Práctica 6: Teoría de Perturbaciones

1.

$$H = \underbrace{\frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2}_{H_0} + \lambda \underbrace{bx}_{V} \quad , \qquad H_0|0_n\rangle = E_n^0|0_n\rangle$$

$$E_n = E_n^0 + \lambda E_n^1 + \lambda^2 E_n^2 + \dots$$

orden en perturbación

$$|n\rangle = \underbrace{|0_n\rangle} + |1_n\rangle + |2_n\rangle + \dots + |i_n\rangle +$$

autoestado

a) la perturbación

$$E_n^1 = \langle\varphi_n|V|\varphi_n\rangle = \langle 0_n|V|0_n\rangle = \langle N|V|N\rangle$$

autoestados del oscilador armónico con hamiltoniano H_0

$$E_n^0 = E_N = \left(N + \frac{1}{2}\right)\hbar\omega$$

$$\langle N|V|N\rangle = \langle N|\lambda b x|N\rangle = \lambda b \langle N|\left(\frac{a+a^+}{2}\right)\left(\frac{2\hbar}{m\omega}\right)^{1/2}|N\rangle$$

$$= \lambda b \frac{\sqrt{2\hbar}}{\sqrt{m\omega}}\,\frac{1}{2}\left(\langle N|a|N\rangle + \langle N|a^+|N\rangle\right)$$

$$= \frac{\lambda b}{\sqrt{2}}\sqrt{\frac{\hbar}{m\omega}}\left(\sqrt{N}\langle N|N-1\rangle + \sqrt{N+1}\langle N|N+1\rangle\right)$$

El estado fundamental del oscilador armónico es $|N=0\rangle$, con $E = \frac{\hbar\omega}{2}$ que corresponde a

$$|0_0\rangle = |N=0\rangle \qquad E_0^0 = E = \frac{\hbar\omega}{2}$$

- Entonces a orden 1:

$$E^1_{N=0} = \langle N=0|V|N=0\rangle = \frac{\lambda b}{\sqrt{2}}\sqrt{\frac{\hbar}{m\omega}}\left(0\langle 0|-1\rangle + \sqrt{1}\underbrace{\langle 0|1\rangle}_{=0}\right) = 0$$

por ortonormalidad

, a orden 1 la energía es nula

Veamos a orden dos que resulta:

$$E_n^2 = \sum_p \frac{|\langle\varphi_p|V|\varphi_n\rangle|^2}{(E_n^0 - E_p^0)} = \sum_p \frac{\left|\frac{\lambda b}{2^{1/2}}\left(\frac{\hbar}{m\omega}\right)^{1/2}\right|^2 |\langle p|a+a^+|N\rangle|^2}{\underbrace{\left(N+\frac{1}{2}\right)\hbar\omega - \left(p+\frac{1}{2}\right)\hbar\omega}_{\hbar\omega(N-p)}} \quad \text{, pero}$$

$$\langle p|a|N\rangle + \langle p|a^+|N\rangle$$

$$\sqrt{N}\langle p|N-1\rangle + \sqrt{N+1}\langle p|N+1\rangle$$

$$\neq 0 \quad p = N-1 \qquad \neq 0 \quad p = N+1$$

Como estamos queriendo ver variación de energía del estado

$$E_n^2 = \left(\frac{\lambda^2 b^2}{2}\frac{\hbar}{m\omega}\right)\frac{(\sqrt{N})^2}{(\hbar\omega)\,1} + \left(\frac{\lambda^2 b^2}{2}\frac{\hbar}{m\omega}\right)\frac{(\sqrt{N+1})^2}{(\hbar\omega)\,-1}$$

$$E_N^{(2)} = \frac{\lambda^2 b^2 \hbar}{2m\omega\,\hbar\omega}\left(N - N + 1\right) = \frac{\lambda^2 b^2}{2m\omega^2}$$

Esta es la expresión genérica de la energía a orden 2 para cualquier autoestado $|N\rangle$ del oscilador armónico. Nótese que no depende del autoestado $|N\rangle$, entonces:

$[b] = [\omega]^2[m]$

$$E^{(2)}_{N=0} = \frac{\lambda^2 b^2}{2m\omega^2} \quad , y \quad E^{(0)}_{N=0} = \frac{\hbar\omega}{2}$$

$$\Delta E = E_0^{(0)} + E_0^{(2)} = \boxed{\frac{\hbar\omega}{2} - \frac{\lambda^2 b^2}{2m\omega^2} = \Delta E}$$

b) Para resolver en forma exacta, notaremos que podemos completar cuadrados

$$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} + \lambda b x + \underbrace{\frac{\lambda^2 b^2}{2m\omega^2} - \frac{\lambda^2 b^2}{2m\omega^2}}_{=0}$$

Completar

$$\frac{m\omega^2 x^2}{2} = A^2 \rightarrow \sqrt{\frac{m}{2}}\,\omega x = A$$

$$\lambda b x = 2AB$$

$$2\sqrt{\frac{m}{2}}\,\omega \cancel{x}\, B = \lambda b \cancel{x}$$

$$B = \frac{\lambda b}{\omega\sqrt{2m}}$$

$$H = \frac{p^2}{2m} + \left(\sqrt{\frac{m}{2}}\,\omega x + \frac{\lambda b}{\sqrt{2m}\,\omega}\right)^2 - \frac{\lambda^2 b^2}{2m\omega^2}$$

$$\frac{p^2}{2m} + \left(\sqrt{\frac{m}{2}}\,\omega\right)^2 \underbrace{\left(x + \frac{\lambda b}{m\omega^2}\right)^2}_{x'}$$

Es un hamiltoniano del mismo oscilador armónico, pero desplazado a $\left(-\frac{\lambda b}{m\omega^2}\right)$ y con un término constante de energía extra

$$H' = \frac{p^2}{2m} + \frac{m\omega^2}{2}\, x'^2 - \frac{\lambda^2 b^2}{2m\omega^2}$$

$$H|N\rangle = E_N|N\rangle = \left(N+\frac{1}{2}\right)\hbar\omega\,|N\rangle \quad , \text{con } H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2}$$

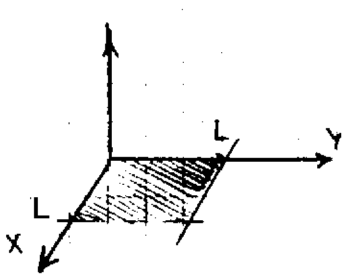
$$H'|N\rangle = \left(H + -\frac{\lambda^2 b^2}{2m\omega^2}\right)|N\rangle = \left[\left(N+\frac{1}{2}\right)\hbar\omega - \frac{\lambda^2 b^2}{2m\omega^2}\right]|N\rangle$$

$$\boxed{E_N = \left(N+\frac{1}{2}\right)\hbar\omega - \frac{\lambda^2 b^2}{2m\omega^2}}$$

Según puede verse todo el efecto de la perturbación quedó metido en el orden dos

2. Potencial 2D (pozo infinito)

$$V = \begin{cases} 0 & \text{en } 0 \le x \le L,\ 0 \le y \le L \\ \infty & \text{en otro caso} \end{cases}$$



$$H = \frac{p^2}{2m} = \frac{p_x^2}{2m} + \frac{p_y^2}{2m}$$

$$\Psi = \left[A e^{ikx} + B.e^{-ikx}\right]\left[C.e^{iky} + D.e^{-iky}\right]$$

$$A + B = 0 \rightarrow A = -B$$
$$A.e^{ikL} + B.e^{-ikL} = 0$$
$$A\left(e^{ikL} - e^{-ikL}\right) = 0$$

$$k_x L = n\pi \qquad k_x = \frac{n\pi}{L}$$

frec. espaciales permitidas

$$|\alpha\rangle = \int d^2x\, |\vec{x}'\rangle\langle\vec{x}'|\alpha\rangle$$
$$|\alpha\rangle = \int d^2x\, \Psi_\alpha(\vec{x}')\, |\vec{x}'\rangle$$
$$|\alpha\rangle = \int d^2p\, \phi_\alpha(\vec{p}')\, |\vec{p}'\rangle$$

$$\Psi(x,y) = N.\ \text{sen}(k_x x).\ \text{sen}(k_y y)$$

$$\int_0^L\int_0^L |N|^2\, \text{sen}^2(kx).\ \text{sen}^2(ky)\, dx\, dy$$

$$|N|^2\, \frac{\pi}{2}\,\frac{\pi}{2} = 1 \rightarrow |N|^2 = \frac{4}{\pi^2} \qquad N = \frac{2}{\pi}$$

$$\hbar k = p \qquad \frac{\hbar n_x \pi}{L} = p_x \qquad \frac{\hbar n_y \pi}{L} = p_y$$

estado fundamental

$$E_n = \frac{\hbar^2 n_x^2 \pi^2}{L^2 2m} + \frac{\hbar^2 n_y^2 \pi^2}{L^2 2m} = \frac{\hbar^2 \pi^2}{m L^2}\left(n_x^2 + n_y^2\right)$$

$$E_1 = \frac{\hbar^2 \pi^2}{m L^2}(1) \longrightarrow \begin{matrix} n_x = 1 & n_y = 0 \\ n_x = 0 & n_y = 1 \end{matrix} \Big\} \text{ deg. } 2 \qquad |\vec{p}_1\rangle = |p_x, 0\rangle, \ |p_y, 0\rangle$$

primer excitado

$$E_2 = \frac{\hbar^2 \pi^2}{m L^2}(2) \longrightarrow n_x = 1 \quad n_y = 1 \ \} \text{ deg. } 1 \qquad |\vec{p}_2\rangle$$

Ahora agregamos la perturbación $V_1 = \begin{cases} \lambda x y & 0 \le x \le L, \ 0 \le y \le L \\ 0 & \text{en otro caso} \end{cases}$

$$H = H_0 + \lambda x y \qquad \text{en} \qquad 0 \le x \le L, \ 0 \le y \le L$$

$$E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)}$$

$E_n^{(1)} = \langle \varphi_n | V | \varphi_n \rangle$; para el fundamental será: $\begin{array}{cc} p_x & p_y \\ \left|\frac{\hbar\pi}{L}, 0\right\rangle ; & \left|0, \frac{\hbar\pi}{L}\right\rangle \end{array}$

$$E_1^{(1)} = \langle \varphi_1 | \lambda x y | \varphi_1 \rangle$$

$$= \lambda \langle \vec{p}_1 | \left(i\hbar \frac{\partial}{\partial p_x}\right)\left(i\hbar \frac{\partial}{\partial p_y}\right) | p_x, 0 \rangle = 0$$

$$\hat{x} = i\hbar \frac{\partial}{\partial p}$$

$$-\hbar^2 \frac{\partial^2}{\partial p_x \partial p_y} \begin{Bmatrix} |p_x, 0\rangle \\ |0, p_y\rangle \end{Bmatrix} = 0$$

3.

$$H_0 = \frac{P_x^2}{2m} + \frac{P_y^2}{2m} + \frac{m\omega^2}{2}(x^2+y^2)$$

a) Como está desacoplado puede verse como

$$H_0 = H_1 + H_2 \;, \text{ con } \quad H_j = \frac{P_j^2}{2m} + \frac{m\omega^2 j^2}{2} \qquad ; j=1,2$$

, entonces su solución es la suma de soluciones H_1, H_2 de los osciladores unidimensionales.

$$H_1 \longrightarrow \quad E_x = \left(N_x + \frac{1}{2}\right)\hbar\omega$$

$$H_2 \longrightarrow \quad E_y = \left(N_y + \frac{1}{2}\right)\hbar\omega \qquad \Rightarrow \qquad H_0 \rightarrow \quad E = \left(N_x + N_y + 1\right)\hbar\omega$$

Estado fundamental	$e_0 = \hbar\omega$	$N_x = N_y = 0$
Primer excitado	$e_1 = 2\hbar\omega$	$N_x = 1, N_y = 0$; $N_x = 0, N_y = 1$ — deg 2
Segundo excitado	$e_2 = 3\hbar\omega$	$N_x = 2$ $N_y = 0$; $N_x = 0$ $N_y = 2$; $N_x = 1$ $N_y = 1$ — deg 3

A partir del fundamental hay degeneración.

b) $\quad V = \delta m\omega^2 xy \qquad . \; \delta \ll 1$

$$H = H_0 + \delta m\omega^2 xy \quad , \quad H \text{ es par} \rightarrow [\Pi, H] = 0$$ pero hay degeneración

* <u>Estado fundamental</u> (N=0)

$$E^{(0)}_{n=0} = e_0$$

$$E^{(1)}_{n=0} = \langle \varphi_n | V | \varphi_n \rangle = \langle N_x=0, N_y=0 | \delta m\omega^2 xy | N_x=0, N_y=0 \rangle$$

$$= \delta m\omega^2 \langle 0,0 | xy | 0,0 \rangle = \delta m\omega^2 \langle 0,0 | yx | 0,0 \rangle$$

, pero $\quad x = \sqrt{\frac{\hbar}{2m\omega}}\,(a + a^+) \quad , \quad y = \sqrt{\frac{\hbar}{2m\omega}}\,(a + a^+)$

$$\delta m\omega^2 \sqrt{\frac{\hbar}{2m\omega}} \langle n_x=0, n_y=0 | y\,(a+a^+) | n_x=0, n_y=0 \rangle \Rightarrow$$

$$\langle 00 | y.a | 00 \rangle + \langle 00 | y a^+ | 00 \rangle$$

$$\sqrt{0}\,\langle 00 | y | {-1}0 \rangle + \sqrt{1}\,\langle 00 | y | 10 \rangle \rightarrow \delta m\omega^2 \langle 00 | xy | 00 \rangle = 0 = E^{(1)}_{n=0}$$

* <u>Primer excitado</u> (N=1) (hay degeneración 2)

$$E^{(0)}_{N=1} = e_1$$

$$E^{(1)}_{N=1} = \langle \varphi_n | V | \varphi_n \rangle$$

$$\langle 1\,0 | \delta m\omega^2 xy | 1\,0 \rangle = 0$$ es nulo porque es la misma situación que el anterior pero reemplazando $x \to y$, $n_x \to n_y$

$$\langle 0\,1 | \delta m\omega^2 xy | 0\,1 \rangle = 0$$ es nulo por la misma cuenta anterior

$$\langle 1\,0 | \delta m\omega^2 xy | 0\,1\rangle = \delta m\omega^2 \overset{\sqrt{\frac{\hbar}{2m\omega}}}{\downarrow} \langle n_x=1, n_y=0 | (a+a^\dagger)y | n_x=0, n_y=1\rangle$$

$$= \delta m\omega^2 \sqrt{\frac{\hbar}{2m\omega}} \cdot \Big[\underbrace{\langle 10 | \underbrace{a\,y}_{y\,a|01\rangle=0} | 01\rangle}_{=0} + \underbrace{\langle 10 | a^\dagger y | 01\rangle}_{1\langle 10|y|11\rangle} \Big] =$$

$$\sqrt{\frac{\hbar}{2m\omega}} \Big[\underset{1\langle 10|10\rangle}{\langle 10 | a | 11\rangle} + \underbrace{\langle 10 | a^\dagger | 11\rangle}_{\langle 10|a^\dagger)=0} \Big]$$

$$= \delta \cancel{m}\omega^{\cancel{2}} \frac{\hbar}{\cancel{2m\omega}} \langle 1\,0 | 1\,0\rangle = \frac{\delta\hbar\omega}{2}\langle 10|10\rangle = (\langle 1|\otimes\langle 0|)(|1\rangle\otimes|0\rangle)\frac{\delta\hbar\omega}{2} = \underbrace{\langle 1|1\rangle\langle 0|0\rangle}_{=1}\frac{\delta\hbar\omega}{2} =$$

$$\langle 0\,1 | \delta m\omega^2 xy | 1\,0\rangle = \delta m\omega^2 \sqrt{\frac{\hbar}{2m\omega}} \Big[\underbrace{\langle 01 | y\,a | 10\rangle}_{\langle 01|y|00\rangle} + \underbrace{\langle 01 | y\,a^\dagger | 10\rangle}_{\langle 01|y|20\rangle = 0} \Big] =$$

$$\sqrt{\frac{\hbar}{2m\omega}} \Big[\underbrace{\langle 01 | a | 00\rangle}_{=0} + \underset{\langle 01|01\rangle}{\langle 01 | a^\dagger | 00\rangle} \Big]$$

$$= \delta \cancel{m}\omega^{\cancel{2}} \frac{\hbar}{\cancel{2m\omega}} \langle 0\,1 | 0\,1\rangle = \frac{\delta\hbar\omega}{2}\underbrace{\langle 01|01\rangle}_{\langle 0|0\rangle\langle 1|1\rangle = 1} = \frac{\delta\hbar\omega}{2}$$

$$V \doteq \begin{array}{cc} |01\rangle & |10\rangle \end{array}$$
$$V \doteq \begin{array}{c} \begin{pmatrix} 0 & \delta\hbar\omega/2 \\ \delta\hbar\omega/2 & 0 \end{pmatrix} \end{array} \begin{array}{l} \langle 01| \\ \langle 10| \end{array}$$

Normalización (de autoestados osc. armónico)

$$\langle N' | N\rangle = \delta_{N'N}$$

$$\langle N_x=1, N_y=0 | N_x=1, N_y=0\rangle = 1$$
$$\langle N_x=0, N_y=1 | N_x=0, N_y=1\rangle = 1$$

diagonalización V

$$\lambda^2 - \frac{\delta^2\hbar^2\omega^2}{4} = 0 \rightarrow \lambda = \pm\frac{\delta\hbar\omega}{2}$$

$$\rightarrow \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ +1 \end{pmatrix}$$

$$|A\rangle = \frac{1}{\sqrt{2}}\left(|0,1\rangle + |1,0\rangle\right)$$

$$|B\rangle = \frac{1}{\sqrt{2}}\left(|0,1\rangle - |1,0\rangle\right)$$

$$E^{(1)}_{N=1} = \pm\frac{\delta\hbar\omega}{2}$$

NOTA

$$\langle 10 | xy | 01\rangle = (\langle 1|\otimes\langle 0|)(x\otimes 1)(1\otimes y)(|0\rangle\otimes|1\rangle) =$$
$$\langle 1|x|0\rangle . \langle 0|y|1\rangle$$

Escribir bien un operador entre estados de diferentes espacios

$$E_{N=0} = E^{(0)}_{N=0} + E^{(1)}_{N=0} = \hbar\omega + 0 = \boxed{\hbar\omega = E_{N=0}}$$

$$E_{N=1} = E^{(0)}_{N=1} + E^{(1)}_{N=1} = 2\hbar\omega + \left(\pm\frac{\delta\hbar\omega}{2}\right) = \begin{cases} 2\hbar\omega + \delta\frac{\hbar\omega}{2} \\ 2\hbar\omega - \delta\frac{\hbar\omega}{2} \end{cases}$$

$$\boxed{E_{N=1} = \begin{cases} \hbar\omega\left(2+\frac{\delta}{2}\right) \\ \hbar\omega\left(2-\frac{\delta}{2}\right) \end{cases}}$$

Autoestados (N=0, N=1) a orden cero

$$|N = N_x+N_y = 0\rangle^{(0)} = |0\,0\rangle$$
$$|N = N_x+N_y = 1\rangle^{(0)} = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$$
$$\frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

c)

$$H_0+V = \frac{P_x^2}{2m} + \frac{P_y^2}{2m} + \frac{m\omega^2}{2}x^2 + \frac{m\omega^2}{2}y^2 + \delta m\omega^2 xy$$

$$\frac{m\omega^2}{2}\left(x^2+y^2+\delta 2xy\right)$$

Para desacoplar necesitaremos un cambio de variables, ensayaremos en el término de acople $x.y$ para disolverlo:

$$\begin{aligned} x &= K_1\alpha + K_2\beta \\ y &= K_3\alpha + K_4\beta \end{aligned} \Rightarrow \begin{aligned} x.y = {} & K_1K_3\alpha^2 + K_2K_4\beta^2 + \\ & K_1K_4\alpha\beta + K_2K_3\beta\alpha \end{aligned}$$

, necesitaré nulos los términos con $\alpha\beta \rightarrow$ $K_1K_4 + K_2K_3 = 0 \rightarrow K_1K_4 = -K_2K_3$
puedo tomar $K_1 = K_2 = 1$ $K_4 = -K_3 = 1 \rightarrow , K_3 = -1$

$$\begin{aligned} x &= \alpha+\beta \\ y &= -\alpha+\beta \end{aligned} \Rightarrow \quad \alpha = \frac{x-y}{2} \qquad \beta = \frac{x+y}{2}$$

Podemos normalizarlos con $\alpha = \dfrac{x-y}{\sqrt{2}}$, $\beta = \dfrac{x+y}{\sqrt{2}}$ $\Rightarrow$

$$P_\alpha = \frac{P_x - P_y}{\sqrt{2}} \quad , P_\beta = \frac{P_x+P_y}{\sqrt{2}} \quad \Rightarrow$$

$$P_\beta^2 + P_\alpha^2 = \frac{P_x^2}{2} - \frac{2P_xP_y}{\sqrt{2}} + \frac{P_y^2}{2} + \frac{P_x^2}{2} + \frac{2P_xP_y}{\sqrt{2}} + \frac{P_y^2}{2} = P_x^2 + P_y^2$$

$$x^2+y^2 = \frac{\alpha^2}{2} + \frac{\beta^2}{2} - \frac{2\alpha\beta}{\sqrt{2}} + \frac{\alpha^2}{2} + \frac{\beta^2}{2} + \frac{2\alpha\beta}{\sqrt{2}} = \alpha^2+\beta^2$$

$$x.y = \left(\frac{\alpha+\beta}{\sqrt{2}}\right)\left(\frac{\beta-\alpha}{\sqrt{2}}\right) = \frac{\beta^2-\alpha^2}{2}$$

$$H_0+V = H = \frac{P_\alpha^2}{2m} + \frac{P_\beta^2}{2m} + \frac{m\omega^2}{2}(\alpha^2+\beta^2) + \delta\frac{m\omega^2}{2}(\beta^2-\alpha^2)$$

$$H = \frac{P_\alpha^2}{2m} + \frac{P_\beta^2}{2m} + \frac{m\omega^2}{2}\alpha^2\left[1-\delta\right] + \frac{m\omega^2}{2}\beta^2\left[1+\delta\right]$$

, ahora ha quedado un H desacoplado en α y β $\Rightarrow$

$$E = {}^{(\alpha)}E_n + {}^{(\beta)}E_n \rightarrow$$

$$E_n = \left(n_\alpha + \frac{1}{2}\right)\hbar\omega_\alpha + \left(n_\beta + \frac{1}{2}\right)\hbar\omega_\beta \qquad , \text{donde} \begin{cases} \omega_\alpha = \omega.(1-\delta)^{1/2} \\ \omega_\beta = \omega.(1+\delta)^{1/2} \end{cases}$$

$$= \left(n_\alpha+\frac{1}{2}\right)\hbar\omega\sqrt{1-\delta} + \left(n_\beta + \frac{1}{2}\right)\hbar\omega\sqrt{1+\delta}$$

$$\boxed{E_n = \hbar\omega\left[n_\alpha\sqrt{1-\delta} + \frac{\sqrt{1-\delta}}{2} + n_\beta\sqrt{1+\delta} + \frac{\sqrt{1+\delta}}{2}\right]} \rightarrow \text{Solución Exacta de las Energías}$$

si $\delta \rightarrow 0 \Rightarrow E_n \rightarrow \hbar\omega\left\{\left(n_\alpha+\frac{1}{2}\right)\left[1-\frac{\delta}{2}+\frac{3\delta^2}{8}\right] + \left(n_\beta+\frac{1}{2}\right)\left[1+\frac{\delta}{2}+\frac{3\delta^2}{8}\right]\right\}$

$$E_n^{(1)} \rightarrow \hbar\omega\left\{ n_\alpha - \frac{n_\alpha\delta}{2} + \frac{1}{2} - \cancel{\frac{\delta}{4}} + n_\beta + n_\beta\frac{\delta}{2} + \frac{1}{2} + \cancel{\frac{\delta}{4}} \right\}$$

a 1er orden $\rightarrow \hbar\omega\left\{n_\alpha + n_\beta + 1 + (n_\beta - n_\alpha)\frac{\delta}{2}\right\}$

A orden cero $\longrightarrow$ $\delta = 0$ $\quad E_n^{(0)} = \hbar\omega\left(n_\alpha + n_\beta + 1\right)$ $\quad E_{00}^{(0)} = \hbar\omega$

A orden uno $\longrightarrow$ δ^1 $\quad E_n^{(1)} = \hbar\omega\left(n_\alpha + n_\beta + 1 + [n_\alpha - n_\beta]\frac{\delta}{2}\right)$

$$E_{n=1}^{(1)} = \begin{cases} \hbar\omega\left(2 + \frac{\delta}{2}\right) \\ \hbar\omega\left(2 - \frac{\delta}{2}\right) \end{cases}$$

Estos resultados coinciden con los que se obtuvieron para el método perturbativo, a primer orden en el factor de perturbación δ.

4.

fundamental no degenerado , estados $|n, l\rangle$

$\vec{E} = E_0\hat{z}$ $\qquad H = H_0 - eE_0 z$, H_0 hamiltoniano del átomo de H

operador $\longrightarrow$ $e\hat{z}$

5.
$$H = \begin{pmatrix} E_1^0 & \lambda\Delta \\ \lambda\Delta & E_2^0 \end{pmatrix} \qquad \lambda = 0 \text{ es el problema no perturbado}$$

a)
$$(E_1^0 - \gamma)(E_2^0 - \gamma) - \lambda^2\Delta^2 = 0$$
$$E_1^0 E_2^0 - \gamma E_2^0 - \gamma E_1^0 + \gamma^2 - \lambda^2\Delta^2 = 0$$
$$\gamma^2 + \gamma \underbrace{(-E_2^0 - E_1^0)}_{b} + \underbrace{E_1^0 E_2^0 - \lambda^2\Delta^2}_{c} = 0$$

$$\gamma_{1,2} = \frac{-b \pm \sqrt{b^2 - 4\,1\,c}}{2}$$

$$\gamma_{1,2} = \frac{E_2^0 + E_1^0}{2} \pm \frac{\sqrt{E_2^{0\,2} + E_1^{0\,2} + 2E_2^0 E_1^0 - 4 E_1^0 E_2^0 + 4\lambda^2\Delta^2}}{2}$$

$$\gamma_{1,2} = \frac{E_2^0 + E_1^0 \pm \sqrt{(E_2^0 - E_1^0)^2 + 4\lambda^2\Delta^2}}{2}$$

Autovalores $\longrightarrow$ $\boxed{\gamma_{1,2} = \frac{E_2^0 + E_1^0}{2} \pm \sqrt{\left(\frac{E_2^0 - E_1^0}{2}\right)^2 + \lambda^2\Delta^2}} = \frac{E_2^0 + E_1^0}{2} \pm \Phi$

• Autovectores de γ_1
$$(E_1^0 - \gamma_1)x_1 + \lambda\Delta x_2 = 0$$
$$\left(\frac{-E_2^0 + E_1^0}{2} - \Phi\right)x_1 + \lambda\Delta x_2 = 0$$
$$(E_1^0 - E_2^0)\left[1 - \frac{2\Phi}{(E_1^0 - E_2^0)}\right]x_1 + \lambda\Delta x_2 = 0$$
$$x_2 = x_1\left(\frac{-E_1^0 + E_2^0}{2} + \Phi\right)\frac{1}{\lambda\Delta}$$
$$V_1 = \begin{pmatrix} x_1 \\ \frac{1}{\lambda\Delta}\left(\frac{E_2^0 - E_1^0}{2} + \Phi\right)x_1 \end{pmatrix}$$
$$x_1^2 + x_1^2 \frac{1}{\lambda^2\Delta^2}\left[\frac{E_2^0 - E_1^0}{2} + \Phi\right]^2 = 1$$
$$x_1^2 = \frac{1}{1 + \frac{(E_2^0 - E_1^0 + 2\Phi)^2}{4\lambda^2\Delta^2}}$$

• Autovector de γ_2
$$(E_1^0 - \gamma_2)x_1 + \lambda\Delta x_2 = 0$$
$$\left(\frac{-E_2^0 + E_1^0}{2} + \Phi\right)x_1 + \lambda\Delta x_2 = 0$$
$$x_2 = x_1\left(\left(\frac{E_2^0 - E_1^0}{2}\right) - \Phi\right)\cdot\frac{1}{\lambda\Delta}$$
$$V_2 = \begin{pmatrix} x_1 \\ \frac{1}{\lambda\Delta}\left(\frac{E_2^0 - E_1^0}{2} - \Phi\right)x_1 \end{pmatrix}$$
$$x_1^2 = \frac{1}{1 + \frac{(E_2^0 - E_1^0 - \Phi 2)^2}{4\lambda^2\Delta^2}} = \frac{4\lambda^2\Delta^2}{4\lambda^2\Delta^2 + [E_2^0 - E_1^0 - 2\Phi]^2}$$

$$\frac{4\lambda^2\Delta^2 + (E_2^0 - E_1^0)^2 - 2(E_2^0 - E_1^0)2\Phi + 4\left([E_2^0 - E_1^0]/2 + \lambda^2\Delta^2\right)}{4\lambda^2\Delta^2}$$
$$-4(E_2^0 - E_1^0)\,[\,\Phi$$

$$V_1 = \begin{pmatrix} \dfrac{2\lambda\Delta}{\left[4\lambda^2\Delta^2 + (E_2^0 - E_1^0 + 2\Phi)^2\right]^{1/2}} \\ \dfrac{2\lambda\Delta}{\frac{1}{\lambda\Delta}\left(\frac{E_2^0-E_1^0}{2} + \Phi\right)\left[4\lambda^2\Delta^2 + (E_2^0 - E_1^0 + 2\Phi)^2\right]^{1/2}} \end{pmatrix}$$

$$V_2 = \begin{pmatrix} \dfrac{2\lambda\Delta}{\left[4\lambda^2\Delta^2 + (E_2^0 - E_1^0 - 2\Phi)^2\right]^{1/2}} \\ \dfrac{2\lambda\Delta}{\frac{1}{\lambda\Delta}\left(\frac{E_2^0-E_1^0}{2} - \Phi\right)\left[4\lambda^2\Delta^2 + (E_2^0 - E_1^0 - 2\Phi)^2\right]^{1/2}} \end{pmatrix}$$

b)

$\lambda|\Delta| \ll |E_1^0 - E_2^0|$ $\Rightarrow$ podemos emplear teoría de perturbaciones

$$H_0 \doteq \begin{array}{cc} |A\rangle & |B\rangle \end{array} \begin{pmatrix} E_1^0 & 0 \\ 0 & E_2^0 \end{pmatrix} \begin{array}{c} \langle A| \\ \langle B| \end{array} \qquad \lambda V = \begin{pmatrix} 0 & \lambda\Delta \\ \lambda\Delta & 0 \end{pmatrix} \Rightarrow H = H_0 + \lambda V$$

autoestado n=2

autoestado n=1

autoestado n=1 a orden cero

$$E_{n=1} = E_1^{(0)} + E_1^{(1)} = \overset{\text{dato}}{E_1^0} + \langle \varphi_1 | V | \varphi_1 \rangle$$

$$E_{n=2} = E_2^{(0)} + E_2^{(1)} = E_2^0 + \langle \varphi_2 | V | \varphi_2 \rangle$$

autoestado n=2 a orden cero

$$V = \begin{pmatrix} 0 & \lambda\Delta \\ \lambda\Delta & 0 \end{pmatrix} = \begin{pmatrix} \langle A|V|A\rangle & \langle A|V|B\rangle \\ \langle B|V|A\rangle & \langle B|V|B\rangle \end{pmatrix} \Rightarrow \begin{cases} E_1^{(1)} = 0 \\ E_2^{(1)} = 0 \end{cases}$$

$$E_{n=1}^{(2)} = \sum_p \frac{|\langle \varphi_p | V | \varphi_1 \rangle|^2}{(E_1^0 - E_p^0)} = \frac{|\langle \varphi_2 | V | \varphi_1 \rangle|^2}{(E_1^0 - E_2^0)} = \frac{|\langle B | V | A \rangle|^2}{(E_1^0 - E_2^0)} = \frac{\lambda^2\Delta^2}{(E_1^0 - E_2^0)}$$

Solo tengo dos niveles (1, 2)

$$E_{n=2}^{(2)} = \frac{|\langle \varphi_1 | V | \varphi_2 \rangle|^2}{E_2^0 - E_1^0} = \frac{\lambda^2\Delta^2}{(E_2^0 - E_1^0)}$$

$$\boxed{E_{n=1} \cong E_1^0 + \frac{\lambda^2\Delta^2}{(E_1^0 - E_2^0)}}$$

$$\boxed{E_{n=2} \cong E_2^0 + \frac{\lambda^2\Delta^2}{(E_2^0 - E_1^0)}}$$

$$|n=1\rangle = |A\rangle + \sum_{\substack{p \neq n \\ n=1 \\ \to p=2}} \frac{\langle \varphi_p | V | \varphi_n \rangle}{(E_n^0 - E_p^0)} |\varphi_p\rangle$$

$$|n=1\rangle = |A\rangle + \frac{\langle \varphi_2 | V | \varphi_1 \rangle}{(E_1^0 - E_2^0)} |B\rangle = |A\rangle + \frac{\lambda\Delta}{(E_1^0 - E_2^0)} |B\rangle$$; luego en forma idem

$$|n=2\rangle = |B\rangle + \frac{\lambda\Delta}{(E_2^0 - E_1^0)} |A\rangle$$

Veamos que se parece la solución exacta a lo hallado aquí:

$$\gamma_1 = \frac{E_2^\circ + E_1^\circ}{2} + \frac{|E_2^\circ - E_1^\circ|}{2}\underbrace{\left(1 + \frac{4\lambda^2\Delta^2}{(E_2^\circ - E_1^\circ)^2}\right)^{1/2}}_{\to\ 1 + \frac{1}{2}\varepsilon \text{ (first order)}}$$

sea $E_1^\circ > E_2^\circ$

$$\gamma_1 \cong \frac{E_2^\circ}{2} + \frac{E_1^\circ}{2} + \frac{E_1^\circ}{2} - \frac{E_2^\circ}{2} + \left(\frac{\cancel{E_1^\circ - E_2^\circ}}{\cancel{2}}\right)\left(\frac{1}{\cancel{2}}\right)\frac{\cancel{4}\lambda^2\Delta^2}{(E_2^\circ - E_1^\circ)^{\cancel{2}}}$$

$$\boxed{\gamma_1 \cong E_1^\circ + \frac{\lambda^2\Delta^2}{(E_1^\circ - E_2^\circ)}}$$

$$\gamma_2 = \frac{E_2^\circ + E_1^\circ}{2} - \frac{|E_2^\circ - E_1^\circ|}{2}\left(1 + \frac{4\lambda^2\Delta^2}{(E_2^\circ - E_1^\circ)^2}\right)^{1/2}$$

$$\gamma_2 \cong \frac{E_2^\circ}{2} + \frac{E_1^\circ}{2} - \frac{E_1^\circ}{2} + \frac{E_2^\circ}{2} - \frac{1}{\cancel{2}}\frac{\cancel{4}\lambda^2\Delta^2}{(E_2^\circ - E_1^\circ)^{\cancel{2}}}\cdot\frac{\cancel{|E_2^\circ - E_1^\circ|}}{\cancel{2}}$$

$$\boxed{\gamma_2 \cong E_2^\circ + \frac{\lambda^2\Delta^2}{(E_2^\circ - E_1^\circ)}}$$

Con lo cual está más que bien para las energías.

$$|n=1\rangle^{(1)} = |A = \phi_1^{(0)}\rangle + \frac{\lambda\Delta}{(E_1^\circ - E_2^\circ)}|B = \phi_2^{(0)}\rangle$$

$$|n=1\rangle^{(1)} = \begin{pmatrix} 1 \\ \frac{\lambda\Delta}{(E_1^\circ - E_2^\circ)} \end{pmatrix}$$

,y en forma idem

$$|n=2\rangle^{(1)} = |B = \phi_2^{(0)}\rangle + \frac{\lambda\Delta}{(E_2^\circ - E_1^\circ)}|A = \phi_1^{(0)}\rangle$$

$$|n=2\rangle^{(1)} = \begin{pmatrix} \frac{\lambda\Delta}{E_2^\circ - E_1^\circ} \\ 1 \end{pmatrix}$$

Para los autovectores, tomaremos aproximación lineal en las autoenergías γ_1, γ_2

$$\gamma_1 \cong E_1^\circ + \frac{\lambda^2\Delta^2}{(E_1^\circ - E_2^\circ)} \to \qquad -\frac{\lambda^2\Delta^2}{(E_1^\circ - E_2^\circ)}\cdot x_1 + \lambda\Delta x_2 = 0$$

$$x_2 = x_1 \cdot \frac{\lambda\Delta}{(E_1^\circ - E_2^\circ)}$$

$$|A\rangle_{\text{aprox. desde el exacto}} = \begin{pmatrix} x_1 \\ x_1 \cdot \frac{\lambda\Delta}{(E_1^\circ - E_2^\circ)} \end{pmatrix}$$

$$x_1 = \frac{1}{\left(1 + \frac{\lambda^2\Delta^2}{(E_2^\circ - E_1^\circ)}\right)^{1/2}} \cong 1 - \frac{\lambda^2\Delta^2}{(E_2^\circ - E_1^\circ)2} \qquad x^2\left(1 + \frac{\lambda^2\Delta^2}{(E_2^\circ - E_1^\circ)^2}\right) = 1$$

orden 2 $\to$ $x_1 \cong 1$ (a orden 1)

$$|A\rangle = \begin{pmatrix} 1 \\ \frac{\lambda\Delta}{E_1^0 - E_2^0} \end{pmatrix}$$

aprox. desde el exacto

$\longrightarrow$ Esto se condice perfectamente con el cálculo exacto; no calcularemos $|B\rangle$ porque sale en forma ídem

c)

$$|E_1^0 - E_2^0| \ll \lambda|\Delta| \rightarrow \frac{|E_1^0 - E_2^0|}{\lambda|\Delta|} \ll 1 \Rightarrow \text{Supongamos que } E_1^0 = E_2^0 = E$$

$\therefore$ aplicaremos teoría de perturbaciones para caso degenerado.

autoestado $n=1$ y $n=2$ tienen igual energía $\Rightarrow$ $|\phi_1^{(0)}\rangle, |\phi_2^{(0)}\rangle$ corresponden a E

$$E^{(1)} = \langle \varphi_1 | V | \varphi_1 \rangle \rightarrow$$

ya tengo la matriz del potencial

$$V \doteq \begin{array}{c} \begin{array}{cc} |\phi_1^{(0)}\rangle & |\phi_2^{(0)}\rangle \end{array} \\ \begin{pmatrix} 0 & \lambda\Delta \\ \lambda\Delta & 0 \end{pmatrix} \end{array} \begin{array}{c} \langle\phi_1^{(0)}| \\ \langle\phi_2^{(0)}| \end{array}$$

$$\alpha^2 - \lambda^2\Delta^2 = 0$$

$$\alpha = \pm\lambda\Delta$$

$$\lambda\Delta x + \lambda\Delta y = 0$$

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

energía $E = E_1^0 = E_2^0$

autoestados

$$|a\rangle = \frac{1}{\sqrt{2}}\left(|\phi_1^{(0)}\rangle - |\phi_2^{(0)}\rangle\right)$$

$$|b\rangle = \frac{1}{\sqrt{2}}\left(|\phi_1^{(0)}\rangle + |\phi_2^{(0)}\rangle\right)$$

Para ver los autovectores realizaremos el correspondiente límite en las autoenergías:

$$\gamma_1 = \frac{E_2^0 + E_1^0}{2} \pm \sqrt{\left(\frac{E_2^0 - E_1^0}{2}\right)^2 + \lambda^2\Delta^2} = \frac{E_2^0 + E_1^0}{2} \pm \lambda\Delta\left(1 + \frac{(E_2^0 - E_1^0)^2}{4\lambda^2\Delta^2}\right)^{1/2}$$

$$(E_1^0 - \gamma_1)x_1 + x_2\lambda\Delta = 0$$

$$\left[\frac{E_1^0 - E_2^0}{2} - \lambda\Delta\left(1 + \frac{(E_2^0 - E_1^0)^2}{4\lambda^2\Delta^2}\right)^{1/2}\right]x_1 + x_2\lambda\Delta = 0$$

$$\left[\frac{E_1^0 - E_2^0}{2\lambda\Delta} - \left(1 + \underbrace{\frac{(E_2^0 - E_1^0)^2}{4\lambda^2\Delta^2}}_{\sim 0}\right)^{1/2}\right]x_1 + x_2 = 0$$

a first order

$$\left(-1 + \frac{E_1^0 - E_2^0}{2\lambda\Delta}\right)x_1 + x_2 = 0$$

$$x_2 = x_1\left(1 - \frac{E_1^0 - E_2^0}{2\lambda\Delta}\right)$$

$$|a\rangle = \begin{pmatrix} x_1 \\ x_1(1-\epsilon) \end{pmatrix} \qquad |b\rangle = \begin{pmatrix} x_1 \\ -x_1(1+\epsilon) \end{pmatrix}$$

$$(E_2^0 - \gamma_2)x_1 + x_2\lambda\Delta = 0$$

$$\frac{E_2^0 - E_1^0}{2} + \lambda\Delta\left(1 + \frac{(E_2^0 - E_1^0)^2}{4\lambda^2\Delta^2}\right)^{1/2} + \lambda\Delta x_2 = 0$$

$$\left(1 + \frac{E_2^0 - E_1^0}{2\lambda\Delta}\right)x_1 + x_2 = 0$$

$$x_1^2 + x_1^2[1-\epsilon]^2 = 1$$

$$x_1^2 = \left(\frac{1}{1+(1-\epsilon)^2}\right) \approx \frac{1}{2(1-\epsilon)}$$

$$x_1 = \frac{1}{\sqrt{2}\sqrt{1-\epsilon}} \approx \frac{1}{\sqrt{2}}\left(1+\frac{\epsilon}{2}\right)$$

$$\underbrace{1+1-2\epsilon+\epsilon^2}_{2(1-\epsilon)}$$

$$|a\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(1+\frac{\varepsilon}{2}\right) \\ \frac{1}{\sqrt{2}}(1-\varepsilon)\left(1+\frac{\varepsilon}{2}\right) \end{pmatrix} \overset{1^{er}\text{orden}}{=} \begin{pmatrix} 1+\frac{\varepsilon}{2} \\ 1-\frac{\varepsilon}{2} \end{pmatrix}\sqrt{2}$$

$1-\varepsilon+\frac{\varepsilon}{2}-\frac{\varepsilon^2}{2}$ ~ 0

$1-\varepsilon/2$

$1+\varepsilon-\frac{\varepsilon}{2}$

$$|b\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(1-\frac{\varepsilon}{2}\right) \\ -\frac{1}{\sqrt{2}}(1+\varepsilon)\left(1-\frac{\varepsilon}{2}\right) \end{pmatrix} = \begin{pmatrix} 1-\frac{\varepsilon}{2} \\ -\left[1+\frac{\varepsilon}{2}\right] \end{pmatrix}\sqrt{2}$$

Entonces fijaros que a orden cero coinciden ($\varepsilon \to 0$)

6.

Sistema de 3 niveles ($N=1, N=2, N=3$)

$$H = \begin{pmatrix} E_1 & 0 & a \\ 0 & E_1 & b \\ a^* & b^* & E_2 \end{pmatrix} \rightarrow \quad H = H_0 + \lambda V$$

$$H_0 = \left(\begin{array}{cc|c} E_1 & 0 & 0 \\ 0 & E_1 & 0 \\ \hline 0 & 0 & E_2 \end{array}\right) + \begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ a^* & b^* & 0 \end{pmatrix}$$

$E_2 > E_1$

$|a|, |b| \ll E_2 - E_1$

Tenemos degeneración de orden 2 para los autoestados 1,2

* <u>diagonalización</u> <u>del</u> <u>H</u> (solución exacta)

$$\begin{vmatrix} E_1-\lambda & 0 & a \\ 0 & E_1-\lambda & b \\ a^* & b^* & E_2-\lambda \end{vmatrix} = (E_1-\lambda)^2(E_2-\lambda) - aa^*(E_1-\lambda) - bb^*(E_1-\lambda) = 0$$

$$[E_1-\lambda]\cdot[E_1-\lambda)(E_2-\lambda) - a^2 - b^2)] = 0$$

$\underline{\lambda_1 = E_1}$ $\qquad (E_1-\lambda)(E_2-\lambda) - [a^2+b^2] = 0$

$$E_1E_2 + \lambda^2 - \lambda(E_2+E_1) - (a^2+b^2) = 0$$

$$\frac{E_2+E_1 \pm \sqrt{(E_2+E_1)^2 - 4\left(E_1E_2 - [a^2+b^2]\right)}}{2}$$

$$E_2^2 + 2E_1E_2 + E_1^2 - 4E_1E_2 + 4(a^2+b^2)$$

$$\underline{\lambda_2, \lambda_3} = \frac{E_2+E_1 \pm \sqrt{(E_2-E_1)^2 - 4(a^2+b^2)}}{2}$$

Como nos informan que $|a|, |b| \ll (E_2 - E_1) \rightarrow$

$$a^2+b^2 = |a|^2+|b|^2 \ll (E_2-E_1) \rightarrow \qquad \frac{a^2+b^2}{E_2-E_1} \ll 1$$

$\Rightarrow$ podemos aproximar la raíz en λ_2, λ_3:

$$(E_2-E_1)\sqrt{1 - \frac{4\,a^2+b^2}{(E_2-E_1)^2}} \cong$$

$$(E_2-E_1)\left(1 - \frac{2(a^2+b^2)}{(E_2-E_1)^2}\right) = E_2 - E_1 - \frac{2(a^2+b^2)}{(E_2-E_1)}$$

$$\boxed{\begin{aligned}\lambda_1 &= E_1\\ \lambda_2 &= E_2 - \frac{a^2+b^2}{(E_2-E_1)}\\ \lambda_3 &= E_1 + \frac{a^2+b^2}{(E_2-E_1)}\end{aligned}}$$

* Teoría de Perturbaciones caso no degenerado

$$E_n^{(2)} = \sum_p \frac{|\langle\varphi_p|V|\varphi_n\rangle|^2}{E_N^0 - E_p^0}$$

$$V \doteq \begin{array}{c} \begin{matrix} |1\rangle & |2\rangle & |3\rangle \end{matrix} \\ \begin{pmatrix} 0 & 0 & a \\ 0 & 0 & b \\ a^* & b^* & 0 \end{pmatrix} \end{array} \begin{matrix} \langle 1| \\ \langle 2| \\ \langle 3| \end{matrix}$$

El sistema tiene 3 niveles (3 autoestados)
⇒ A orden dos los autovalores serán

$$E_1^{(2)} = \frac{|\langle\varphi_2|V|\varphi_1\rangle|^{2} \rightarrow =0^2}{(E_1^0 - E_2^0) \rightarrow =0} + \frac{|\langle\varphi_3|V|\varphi_1\rangle|^2}{(E_1^0-E_3^0)} = \frac{a^2}{E_1-E_2}$$

$$E_2^{(2)} = \frac{|\langle\varphi_1|V|\varphi_2\rangle|^{2} \rightarrow =0^2}{(E_2^0 - E_1^0) \rightarrow =0} + \frac{|\langle\varphi_3|V|\varphi_2\rangle|^2}{(E_2^0-E_3^0)} = \frac{b^2}{E_1-E_2}$$

$$E_3^{(2)} = \frac{|\langle\varphi_1|V|\varphi_3\rangle|^2}{(E_3^0-E_1^0)} + \frac{|\langle\varphi_2|V|\varphi_3\rangle|^2}{(E_3^0-E_2^0)} = \frac{a^2}{E_2-E_1} + \frac{b^2}{E_2-E_1} = \frac{a^2+b^2}{E_2-E_1}$$

Nótese que a orden uno es $E_n^{(1)} = \langle\varphi_n|V|\varphi_n\rangle = 0$ con $n=1,2,3$ [traza nula]

* Teoría de Perturbaciones caso degenerado
Usaremos la expresión

$$E_N^{(2)} = \sum_{p\neq N} \frac{|\langle\varphi_p|V|\varphi_N\rangle|^2}{E_D^{(0)} - E_p^{(0)}}$$

donde N es autoestado degenerado
p son los autoestados no degenerados

$$V = \begin{array}{c} \begin{matrix} |1\rangle & |2\rangle & |3\rangle \end{matrix} \\ \left(\begin{array}{cc|c} 0 & 0 & a \\ 0 & 0 & b \\ \hline a^* & b^* & 0 \end{array}\right) \end{array} \begin{matrix} \langle 1| \\ \langle 2| \\ \langle 3| \end{matrix}$$

$$E_1^{(2)} = \frac{|a^*|^2}{E_1-E_2} = \frac{a^2}{E_1-E_2}$$

$$E_2^{(2)} = \frac{|b^*|^2}{E_1-E_2} = \frac{b^2}{E_1-E_2}$$

Comparando esto con la solución exacta vemos que no está bien. Obtenemos lo mismo que en el caso anterior.

EL problema aquí es que el subbloque $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ de la matriz correspondiente a la parte del V que está degenerado no puede diagonalizarse simplemente. Es decir, al diagonalizarla obtenemos $\lambda_1 = \lambda_2 = 0$ ⇒ No se ha roto la degeneración.
En este caso definiremos una nueva matriz:

$$M_{nm} = \sum_{k\notin D} \frac{\langle n|V|k\rangle\langle k|V|m\rangle}{E_D^{(0)} - E_k}$$

donde n,m son degenerados
k es no degenerados

Luego de obtenida esta matriz deberemos realizar la diagonalización pertinente que nos dará los autovalores ≠ necesarios para romper la degeneración.

$$M_{12} = \frac{\langle 1|V|3\rangle\langle 3|V|2\rangle}{E_1 - E_2} = \frac{a.b^*}{E_1-E_2} \qquad M_{11} = \frac{\langle 1|V|3\rangle\langle 3|V|1\rangle}{E_1 - E_2} = \frac{a.a^*}{E_1-E_2}$$

$$M_{21} = \frac{\langle 2|V|3\rangle\langle 3|V|1\rangle}{E_1 - E_2} = \frac{b.a^*}{E_1-E_2} \qquad M_{22} = \frac{\langle 2|V|3\rangle\langle 3|V|2\rangle}{E_1 - E_2} = \frac{b.b^*}{E_1-E_2}$$

$$M \doteq \frac{1}{E_1-E_2}\begin{pmatrix} a^2 & a.b^* \\ b.a^* & b^2 \end{pmatrix}$$

diagonalizaremos M : $\lambda = \dfrac{\gamma}{E_1-E_2} \Rightarrow$

$$(a^2-\gamma)(b^2-\gamma) - b.a^*.ab^* = 0$$

$$a^2b^2+\gamma^2-\gamma(b^2+a^2) - a^2b^2 = 0$$

$$\gamma - \gamma(a^2+b^2) = 0$$

$$\gamma = (a^2+b^2)$$

$$\frac{\gamma_1}{E_1-E_2} = \lambda_1 = \boxed{0 = E_1^{(2)}}$$

$$\frac{\gamma_2}{E_1-E_2} = \lambda_2 = \boxed{\frac{a^2+b^2}{E_1-E_2} = E_2^{(2)}}$$

Ahora hemos obtenido la expresión correcta. Vemos que no era apropiado calcular los corrimientos a orden 2 con la teoría del caso no degenerado; en ese cálculo se vio que la energía se repartió equitativamente en los dos autoestados (mismo autovalor).

8.

a) $H = \underbrace{H_0}_{AL^2 + BL_z} + \underbrace{\lambda V}_{CL_y}$ es una perturbación al H_0

$B \gg C$

autoestados de H_0 serán: $|l,m\rangle$

$$H_0|l,m\rangle = AL^2|l,m\rangle + BL_z|l,m\rangle$$
$$= A\hbar^2 l(l+1)\ |l,m\rangle + Bm\hbar|l,m\rangle$$
$$H_0|l,m\rangle = (A\hbar l(l+1) + B.m)\hbar\ |l,m\rangle$$
$$E^{(0)}_{l,m} = \hbar^2 A\, l(l+1) + Bm\hbar$$

$-l \leq m \leq l$

$$E^{(0)}_{10} = 2\hbar^2 A$$
$$E^{(0)}_{1-1} = 2\hbar^2 A - B\hbar$$
$$E^{(0)}_{11} = 2\hbar^2 A + B\hbar$$

* orden uno

$$E^{(1)}_{N=lm} = \langle\varphi_{lm}|CL_y|\varphi_{lm}\rangle \rightarrow E^{(1)}_{lm} = \langle l\,m\,|\,CL_y\,|\,l\,m\rangle \qquad \frac{L_+ - L_-}{2i}$$
$$E^{(1)}_{lm} = \frac{C}{2i}\left(\langle lm|L_+|lm\rangle - \langle lm|L_-|lm\rangle\right)$$
$$E^{(1)}_{lm} = \frac{C}{2i}\left(\hbar\sqrt{(l-m)(l+m+1)}\,\langle l,m|l,m+1\rangle - \hbar\sqrt{(l+m)(l-m+1)}\,\langle l,m|l,m-1\rangle\right)$$
$$E^{(1)}_{lm} = 0 \longrightarrow$$ A 1er orden las energías son nulas para todo autoestado $|l,m\rangle$

* orden dos

$$E^{(2)}_{N=l,m} = \sum_{P} \frac{|\langle\varphi_P|V|\varphi_N\rangle|^2}{(E^0_N - E^0_P)} = \sum_{l'm'} \frac{|\langle l'm'|V|lm\rangle|^2}{(E^0_{lm} - E^0_{l'm'})}$$

$$\langle l'm'|V|lm\rangle = $$
$$= \frac{C}{2i}\left[\langle l'm'|L_+|lm\rangle - \langle l'm'|L_-|lm\rangle\right] =$$
$$\frac{C}{2i}\left[\hbar\sqrt{(l-m)(l+m+1)}\,\underbrace{\langle l',m'|l,m+1\rangle}_{\delta_{ll'}\cdot\delta_{m+1,m'}} - \hbar\sqrt{(l+m)(l-m+1)}\,\underbrace{\langle l'm'|l,m-1\rangle}_{\delta_{ll'}\delta_{m-1,m'}}\right]$$

$$E^{(0)}_{lm} - E^{(0)}_{l'm'} = \hbar^2 A\, l(l+1) + Bm\hbar - \hbar^2 A\, l'(l'+1) - Bm'\hbar$$
$$= \hbar^2 A\,(l^2 + l - l'^2 - l') + \hbar B(m - m')$$

si $l \neq l' \rightarrow \langle l',m'|V|l,m\rangle = 0 \Rightarrow l = l' \Rightarrow \sum_{lm'}$

luego m' tendrá dos valores $m' = \begin{cases} m+1 \\ m-1 \end{cases} \Rightarrow$

$$E^{(2)}_{\ell,m} = \frac{|\langle \ell, m+1 | V | \ell m\rangle|^2}{(E^0_{\ell m} - E^0_{\ell, m+1})} + \frac{|\langle \ell, m-1 | V | \ell, m\rangle|^2}{(E^0_{\ell m} - E^0_{\ell, m-1})}$$

$$= \frac{\left|\frac{C}{2i}\left(\hbar\sqrt{(\ell-m)(\ell+m+1)}\right)\right|^2}{\hbar B(m-m-1)} + \frac{\left|\frac{C}{2i}\left(\hbar\sqrt{(\ell+m)(\ell-m+1)}\right)\right|^2}{\hbar B(m-m+1)}$$

$$\frac{\frac{C^2}{4}\cdot\hbar^{\cancel{2}}(\ell-m)(\ell+m+1)}{-\cancel{\hbar}B} + \frac{+\frac{C^2}{4}\hbar^{\cancel{2}}(\ell+m)(\ell-m+1)}{\cancel{\hbar}B}$$

$$E^{(2)}_{\ell,m} = \frac{C^2\hbar}{4B}\Big[-(\ell^2+\cancel{\ell m}+\ell-\cancel{m\ell}-m^2-m) + (\ell^2-\cancel{\ell m}+\ell+\cancel{m\ell}-m^2+m)\Big]$$

$$E^{(2)}_{\ell,m} = +\frac{C^2\hbar\,\cancel{2}m}{\cancel{4}_2 B} = \boxed{+\frac{\hbar C^2 \cdot m}{2B}}$$

b)

$$\langle n'\ell' m'_\ell m'_s | 3z^2 - r^2 | n, m, m_\ell, m_s\rangle$$

$$\langle n'\ell' m'_\ell m'_s | x\,y | n, m, m_\ell, m_s\rangle$$

Es conveniente pasar los operadores en función de armónicos esféricos, que son autofunciones de n, ℓ

$$3z^2 - r^2 = 3r^2\cos^2\theta - r^2 = r^2(3\cdot\cos\theta - 1)$$

$$x\cdot y = r\cdot\cos\phi\cdot\operatorname{sen}\theta\cdot r\cdot\operatorname{sen}\phi\cdot\operatorname{sen}\theta = r^2\operatorname{sen}^2\theta\,(\cos\phi + \operatorname{sen}\phi)$$

Son elementos para un átomo de un electrón (tipo alcalino) con spin

$$\Pi\vec{X}\Pi^{-1} = -\vec{X} \qquad H = \frac{P_x^2+P_y^2+P_z^2}{2m} - \frac{e^2}{\sqrt{x^2+y^2+z^2}}$$

$$\Pi\hat{H}\Pi^{-1} = \Pi \Rightarrow$$

$[\Pi, H] = 0$ el hamiltoniano conmuta con paridad $\Rightarrow$ y los $|n,\ell,m_\ell,m_s\rangle$ son autoestados no degenerados de H $\Rightarrow$ $|n,\ell,m_\ell,m_s\rangle$ son autoestados de paridad $\Rightarrow$

$$\Pi|n,\ell,m_\ell,m_s\rangle = \pm|n,\ell,m_\ell,m_s\rangle$$

9. efecto Zeeman cuadrático ; es decir energías del estado fundamental del átomo de H (a 1^{er} orden), pero el cuadrado

$$\langle \vec{x} | \varphi_0 \rangle = \frac{1}{\sqrt{\pi a_0^3}} e^{-\frac{r}{a_0}}$$

$$V = \frac{e^2 A^2}{2 m_e c^2}$$

$$E^{(1)}_{\text{Fundamental}} = \langle \varphi_0 | V | \varphi_0 \rangle$$

$$E^{(1)}_f = \langle \varphi_0 | \frac{e^2 A^2}{2 m_e c^2} | \varphi_0 \rangle \qquad 5.3.18$$

$$-\frac{1}{2}$$

$$E^{(1)}_f = \int \langle \varphi_0 | \vec{x} \rangle \langle \vec{x} | \frac{e^2 A^2}{2 m_e c^2} | \varphi_0 \rangle \, d^3x$$

$$E^{(1)}_f = \frac{e^2 A^2}{2 m_e c^2} \int \frac{1}{\pi a_0^3} e^{-\frac{2r}{a_0}} . r^2 \operatorname{sen}\theta . dr . d\theta . d\varphi$$

$$E^{(1)}_f = \frac{e^2 A^2}{\cancel{2} m_e c^2} \cdot \frac{\cancel{2}\cancel{\pi}}{\cancel{\pi} a_0^3} \int_0^{\pi} \operatorname{sen}\theta . d\theta \int_0^{+\infty} e^{-(2/a_0).r} r^2 . dr$$

$$E^{(1)}_f = \frac{e^2 A^2}{m_e c^2} (-\cancel{2}) . \frac{\cancel{2}! . a_0^3}{2^{\cancel{3}}} = \frac{-e^2 A^2 a_0^2}{2 m_e c^2}$$

hacer nuevamente

A no puede ser constante

10. $\langle x | \tilde{0} \rangle = e^{-\beta |x|} \equiv \Psi_{\tilde{0}}(x)$ oscilador armónico unidimensional

$$H = \frac{P^2}{2m} + \frac{m\omega^2 x^2}{2}$$

función de onda para el estado de prueba

Utilizaremos el método variacional para aproximar E_0 ∴

$$E(\Psi) = \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} \geq E_0 \quad \Rightarrow$$

donde $|\Psi\rangle$ es algún estado de prueba aproximación de autoestados de H

$$\langle \Psi | \Psi \rangle = \langle \tilde{0} | \tilde{0} \rangle = \int dx \, \langle \tilde{0} | x \rangle \langle x | \tilde{0} \rangle = \int_{-\infty}^{+\infty} e^{-2\beta |x|} . dx$$

$$= \int_{-\infty}^{0} e^{+2\beta x} . dx + \int_0^{\infty} e^{-2\beta x} dx = 2 \int_0^{\infty} e^{-2\beta x} . dx$$

$$\langle \tilde{0} | \tilde{0} \rangle = 2 \cdot \frac{e^{-2\beta x}}{-2\beta} \Big]_0^{\infty} = -\frac{2}{-2\beta} = \frac{1}{\beta}$$

$$\langle \tilde{0} | H | \tilde{0} \rangle = \underbrace{\langle \tilde{0} | \frac{P^2}{2m} | \tilde{0} \rangle}_{①} + \underbrace{\langle \tilde{0} | \frac{m\omega^2 x^2}{2} | \tilde{0} \rangle}_{②}$$

① $\int_{-\infty}^{+\infty} dx \, \langle\tilde{0}| -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}|x\rangle\langle x|\tilde{0}\rangle = -\frac{\hbar^2}{2m}\int_{-\infty}^{+\infty} dx \left(\frac{\partial^2}{\partial x^2} e^{-\beta|x|}\right) e^{-\beta|x|}$

$$= -\frac{\hbar^2}{2m}\left[\int_{-\infty}^{0} e^{\beta x}\beta^2 e^{\beta x}\, dx + \int_{0}^{\infty} e^{-\beta x}(-\beta)^2 e^{-\beta x}\, dx\right]$$

$$\int_{-\infty}^{0} e^{2\beta x}\beta^2\, dx + \int_{0}^{\infty} e^{-2\beta x}\beta^2\, dx$$

① $= -\frac{\hbar^2}{2m}\left[2\int_{0}^{\infty} e^{-2\beta x}\beta^2\, dx\right] = -\frac{\hbar^2}{m}\beta^{\cancel{2}}\left.\frac{e^{-2\beta x}}{(-2\cancel{\beta})}\right]_0^{\infty} = -\frac{\hbar^2\beta}{2m}$

② $\int_{-\infty}^{+\infty}\frac{m\omega^2}{2}\, dx \, \langle\tilde{0}|x\rangle x^2\langle x|\tilde{0}\rangle = \frac{m\omega^2}{2}\int_{-\infty}^{+\infty} e^{-2\beta|x|}x^2\, dx$

$$= \frac{m\omega^2}{2}\left[\int_{-\infty}^{0} e^{2\beta x}x^2\, dx + \int_{0}^{\infty} e^{-2\beta x}x^2\, dx\right]$$

$$= \frac{m\omega^2}{\cancel{2}}\cancel{2}\int_{0}^{\infty} e^{-2\beta x}x^2\, dx = m\omega^2\cdot\frac{2!}{(-2\beta)^3}$$

$$= -\frac{m\omega^2}{4\beta^3}$$

$$E(\tilde{0})_{[\beta]} = \left(-\frac{\hbar^2\beta}{2m} - \frac{m\omega^2}{4\beta^3}\right)\cdot\left(\frac{1}{\beta}\right)^{-1} = -\frac{\hbar^2\beta^2}{2m} - \frac{m\omega^2}{4\beta^2}$$

Quiero mínimo de $E(\beta)$ ⇒

$$\frac{dE}{d\beta} = -\frac{\hbar^2\beta}{m} - \frac{m\omega^2(-2)}{4\beta^3} = 0 \quad\to$$

$$\frac{\hbar\beta}{m} = \frac{m\omega^2}{2\beta^3} \quad\to\quad \beta^4 = \frac{m^2\omega^2}{2\hbar^2} \quad\to\quad \beta = \pm\sqrt{\frac{m\omega}{\hbar}}\cdot\frac{1}{2^{1/4}}$$

$$E(\tilde{0})_{\beta_{min}} = -\frac{\hbar^{\cancel{2}}}{2\cancel{m}}\frac{1}{\sqrt{2}}\frac{\cancel{m}\omega}{\cancel{\hbar}} - \frac{\cancel{m}\omega^{\cancel{2}}}{\underset{2\sqrt{2}}{\cancel{4}}}\cdot\frac{\cancel{\sqrt{2}}\,\hbar}{\cancel{m\omega}}$$

$$E(\tilde{0})_{\beta_{min}} = -\frac{\hbar\omega}{2\sqrt{2}} - \frac{\hbar\omega}{2\sqrt{2}} = \boxed{-\frac{\hbar\omega}{\sqrt{2}} \approx E_0} \longrightarrow$$

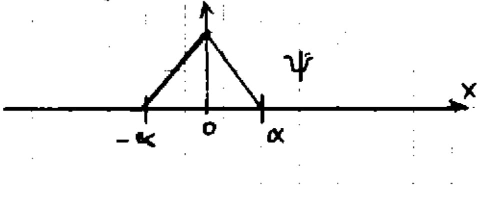
el signo está switcheado porque habría que usar el valor (−) en la forma de β, o en la de β^2

, lo cual está bastante bien porque la $E_0 = \frac{\hbar\omega}{2}$

11.

$$\frac{d^2\psi}{dx^2} + (\lambda - |x|)\psi = 0 \qquad \text{con } \psi \underset{|x|\to\infty}{\to} 0$$

usando método variacional con $\psi = \begin{cases} c(\alpha-|x|) & |x|<\alpha \\ 0 & |x|>\alpha \end{cases}$



El método variacional permite estimar E_0 con

$$E_n(\Psi) = \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle} \geq E_0 \quad , \text{ donde } H \text{ cumple: } \quad H|\Psi\rangle_n = E_n|\Psi\rangle_n$$

$$-\frac{\hbar^2}{2m}\left(\frac{d^2\Psi}{dx^2} + [\lambda - |x|]\Psi\right) = 0$$

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + \frac{\hbar^2}{2m}[\lambda - |x|]\Psi = 0$$

ahora Ψ puedo considerarla como función de onda de un hamiltoniano dado por

$$H = \frac{p^2}{2m} + V \quad , \text{con} \quad V = \frac{\hbar^2|x|}{2m}$$

$$H\Psi = \frac{\lambda\hbar^2}{2m}\Psi \rightarrow \frac{\lambda\hbar^2}{2m} \text{ autovalor de energía (autoenergía)}$$

$$\underbrace{\left(\frac{p^2}{2m} + \frac{|x|\hbar^2}{2m}\right)}_{\equiv H}\Psi = \left(\lambda\frac{\hbar^2}{2m}\right)\Psi$$

$$\langle\Psi|\Psi\rangle = \int_{-\alpha}^{+\alpha} dx\ c(\alpha - |x|)\,c(\alpha - |x|) = c^2 \cdot \int_{-\alpha}(\alpha^2 - 2\alpha|x| + x^2)$$

$\int dx \langle\Psi|x\rangle\langle x|\Psi\rangle$

$$= c^2\alpha^2 2\alpha + \frac{x^3}{3}c^2\Big|_{-\alpha}^{\alpha} - 2c^2\alpha\left(-\int_{-\alpha}^{0} x\,dx + \int_0^{\alpha} x\,dx\right)$$

$$= 2\alpha^3c^2 + \frac{1}{3}c^2 2\alpha^3 - 2c^2\alpha \cancel{2}\frac{x^2}{\cancel{2}}\Big|_0^{\alpha} = \frac{2}{3}c^2\alpha^3$$

$$\langle\Psi|H|\Psi\rangle = \underbrace{\int_{-\alpha}^{+\alpha} dx \langle\Psi|\frac{p^2}{2m}|x\rangle\langle x|\Psi\rangle}_{①} + \underbrace{\int_{-\alpha}^{+\alpha} dx\ \langle\Psi|\frac{|x|\hbar^2}{2m}|x\rangle\langle x|\Psi\rangle}_{②}$$

② $$\int_{-\alpha}^{0} dx \cdot \frac{-x\hbar^2}{2m}\cdot c^2(\alpha + x)^2 + \int_0^{\alpha} dx \cdot \frac{x\hbar^2}{2m}c^2(\alpha - x)^2$$

$-x = x'$, $-dx = dx'$

$$-\int_{\alpha}^{0} dx' \cdot \frac{x'\hbar^2}{2m} c^2(\alpha - x')^2 + \int_0^{\alpha} dx \cdot \frac{x\hbar^2}{2m}c^2(\alpha - x)^2 = \frac{c^2\cancel{2}\hbar^2}{\cancel{2}m}\int_0^{\alpha} dx\,(\alpha - x)^2 x$$

$$= \frac{c^2\hbar^2}{m}\left[\alpha^2\frac{x^2}{2}\Big|_0^{\alpha} - 2\alpha\frac{1}{3}x^3\Big|_0^{\alpha} + \frac{x^4}{4}\Big|_0^{\alpha}\right] = \frac{\hbar^2c^2}{m}\left(\alpha^4\left[\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right]\right) = \frac{\hbar^2c^2\alpha^4}{12m}$$

hay que hacer

① $$\int_{-\alpha}^{+\alpha} dx \langle\Psi|-\frac{\hbar^2}{2m}\partial_{xx}|x\rangle \cdot c(\alpha - |x|) = -\frac{\hbar^2}{2m}\int_{-\alpha}^{+\alpha} dx \cdot \frac{\partial}{\partial x}\left[\left(\frac{\partial}{\partial x}\Psi\right)\right]\Psi = -\frac{\hbar^2}{2m}\int_{-\alpha}^{+\alpha} dx\ \frac{\partial}{\partial x}[\delta(x)]\Psi$$

$$= -\frac{\hbar^2 2\alpha c}{2m}\Psi'(x)$$

$$= \frac{\pm\hbar^2\cancel{2}\alpha cc}{\cancel{2}m} = \pm\frac{\hbar^2\alpha c^2}{m}$$

$$= \frac{\pm\hbar^2\alpha\cancel{c^2}/m + \hbar^2c^2\alpha^4/12m}{(2/3)\cancel{c^2}\alpha^3}$$

$$= \pm\frac{\hbar^2 3}{2\alpha^2 m} + \frac{\hbar^2\alpha}{8m}$$

$\int dx\,\delta(x)f(x)$

$f'(x_0)$

definición de una delta de Dirac

$$\frac{\partial}{\partial x}\Psi = -c\frac{x}{|x|}$$

$$\int_{-\infty}^{+\infty} dx\,\delta(x) = 1 \Rightarrow \int_{-\alpha}^{+\alpha} dx\ c\frac{x}{|x|} \cdot N = N \cdot \text{ Area} =$$

$1 = N c\, 2\alpha$

$$N = \frac{1}{2\alpha c}$$

$2c\alpha$

$$\frac{\partial_x\Psi}{2\alpha c} = \delta(x)$$

$$\frac{dE_n(\alpha)}{d\alpha} = \frac{\hbar^2}{8m} \pm \frac{3\hbar^2}{m\alpha^3} = 0 \rightarrow \frac{1}{8} = \frac{3}{\alpha^3} \qquad \begin{array}{l}\alpha^3 = \pm 24 \\ \alpha = 3^{1/3} \cdot 8^{1/3}\end{array}$$

$$E_0 \equiv \frac{\lambda \hbar^2}{2m} = \frac{\hbar^2 \alpha}{8m} - \frac{\hbar^2 3}{\alpha^2 2m} = \frac{\hbar^2}{2m}\left(\frac{\alpha}{4} - 3\frac{1}{\alpha^2}\right)$$

$$\cong \frac{\hbar^2}{2m}\left(-\frac{\alpha}{4} - \frac{3}{\alpha^2}\right) = \frac{\hbar^2}{2m}\left(-1{,}0816\right) \Rightarrow \boxed{\lambda \cong \pm 1{,}0816}$$

12. Oscilador armónico 1D de frecuencia ω_0. $t<0$ fundamental $(|0\rangle$; $t=0$ perturbación:

$$H_0 = \frac{p^2}{2m} + \frac{m\omega_0^2 x^2}{2} \qquad V(t) = F_0 . x . \cos(\omega t)$$

Hasta $t=0$ tenemos un hamiltoniano H_0 y

$$H_0|n\rangle = E_n|n\rangle \quad , \text{con} \quad E_n = \hbar\omega\left(n+\frac{1}{2}\right)$$

luego $t \geq 0$ se "prende" una perturbación $V(t)$; con lo cual el hamiltoniano es:

$$H = H_0 + V(t)$$

Queremos evaluar $\langle x \rangle$ en función del tiempo usando teoría de perturbaciones. En realidad se está queriendo hallar:

$$\langle x \rangle = {}_I\langle \alpha, t_0, t | x_I | \alpha, t_0, t\rangle_I = {}_I\langle \alpha, t_0, t | U_I^+ \, x_I \, U_I | \alpha, t_0, t_0\rangle_I$$

$$= {}_I\langle \alpha, t_0, t_0 | U_I^+ e^{+i\frac{H_0 t}{\hbar}} x \, e^{-i\frac{H_0 t}{\hbar}} U_I |\alpha, t_0, t_0\rangle_I$$

pero si tomamos $t_0 = 0 \rightarrow$ en nuestro sistema $|\alpha, t_0, t_0\rangle = |0\rangle$

$$\langle x \rangle = \langle 0 | U_I^+ e^{\frac{iH_0 t}{\hbar}} x \, e^{-\frac{iH_0 t}{\hbar}} U_I |0\rangle$$

Como el sistema estaba inicialmente en un autoestado, partiremos de allí en $t=0$

$$t=0 \qquad \langle x \rangle = \langle 0 | x | 0 \rangle$$

$$V_{n0} = \langle n | V(t) | 0 \rangle = \langle n | F_0 . x . \cos(\omega t) | 0 \rangle = F_0 . \cos(\omega t) \langle n | x | 0 \rangle$$

$$x = \frac{(a+a^+)}{2}\sqrt{\frac{2\hbar}{m\omega_0}} \Rightarrow \qquad V_{n0} = F_0 . \cos(\omega t)\sqrt{\frac{\hbar}{2m\omega}}\,\delta_{n1}$$

$$\langle N | (a+a^+)\sqrt{\frac{\hbar}{2m\omega_0}} |0\rangle = \sqrt{\frac{\hbar}{2m\omega_0}}\left(\underbrace{\langle n|a|0\rangle}_{=0} + \langle n|a^+|0\rangle\right) = \sqrt{\frac{\hbar}{2m\omega_0}}\left(\langle n|1\rangle\right) = \sqrt{\frac{\hbar}{2m\omega_0}}\,\delta_{n1}$$

$$\Rightarrow V_{10} = F_0 . \cos(\omega t)\sqrt{\frac{\hbar}{2m\omega_0}} \quad \text{y} \quad V_{n0} = 0 \quad \text{con } n \neq 1$$

el $V(t)$ sólo me conecta el $|0\rangle$ con el $|1\rangle$

$$C_n^{(0)}(t) = \langle n | 1 | 0 \rangle = 0 \quad n \neq 0 \; ; \quad C_0^{(0)}(t) = 1 = \langle 0 | 1 | 0 \rangle$$

A orden cero no hay transición

$$C_n^{(1)}(t) = -\frac{i}{\hbar}\int_0^t e^{\frac{it'}{\hbar}(E_n - E_m)} \langle n | V(t) | m \rangle \, dt' = 0 \quad \text{salvo si } \begin{array}{l} n=1 \\ m=0 \end{array}$$

$$C_1^{(1)}(t) = -\frac{i}{\hbar}\int_0^t e^{\frac{it'}{\hbar}(E_1-E_0)} < 1|V(t)|0> . dt'$$

$$C_1^{(1)}(t) = -\frac{i}{\hbar}\int_0^t e^{\frac{it'}{\hbar}(E_1-E_0)} \cdot F_0 . \cos(\omega t')\sqrt{\frac{\hbar}{2m\omega}} . dt'$$

$$-\frac{i}{\hbar}F_0\sqrt{\frac{\hbar}{2m\omega_0}}\int_0^t e^{\frac{it'\hbar\omega_0}{\hbar}} . \cos(\omega t') . dt' = \mathcal{U}(t)$$

$E_1 - E_0 = \hbar\omega\left[\left(1+\frac{1}{2}\right)-\frac{1}{2}\right] = \hbar\omega_0$

$E_1 + E_0 = \hbar\omega_0 2$

$$-\frac{1}{\hbar}F_0\sqrt{\frac{\hbar}{2m\omega_0}}\, i\left[\int_0^t \cos(\omega_0 t') . \cos(\omega t')\, dt' + i\int_0^t \operatorname{sen}(\omega_0 t') . \cos(\omega t') dt'\right]$$

$$\mathcal{U} = \frac{-F_0}{\sqrt{\hbar 2m\omega_0}}\left[i\int_0^t \cos(\omega_0 t')\cos(\omega t') dt' - \int_0^t \operatorname{sen}(\omega_0 t') , \cos(\omega t') . dt'\right]$$

$\omega t' = z$

$dt' = \frac{dz}{\omega}$

Pero $C_1^{(0)}(t) = 0$ y $C_0^{(1)}(t) = 0$

$C_0^{(0)}(t) = 1$ $\quad$ $C_1^{(1)}(t) = \mathcal{U}$

Luego
$$|0, t>_I = \sum_n |n><n|0, t>_I = \sum_n c_n(t)\, |n>$$

$$\cong \left(C_0^{(0)}(t) + C_0^{(1)}(t)\right)|0> + \left(C_1^{(0)}(t) + C_1^{(1)}(t)\right)|1>$$

$$|0,t>_I \cong 1\,|0> + \mathcal{U}\,|1>$$

$${}_I<0,t|x_I|0,t>_I = {}_I<0,t|e^{+\frac{iH_0t}{\hbar}}\, x\, e^{-\frac{iH_0t}{\hbar}}|0,t>_I$$

$$= \sqrt{\frac{\hbar}{2m\omega_0}}\left[\left(<0| + <1|\mathcal{U}^*\right)e^{\frac{iH_0t}{\hbar}}(a+a^+)e^{-\frac{iH_0t}{\hbar}}\left(|0> + \mathcal{U}|1>\right)\right.$$

$$\left(<0| + <1|\mathcal{U}^*\right)e^{\frac{iE_0t}{\hbar} - \frac{iE_1t}{\hbar}}(a+a^+)e^{-\frac{iE_0t}{\hbar} - \frac{iE_1t}{\hbar}}\left(|0>+\mathcal{U}|1>\right)$$

$$\sqrt{\frac{\hbar}{2m\omega_0}}\left[\left(<0| + <1|\mathcal{U}^*\right)\underbrace{e^{it2\hbar\omega_0} . e^{-it2\hbar\omega_0}}_{=1}(a+a^+)\left(|0> + \mathcal{U}|1>\right)\right.$$

$$\left[\left(<0| + <1|\mathcal{U}^*\right)a\left(|0>+\mathcal{U}|1>\right) + \left(<0|+<1|\mathcal{U}^*\right)a^+\left(|0>+\mathcal{U}|1>\right)\right]$$

$$\left[\left(<0| + <1|\mathcal{U}^*\right)\left(\mathcal{U}|0>\right) + \left(<0|+<1|\mathcal{U}^*\right)\left(|1> + \mathcal{U}\sqrt{2}|2>\right)\right]$$

$$< x >_I = \sqrt{\frac{\hbar}{2m\omega_0}}\left[\mathcal{U} + \mathcal{U}^*\right] = \sqrt{\frac{\hbar}{2m\omega_0}} . 2\operatorname{Re}(\mathcal{U})$$

$$\int_0^t \operatorname{sen}\omega_0 t' . \cos\omega t' . dt' = \frac{[\omega_0-\omega] . \cos((\omega_0+\omega)t) + [\omega_0+\omega] . \cos((\omega_0-\omega)t) - 2\omega_0}{2(\omega_0^2-\omega^2)}$$

$$<x>_I = \sqrt{\frac{\hbar}{2m\omega_0}}\, \cancel{2} . \frac{F_0}{\sqrt{2m\omega_0 . \hbar}} . \frac{1}{\cancel{2}(\omega_0^2-\omega^2)} . \left(\frac{[\omega_0-\omega] . \cos([\omega_0+\omega]t) + [\omega_0+\omega]\cos([\omega_0-\omega]t) - 2\omega_0}{}\right.$$

$$= \frac{F_0}{2m\omega_0} . \frac{1}{[\omega_0^2-\omega^2]}$$

13.

$$H_0 = \frac{P^2}{2m} + \frac{m\omega^2 x^2}{2} \qquad t<0 \quad \text{fundamental } |0\rangle$$

con $t>0$ fuerza $\quad F(t) = F_0 . e^{-t/\tau}$

a) Esta fuerza vendrá de un potencial

$$V(t) = \int_0^x F_0 . e^{-t/\tau} . dx' = F_0 . e^{-t/\tau} x$$

$$H = H_0 + F_0 . x . e^{-t/\tau}$$

$$P^{(1)}_{|0\rangle \to |1\rangle} = | c_1^{(0)}(t) + c_1^{(1)}(t) |^2$$

$$|\langle 1 | 0, t_0 = 0, t \rangle_I^{(1)}|^2$$

$$V_{n0} = \langle n | V(t) | 0 \rangle = \langle n | F_0 e^{-t/\tau} x | 0 \rangle = F_0 . e^{-t/\tau} \langle n | x | 0 \rangle = F_0 . e^{-t/\tau} \sqrt{\frac{\hbar}{2m\omega}} \delta_{n1}$$

$$V_{n0} = 0 \quad \text{si } n \neq 1$$

$$V_{n1} = F_0 . e^{-t/\tau} . \sqrt{\frac{\hbar}{2m\omega}} \quad \text{si } n = 1$$

$$c_1^{(0)}(t) = \langle 1 | 1 | 0 \rangle = 0$$

$$c_1^{(1)}(t) = -\frac{i}{\hbar} \int_0^t e^{i \frac{t'}{\hbar}(E_1 - E_0)} \langle 1 | V(t) | 0 \rangle . dt' \qquad E_1 = \frac{3}{2}\hbar\omega \quad E_0 = \frac{1}{2}\hbar\omega$$

$$= -\frac{i}{\hbar} \int_0^t e^{it'\omega} F_0 e^{-t'/\tau} \sqrt{\frac{\hbar}{2m\omega}} . dt'$$

$$c_1^{(1)}(t) = -\frac{i F_0}{\sqrt{2m\hbar\omega}} \int_0^t e^{it'\omega} . e^{-t'/\tau} dt'$$

$$\int_0^t e^{(i\omega - 1/\tau)t'} dt' = \int_0^{(i\omega - \frac{1}{\tau})t} \frac{e^z \, dz}{i\omega - 1/\tau}$$

$$(i\omega - 1/\tau) t' = z \qquad dt' = \frac{dz}{(i\omega - 1/\tau)} \qquad = e^z \Big|_0^{(i\omega - \frac{1}{\tau})t} \cdot \frac{1}{(i\omega - 1/\tau)}$$

$$\frac{-i F_0}{\sqrt{2m\hbar\omega}} e^{(i\omega - \frac{1}{\tau})t} \cdot \frac{1}{(i\omega - \frac{1}{\tau})}$$

$$\frac{-i F_0}{\sqrt{2m\hbar\omega}} e^{i\omega t} . e^{-\frac{t}{\tau}} \frac{(-i\omega - 1/\tau)}{(\omega^2 + 1/\tau^2)}$$

$$c_1^{(1)}(t) = \frac{F_0}{\sqrt{2m\hbar\omega}} e^{i\omega t} . e^{-t/\tau} \frac{(-\omega + i/\tau)}{(\omega^2 + 1/\tau^2)}$$

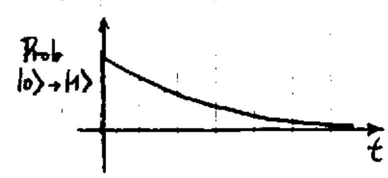
$$C_1^{(1)}(t) = \frac{F_0\, e^{-t/T}}{\sqrt{2m\hbar\omega}}\cdot\frac{1}{\left(\omega^2+\frac{1}{T^2}\right)}\left(-\cos(\omega t)\,.\,\omega + \frac{i}{T}\cos(\omega t) - i\omega\,.\,\text{sen}(\omega t) + \left(-\frac{1}{T}\right)\text{sen}(\omega t)\right)$$

$$\left[-\omega\,.\cos(\omega t) - \frac{\text{sen}(\omega t)}{T} + i\left(\frac{\cos(\omega t)}{T} - \omega\,.\,\text{sen}(\omega t)\right)\right]$$

$$\left|C_1^{(1)}(t)\right|^2 = \_\_\_\_\,.\,\_\_\_\_\left[\omega^2\cos^2(\omega t) + \frac{\text{sen}^2(\omega t)}{T^2} + \frac{2\omega}{T}\cos(\omega t)\,\text{sen}(\omega t) + \frac{\cos^2(\omega t)}{T^2} + \omega^2\,\text{sen}^2(\omega t) - \frac{2\omega}{T}\cos(\omega t)\,\text{sen}(\omega t)\right]$$

$$= \frac{F_0^2\, e^{-\frac{2t}{T}}}{2m\hbar\omega}\cdot\frac{1}{\left(\omega^2+\frac{1}{T^2}\right)^{\cancel{2}}}\cancel{\left[\omega^2+\frac{1}{T^2}\right]}$$

$$\boxed{\text{Prob}^{(1)}_{|0\rangle\to|1\rangle}(t) = \frac{F_0^2\, e^{-\frac{2t}{T}}}{2m\hbar\omega\left(\omega^2+\frac{1}{T^2}\right)}}$$



En el límite $t\to\infty$ es $\text{Prob}^{(1)}_{|0\rangle\to|1\rangle} = 0$

- Es razonable que esto ocurra porque si bien la fuerza es espacialmente uniforme, se apaga exponencialmente $\Rightarrow$ a medida que pasa t la perturbación es más débil y la prob. de saltar de estado decae.
No olvidemos que el responsable del cambio de autoestado es la perturbación.

b) A orden cero, seguro que no $\Leftarrow$ $\langle n|\,1\,|0\rangle = C_n^{(0)}(t) = 0$ con $n\neq 0$

A orden uno, tampoco $\Leftarrow$ $C_n^{(1)}(t) = -\frac{i}{\hbar}\int_0^t e^{\frac{i}{\hbar}t'(E_n-E_0)}\underbrace{\langle n|V(t')|0\rangle}\,dt'$

$V_{n0} = 0$ con $n\neq 1$

$\Leftarrow$ $C_n^{(2)}(t) = \left(-\frac{i}{\hbar}\right)^2\sum\int\int\ldots\underbrace{\langle n|V|m\rangle}\ldots\underbrace{\langle m|V|0\rangle}_{\neq 0 \text{ solo si } m=1}$

$\langle n|\,V(t)\,|m\rangle = F_0\, e^{-t/T}\langle n|x|m\rangle$

$\sqrt{\frac{\hbar}{2m\omega}}\,.\,F_0\,.\,e^{-t/T}\,.\left(\langle n|a|m\rangle + \langle n|a^+|m\rangle\right)$

$\left(\langle n|\sqrt{m}\,|m-1\rangle + \langle n|\sqrt{m+1}\,|m+1\rangle\right)$

$\left(\sqrt{m}\,.\,\delta_{n,m-1} + \sqrt{m+1}\,\delta_{n,m+1}\right)$

con $m=1$ $\left(\delta_{n0} + \sqrt{2}\,\delta_{n2}\right)$

con $n=2$ $\to$ $\langle 2|\,V(t)\,|1\rangle \neq 0 \to C_2^{(2)}(t)\neq 0$

Recién a orden dos hay alguna probabilidad de hallar al oscilador en un estado excitado de mayor energía

14.

$$H_0 = \begin{pmatrix} E_1^0 & 0 \\ 0 & E_2^0 \end{pmatrix} \qquad V(t) = \begin{pmatrix} 0 & \lambda\cos(\omega t) \\ \lambda\cos(\omega t) & 0 \end{pmatrix} \qquad \lambda \in \mathbb{R}$$

a) En $t=0$ el sistema se halla en $\begin{pmatrix}1\\0\end{pmatrix}$; con energía E_1^0

$$|1\rangle \equiv \begin{pmatrix}1\\0\end{pmatrix} \qquad |2\rangle \equiv \begin{pmatrix}0\\1\end{pmatrix} \qquad \begin{array}{ccc} & |1\rangle & |2\rangle \\ \langle 1| & & \\ \langle 2| & & \end{array}$$

$$\underset{|1\rangle\to|2\rangle}{\text{Prob}}(t) = \left| C_{|2\rangle}^{(0)}(t) + C_{|2\rangle}^{(1)}(t) + C_{|2\rangle}^{(2)}(t) + \dots \right|^2$$

$$\begin{aligned} \langle 1|V(t)|1\rangle &= 0 \\ \langle 1|V(t)|2\rangle &= \lambda\cos(\omega t) \\ \langle 2|V(t)|1\rangle &= \lambda\cos(\omega t) \\ \langle 2|V(t)|2\rangle &= 0 \end{aligned} \Rightarrow$$

La perturbación saca de un estado y manda al otro

$$C_{|2\rangle}^{(0)}(t) = \langle 2|1|1\rangle = (0\ 1)\begin{pmatrix}1\\0\end{pmatrix} = 0$$

$$C_{|2\rangle}^{(1)}(t) = \frac{-i}{\hbar}\int_0^t e^{i\frac{t'}{\hbar}\underbrace{(E_2-E_1)}_{E_2^0-E_1^0}} \cdot \lambda\cos(\omega t')\, dt'$$

$$= \frac{-i}{\cancel{\hbar}}\lambda \cdot \cancel{\hbar}\omega \left[\frac{e^{i\frac{t}{\hbar}(E_2^0-E_1^0)} \cdot i\,(E_2^0-E_1^0)(\text{sen}(\omega t) - \cos(\omega t))}{\hbar^2\omega^2 + (E_2^0-E_1^0)^2} \right.$$

- **Cálculo integral**

$$\int_0^t \underbrace{e^{i\frac{t'}{\hbar}\Delta E}}_{u} \cdot \underbrace{\cos(\omega t')\, dt'}_{dv} = \text{sen}(\omega t') \cdot e^{it'\Delta E/\hbar} \frac{i\Delta E}{\omega\hbar}\Big|_0^t - \int_0^t \text{sen}(\omega t') \cdot e^{it'\frac{\Delta E}{\hbar}} \frac{i\Delta E}{\omega\hbar} \cdot dt'$$

$$= \text{sen}(\omega t)\, e^{it'\frac{\Delta E}{\hbar}} \cdot \frac{i\Delta E}{\hbar\omega} - \frac{i\Delta E}{\omega\hbar} \cdot \left[-\cos(\omega t) \cdot e^{it'\frac{\Delta E}{\hbar}} \Big|_0^t - \int \cos(\omega t') \cdot e^{it'\frac{\Delta E}{\hbar}} \frac{i\Delta E}{\omega\hbar} \cdot dt' \right]$$

$u = e^{i\frac{t'\Delta E}{\hbar}}$ $\quad dv = \cos(\omega t)$

$du = e^{i\frac{t'\Delta E}{\hbar}} \cdot \frac{i\Delta E}{\hbar}$ $\quad v = \text{sen}(\omega t) \cdot \frac{1}{\omega}$

$dv = \text{sen}(\omega t)$

$v = -\cos(\omega t) \frac{1}{\omega}$

$$\int u\, dv = uv\Big| - \int v\, du$$

$$\int = -\frac{i\Delta E}{\omega\hbar}\left[-\cos(\omega t) e^{it\frac{\Delta E}{\hbar}} + 1 + \frac{i\Delta E}{\omega\hbar}\int \cos(\omega t') e^{it'\frac{\Delta E}{\hbar}} \right.$$

$$\left(\int\ \right)\left(1 - \frac{\Delta E^2}{\omega^2\hbar^2}\right) = \text{sen}(\omega t)\, e^{it\frac{\Delta E}{\hbar}} \frac{i\Delta E}{\hbar\omega} - \cos(\omega t)\, e^{it\frac{\Delta E}{\hbar}} \frac{i\Delta E}{\omega\hbar}$$

$$\int = \frac{\left(e^{it\frac{\Delta E}{\hbar}} \cdot \frac{i\Delta E}{\hbar\omega}\left(\text{sen}(\omega t) - \cos(\omega t)\right)\right)\hbar^2\omega^2}{\omega^2\hbar^2 - (\Delta E)^2}$$

$$C_{|2\rangle}^{(1)}(t) = \frac{\lambda\omega\,(E_2^0-E_1^0)}{\hbar^2\omega^2 - (E_2^0-E_1^0)^2} \cdot [\text{sen}(\omega t) - \cos(\omega t)]\, e^{i\frac{t}{\hbar}(E_2^0-E_1^0)}$$

$$\boxed{\left|C_{|2\rangle}^{(1)}(t)\right|^2 = \frac{\lambda^2\omega^2(E_2^0-E_1^0)^2}{[\hbar^2\omega^2 - (E_2^0-E_1^0)^2]^2}\left[1 - 2\cos(\omega t)\,\text{sen}(\omega t)\right]}$$

b) Porque cuando $E_1^0 - E_2^0 \approx \pm\hbar\omega$ se tiene el denominador tendiendo a cero mucho más rápido que el numerador con lo cual: $C_{|2\rangle}^{(1)}(t) \to \infty$ y no puede ser esto así porque no tiene sentido tener una probabilidad tan grande

15. dos objetos de spin $\frac{1}{2}$

$$t<0 \qquad H_0 = 0 \qquad \text{estado } |+-\rangle$$
$$t>0 \qquad H = \left(\frac{4\Delta}{\hbar^2}\right)\vec{S}_1\cdot\vec{S}_2$$

a) Aquí la perturbación es el hamiltoniano H.

$$|+\rangle_1 \; |-\rangle_1 \; |+\rangle_2 \; |-\rangle_2 \qquad H\begin{pmatrix} & \end{pmatrix}$$

$$H = \left(\frac{4\Delta}{\hbar^2}\right)\left(S_1^x\otimes S_2^x + S_1^y\otimes S_2^y + S_1^z\otimes S_2^z\right)$$

$$H = \frac{4\Delta}{\hbar^2}\Big[\frac{\hbar^2}{4}\big(|+\rangle_1\langle -| + |-\rangle_1\langle +|\big)\otimes\big(|+\rangle_2\langle -| + |-\rangle_2\langle +|\big) +$$
$$\frac{\hbar^2}{4}i^2\big(-|+\rangle\langle -| + |-\rangle\langle +|\big)_1\otimes\big(-|+\rangle\langle -| + |-\rangle\langle +|\big)_2 +$$
$$\frac{\hbar^2}{4}\big(|+\rangle\langle +| - |-\rangle\langle -|\big)_1\otimes\big(|+\rangle\langle +| - |-\rangle\langle -|\big)_2\Big]$$

Nota

$$S_z \doteq \frac{\hbar}{2}\begin{array}{c} \begin{array}{cc} |+\rangle & |-\rangle \end{array} \\ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{array}\begin{array}{c} \langle +| \\ \langle -| \end{array}$$

$$S_z \doteq \frac{\hbar}{2}\begin{pmatrix} |+\rangle\langle +| & 0 \\ 0 & -|-\rangle\langle -| \end{pmatrix}$$

$$= \Delta\Big[\, |+\rangle_1\langle -|+\rangle_2\langle -| + |+\rangle_1\langle -|-\rangle_2\langle +| + |-\rangle_1\langle +|+\rangle_2\langle -|$$
$$+ |-\rangle_1\langle +|-\rangle_2\langle +| - \big(+|+\rangle_1\langle -|+\rangle_2\langle -| - |-\rangle_1\langle +|+\rangle_2\langle -|$$
$$- |+\rangle_1\langle -|-\rangle_2\langle +| + |-\rangle_1\langle +|-\rangle_2\langle +|\big)$$
$$+ |+\rangle_1\langle +|+\rangle_2\langle +| - |+\rangle_1\langle +|-\rangle_2\langle -| - |-\rangle_1\langle -|+\rangle_2\langle +|$$
$$+ |-\rangle_1\langle -|-\rangle_2\langle -|\,\Big]$$

$$= \Delta\Big[\, \cancel{|++\rangle\langle --|} + |+-\rangle\langle -+| + |-+\rangle\langle +-| + \cancel{|--\rangle\langle ++|} - \cancel{|++\rangle\langle --|}$$
$$+ |-+\rangle\langle +-| + |+-\rangle\langle -+| - \cancel{|--\rangle\langle ++|} + |++\rangle\langle ++| - |+-\rangle\langle +-|$$
$$- |-+\rangle\langle -+| + |--\rangle\langle --|\,\Big]$$

$$H \doteq \Delta\;\begin{array}{c} \begin{array}{cccc} |++\rangle & |+-\rangle & |-+\rangle & |--\rangle \end{array} \\ \left(\begin{array}{c|cc|c} 1 & 0 & 0 & 0 \\ \hline 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \end{array}\begin{array}{c} \langle ++| \\ \langle +-| \\ \langle -+| \\ \langle --| \end{array}$$

Resulta la matriz diagonal por bloques, entonces tenemos dos energías a la vista. Y hay que diagonalizar el bloque central.

$$\left|\begin{pmatrix} -1-\lambda & 2 \\ 2 & -1-\lambda \end{pmatrix}\right| = (1+\lambda)^2 - 4 = 0 = 1 + 2\lambda + \lambda^2 - 4 = 0$$
$$\lambda^2 + 2\lambda - 3 = 0$$
$$\frac{-2 \pm\sqrt{4+4\cdot 3}}{2} = \frac{-2\pm 4}{2} = \begin{cases} 1 \\ -3 \end{cases}$$

$\underline{\lambda = 1}$: $-2x + 2y = 0$, $x = y$, $v_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\underline{\lambda = -3}$: $2x + 2y = 0$, $v_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Entonces los autoestados serán

$|1\rangle = |++\rangle$ con energía Δ

$|2\rangle = \frac{1}{\sqrt{2}}\left(|+-\rangle + |-+\rangle \right)$ con energía Δ

$|3\rangle = \frac{1}{\sqrt{2}}\left(|+-\rangle - |-+\rangle \right)$ con energía -3Δ

$|4\rangle \equiv |--\rangle$ con energía Δ

ahora estos son ortogonales

$$H \doteq \Delta \begin{array}{c} \begin{array}{cccc} |1\rangle & |2\rangle & |3\rangle & |4\rangle \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

diagonalizado like como

En $t<0$ el sistema se halla en $|+-\rangle$, que no es autoestado sino CL de los autoestados $|2\rangle, |3\rangle$

$$|+-\rangle = \frac{1}{\cancel{2}}\frac{\cancel{2}^{\,1}}{\sqrt{2}}\left(|2\rangle + |3\rangle\right) \qquad \frac{\sqrt{2}}{\cancel{2}}\,\frac{\cancel{2}}{\sqrt{\cancel{2}}}|+-\rangle = |2\rangle + |3\rangle$$

Lo evolucionaremos en el tiempo:

$$U|+-\rangle = \frac{1}{\sqrt{2}}\left(U|2\rangle + U|3\rangle\right) = \frac{1}{\sqrt{2}}\left[e^{-i\Delta t/\hbar}|2\rangle + e^{i3\Delta t/\hbar}|3\rangle \right]$$

$$\text{Prob}_{|++\rangle} = \left|\langle ++|U|+-\rangle\right|^2 = \frac{1}{2}\left|\langle 1| e^{-i\Delta t/\hbar}|2\rangle + \langle 1| e^{i3\Delta t/\hbar}|3\rangle\right|^2 = 0$$

$$\text{Prob}_{|--\rangle} = \left|\langle --|U|+-\rangle\right|^2 = \frac{1}{2}\left|\langle 4| e\,|2\rangle + \langle 4| e\,|3\rangle\right|^2 = 0$$

$$\text{Prob}_{|+-\rangle} = \left|\langle +-|U|+-\rangle\right|^2 = \frac{1}{4}\left|\left(\langle 2| + \langle 3|\right)\left(e^{-i\Delta t/\hbar}|2\rangle + e^{i3\Delta t/\hbar}|3\rangle\right)\right|^2$$

$$= \frac{1}{4}\left| e^{-i\Delta t/\hbar} + e^{i3\Delta t/\hbar}\right|^2$$

$$= \frac{1}{4}\left[\left(\cos\left(\frac{\Delta t}{\hbar}\right) + \cos\left(\frac{3\Delta t}{\hbar}\right)\right)^2 + \left(-\operatorname{sen}\left(\frac{\Delta t}{\hbar}\right) + \operatorname{sen}\left(\frac{3\Delta t}{\hbar}\right)\right)^2\right]$$

$$= \frac{1}{4}\left[\cos^2\left(\frac{\Delta t}{\hbar}\right) + \cos^2\left(\frac{3\Delta t}{\hbar}\right) + \operatorname{sen}^2\left(\frac{\Delta t}{\hbar}\right) + \operatorname{sen}^2\left(\frac{3\Delta t}{\hbar}\right) + 2\cos\left(\frac{\Delta t}{\hbar}\right)\cos\left(\frac{3\Delta t}{\hbar}\right) - 2\operatorname{sen}\left(\frac{\Delta t}{\hbar}\right)\cdot\operatorname{sen}\left(\frac{3\Delta t}{\hbar}\right)\right]$$

$$\boxed{\text{Prob}_{|+-\rangle} = \left[1 + \cos\left(\frac{4\Delta t}{\hbar}\right)\right]\frac{1}{2}}$$

$$\text{Prob}_{|-+\rangle} = \left|\langle -+|U|+-\rangle\right|^2 = \frac{1}{4}\left|\left(\langle 2| - \langle 3|\right)\left(e^{-i\Delta t/\hbar}|2\rangle + e^{i\Delta t 3/\hbar}|3\rangle\right)\right|^2$$

$$= \frac{1}{4}\left| e^{-i\Delta t/\hbar} - e^{i3\Delta t/\hbar}\right|^2$$

$$= \frac{1}{4}\left|1 - e^{i4\Delta t/\hbar}\right|^2 \left|e^{-i\Delta t/\hbar}\right|^2$$

$$= \frac{1}{4}\left[\left(1 - \cos\left(\frac{4\Delta t}{\hbar}\right)\right)^2 + \left(-\operatorname{sen}\left(\frac{4\Delta t}{\hbar}\right)\right)^2\right]$$

$$= \frac{1}{4}\left(2 - 2\cos\left(\frac{4\Delta t}{\hbar}\right)\right) = \boxed{\frac{1}{2}\left[1 - \cos\left(\frac{4\Delta t}{\hbar}\right)\right] = \text{Prob}_{|-+\rangle}}$$

debería ser

$\frac{1}{2} - \frac{\cos}{2}\left(\frac{4\Delta t}{\hbar}\right)$

$\frac{1}{2} + \frac{1}{2}\cos + \square = 1$

b) Supongamos válida la teoría de perturbaciones, y consideremos

$$H = \left(\frac{4\Delta}{\hbar^2}\right)\vec{S}_1 . \vec{S}_2$$

la perturbación; entonces la matriz de H es la matriz de "V", leeremos de ahí los V_{nm}. Nuestro estado inicial será el $|+-\rangle$, y como queremos probabilidades de $|++\rangle, |+-\rangle, |--\rangle, |-+\rangle$ (base vieja) emplearemos dicha base

$$C^{(0)}_{|++\rangle}(t) = \langle ++ | 1 | +- \rangle = 0$$

$$C^{(0)}_{|--\rangle}(t) = \langle -- | 1 | +- \rangle = 0$$

$$C^{(0)}_{|+-\rangle}(t) = \langle +- | 1 | +- \rangle = \left(\frac{\langle 2| + \langle 3|}{\sqrt{2}}\right)\left(\frac{|2\rangle + |3\rangle}{\sqrt{2}}\right) = \frac{1}{2} + \frac{1}{2} = 1$$

$$C^{(0)}_{|-+\rangle}(t) = \langle -+ | 1 | +- \rangle = \frac{\langle 2| - \langle 3|}{\sqrt{2}}\left(\frac{|2\rangle + |3\rangle}{\sqrt{2}}\right) = \frac{1}{2} - \frac{1}{2} = 0$$

$$C^{(1)}_{|++\rangle}(t) = -\frac{i}{\hbar}\int_0^t e^{i\frac{t'}{\hbar}\underbrace{(E_{|++\rangle} - E_{|+-\rangle})}_{=0}} . \underbrace{\langle ++ | V | +- \rangle}_{=0} . dt' = 0$$

$$C^{(1)}_{|--\rangle}(t) = -\frac{i}{\hbar}\int_0^t e^{i\frac{t'}{\hbar}\underbrace{(E_{|--\rangle} - E_{|+-\rangle})}_{=0}} \underbrace{\langle -- | V | +- \rangle}_{=0} . dt' = 0$$

$$C^{(1)}_{|+-\rangle}(t) = -\frac{i}{\hbar}\int_0^t e^{i\frac{t'}{\hbar}\underbrace{(E_{|+-\rangle} - E_{|+-\rangle})}_{=0}} \underbrace{\langle +- | V | +- \rangle}_{-1.\Delta} . dt' =$$

$$-\frac{i}{\hbar} - \Delta . t = \frac{i\Delta t}{\hbar}$$

$$\frac{1}{2}\left[\left(\langle 2| + \langle 3|\right) V \left(|2\rangle + |3\rangle\right)\right]$$

$$\frac{1}{2}\left[1 - 3\right] = -1$$

da lo mismo el cálculo en cualquier base

Las energías son nulas porque corresponden al H_0 (sin perturbar) que es nula.

$$C^{(1)}_{|-+\rangle}(t) = -\frac{i}{\hbar}\int_0^t e^{i\frac{t'}{\hbar}(\overbrace{E_{|-+\rangle}}^{=0} - \overbrace{E_{|+-\rangle}}^{=0})} . \underbrace{\langle -+ | V | +- \rangle}_{=2.\Delta} . dt'$$

$$C^{(1)}_{|-+\rangle}(t) = -\frac{i}{\hbar} 2\Delta t$$

$$\frac{1}{2}\left[\langle 2| - \langle 3| V (|2\rangle + |3\rangle)\right]$$

$$\frac{1}{2}\left[1 + 3\right] = 2$$

$$P^{(1)}_{|+-\rangle} = \left|1 + \frac{i\Delta t}{\hbar}\right|^2 = 1 + \frac{\Delta^2 t^2}{\hbar^2}$$

$$P^{(1)}_{|-+\rangle} = \left|0 + -\frac{2i\Delta t}{\hbar}\right|^2 = \frac{4\Delta^2 t^2}{\hbar^2}$$

‡ Cuando evaluamos la prob. de que continúe en el mismo estado da > 1 → lo que se hace es evaluarla restando $1 - \Sigma$ prob. otros estados

Aproximemos la Prob exacta con $\frac{\Delta}{\hbar} \ll 1 \rightarrow \quad \cos\left(\frac{4\Delta t}{\hbar}\right) \cong 1 - \left(\frac{4\Delta t}{\hbar}\right)^2 . \frac{1}{2}$

$$P_{|+-\rangle} \cong \frac{1}{2}\left(1 + 1 - \frac{1}{2} . \frac{16\Delta^2 t^2}{\hbar^2}\right) = 1 - \frac{4\Delta^2 t^2}{\hbar^2}$$

$$P_{|-+\rangle} \cong \frac{1}{2}\left(1 - 1 + \frac{1}{2} \frac{16\Delta^2 t^2}{\hbar^2}\right) = \frac{4\Delta^2 t^2}{\hbar^2}$$

Comparando con la prob. del método perturbativo (nota ‡) vemos que coinciden:

$$P^{(1)}_{|+-\rangle} = 1 - P^{(1)}_{|-+\rangle} = 1 - \frac{4\Delta^2 t^2}{\hbar^2}$$

$$P^{(1)}_{|-+\rangle} = \frac{4\Delta^2 t^2}{\hbar^2}$$

Obviamente los resultados son correctos mientras aseguremos

$$\frac{\Delta}{\hbar} \ll 1$$