}dV + \underbrace{T.\frac{\partial S}{\partial T}\Big|_V}_{\frac{\partial E}{\partial T}\big|_{V,N}} dT + \sum_i u_i dN_i$$

* $F = U - TS$

$dF = dU - T.dS - S\,dT$

$dU = TdS - pdV + \sum_i u_i dN_i$

$dF = dU - S.dT - T.dS$

$dU - T.dS = dF + S.dT$

$-S.dT - p.dV + \sum_i u_i dN_i = dF$

$dE - T.dS = -p.dV + \sum_i u_i dN_i$

$dF + S.dT = -p.dV + \sum_i u_i dN_i$

$dF = -S.dT - p.dV + \sum_i u_i dN_i$

$dF = dF(T, V, N_i)$

* $G = U - ST + PV$

$dG = dU - S.dT - T.dS + p.dV + V.dP$

$dG = dU - T.dS + p.dV - S\,dT + V.dP$

$dG = \sum_i u_i dN_i - S.dT + V.dP \longrightarrow dG = dG(T, P, N_i)$

$f = f(x_1, \dots, x_n)$ con $f(\lambda x_1, \dots, \lambda x_n) = \lambda f(x_1, \dots, x_n) \rightarrow$ es homogénea de 1er orden

1. $\lambda x_i \equiv U_i \rightarrow \frac{\partial U_i}{\partial x_i} = \lambda$

$$\frac{\partial f(U_1,\dots,U_n)}{\partial x_i} = \frac{\partial f(U_1,\dots,U_n)}{\partial U_i}.\frac{\partial U_i}{\partial x_i} = \frac{\partial f(x_1,\dots,x_n)}{\partial x_i}.\lambda \Rightarrow \frac{\partial f(x)}{\partial x_i} = \frac{\partial f(U)}{\partial U_i}$$

$$df = \sum_i^N \frac{\partial f(x_1 \dots x_n)}{\partial x_i}\Big|_{x_j} dx_i = \sum_i^N \frac{\partial f(U_1,\dots,U_N)}{\partial U_i}\Big|_{U_j}.dx_i \Rightarrow$$

$$\int df = \int \sum_i^N \frac{\partial f(U)}{\partial U_i}\Big|_{U_j} dx_i = \sum_i^N \frac{\partial f(U)}{\partial U_i}\Big|_{U_j} \int dx_i$$

$$\Rightarrow \boxed{f = \sum_i^N \frac{\partial f(x)}{\partial x_i}\Big|_{x_j} x_i}$$

Como $dE = T.dS - P.dV + \sum_i^N \mu_i dN_i$ y S, V, N_i son extensivas y $E(\lambda S, \lambda V, \lambda N_i) = \lambda E(S, V, N_i)$

podemos utilizar el mismo trick

$$E = TS - P.V + \sum_i^M \mu_i N_i$$ ↘ diferenciando implícitamente

$$dE = \underbrace{T.dS - p.dV + \sum_i^M \mu_i dN_i}_{} + S.dT - V.dp + \sum_i^M N_i d\mu_i$$

$$dE = dE + S.dT - V.dP + \sum_i^M N_i d\mu_i$$

$$0 = S.dT - V.dP + \sum_i^M N_i d\mu_i$$

$$\boxed{\sum_i^M N_i d\mu_i = -S.dT + V.dp}$$

Relación de Gibbs - Duhem

3. Para un sistema de un solo componente

$$N.d\mu = -S.dT + V.dP$$

$$d\mu = -\frac{S}{N}.dT + \frac{V}{N}.dP \rightarrow d\mu = -s.dT + v.dP$$

Pero, recordando que Gibbs es $G = U - TS + PV$

$$dG = dU - T.dS - S.dT + P.dV + V.dP$$

$$dG = dU - T.dS + p\,dV - S\,dT + V\,dP$$

$$dG = \sum_i^M \mu_i dN_i - S.dT + V.dP$$

sistema de un solo componente $dN = 0$ →

$$dG = -S.dT + V.dP$$

↓ ← dividiendo por el # de moles

$$dg = -s.dT + v.dp$$

$$\Rightarrow \boxed{dg = \frac{dG}{N} = d\mu}$$ (sistemas de 1 componente)

6.

$$U = \left(\frac{v_0.\theta}{R^2}\right)\frac{S^3}{N.V} \rightarrow U = U(S,V,N) \rightarrow \left.\frac{\partial U}{\partial S}\right|_{V,N} = T$$

Son ecuaciones de estado →

$$T = \left(\frac{v_0.\theta}{R^2}\right)\frac{3S^2}{N.V} \qquad +P = +\left(\frac{v_0\theta}{R^2}\right)\frac{S^3}{NV^2}.2 \qquad \mu = -\left(\frac{v_0\theta}{R^2}\right)\frac{S^3}{V.N^2}.2$$

$$\left(\frac{R^2}{v_0\theta}\right)\frac{T}{3}\frac{1}{S} = \frac{S}{NV} \qquad \left(\frac{R^2}{v_0\theta}\right)\frac{P}{2}\frac{V}{S^2} = \frac{S}{NV} \qquad -\left(\frac{R^2}{v_0\theta}\right)\frac{\mu}{2}\frac{N}{S^2} = \frac{S}{NV}$$

$$\frac{T}{3} = \frac{PV}{2S} = \frac{\mu N}{2S} \qquad PV = -\mu N \qquad \frac{\mu}{P} = -\frac{V}{N} = -v$$

$$\frac{T}{3S} = \frac{PV}{2S^2} \rightarrow PV = \left(\frac{2}{3}\right)S\,T$$

las relaciones pedidas ⟶ $\boxed{Pv = -\mu}$ $\boxed{T = (3/2)\mu}$

↓

$$S = \left(-\frac{R^2 N^2 V\mu}{2v_0\theta}\right)^{1/3} \rightarrow P.V = -\frac{2}{3}.\frac{R^{2/3}N^{2/3}V^{1/3}}{2^{1/3}(v_0\theta)^{1/3}}\mu^{1/3}.T$$

$$P.V^{2/3} = +\frac{(2RN)^{2/3}}{3(v_0\theta)^{1/3}}.\left(\frac{PV}{N}\right)^{1/3}T \Rightarrow$$

$$P^{2/3} V^{1/3} = \frac{(2R)^{2/3} N^{1/3}}{3\,(\sqrt{v_0}\,\theta)^{1/3}} \cdot T$$

$$P^2 V = \frac{4}{27} \cdot \left(\frac{R^2}{\sqrt{v_0}\,\theta}\right) N\, T^3$$ ← Ecuación de estado P, V, T, N para este sistema.

7.

$$f(x,y,z) = 0 \quad \rightarrow \quad df = 0$$

$$\left.\frac{\partial f}{\partial x}\right|_{yz} dx + \left.\frac{\partial f}{\partial y}\right|_{xz} dy + \left.\frac{\partial f}{\partial z}\right|_{xy} dz = 0 \qquad [1]$$

Si se puede despejar cualquier variable en función de las otras dos, entonces:

$$x = \omega_1(y,z) \rightarrow f([\omega_1(y,z)], y, z) = 0 \rightarrow$$

$$0 = \frac{\partial f}{\partial \omega_1}\frac{\partial \omega_1}{\partial y} + \frac{\partial f}{\partial y} \rightarrow \left.\frac{\partial f}{\partial x}\right|_{yz} \left.\frac{\partial x}{\partial y}\right|_z = -\left.\frac{\partial f}{\partial y}\right|_{xz}$$

$$0 = \frac{\partial f}{\partial \omega_1}\frac{\partial \omega_1}{\partial z} + \frac{\partial f}{\partial z}$$

$$y = \omega_2(x,z)$$

$$f(x, \omega_2, z) = 0$$

$$0 = \frac{\partial f}{\partial \omega_2}\frac{\partial \omega_2}{\partial x} + \frac{\partial f}{\partial x} \rightarrow \left.\frac{\partial f}{\partial y}\right|_{xz} \left.\frac{\partial y}{\partial x}\right|_z = -\left.\frac{\partial f}{\partial x}\right|_{yz}$$

$$0 = \frac{\partial f}{\partial \omega_2}\frac{\partial \omega_2}{\partial z} + \frac{\partial f}{\partial z} \qquad \left.\frac{\partial f}{\partial \omega_2}\right|_{xz} \left.\frac{\partial \omega_2}{\partial z}\right|_x = -\left.\frac{\partial f}{\partial z}\right|_x$$

$$z = \omega_3(x,y)$$

$$f(x, y, \omega_3) = 0$$

$$0 = \frac{\partial f}{\partial \omega_3}\frac{\partial \omega_3}{\partial x} + \frac{\partial f}{\partial x} \qquad \left.\frac{\partial f}{\partial y}\right|_{xz} \left.\frac{\partial y}{\partial z}\right|_x = -\left.\frac{\partial f}{\partial z}\right|_x$$

$$0 = \frac{\partial f}{\partial \omega_3}\frac{\partial \omega_3}{\partial y} + \frac{\partial f}{\partial y}$$

b)

$$\left.\frac{\partial x}{\partial y}\right|_{z,f} = \frac{-\left.\frac{\partial f}{\partial y}\right|_{xz}}{\left.\frac{\partial f}{\partial x}\right|_{yz}} \qquad \left.\frac{\partial y}{\partial x}\right|_z = \frac{\left.\frac{\partial f}{\partial x}\right|_{yz}}{-\left.\frac{\partial f}{\partial y}\right|_{xz}}$$

tomando $dz = 0$ en [1]

Comparando ambas expresiones se llega a

$$\Rightarrow \boxed{\left(\frac{\partial x}{\partial y}\right)_{z,f} = \frac{1}{\left(\frac{\partial y}{\partial x}\right)_{z,f}}}$$

a)

$$\left.\frac{\partial x}{\partial y}\right|_z \left.\frac{\partial y}{\partial z}\right|_x = \left(-\frac{\partial f}{\partial y}\right)_{xz}\left(\frac{\partial x}{\partial f}\right)_{yz} \cdot \left(-\frac{\partial f}{\partial z}\right)_{xy}\left(\frac{\partial y}{\partial f}\right)_{xz}$$

$$= \left(\frac{\partial f}{\partial y}\right)_z \cdot \left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial y}{\partial f}\right)_{xz} = \left(\frac{\partial f}{\partial y}\right)_z \left(\frac{\partial y}{\partial f}\right)_{xz} \left(\frac{\partial x}{\partial z}\right)_y$$

$$\Rightarrow \boxed{\left(\frac{\partial x}{\partial y}\right)_z \cdot \left(\frac{\partial y}{\partial z}\right)_x = \left(\frac{\partial x}{\partial z}\right)_y}$$

c)

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = \left[\left(-\frac{\partial f}{\partial y}\right)_z \left(-\frac{\partial f}{\partial z}\right)_x \left(-\frac{\partial f}{\partial x}\right)_y\right]\left[\left(\frac{\partial f}{\partial x}\right)_{yz}\left(\frac{\partial f}{\partial y}\right)_{xz}\left(\frac{\partial f}{\partial z}\right)_{xy}\right]^{-1}$$

$$= \frac{-\left(\frac{\partial f}{\partial y}\right)_z \left(\frac{\partial f}{\partial z}\right)_x \left(\frac{\partial f}{\partial x}\right)_y}{\left(\frac{\partial f}{\partial x}\right)_{yz}\left(\frac{\partial f}{\partial y}\right)_{xz}\left(\frac{\partial f}{\partial z}\right)_{xy}} \Rightarrow$$

$$\boxed{\left(\frac{\partial x}{\partial y}\right)_z \cdot \left(\frac{\partial y}{\partial z}\right)_x \cdot \left(\frac{\partial z}{\partial x}\right)_y = -1}$$

$$\omega = \omega(x, z)$$

$$dx = \left(\frac{\partial x}{\partial y}\right)_z dy + \left(\frac{\partial x}{\partial z}\right)_y dz$$

$$dx = \left(\frac{\partial x}{\partial y}\right)_z \cdot \left(\frac{\partial \omega}{\partial x}\Big|_z dx + \frac{\partial \omega}{\partial z}\Big|_x dz\right)$$

$$\left(\frac{\partial x}{\partial \omega}\right)_z = \left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial \omega}\right)_z + \left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial \omega}\right)_z$$

$$d\omega = \frac{\partial \omega}{\partial y}\Big|_x dy + \frac{\partial \omega}{\partial x}\Big|_y dx$$

$$d\omega - \frac{\partial \omega}{\partial x}\Big|_y dx = \frac{\partial \omega}{\partial y}\Big|_x dy$$

$$\frac{\partial \omega}{\partial y}\Big|_z - \frac{\partial \omega}{\partial x}\Big|_y \frac{\partial x}{\partial y}\Big|_z = \frac{\partial \omega}{\partial y}\Big|_x$$

1. $dw = A.dx + B.dy$ com $A = \frac{\partial \omega}{\partial x}\Big|_y$, $B = \frac{\partial \omega}{\partial y}\Big|_x$

Pero $\frac{\partial^2 \omega}{\partial y \partial x} = \frac{\partial^2 \omega}{\partial x \partial y} \Rightarrow$

$$\frac{\partial}{\partial y}(A)\Big]_x = \frac{\partial}{\partial x}(B)\Big]_y \Rightarrow \boxed{\frac{\partial A}{\partial y}\Big|_x = \frac{\partial B}{\partial x}\Big|_y}$$

2. $dU = T.dS - p.dV$

$$\frac{\partial}{\partial V}\left(\frac{\partial U}{\partial S}\Big|_V\right)\Big|_S = \frac{\partial}{\partial S}\left(\frac{\partial U}{\partial V}\Big|_S\right)\Big|_V$$

$$\boxed{\frac{\partial T}{\partial V}\Big|_S = -\frac{\partial P}{\partial S}\Big|_V}$$

$A = U - TS \Rightarrow \quad dA = dU - T.dS - S.dT = -p.dV - S.dT$

$$\frac{\partial}{\partial T}\left(\frac{\partial A}{\partial V}\Big|_T\right)_V = \frac{\partial}{\partial V}\left(\frac{\partial A}{\partial T}\Big|_V\right)_T$$

$$\boxed{-\frac{\partial P}{\partial T}\Big|_V = -\frac{\partial S}{\partial V}\Big|_T}$$

$G = U - TS + PV \rightarrow dG = \underbrace{dU - T.dS + p\,dV}_{0} - S.dT + V.dP = -S.dT + V.dP$

$$\frac{\partial}{\partial P}\left(\frac{\partial G}{\partial T}\Big|_P\right)_T = \frac{\partial}{\partial T}\left(\frac{\partial G}{\partial P}\Big|_T\right)_P$$

$$\boxed{-\frac{\partial S}{\partial P}\Big|_T = \frac{\partial V}{\partial T}\Big|_P}$$

$H = U + PV \rightarrow \quad dH = dU + P.dV + V.dP = T.dS + V.dP$

$$\frac{\partial}{\partial P}\left(\frac{\partial H}{\partial S}\Big|_P\right)_S = \frac{\partial}{\partial S}\left(\frac{\partial H}{\partial P}\Big|_S\right)_P$$

$$\boxed{\frac{\partial T}{\partial P}\Big|_S = \frac{\partial V}{\partial S}\Big|_P}$$

3. $*\ \frac{\partial}{\partial V}[C_V]\Big|_{T,N} =$

$$\frac{\partial}{\partial V}\left(\frac{\partial U}{\partial T}\Big|_{V,N}\right)_{T,N} =$$

$$\frac{\partial}{\partial V}\left[T.\frac{\partial S}{\partial T}\Big|_{V,N}\right]_{T,N} =$$

$$T.\frac{\partial}{\partial V}\left(\frac{\partial S}{\partial T}\Big|_{V,N}\right)_{T,N} =$$

$$T.\frac{\partial}{\partial T}\left(\frac{\partial S}{\partial V}\Big|_{T,N}\right)_{V,N} = T.\frac{\partial}{\partial T}\left(\frac{\partial P}{\partial T}\Big|_{V,N}\right)_{V,N}$$

$$\Rightarrow \boxed{\frac{\partial C_V}{\partial V}\Big|_{T,N} = T.\frac{\partial^2 P}{\partial T^2}\Big|_{V,N}}$$

• auxiliar

$C_V = \frac{1}{N}\frac{\partial Q}{\partial T}\Big|_V$

$dQ = dU + P.dV \Rightarrow V$ constante

$C_V = \frac{1}{N}\frac{\partial U}{\partial T}\Big|_V$

$dU = T.dS - P.dV$

$= T\left(\frac{\partial S}{\partial T}\Big|_V dT + \frac{\partial S}{\partial V}\Big|_T dV\right) - P.dV$

$dU = T.\frac{\partial S}{\partial T}\Big|_V dT + \left(\frac{\partial S}{\partial V}\Big|_T - P\right).dV$

$*\ \frac{\partial}{\partial P}[C_P]\Big|_{T,N} =$

$$\frac{\partial}{\partial P}\left(\frac{\partial U}{\partial T}\Big|_P + P.\frac{\partial V}{\partial T}\Big|_P\right)\Big|_{T,N} =$$

$$\frac{\partial}{\partial P}\left(\frac{\partial U}{\partial T}\Big|_P\right)_{T,N} + \frac{\partial}{\partial P}\left(P.\frac{\partial V}{\partial T}\Big|_P\right)_{T,N} =$$

$$\frac{\partial}{\partial P}\left(\frac{\partial U}{\partial T}\Big|_P\right)_{T,N} + \frac{\partial V}{\partial T}\Big|_P + P.\frac{\partial}{\partial P}\left(\frac{\partial V}{\partial T}\Big|_P\right)_{T,N} =$$

$$\frac{\partial}{\partial P}\left[T.\frac{\partial S}{\partial T}\Big|_P - P.\frac{\partial V}{\partial T}\Big|_P\right]_{T,N} + P.\frac{\partial}{\partial P}\left(\frac{\partial V}{\partial T}\Big|_P\right)_{T,N} + \frac{\partial V}{\partial T}\Big|_P =$$

$$T.\frac{\partial}{\partial P}\left[\frac{\partial S}{\partial T}\Big|_P\right]_{T,N} =$$

$$T.\frac{\partial}{\partial T}\left(\frac{\partial S}{\partial P}\Big|_T\right)_{P,N} = -T.\frac{\partial}{\partial T}\left(\frac{\partial V}{\partial T}\Big|_P\right)_{P,N}$$

$$\Rightarrow \boxed{\left(\frac{\partial C_P}{\partial P}\right)_{T,N} = -T\left(\frac{\partial^2 V}{\partial T^2}\right)_{T,N}}$$

Considero C_P como calor específico ($\hookrightarrow c_P = \frac{C_P}{N}$) $\quad C_P = \frac{\partial Q}{\partial T}\Big|_P$

$dQ = dU + P.dV$

$dV = \frac{\partial V}{\partial P}\Big|_T dP + \frac{\partial V}{\partial T}\Big|_P dT$

$dQ = dU + P.\frac{\partial V}{\partial P}\Big|_T dP + P.\frac{\partial V}{\partial T}\Big|_P dT$

$dU = \frac{\partial U}{\partial P}\Big|_T dP + \frac{\partial U}{\partial T}\Big|_P dT$

$C_P = \frac{\partial Q}{\partial T}\Big|_P = \frac{\partial U}{\partial T}\Big|_P + P.\frac{\partial V}{\partial T}\Big|_P$

$dU = T.dS - P.dV$

$T\left(\frac{\partial S}{\partial T}\Big|_P dT + \frac{\partial S}{\partial P}\Big|_T dP\right) - P\left(\frac{\partial V}{\partial T}\Big|_P dT + \frac{\partial V}{\partial P}\Big|_T dP\right)$

$\frac{\partial U}{\partial T}\Big|_P = T.\frac{\partial S}{\partial T}\Big|_P - P.\frac{\partial V}{\partial T}\Big|_P$

$*\ C_P - C_V = \frac{\partial U}{\partial T}\Big|_P + P.\frac{\partial V}{\partial T}\Big|_P - \frac{\partial U}{\partial T}\Big|_V = T.\frac{\partial S}{\partial T}\Big|_P - T.\frac{\partial S}{\partial T}\Big|_V = T.\left(\frac{\partial S}{\partial T}\Big|_P - \frac{\partial S}{\partial T}\Big|_V\right)$

$$= T.\left(\frac{\partial S}{\partial V}\Big|_T.\frac{\partial V}{\partial T}\Big|_P\right) = T.\frac{\partial P}{\partial T}\Big|_V.\frac{\partial V}{\partial T}\Big|_P = V.T.\frac{\alpha^2}{\kappa_T}$$

$$= -T.\left(-V\,\alpha\right.$$

usando lo de abajo

$$\boxed{C_P - C_V = V.T.\frac{\alpha^2}{\kappa_T}}$$

$$VT\frac{\alpha^2}{\kappa_T} = -\frac{VT}{V^2}\frac{\frac{\partial V}{\partial T}\Big|_P.\frac{\partial V}{\partial T}\Big|_P V}{\left(\frac{\partial V}{\partial P}\right)_T} = -T.\frac{\frac{\partial V}{\partial T}\Big|_P}{\frac{\partial T}{\partial V}\Big|_P.\frac{\partial V}{\partial P}\Big|_T} = -T.\frac{\partial V}{\partial T}\Big|_P.\frac{\partial P}{\partial T}\Big|_V$$

$\frac{\partial T}{\partial P}\Big|_V$

8.

$U = U(S,V,N)$ (en general) $\qquad H = U + P.V$

$\left.\frac{\partial U}{\partial V}\right|_{S,N} = 0 \rightarrow$ en particular será $\left.\frac{\partial U}{\partial V}\right|_T = 0$ $\qquad H = H(S,P)$, $H = H(T,P)$

$\left.\frac{\partial H}{\partial P}\right|_T = 0$

Puedo ver a la energía interna como función de T,V con lo cual:

$$dU = \left.\frac{\partial U}{\partial T}\right|_V dT + \underbrace{\left.\frac{\partial U}{\partial V}\right|_T}_{=0} . dV \Rightarrow dU = \left.\frac{\partial U}{\partial T}\right|_V . dT$$

$$dU = C_v.N.dT$$

$$\boxed{U = C_v.N.T + U_0}$$

EL diferencial de energía interna se escribirá:

$$dU = dQ - p\,dV + \mu.dN$$

$$dU = \underbrace{T}_{\left.\frac{\partial U}{\partial S}\right|_{V,N}}.dS - \underbrace{p}_{\left.\frac{\partial U}{\partial V}\right|_{S,N}}.dV + \underbrace{\mu}_{\left.\frac{\partial U}{\partial N}\right|_{S,V}}.dN$$

$\left.\frac{\partial U}{\partial V}\right|_{S,N} = 0 = -P$ significa que la presión es nula si S,N se mantienen constantes.

Con $dN = 0 \rightarrow dU = T.\left(\left.\frac{\partial S}{\partial T}\right|_V dT + \left.\frac{\partial S}{\partial V}\right|_T dV\right) - p.dV$

$$dU = \underbrace{T.\left.\frac{\partial S}{\partial T}\right|_V}_{\left.\frac{\partial U}{\partial T}\right|_V} . dT + \underbrace{\left[T\left.\frac{\partial S}{\partial V}\right|_T - P\right]}_{\left.\frac{\partial U}{\partial V}\right|_T = 0} . dV \qquad P = T.\left.\frac{\partial S}{\partial V}\right|_T$$

relación de Maxwell $\rightarrow$ $P = T.\left.\frac{\partial P}{\partial T}\right|_V$

$$\int \left.\frac{\partial T}{T}\right|_V = \int \left.\frac{\partial P}{P}\right|_V$$

$$\ln\left(\frac{T}{T_0}\right) = \ln\left(\frac{P}{P_0}\right)$$

$\left.\frac{\partial S}{\partial V}\right|_T = \frac{P}{T} = \frac{k}{V}$ ← uso la ecuación de estado

$\int \partial S|_T = k\int \left.\frac{\partial V}{V}\right|_T \rightarrow S|_T = k.\ln\left(\frac{V}{V_0}\right) + C_1(T)$

$\left.\frac{\partial S}{\partial T}\right|_V = \frac{C_v.N}{T} \rightarrow \int \partial S|_V = C_v N \int \left.\frac{\partial T}{T}\right|_V$

$S|_V = C_v N \ln\left(\frac{T}{T_0}\right) + C_2(V)$

$T = P.C_2(V) \Rightarrow P = T.C_3(V)$

* $H = H(T,P)$ $\qquad dH = \left.\frac{\partial H}{\partial T}\right|_P dT + \underbrace{\left.\frac{\partial H}{\partial P}\right|_T}_{=0} dP$ $\qquad$ Considero un solo tipo de moles $dN = 0$

$$dH = \left.\frac{\partial H}{\partial T}\right|_P . dT$$

$dH = dU + P.dV + V.dP$

$dH = C_v N dT + P.\left(\left.\frac{\partial V}{\partial T}\right|_P dT + \left.\frac{\partial V}{\partial P}\right|_T dP\right) + V.dP$

$$dH = \left[C_v N + P.\left.\frac{\partial V}{\partial T}\right|_P\right].dT + \underbrace{\left[V + P.\left.\frac{\partial V}{\partial P}\right|_T\right]}_{=0} dP$$

$$V = -\left.\frac{\partial V}{\partial P}\right|_T . P$$

$$\int -\left.\frac{\partial P}{P}\right|_T = \int \frac{1}{V}.\partial V$$

$$\ln\left(\frac{P_0}{P}\right) = \ln\left(\frac{V}{V_0}\right) \rightarrow P = \frac{1}{V}.C_4(T)$$

Comparando $\left\{\begin{array}{l} P = T.C_3(V) \\ P = \frac{1}{V}.C_4(T)\end{array}\right\} \Rightarrow C_3 = \frac{k}{V}, \; C_4 = k.T$

$$\boxed{P = \frac{T}{V}k} \Rightarrow \boxed{P.V = kT} \qquad \boxed{S = k.\ln\left(\frac{V}{V_0}\right) + C_v.N.\ln\left(\frac{T}{T_0}\right)}$$

con $k \equiv$ Constante de Boltzmann

9.

$$P.V = nRT \rightarrow P.v = R.T \leftarrow \text{Ecuación de estado del gas ideal por mol}$$

$$\alpha = \frac{1}{V}\cdot\frac{\partial V}{\partial T}\Big|_P = \frac{1}{v}\cdot\frac{\partial v}{\partial T}\Big|_P \rightarrow \alpha = \frac{1}{v}\cdot\frac{\partial}{\partial T}\left(\frac{R.T}{P}\right)\Big|_P =$$

$$\alpha = \frac{R}{v.P} \Rightarrow \boxed{\alpha = \frac{1}{T}}$$

$$\varkappa_T = -\frac{1}{v}\cdot\frac{\partial v}{\partial P}\Big|_T = -\frac{1}{v}\frac{\partial}{\partial P}\left(\frac{RT}{P}\right)\Big|_T \Rightarrow \varkappa_T = -\frac{1}{v}\left(-\frac{R.T}{P^2}\right) =$$

$$\boxed{\varkappa_T = \frac{1}{P}}$$

$$C_P - C_V = V.T\frac{\alpha^2}{\varkappa_T} = V.T\frac{1}{T^2}.P = \frac{P.V}{T} = n.R$$

capacidades caloríficas $\rightarrow C_P - C_V = n.R$

$c_p - c_v = R$ valores por mol

Usando teoría cinética de los gases [para el gas ideal] se ve que:

Energía total $\rightarrow U = \sum_{i=1}^{N} \frac{1}{2} m V_i^2$

Energía promedio $\rightarrow \langle u \rangle = \frac{\sum^N \frac{1}{2} m v_i^2}{N} = \frac{1}{2} m \sum_i^N v_i^2/N = \frac{1}{2} m \langle v^2 \rangle$

Según cálculo de presión $\longrightarrow P.V = m.N\frac{\langle v^2\rangle}{3}$

$$\langle u \rangle = \frac{m}{2}\langle v^2\rangle = \frac{3PV}{m.N}\cdot\frac{m}{2} = \frac{3}{2}\frac{PV}{N} = \frac{3}{2}kT$$

$$\langle u \rangle = \frac{3}{2}kT$$

$$U = \frac{3}{2}kNT = \frac{3}{2}nRT$$

Nota

$\frac{1}{n}\frac{\partial U}{\partial T}\Big|_V = C_v \leftarrow$ calor específico por mol

$\frac{1}{N}\frac{\partial U}{\partial T}\Big|_V = c_v \leftarrow$ calor específico por molécula

$$\frac{\partial U}{\partial T}\Big|_{V,N} = C_v.n = \frac{3}{2}kN \rightarrow C_v = \frac{3}{2}\frac{kN}{n} = \frac{3}{2}R$$

$$\Rightarrow C_p = R + C_v = \frac{5}{2}R \qquad \boxed{C_v = \frac{3}{2}R} \qquad \boxed{C_p = \frac{5}{2}R}$$

10.

$$\alpha = \frac{1}{V}\cdot\frac{\partial V}{\partial T}\Big|_P \qquad \varkappa_T = \frac{1}{V}\cdot\frac{\partial V}{\partial P}\Big|_T \qquad P = \frac{NRT}{V - bN} - \frac{N^2 a}{V^2}$$

obs. N son los moles aquí

No se puede despejar directamente V en la ecuación de estado para despejar

$$\frac{\partial V}{\partial P}\Big|_T = \frac{1}{\frac{\partial P}{\partial V}\Big|_T} \rightarrow \frac{\partial P}{\partial V}\Big|_T = NRT\cdot\frac{-1}{(V-bN)^2} + \frac{N^2 a\,2}{V^3}$$

$$\varkappa_T = \frac{1}{V}\cdot\frac{1}{\frac{2aN^2}{V^3} - \frac{NRT}{(V-bN)^2}} = \frac{1}{\frac{2N^2 a}{V^2} - \frac{NRT.V}{V(V-bN)^2}}$$

Pero, en Van der Waals puede ponerse:

$$\left(P+\frac{N^2a}{V^2}\right)(V-bN)=NRT \quad \rightarrow$$

$$\varkappa_T = \frac{1}{\frac{2N^2a}{V^2} - \left(P+\frac{N^2a}{V^2}\right)\frac{(V-bN).V}{(V-bN)^2}} = \frac{1}{\frac{2N^2a}{V^2} - P.V - \frac{N^2a}{V(V-bN)}}$$

$$\varkappa_T = \frac{1}{\frac{N^2a}{V}\left[\frac{2}{V}-\frac{1}{(V-bN)}\right]-P.V} = \frac{1}{\frac{N^2a}{V}\left(\frac{V-bN-V}{V(V-bN)}\right)-P.V}$$

$$\varkappa_T = \frac{-1}{P.V+\frac{N^3a.b}{V^2(V-bN)}} = \boxed{\frac{-V^2(V-bN)}{PV^3(V-bN)+N^3ab}=\varkappa_T}$$

$$\alpha = \frac{1}{V}\cdot\frac{1}{\left.\frac{\partial T}{\partial V}\right|_P} = \frac{1}{V}\cdot\frac{1}{\left.\frac{\partial}{\partial V}\left(\left[P+\frac{N^2a}{V^2}\right]\frac{(V-bN)}{NR}\right)\right|_T} = \frac{1}{V}\cdot\frac{1}{\left(\frac{N^2a\,2}{V^3}\right)\left(\frac{V-bN}{NR}\right)+\left[P+\frac{N^2a}{V^2}\right]\frac{1}{NR}}$$

$$\alpha = \frac{1}{\frac{2aN^2}{V^2}\cdot\frac{bN-V}{NR}+\left(PV+\frac{N^2a}{V}\right)\frac{1}{NR}}$$

$$\alpha = \frac{1}{\frac{1}{NR}\left[\frac{2aN^2}{V^2}(bN-V)+\left(\frac{PV^2+N^2a}{V}\right)\right]} = \frac{NRV^2}{2aN^2(bN-V)+PV^3+N^2aV}$$

$$\alpha = \frac{NRV^2}{P.V^3+N^2a\left(V-2(V-bN)\right)}$$

$$\boxed{\alpha = \frac{NRV^2}{P.V^3-N^2a(V-bN)}}$$

11.

$$\tau|_T = -k.x|_T \qquad \text{con } k=k(T) \quad , \quad \tau \equiv \frac{\text{fuerza}}{\text{longitud}}$$

tensión — elongación por unidad de masa

$$A = U - T.S \rightarrow dA = dU - T.dS - S.dT$$
$$dA = +\tau.dx - S.dT$$

$\psi = \psi(T,\tau,x) \leftarrow$ ecuación de estado

obs. Notar que $W=\tau.dx$ en comparación a $W=-p.dV$

$$dS = \left.\frac{\partial S}{\partial T}\right|_x dT + \left.\frac{\partial S}{\partial x}\right|_T dx$$
$$dS = \frac{1}{T}.dU + \frac{\tau}{T}.dx$$

$$dU = dQ + \tau.dx$$
$$dU = T.dS + \tau.dx$$

relación de Maxwell:

$$\frac{\partial}{\partial T}(\tau)\Big|_x = \frac{\partial}{\partial x}(-S)\Big|_T$$
$$-x\frac{\partial k(T)}{\partial T} = -\left.\frac{\partial S}{\partial x}\right|_T$$
$$+\frac{x^2}{2}k'(T) + C_2(T) = S$$

$$\left.\frac{\partial A}{\partial T}\right|_x = -S$$
$$S = +k'(T)\frac{x^2}{2} - C'(T)$$

$$-C_2(T) = C'(T)$$

$$\left.\frac{\partial A}{\partial x}\right|_T = -k(T).x$$
$$A = -k(T).\frac{x^2}{2} + C(T)$$

$$U = k(T)\frac{x^2}{2} + C(T) - T.k'(T)\frac{x^2}{2} - T.C'(T)$$

$$\left.\frac{\partial U}{\partial T}\right|_x = k'(T)\frac{x^2}{2} + C'(T) - k'(T)\frac{x^2}{2} - T.k''(T)\frac{x^2}{2} - C'(T) - T.C''(T)$$

$$\left.\frac{\partial U}{\partial T}\right|_x = -T.k''(T)\frac{x^2}{2} - T.C''(T) = T.\left.\frac{\partial S}{\partial T}\right|_x \rightarrow$$

$$\left.\frac{\partial S}{\partial T}\right|_x = -k''(T)\frac{x^2}{2} - C''(T)$$

$$dU = T.dS - \tau.dx$$

$$dU = T.\left.\frac{\partial S}{\partial T}\right|_x dT + \left(T.\left.\frac{\partial S}{\partial x}\right|_T - \tau\right).dx$$

$$\boxed{S = +k'(T).\frac{x^2}{2} - C'(T)}$$

$$\boxed{A = -k(T).\frac{x^2}{2} + C(T)}$$

$$U = \frac{x^2}{2}\left(-k(T) + T.k'(T)\right) + \underbrace{C(T) - T.C'(T)}$$

$$\boxed{U = [-k(T) + T.k'(T)]\frac{x^2}{2} + C_2(T)}$$

12.

1. $\frac{A}{M} = \frac{k}{2}x^2 = \frac{k}{2}\frac{L^2}{M^2}$ Tensión ahora es $f \rightarrow$ $[f] = \frac{kg.l}{s^2}$

tiene unidades de fuerza

$$\frac{dA}{M} = +\frac{f}{M}dx - \frac{S}{M}.dT \rightarrow \frac{1}{M}.\left.\frac{\partial A}{\partial x}\right|_T = \frac{1}{M}.kxM = \frac{f}{M} \rightarrow \boxed{f = +k.xM}$$

$\left.\frac{\partial A}{\partial x}\right|_T = f$

$$\frac{A}{M} = \frac{h}{2}(x-x_0)^2 + C \qquad \frac{1}{M}\left.\frac{\partial A}{\partial x}\right|_T = \frac{h}{M}(x-x_0) = -\frac{f}{M} \rightarrow \boxed{f = +h(x-x_0)M}$$

Estas dos recuadradas son las ecuaciones de estado porque se tiene $f = f(T, x)$ y como dice el enunciado k, x_0, h pueden depender de T

2. $\mu = \left.\frac{\partial A}{\partial M}\right|_{T, L = xM}$ $\qquad \mu = -\frac{k L^2}{2M^2} = \boxed{-\frac{kx^2}{2}}$

$$\frac{\partial}{\partial M}\left(\frac{kM}{2}\frac{L^2}{M^2}\right) = -\frac{kL^2}{2M^2}$$

$$\mu = \boxed{-\frac{h}{2}(x^2 - x_0^2) + C}$$

$$\frac{\partial}{\partial M}\left(\frac{hM}{2}\left(\frac{L}{M} - x_0\right)^2 + CM\right) = \frac{h}{2}(x-x_0)^2 + \frac{hM}{2}2(x-x_0)\cdot\frac{-L}{M^2} + C = \frac{h}{2}(x-x_0)^2 - h(x-x_0)x + C = -\frac{h}{2}(x^2 - x_0^2) + C$$

$$\frac{h}{2}x^2 + \frac{h}{2}x_0^2 - hx^2 + hx_0x - hx_0x$$

3. $\mu = \frac{A}{M} - \frac{f}{M}x$ $\qquad \mu = \frac{kx^2}{2} - kx^2 = \boxed{-\frac{kx^2}{2}}$ vale

$\left[\frac{A}{M}\right] = \frac{m^2}{kg.s^2}$

para que tenga las unidades correctas

$$\mu = \frac{h}{2}(x-x_0)^2 + C - h(x-x_0)x = \boxed{-\frac{hx^2}{2} + \frac{hx_0^2}{2} + C} \text{ vale}$$

$$\frac{h}{2}x^2 + \frac{hx_0^2}{2} - hxx_0 + C - hx^2 + hx_0x$$

4. Sano y roto para el resorte es como un cambio de fase → en la transición → $\mu_s = \mu_R$

$$-\frac{kx^2}{2} = -\frac{h}{2}(x^2 - x_0^2) = -\frac{hx^2}{2} + \frac{hx_0^2}{2}$$

$$\left(\frac{h-k}{2}\right)x^2 = \frac{x_0^2}{2} \rightarrow x_{\text{ruptura}} = \frac{x_0}{\sqrt{h-k}}$$

La tensión que a T fija rompe el resorte será

$$f_{\text{ruptura}} = -\frac{k.M.x_0}{\sqrt{h-k}} = -\frac{k.L_0}{\sqrt{h-k}}$$

14.

$$S = R\cdot\frac{V_0}{V}\cdot\left(\frac{T}{T_0}\right)^a \qquad \underset{\substack{\text{isot.}\\ \text{a}\\ T_0}}{W} = R\cdot T_0\cdot\ln\left(\frac{V}{V_0}\right) \qquad \underset{\substack{\text{isot.}\\ \text{a}\\ T_0}}{W} = \int_{V_0}^{V} -P\cdot dV$$

$S = S(V,T)$

$$dU = T\cdot dS - P\cdot dV$$

$$dU = \underbrace{\left[T\cdot\frac{\partial S}{\partial V}\Big|_T - P\right]}_{\frac{\partial U}{\partial V}\big|_T}\cdot dV + \underbrace{T\cdot\frac{\partial S}{\partial T}\Big|_V}_{\frac{\partial U}{\partial T}\big|_V}\cdot dT$$

$A = U - T\cdot S$

$dA = dU - T\cdot dS - S\,dT = -S\cdot dT - P\cdot dV \longrightarrow$ sea $T = T_0 \rightarrow \quad dA = -P\cdot dV \rightarrow$

$$S = -\frac{\partial A}{\partial T}\Big|_V \qquad P = -\frac{\partial A}{\partial V}\Big|_T \qquad\qquad dA = W_{\text{isot. a } T_0} \Rightarrow$$

$$-\frac{RV_0}{V}\cdot\frac{T^a}{T_0^a} = \frac{\partial A}{\partial T}\Big|_V \qquad\qquad A(V,T_0) = R\cdot T_0\,\ln\left(\frac{V}{V_0}\right)$$

$$-\frac{RV_0}{V}\cdot\frac{1}{T_0^a}\cdot\frac{T^{a+1}}{(a+1)} + C_1(V) = A(V,T) \rightarrow \quad A(V,T_0) = -\frac{R\,V_0}{V}\cdot\frac{T_0}{(a+1)} + C_1(V) = R\,T_0\,\ln\left(\frac{V}{V_0}\right)$$

$$\frac{\partial S}{\partial V}\Big|_T = \frac{\partial P}{\partial T}\Big|_V \qquad P = -\frac{\partial A}{\partial V}\Big|_T = -\frac{RV_0}{V^2}\,\frac{T^{a+1}}{T_0^a(a+1)} - C_1'(V) \qquad C_1(V) = R\,T_0\left(\frac{-V_0}{V(a+1)} + \ln\left[\frac{V}{V_0}\right]\right)$$

$$-RV_0\left(\frac{T}{T_0}\right)^a\frac{1}{V^2} = \frac{\partial P}{\partial T}\Big|_V$$

$$\boxed{A(V,T) = -\frac{RV_0}{V}\cdot\frac{T^{a+1}}{T_0^a(a+1)} - \frac{R\,T_0\,V_0}{V(a+1)} + RT_0\ln\left(\frac{V}{V_0}\right)}$$

$$-\frac{R\,V_0}{T_0^a}\,\frac{T^{a+1}}{(a+1)}\cdot\frac{1}{V^2} + C_2(V) = P$$

$$P(V,T) = \frac{R\,V_0\,T^{a+1}}{T_0^a\cdot(a+1)\cdot V^2} - C_1'(V)$$

$$\boxed{P(V,T) = \frac{R\cdot V_0\,T^{a+1}}{V^2\cdot T_0^a\cdot(a+1)} + RT_0\left[\frac{-V_0}{V^2(a+1)} - \frac{1}{V/V_0}\right]}$$

$$W = \int_{V_0}^{V} -P\cdot dV = -\frac{R\,T^{a+1}\cdot V_0}{T_0^a\,(a+1)}\int\frac{dV}{V^2} + \frac{R\,T_0\,V_0}{(a+1)}\int\frac{dV}{V^2} + RT_0V_0\int\frac{1}{V}\,dV$$

$$W = \frac{-RT^{a+1}V_0}{T_0^a(a+1)}\left(-\frac{1}{V} + \frac{1}{V_0}\right) + \frac{RT_0V_0}{(a+1)}\left(-\frac{1}{V} + \frac{1}{V_0}\right) + RT_0V_0\,\ln\left(\frac{V}{V_0}\right)$$

si $T = T_0 \rightarrow$ resulta el $W_{isot.}$ del enunciado

$$W = \frac{RV_0}{(a+1)}\left(-\frac{T^{a+1}}{T_0^a} + T_0\right)\left(\frac{V - V_0}{V_0\,V}\right) + R\,T_0V_0\,\ln\left(\frac{V}{V_0}\right)$$

$$\boxed{W = \frac{R}{a+1}\left(\frac{T_0^{a+1} - T^{a+1}}{T_0^a}\right)\left(\frac{V - V_0}{V}\right) + R\,T_0V_0\,\ln\left(\frac{V}{V_0}\right)}$$