Práctica 2 : Dinámica Cuántica

1. $$\hat{H} = -\left(\frac{eB}{mc}\right)\hat{S}_z = \omega \hat{S}_z$$

a) $\hat{H} \neq H(t)$

$$|S_z;+\rangle = |+\rangle$$
$$|S_z;-\rangle = |-\rangle$$

$$\hat{H} = \omega \frac{\hbar}{2}\left(|+\rangle\langle+| - |-\rangle\langle-| \right)$$

$$\hat{H}\,|+\rangle = \underbrace{\frac{\omega\hbar}{2}}_{\equiv E_+}|+\rangle \qquad \hat{H}\,|-\rangle = \underbrace{-\frac{\omega\hbar}{2}}_{\equiv E_-}|-\rangle$$

$$\boxed{\text{autovalores} \quad +\frac{\hbar\omega}{2}, -\frac{\hbar\omega}{2}}$$

b) $$|\alpha, t=0\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right) = |S_x;+\rangle$$

en t valdrá otra cosa y para ello usamos el operador $\mathcal{U}(t,t_0)$ (evolución)

⇒ $$e^{-i\frac{\hat{H}}{\hbar}t-\overset{0}{t_0}}|\alpha\rangle = e^{-i\frac{\hat{H}}{\hbar}t}\left(\frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle\right) =$$

$$e^{-i\frac{\cancel{\hbar}\omega}{2\cancel{\hbar}}t}\frac{1}{\sqrt{2}}|+\rangle + e^{+i\frac{\cancel{\hbar}\omega}{2\cancel{\hbar}}t}\frac{1}{\sqrt{2}}|-\rangle$$

$$\boxed{|\alpha, 0; t\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\frac{\omega t}{2}}|+\rangle + e^{+i\frac{\omega t}{2}}|-\rangle\right)}$$

c) $$|S_x;+\rangle = \frac{|+\rangle}{\sqrt{2}} + \frac{|-\rangle}{\sqrt{2}} \qquad |S_x;-\rangle = \frac{|+\rangle}{\sqrt{2}} - \frac{|-\rangle}{\sqrt{2}}$$

La probabilidad total ↘
se conserva ⇒
debería ser
la misma la
total
$P_{(+)} + P_{(-)} = 1$
∀t

$$|\langle s_x;+|\alpha,0;t\rangle|^2 = \left|\left(\frac{\langle+| + \langle-|}{\sqrt{2}}\right)\left(\frac{e^{-i\frac{\omega t}{2}}|+\rangle + e^{+i\frac{\omega t}{2}}|-\rangle}{\sqrt{2}}\right)\right|$$

$$= \left|\frac{e^{-i\frac{\omega t}{2}} + e^{+i\frac{\omega t}{2}}}{2}\right| = \left|\frac{\cancel{2}\cos(\omega t/2)}{\cancel{2}}\right|^2$$

$$\boxed{P_{(s_x+)} = \cos^2\left(\frac{\omega t}{2}\right)}$$

$$|\langle s_x;-|\alpha,0;t\rangle|^2 = \left|\left(\frac{\langle+| - \langle-|}{\sqrt{2}}\right)\left(\frac{e^{-i\frac{\omega t}{2}}|+\rangle + e^{i\frac{\omega t}{2}}|-\rangle}{\sqrt{2}}\right)\right|^2$$

$$= \left|\frac{e^{-i\frac{\omega t}{2}} - e^{i\frac{\omega t}{2}}}{2}\right|^2 = \left|\frac{\cancel{2}i\,\mathrm{sen}(\omega t/2)}{\cancel{2}}\right|^2$$

$$\boxed{P_{(s_x-)} = \mathrm{sen}^2\left(\frac{\omega t}{2}\right)}$$

(d) $\langle S_x \rangle = \langle \alpha, 0; t | S_x | \alpha, 0, t \rangle$

$$\langle S_x \rangle = \left(\frac{\langle +| e^{\frac{i\omega t}{2}} + e^{\frac{-i\omega t}{2}} \langle -|}{\sqrt{2}} \right) \left| \frac{\hbar}{2} \left(|+\rangle\langle -| + |-\rangle\langle +| \right) \right| \left(\frac{e^{\frac{-i\omega t}{2}} |+\rangle + e^{\frac{i\omega t}{2}} |-\rangle}{\sqrt{2}} \right)$$

$$= \frac{\hbar}{4} \left(\langle +| e^{\frac{i\omega t}{2}} + \langle -| e^{\frac{-i\omega t}{2}} \right) \left(e^{\frac{i\omega t}{2}} |+\rangle + e^{\frac{-i\omega t}{2}} |-\rangle \right)$$

$$= \frac{\hbar}{4} \left(e^{i\omega t} + e^{-i\omega t} \right) = \frac{\hbar}{2} . \cos(\omega t)$$

$$\boxed{\langle S_x \rangle = \frac{\hbar}{2} . \cos(\omega t)}$$

e) $\hat{n}$ $\quad |\alpha, 0; t\rangle$ es autoestado de $S.\hat{n}(t)$

$$\Rightarrow \quad \hat{S}.\hat{n}(t) \; |\alpha, 0; t\rangle = \square \; |\alpha, 0; t\rangle$$

$$\downarrow$$

$$\frac{1}{\sqrt{2}} \left(e^{\frac{-i\omega t}{2}} |+\rangle + e^{\frac{i\omega t}{2}} |-\rangle \right) \Rightarrow$$ pasando a notación algebraica es:

$$\hat{S}.\hat{n} \doteq \begin{pmatrix} \frac{\hbar}{2} \cos\beta & \frac{\hbar}{2} . \operatorname{sen}\beta . e^{-i\alpha} \\ \frac{\hbar}{2} \operatorname{sen}\beta . e^{i\alpha} & -\frac{\hbar}{2} . \cos\beta \end{pmatrix} \qquad |\alpha, 0; t\rangle = \begin{pmatrix} \frac{e^{\frac{-i\omega t}{2}}}{\sqrt{2}} \\ \frac{e^{\frac{i\omega t}{2}}}{\sqrt{2}} \end{pmatrix}$$

donde $\beta, \alpha = \beta, \alpha(t) \Rightarrow$

$$\frac{\hbar}{2\sqrt{2}} \begin{pmatrix} \cos\beta . e^{-\frac{i\omega t}{2}} + \operatorname{sen}\beta . e^{-i\alpha + \frac{i\omega t}{2}} \\ \operatorname{sen}\beta . e^{i\alpha} e^{\frac{-i\omega t}{2}} - \cos\beta . e^{\frac{i\omega t}{2}} \end{pmatrix} = \square \begin{pmatrix} \frac{e^{\frac{-i\omega t}{2}}}{\sqrt{2}} \\ \frac{e^{\frac{i\omega t}{2}}}{\sqrt{2}} \end{pmatrix}$$

$$\begin{cases} \frac{\hbar}{2} \left(\cos\beta . e^{-\frac{i\omega t}{2}} + \operatorname{sen}\beta . e^{-i\alpha + \frac{i\omega t}{2}} \right) = \square . e^{-\frac{i\omega t}{2}} \\ \frac{\hbar}{2} \left(\operatorname{sen}\beta . e^{i\alpha - \frac{i\omega t}{2}} - \cos\beta . e^{\frac{i\omega t}{2}} \right) = \square . e^{\frac{i\omega t}{2}} \end{cases}$$

2 ecuaciones con dos incógnitas $\alpha(t), \beta(t)$

en $t=0$ es

$$(\hbar/2)(\cos\beta_0 + \operatorname{sen}\beta_0 e^{-i\alpha_0}) = \square$$
$$(\hbar/2)(\operatorname{sen}\beta_0 . e^{i\alpha_0} - \cos\beta_0 \quad) = \square$$

$$\frac{\hbar}{2} \operatorname{sen}\beta_0 (e^{-i\alpha_0} + e^{i\alpha_0}) = 2\square$$

$$\frac{\hbar}{2} \operatorname{sen}\beta_0 (\cos\alpha_0) \cancel{2} = \cancel{2} \square \rightarrow \boxed{\frac{\hbar}{2} \operatorname{sen}\beta_0 . \cos\alpha_0}$$

autovalor ↓

$$\cos\beta + e^{-i\alpha} \operatorname{sen}\beta . e^{i\omega t} = \square \frac{2}{\hbar}$$

$$\operatorname{sen}\beta . e^{i\alpha} . e^{-i\omega t} - \cos\beta = \square . \frac{2}{\hbar}$$

$$e^{-(-i\alpha + i\omega t)}$$

$$2\left(\frac{2\square}{\hbar}\right) = \operatorname{sen}\beta \left(e^{-i\alpha + i\omega t} + e^{i\alpha - i\omega t} \right) = \operatorname{sen}\beta . 2 . \cos(\omega t - \alpha)$$

$$0 = -2\cos\beta + \text{sen}\,\beta\left(e^{i\alpha - i\omega t} - e^{-i\alpha + i\omega t}\right)$$

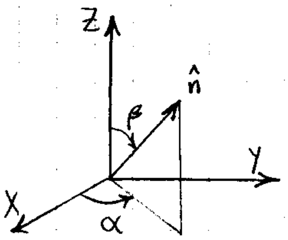
$$+\,\text{sen}\,\beta\,.\left[e^{i(\alpha-\omega t)} - e^{-i(\alpha-\omega t)}\right]$$

$$0 = -2\cos\beta + \text{sen}\,\beta\,.\,2i\,.\,\text{sen}\,(\alpha - \omega t)$$

← Un $\# \in \mathbb{C}$ que es nulo tiene parte real e imaginaria nulas

$$\frac{4\square}{\hbar} = 2\,\text{sen}\,\beta\,.\ \cos(\omega t - \alpha)$$

$$-2\cos\beta = 0 \longrightarrow \beta = \pi/2\,,\ 3/2\,\pi$$



$$\underbrace{\text{sen}\,\beta}_{\neq 0}.\,2i\,.\,\underbrace{\text{sen}\,(\alpha-\omega t)}_{=0} = 0$$

$$\alpha - \omega t = 0, \pi, 2\pi$$

$$\longleftarrow \quad \alpha = \omega t$$

$$\hat{n} = \text{sen}\,\beta.\cos\alpha\ \hat{x} + \text{sen}\,\beta.\,\text{sen}\,\alpha\ \hat{y} + \cos\beta.\,\hat{z}$$

$$\boxed{\hat{n} = \cos(\omega t)\,\hat{x} + \text{sen}(\omega t)\,\hat{y}}$$ ← el versor pedido

$$S\hat{n} \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & e^{-i\omega t + i\frac{\omega t}{2}} \\ e^{i\omega t - i\frac{\omega t}{2}} & 0 \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} 0 & e^{-i\frac{\omega t}{2}} \\ e^{i\frac{\omega t}{2}} & 0 \end{pmatrix}$$

$$S\hat{n}\,(t=0) \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow S\hat{n}\,(t=0)\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \square\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$\frac{4\square}{\hbar} = 2.\,\text{sen}\,\frac{\pi}{2}\,.\ .\cos(\omega t - \omega t) = 4 \Rightarrow \boxed{\square = \hbar/2}$$

$$\frac{\hbar}{2}\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \square\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

2.

$$\hat{H} = H_{11}\,|1\rangle\langle 1| + H_{22}\,|2\rangle\langle 2| + H_{12}\,|1\rangle\langle 2|$$

Sistema dos niveles ; base $= \{|1\rangle, |2\rangle\}$

$\hat{H}$ es hermítico $\Rightarrow$

$$H = H^{+} = H^{t*}$$

si $(H_{ij}) \in \mathbb{R} \Rightarrow H = H^{t}$

$$\hat{H}\,|1\rangle = H_{11}\,|1\rangle$$
$$\hat{H}\,|2\rangle = H_{22}\,|2\rangle + H_{12}\,|1\rangle$$

$$\hat{H} \doteq \begin{pmatrix} \langle 1|H|1\rangle & \langle 1|H|2\rangle \\ \langle 2|H|1\rangle & \langle 2|H|2\rangle \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ 0 & H_{22} \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} H_{11} & H_{12} \\ 0 & H_{22} \end{pmatrix} \neq \begin{pmatrix} H_{11} & 0 \\ H_{12} & H_{22} \end{pmatrix}$$

- Cálculo de autovalores

$$H - \lambda\mathbb{1} = \begin{vmatrix} H_{11}-\lambda & H_{12} \\ 0 & H_{22}-\lambda \end{vmatrix} = (H_{11}-\lambda)(H_{22}-\lambda) = 0$$

$$H_{11}H_{22} - \lambda H_{22} - \lambda H_{11} + \lambda^2 = 0$$

$$\lambda^2 - \underbrace{(H_{22}+H_{11})}_{b}\lambda + \underbrace{H_{11}.H_{22}}_{c} = 0$$

$$(H_{22}+H_{11})^2 - 4H_{11}H_{22}$$
$$H_{22}^2 + H_{11}^2 + 2H_{22}H_{11} - 4H_{11}H_{22}$$
$$(H_{22}-H_{11})^2$$

$$\lambda_{1,2} = \frac{b \pm \sqrt{b^2 - 4c}}{2}$$

$$\lambda_{1,2} = \frac{H_{22}+H_{11} \pm (H_{22}-H_{11})}{2}$$

$$\left.\begin{array}{l}\lambda_1 = H_{22}\\ \lambda_2 = H_{11}\end{array}\right\}\text{ son reales}$$

no me da información

EL principio que está violando es que el hamiltoniano H no es hermítico pues su matriz No es simetrica

b) Sea $\hat{H} = H_{12}\,|1\rangle\langle 2|$

$$i\hbar\frac{\partial}{\partial t}\hat{U}(t,t_0) = \hat{H}\,\hat{U}(t,t_0)$$

$$i\hbar\frac{\partial}{\partial t}\hat{U} - H(t)\,\hat{U} = 0$$

$$\frac{\partial}{\partial t}\hat{U}(t,t_0) = \frac{H(t)}{i\hbar}\hat{U}(t,t_0)$$

$$\frac{\partial}{\partial t}\hat{U}(t,t_0)|\alpha,t_0\rangle = \frac{H(t)}{i\hbar}\hat{U}(t,t_0)|\alpha,t_0\rangle$$

$$\frac{\partial}{\partial t}U = \frac{H(t)}{i\hbar}U$$

$$\frac{\partial U}{\partial t} = \frac{H(t)}{i\hbar}U$$

$$\int\frac{1}{U}dU = \int\frac{H(t')}{i\hbar}dt'$$

$$\ln U = \int_{t_0}^{t}\frac{H_{12}(t')}{i\hbar}dt' = -\frac{i}{\hbar}\int_{t_0}^{t}\hat{H}(t').dt'$$

$$\boxed{U(t,t_0) = e^{-\frac{i}{\hbar}\int_{t_0}^{t}\hat{H}(t').dt'}}$$

Cuando se dice "resolver el problema dependiente del tiempo" no significa $H_{12} = H_{12}(t)$ sino simplemente hacer la evolución temporal $\Rightarrow H_{12} \equiv$ constante

Ahora quero evolucionar un ket $|\alpha\rangle$ genérico $\Rightarrow$

$$|\alpha\rangle = \sum_{a'}(\langle a'|\alpha\rangle)|a'\rangle$$

$$|\alpha\rangle = (\langle 1|\alpha\rangle)|1\rangle + (\langle 2|\alpha\rangle)|2\rangle \longrightarrow \text{el ket más general}$$

$$U(t,t_0)\,|\alpha,t_0\rangle = |\alpha,\overset{=0}{t_0};t\rangle$$

$$e^{-i\frac{\hat{H}}{\hbar}(t-t_0)}\,|\alpha,\underset{=0}{t_0}\rangle = e^{-\frac{i}{\hbar}H_{12}|1\rangle\langle 2|\,t}\;|\alpha,t_0=0\rangle$$

$$= \sum_{n=0}^{\infty}\frac{1}{n!}\left(-\frac{i\,H_{12}|1\rangle\langle 2|\,t}{\hbar}\right)^{n}|\alpha,t_0=0\rangle$$

$n=1$

$$H_{12}\,|1\rangle\langle 2|)\,(c_1|1\rangle + c_2|2\rangle)$$

$$H_{12}\,c_2|1\rangle$$

$n=2$

$$(H_{12}\,|1\rangle\langle 2|)(H_{12}\,|1\rangle\langle 2|)(c_1|1\rangle + c_2|2\rangle)$$

$$(H_{12}|1\rangle\langle 2|)\,H_{12}c_2|1\rangle = H_{12}^{2}c_2\,|1\rangle\langle 2|1\rangle = 0$$

$\forall n \geq 2$ da nulo

Con el tiempo un autoestado cae en el ket nulo lo cual no parece muy católico

3. $[x(t), x(0)]$

particula libre en 1D

$$H = \frac{p^2}{2m} \qquad U = e^{-i\frac{Ht}{\hbar}}$$

$$x(t) = U^\dagger x^s U$$

$$x(t) = \left(e^{-i\frac{Ht}{\hbar}}\right)^\dagger x^s e^{-i\frac{Ht}{\hbar}}$$

$$\frac{dx(t)}{dt} = \frac{1}{i\hbar}\underbrace{[x^H, H]}_{F(p)} = \frac{1}{i\hbar}\cdot\left(i\hbar\frac{\partial H}{\partial p}\right) = \frac{\partial}{\partial p}\left(\frac{p^2}{2m}\right) = \frac{\cancel{2}p}{\cancel{2}m}$$

$$\frac{dp(t)}{dt} = \frac{1}{i\hbar}[p^H, H] = 0 \rightarrow p = \text{constante} \rightarrow \frac{dx}{dt} = \frac{p}{m}$$

en el tiempo

$\Rightarrow p^{(s)} = p^{(H)}$

$$x - x_0 = \frac{p}{m}\cdot t$$

$$x = x_0 + \frac{p}{m}\cdot t$$

$$[x(t), x(0)] = [\overset{X^{sd}}{x(0)} + (p/m)\cdot t, x(0)]$$

$$= [(p/m)\cdot t, x(0)] = \frac{t}{m}[p, x(0)] = \frac{t}{m}\cdot(-i\hbar)$$

son los operadores en Schrödinger cuyo conmutador manejo

$$\boxed{[(p/m)\cdot t, x(0)] = -\frac{t}{m}i\hbar}$$

NOTA

a $t=0$ $x^H = x^S \Rightarrow$

$[x, x]_{t=0} = 0$

debería ser un chequeo de consistencia

4.

$$H = \frac{\hat{P}^2}{2m} + V(\hat{x})$$

Particula en 3D

a) $[\vec{x}\cdot\vec{p}, H]$

$$H = \frac{P_x^2 + P_y^2 + P_z^2}{2m} + V(\vec{x})$$

$$[x\cdot p_x + y\cdot p_y + z\cdot p_z, H]$$

$$[x\cdot p_x, H] + [y\cdot p_y, H] + [z\cdot p_z, H]$$

$$[x\cdot p_x, \frac{p_x^2}{2m} + V(\vec{x})] + [y\cdot p_y, \frac{p_y^2}{2m} + V(\vec{x})] + [z\cdot p_z, \frac{p_z^2}{2m} + V(\vec{x})]$$

$$\left[x\cdot p_x, \frac{p_x^2}{2m}\right] + [x\cdot p_x, V(\vec{x})]$$

$$-\frac{1}{2m}[p_x^2, x\cdot p_x] - [V(\vec{x}), x\,p_x]$$

$$-\frac{1}{2m}\left(x\underbrace{[p_x^2, p_x]}_{=0} + [p_x^2, x]p_x\right) - \left(x[V(\vec{x}), p_x] + \underbrace{[V(\vec{x}), x]}_{=0}p_x\right)$$

$$-[x, p_x^2]\,p_x$$

$$-i\hbar\frac{\partial p_x^2}{\partial p_x}\cdot p_x$$

$$-\frac{1}{2m}(-i\hbar 2p_x^2) \qquad -x[V(\vec{x}), p_x]$$

$$-V(\vec{x})i\hbar\frac{\partial\psi}{\partial x} + \underbrace{i\hbar\frac{\partial}{\partial x}(V(\vec{x})\cdot\psi)}_{+i\hbar\psi\frac{\partial V}{\partial x} + i\hbar\frac{\partial\psi}{\partial x}}$$

$$+i\hbar\frac{d}{dt}(x\cdot p_x) = \frac{1}{\cancel{2}m}i\hbar\cancel{2}p_x^2 - x\left(i\hbar\frac{\partial V(\vec{x})}{\partial x}\right) = \frac{i\hbar}{m}p_x^2 - x\cdot i\hbar\frac{\partial V(x)}{\partial x}$$

$$\Rightarrow \quad \frac{d}{dt}(X.p_x) = \frac{p_x^2}{m} - x.\frac{\partial V(\vec{x})}{\partial x}$$ ⇒ análogamente podemos hallar

$$\frac{d}{dt}(y.p_y) = \frac{p_y^2}{m} - y.\frac{\partial V(\vec{x})}{\partial y}$$

$$\frac{d}{dt}(z.p_z) = \frac{p_z^2}{m} - z.\frac{\partial V(\vec{x})}{\partial z}$$ ⇒ Juntando todo vectorialmente

$$\frac{d}{dt}\left(x p_x + y p_y + z p_z\right) = \frac{p_x^2+p_y^2+p_z^2}{m} - \left(x.\partial_x V + y.\partial_y V + z.\partial_z V\right)$$

$$\frac{d}{dt}\left(\vec{x}.\vec{p}\right) = \frac{|\vec{p}|^2}{m} - \left(\vec{x}.\vec{\nabla} V\right)$$ ⇒ Tomando valor medio respecto de un estado de Heisenberg

$$\langle\alpha|\frac{d}{dt}(\vec{x}.\vec{p})|\alpha\rangle = \langle\alpha|\frac{p^2}{m}|\alpha\rangle - \langle\alpha|\vec{x}.\vec{\nabla}V|\alpha\rangle$$

$$\boxed{\frac{d}{dt}\langle\vec{x}.\vec{p}\rangle = \frac{1}{m}\langle p^2\rangle - \langle\vec{x}.\vec{\nabla}V\rangle}$$

$$\frac{d}{dt}\langle\vec{x}.\vec{p}\rangle_{|\alpha\rangle} = \frac{d}{dt}\langle\alpha,t_0;t|\vec{x}.\vec{p}|\alpha,t_0,t\rangle = \frac{d}{dt}\left(\langle\alpha,t_0|\mathcal{U}^\dagger\right)\vec{x}\vec{p}\left(\mathcal{U}|\alpha,t_0\rangle\right)$$

ahora si $|\alpha,t_0\rangle = |E'\rangle$ autoestado de H ⇒

No depende del tiempo

$$\frac{d}{dt}\langle\vec{x}.\vec{p}\rangle = \frac{d}{dt}\langle E',t_0|e^{-i\frac{E'(t-t_0)}{\hbar}} xp\, e^{+i\frac{E'(t,t_0)}{\hbar}}|E',t_0\rangle = \frac{d}{dt}\overbrace{\langle E',t_0|\vec{x}\vec{p}|E',t_0\rangle}=0$$

Necesito tomar < > respecto a autoestado de H

c) $\vec{X}.\vec{p} = X.p_x + Y.p_y + Z.p_z$ ⇒ Hagamos cálculo para el 1er sumando.

X, p son hermíticos ⇒ $x^+ = x$, $p^+ = p$ ⇒ $(X.p)^+ = p^+x^+ = p.X$

$(X.p)^+ \neq X.p$ ⇒ $\boxed{X.p_x \text{ No es hermítico}}$

Generalizando : $\vec{X}.\vec{p}$ no es hermítico.

$$[\vec{p}.\vec{x}, H] = [\vec{x}.\vec{p} - 3i\hbar\hat{\mathbb{I}}, H]$$ $$X.p_x + Y.p_y + Z.p_z = 3i\hbar + p_x.X + p_y.Y + p_z.Z$$

$$[\vec{p}.\vec{x}, H] = [\vec{x}.\vec{p}, H] - [3i\hbar, H] = [\vec{x}.\vec{p}, H] = \frac{i\hbar p^2}{2m} - i\hbar\,\vec{x}.\vec{\nabla}V(\vec{x})$$

Esto parece estar bien, pero hagamos la cuenta mejor

$$[\vec{p}.\vec{x}, H] \equiv \left[p_x.X, \frac{p_x^2}{2m} + V(x)\right] = \left[p_x.X, \frac{p_x^2}{2m}\right] + \left[p_x.X, V(x)\right]$$

$$-\frac{1}{2m}[p_x^2, p_x.X] - [V(x), p_x.X]$$

$$-\frac{1}{2m}\left(p_x[p_x^2, X] + \underbrace{[p_x^2, p_x]}_{=0}X\right) - \left(p_x\underbrace{[V(x), X]}_{=0} + [V(x), p_x]X\right)$$

$$= \frac{1}{2m}\, p_x [x, p_x^2] \quad - \quad [V(x), p_x]\, x$$

$$[p_x.x, H] = \frac{1}{2m}\, p_x\, (i\hbar\, 2p_x) - x\left(i\hbar \frac{\partial V(x)}{\partial x}\right) = i\hbar \frac{p_x^2}{m} - i\hbar\, x. \frac{\partial V(x)}{\partial x}$$

$$\Rightarrow [\vec p.\vec x, H] = i\hbar\left(\frac{|\vec p|^2}{m} - \vec x.\vec\nabla V(\vec x)\right)$$ $\rightsquigarrow$ Se puede ver que los conmutadores son iguales.

$(x.p_x)^+ = p_x^+.x^+ = p_x.x$

$(p_x.x)^+ = x.p_x$

$\Rightarrow \quad x.p_x + p_x.x = (p_x x)^+ + (x.p_x)^+$

$p_x.x + x.p_x = (p_x.x + x.p_x)^+ \Rightarrow$

es hermítico → $\boxed{\hat{\square} = \vec p.\vec x + \vec x.\vec p}$ →

para que tenga correlato clásico podemos tomar $\frac{\hat{\square}}{2} = \frac{\hat p \hat x + \hat x \hat p}{2}$ que en el límite clásico es $\vec p.\vec x$

b) $V(\lambda \vec x) = \lambda^\alpha V(\vec x)$ →

$\alpha = -1$ Coulomb

$\alpha = 2$ osc. arm.

$$\left\langle \frac{p^2}{m} \right\rangle = \langle \vec x . \vec\nabla V \rangle$$

en 1D $$\left\langle \frac{p_x^2}{m} \right\rangle = \left\langle x. \frac{\partial V}{\partial x} \right\rangle$$

$V = -\frac{q}{r}$, $V = \frac{m x^2 \omega}{2}$

$V = \underbrace{-1}_{\lambda} . \frac{q}{r}$ $\quad V = m\omega \left(\frac{x}{\sqrt 2}\right)^2$

$\frac{1}{\sqrt 2} = \lambda$

$$= \langle x.\ m\omega x \rangle = \langle x^2 m\omega \rangle$$

$$\frac{\partial V(x)}{\partial x} = \frac{1}{\lambda^\alpha} \frac{\partial V(\lambda x)}{\partial x}$$

6. $|a'\rangle, |a''\rangle$; $\hat{A}$ hermítico $\quad A|a'\rangle = a'|a'\rangle \quad a' \neq a''$

$A|a''\rangle = a''|a''\rangle$

$$\hat{H} = \delta\, |a'\rangle\langle a''| + \delta\, |a''\rangle\langle a'| \qquad \delta \in \mathbb{R}$$

a)

$$H \doteq \begin{pmatrix} 0 & \delta \\ \delta & 0 \end{pmatrix} \qquad \text{base } \{|a'\rangle, |a''\rangle\}$$

$$\begin{vmatrix} -\lambda & \delta \\ \delta & -\lambda \end{vmatrix} = \lambda^2 - \delta^2 = 0 \rightarrow \lambda = \begin{cases} +\delta \\ -\delta \end{cases}$$

$-\delta x_1 + \delta x_2 = 0$

$|1\rangle = \alpha \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\delta x_1 + \delta x_2 = 0$

$|2\rangle = \alpha \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

normalización $\rightarrow \alpha = \frac{1}{\sqrt{2}}$

$$H\left(\frac{|a'\rangle}{\sqrt{2}} + \frac{|a''\rangle}{\sqrt{2}}\right) = \frac{\delta\,|a''\rangle}{\sqrt{2}} + \frac{\delta\,|a'\rangle}{\sqrt{2}} = +\delta\left(\frac{|a'\rangle + |a''\rangle}{\sqrt{2}}\right)$$

$$H\left(\frac{|a'\rangle}{\sqrt{2}} - \frac{|a''\rangle}{\sqrt{2}}\right) = \frac{\delta\,|a''\rangle}{\sqrt{2}} - \frac{\delta\,|a'\rangle}{\sqrt{2}} = -\delta\left(\frac{|a'\rangle - |a''\rangle}{\sqrt{2}}\right)$$

Autovalores de energía ⇝ $\boxed{+\delta \ ; -\delta}$

b) $|a'\rangle$ en $t=0 \qquad |a', t=0\rangle$

$$H \neq H(t) \rightarrow \quad U(t,t_0) = e^{-i\frac{\hat{H}}{\hbar}t}$$

$$\hat{U}|a'\rangle = e^{-i\frac{\hat{H}t}{\hbar}}|a'\rangle$$

$$\hat{A}\hat{H} - \hat{H}\hat{A}\ |a'\rangle = \hat{A}\delta|a''\rangle - \hat{H}a'|a'\rangle = \delta a''|a''\rangle - a'\delta|a''\rangle = \delta|a''\rangle(a''-a') \neq 0$$

$\rightarrow [\hat{A}, \hat{H}] \neq 0$ no tienen base en común

$n=1 \quad \hat{H}|a'\rangle = \delta|a''\rangle$

$n=2 \quad \hat{H}^2|a'\rangle = \hat{H}(\delta|a''\rangle) = \delta^2|a'\rangle$

$n=3 \quad \hat{H}^3|a'\rangle = \hat{H}(\delta^2|a'\rangle = \delta^3|a''\rangle$

$n=4 \quad H^4|a'\rangle = \hat{H}(\delta^3|a''\rangle = \delta^4|a'\rangle$

$$H^n|a'\rangle = \begin{cases} \delta^n|a'\rangle & n \text{ par} \\ \delta^n|a''\rangle & n \text{ impar} \end{cases}$$

Acá, sin embargo resultaría más conveniente evaluar $|a'\rangle$ en términos de los autovectores de H

$$|a',0;t\rangle = \sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{-iHt}{\hbar}\right)^n|a'\rangle = \left(\sum_{n=0}^{\infty}\frac{1}{(2n)!}\left(\frac{-iHt}{\hbar}\right)^{2n} + \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}\left(\frac{-iHt}{\hbar}\right)^{2n+1}\right)|a'\rangle$$

$$\sum_{n=0}^{\infty}\frac{1}{(2n)!}\left(\frac{-it}{\hbar}\right)^{2n}H^{2n}|a'\rangle + \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}\left(\frac{-it}{\hbar}\right)^{2n+1}H^{2n+1}|a'\rangle$$

$$\sum_{n=0}^{\infty} \text{''}\ \delta^{2n}|a'\rangle + \sum_{n=0}^{\infty} \text{''}\ \delta^{2n+1}|a''\rangle$$

$$\sum_{n=0}^{\infty}\frac{1}{(2n)!}\left(\frac{-it\delta}{\hbar}\right)^{2n}|a'\rangle + \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}\left(\frac{-it\delta}{\hbar}\right)^{2n+1}|a''\rangle$$

$(-i)^{2n} = [(-i)^2]^n = (-1)^n$

$(-i)^{2n+1} = (-1)^n \cdot i$

$$\sum \frac{(-1)^n}{(2n)!}\left(\frac{t\delta}{\hbar}\right)^{2n}|a'\rangle + \sum -i\frac{(-1)^n}{(2n+1)!}\left(\frac{t\delta}{\hbar}\right)^{2n+1}|a''\rangle$$

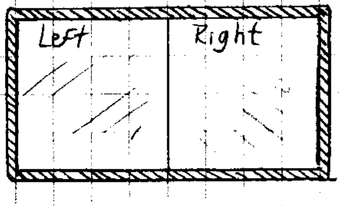
$$\boxed{|a',0;t\rangle = \cos\left(\frac{t\delta}{\hbar}\right)|a'\rangle - i \cdot \operatorname{sen}\left(\frac{t\delta}{\hbar}\right)|a''\rangle}$$

con $\cos^2\left(\frac{t\delta}{\hbar}\right) + \text{sen}^2\left(\frac{t\delta}{\hbar}\right) = 1$ conservación de la norma

c) $$|\langle a' | a', 0; t\rangle|^2 = \left|\cos\left(\frac{\delta t}{\hbar}\right)\langle a' | a'\rangle\right|^2 = \cos^2\left(\frac{\delta t}{\hbar}\right)$$

$$\boxed{\mathcal{P}(|a'\rangle) = \cos^2\left(\frac{\delta t}{\hbar}\right)}$$

7.



$$\left.\begin{matrix}|R\rangle \\ |L\rangle\end{matrix}\right\} \text{autoestados de posición}$$

$$|\alpha\rangle = \underbrace{(\langle R|\alpha\rangle)}\,|R\rangle + \underbrace{(\langle L|\alpha\rangle)}\,|L\rangle$$

podemos verlas como funciones de onda

$$H = \underset{\in\mathbb{R}}{\Delta}\left(|L\rangle\langle R| + |R\rangle\langle L|\right)$$

a)
$$H|R\rangle = \Delta|L\rangle\langle R|R\rangle = \Delta|L\rangle$$
$$H|L\rangle = \Delta|R\rangle\langle L|L\rangle = \Delta|R\rangle$$

$$H \doteq \begin{pmatrix} 0 & \Delta \\ \Delta & 0 \end{pmatrix} \doteq \Delta\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{vmatrix} -\lambda & \Delta \\ \Delta & -\lambda \end{vmatrix} = \lambda^2 - \Delta^2 = 0 \rightarrow \boxed{\lambda = \begin{cases} +\Delta \\ -\Delta \end{cases}}$$

autovalores

$\Delta x_1 + \Delta x_2 = 0 \rightarrow v_{-\Delta} = \alpha\begin{pmatrix}1\\-1\end{pmatrix}$

$-\Delta x_1 + \Delta x_2 = 0 \rightarrow$

$v_{+\Delta} = \alpha\begin{pmatrix}1\\1\end{pmatrix}$

$$|r\rangle_{-\Delta} \equiv \frac{|R\rangle}{\sqrt{2}} - \frac{|L\rangle}{\sqrt{2}}$$

$$|l\rangle_{+\Delta} \equiv \frac{|R\rangle}{\sqrt{2}} + \frac{|L\rangle}{\sqrt{2}}$$

b)
$$|\alpha, t=0\rangle = (\langle R|\alpha\rangle)|R, t=0\rangle + (\langle L|\alpha\rangle)|L, t=0\rangle$$

$$\mathcal{U} = e^{-iHt/\hbar} = e^{-i\Delta(|L\rangle\langle R| + |R\rangle\langle L|)t/\hbar}$$

$$\mathcal{U}|\alpha, 0\rangle = |\alpha, 0; t\rangle$$
$$= e^{-i\Delta(|L\rangle\langle R| + |R\rangle\langle L|)t/\hbar}\left\{(\langle R|\alpha\rangle)|R\rangle + (\langle L|\alpha\rangle)|L\rangle\right\}$$

Conviene poner $|\alpha, 0\rangle$ en función de la base de autoestados de H

$$|r\rangle + |l\rangle = \sqrt{2}\,|R\rangle \rightarrow |R\rangle = \frac{|r\rangle + |l\rangle}{\sqrt{2}}$$

$$|r\rangle - |l\rangle = -\sqrt{2}\,|L\rangle \rightarrow |L\rangle = \frac{-|r\rangle + |l\rangle}{\sqrt{2}}$$

$$\mathcal{U}|\alpha,0\rangle = e^{-iHt/\hbar}\left\{\left(\left(\langle R|\alpha\rangle\right)\frac{|r\rangle+|\ell\rangle}{\sqrt{2}} + \left(\langle L|\alpha\rangle\right)\frac{|\ell\rangle-|r\rangle}{\sqrt{2}}\right\}$$

$$= \sum_{n=0}^{\infty}\frac{1}{n!}\left(-i\frac{Ht}{\hbar}\right)^n \left\{ \#_1\left(|r\rangle+|\ell\rangle\right) + \#_2\left(|\ell\rangle-|r\rangle\right)\right\}$$

$$\left\{\#_1|r\rangle + \#_1|\ell\rangle + \#_2|\ell\rangle - \#_2|r\rangle\right\}$$

$$= \sum_{n=0}^{\infty}\frac{1}{n!}\left(-\frac{iHt}{\hbar}\right)^n\left\{(\#_1-\#_2)|r\rangle + (\#_1+\#_2)|\ell\rangle\right\}$$

$$H|r\rangle = -\Delta|r\rangle$$
$$= \frac{1}{\sqrt{2}}\Delta|L\rangle - \frac{1}{\sqrt{2}}\Delta|R\rangle = -\Delta|r\rangle$$
$$H|\ell\rangle = \Delta|\ell\rangle$$
$$= \frac{1}{\sqrt{2}}\Delta|L\rangle + \frac{1}{\sqrt{2}}\Delta|R\rangle = \Delta|\ell\rangle$$

$$= \sum \frac{1}{n!}\left(-\frac{i(-\Delta)t}{\hbar}\right)^n(\#_1-\#_2)\overset{|r\rangle}{\downarrow} + \frac{1}{n!}\left(-\frac{i\Delta t}{\hbar}\right)(\#_1+\#_2)|\ell\rangle$$

$$= e^{+\frac{i\Delta t}{\hbar}}(\#_1-\#_2)|r\rangle + e^{-\frac{i\Delta t}{\hbar}}(\#_1+\#_2)|\ell\rangle$$

$$|\alpha,0;t\rangle = e^{+\frac{i\Delta t}{\hbar}}(\#_1-\#_2)\left(\frac{|R\rangle-|L\rangle}{\sqrt{2}}\right) + e^{-\frac{i\Delta t}{\hbar}}(\#_1+\#_2)\left(\frac{|R\rangle+|L\rangle}{\sqrt{2}}\right)$$

$$|\alpha,0;t\rangle = \left(\frac{e^{+\frac{it\Delta}{\hbar}}(\#_1-\#_2) + e^{-\frac{it\Delta}{\hbar}}(\#_1+\#_2)}{\sqrt{2}}\right)|R\rangle +$$
$$\left(\frac{e^{-\frac{it\Delta}{\hbar}}(\#_1+\#_2) - e^{+\frac{it\Delta}{\hbar}}(\#_1-\#_2)}{\sqrt{2}}\right)|L\rangle$$

$$= \left(\frac{\#_1(e^{+}+e^{-}) - \#_2(e^{+}-e^{-})}{\sqrt{2}}\right)|R\rangle +$$
$$\left(\frac{\#_1(e^{-}-e^{+}) + \#_2(e^{-}+e^{+})}{\sqrt{2}}\right)|L\rangle$$

$$= \left[\frac{2}{\sqrt{2}}\#_1\cdot\cos\left(\frac{t\Delta}{\hbar}\right) - \frac{2i}{\sqrt{2}}\#_2\,\mathrm{sen}\left(\frac{t\Delta}{\hbar}\right)\right]|R\rangle +$$
$$\left[-\frac{2i}{\sqrt{2}}\#_1\,\mathrm{sen}\left(\frac{t\Delta}{\hbar}\right) + \frac{2}{\sqrt{2}}\#_2\cos\left(\frac{t\Delta}{\hbar}\right)\right]|L\rangle$$

$$\boxed{\begin{aligned}|\alpha,0;t\rangle &= \left[\cancel{\sqrt{2}}\frac{\langle R|\alpha\rangle}{\cancel{\sqrt{2}}}\cdot\cos\left(\frac{t\Delta}{\hbar}\right) - i\cancel{\sqrt{2}}\frac{\langle L|\alpha\rangle}{\cancel{\sqrt{2}}}\,\mathrm{sen}\left(\frac{t\Delta}{\hbar}\right)\right]|R\rangle\\ &\left[\langle L|\alpha\rangle\cdot\cos\left(\frac{t\Delta}{\hbar}\right) - i\cdot\langle R|\alpha\rangle\,\mathrm{sen}\left(\frac{t\Delta}{\hbar}\right)\right]|L\rangle\end{aligned}}$$

c) En $t=0$ la partícula se halla en un estado Right $|R\rangle$

$$|\alpha,t=0\rangle = |R\rangle = \frac{|r\rangle}{\sqrt{2}} + \frac{|\ell\rangle}{\sqrt{2}} \Rightarrow$$

$$|\alpha,t=0;t\rangle = e^{-i\frac{Ht}{\hbar}}|R\rangle = \frac{e^{-i\frac{(-\Delta)t}{\hbar}}}{\sqrt{2}}|r\rangle + \frac{e^{-i\frac{(\Delta)t}{\hbar}}}{\sqrt{2}}|\ell\rangle$$

$$\neq e^{-i\frac{\Delta t}{\hbar}}|L\rangle \qquad = \frac{e^{+}}{2}\left(|R\rangle-|L\rangle\right) + \frac{e^{-}}{2}\left(|R\rangle+|L\rangle\right)$$

Remark

Uno estaría tentado a pensar que vale esto pero no es cierto porque switchean los valores

$$H|R\rangle = \Delta|L\rangle$$
$$H^2|R\rangle = \Delta^2|R\rangle$$
$$H^3|R\rangle = \Delta^3|L\rangle$$
$$\vdots$$

$$P(\text{left}) = |\langle L|\alpha,0;t\rangle|^2$$

$$\left|\langle L|\left(\frac{e^+}{2}\left[|R\rangle - |L\rangle\right] + \frac{e^-}{2}\left[|R\rangle + |L\rangle\right]\right)\right|^2$$

$$\left|-\frac{e^{+\frac{i\Delta t}{\hbar}}}{2}\cdot\langle L|L\rangle + \frac{e^{-\frac{i\Delta t}{\hbar}}}{2}\langle L|L\rangle\right|^2 = \left|\frac{-e^{+\frac{i\Delta t}{\hbar}} + e^{-\frac{i\Delta t}{\hbar}}}{2}\right|^2$$

$$= \left|\frac{-\cancel{2}i\,\text{sen}\,(\Delta t.\hbar^{-1})}{\cancel{2}}\right|^2 = \boxed{\text{sen}^2\left(\frac{\Delta t}{\hbar}\right) = P_{(\text{left})}}$$

d)

8. a)

$$\langle m|x|n\rangle \;,\; \langle m|p|n\rangle \;,\; \langle m|\{x,p\}|n\rangle \;,\; \langle m|x^2|n\rangle \;,\; \langle m|p^2|n\rangle$$

$$\sqrt{\frac{2\hbar}{m\omega}}\, a - \frac{ip}{m\omega} = x$$

$$\sqrt{\frac{2\hbar}{m\omega}}\, a^{+} + \frac{ip}{m\omega} = x$$

$$2x = \sqrt{\frac{2\hbar}{m\omega}}\left(a + a^{+}\right)$$

$$x = \sqrt{\frac{\hbar}{2m\omega}}\left(a + a^{+}\right)$$

$$(a^{+} - a)\sqrt{\frac{2\hbar}{m\omega}} + \frac{2ip}{m\omega} = 0$$

$$(a - a^{+})\sqrt{\frac{2\hbar}{m\omega}} = \frac{2ip}{m\omega}$$

$$-i\,(a - a^{+})\sqrt{\frac{m\omega\hbar}{2}} = p$$

$$i\,(a^{+} - a)\cdot\sqrt{\frac{m\omega\hbar}{2}} = p$$

$$* \;\langle m|x|n\rangle =$$

$$\langle m|\sqrt{\frac{\hbar}{2m\omega}}\left(a + a^{+}\right)|n\rangle =$$

$$\sqrt{\frac{\hbar}{2m\omega}}\,\langle m|a + a^{+}|n\rangle =$$

$$\left(\frac{\hbar}{2m\omega}\right)^{1/2}\left(\langle m|\sqrt{n+1}\,|n+1\rangle + \langle m|\sqrt{n}\,|n-1\rangle\right)$$

$$= \boxed{\left(\frac{\hbar}{2m\omega}\right)^{1/2}\left[\sqrt{n+1}\,\underbrace{\langle m|n+1\rangle}_{\delta_{m,n+1}} + \sqrt{n}\,\underbrace{\langle m|n-1\rangle}_{\delta_{m,n-1}}\right]}$$

se puede escribir con δ de Kronecker; Lo cual haremos a partir de ahora

$$* \;\langle m|p|n\rangle =$$

$$= i\left(\frac{m\omega\hbar}{2}\right)^{1/2}\left(\langle m|a^{+} - a|n\rangle\right)$$

$$= \quad " \quad \left(\langle m|a^{+}|n\rangle - \langle m|a|n\rangle\right)$$

$$= i\left(\frac{m\omega\hbar}{2}\right)^{1/2}\left(\sqrt{n+1}\,\langle m|n+1\rangle - \sqrt{n}\,\langle m|n-1\rangle\right)$$

$$\boxed{\langle m|p|n\rangle = i\left(\frac{m\omega\hbar}{2}\right)^{1/2}\left(\sqrt{n+1}\,\delta_{m,n+1} - \sqrt{n}\,\delta_{m,n-1}\right)}$$

$$* \;\langle m|\, x.p + p.x\,|n\rangle$$

$$xp = (a + a^{+})\, i\,(a^{+} - a)\sqrt{\frac{\cancel{m\omega}\hbar}{2}}\sqrt{\frac{\hbar}{2\cancel{m\omega}}} = i\frac{\hbar}{2}\left(a a^{+} + a^{+}a^{+} - a.a - a^{+}a\right)$$

$$a a^{+} - a^{+}a$$

$$[a, a^{+}] = 1$$

$$a.a = \frac{m\omega}{2\hbar}\left(x + \frac{ip}{m\omega}\right)\left(x + \frac{ip}{m\omega}\right) = \frac{m\omega}{2\hbar}\left(x^2 + \frac{ipx}{m\omega} + \frac{ixp}{m\omega} + \frac{i^2p^2}{m^2\omega^2}\right)$$

$$\Rightarrow \quad a^{+}a^{+} - aa$$

$$a^{+}a^{+} = \frac{m\omega}{2\hbar}\left(x - \frac{ip}{m\omega}\right)\left(x - \frac{ip}{m\omega}\right) = \frac{m\omega}{2\hbar}\left(x^2 - \frac{ipx}{m\omega} - \frac{ixp}{m\omega} + \frac{i^2p^2}{m^2\omega^2}\right)$$

$$a^{+}a^{+} - aa = \left(-\frac{\cancel{2}ipx}{\cancel{m\omega}} - \frac{\cancel{2}ixp}{\cancel{m\omega}}\right)\frac{\cancel{m\omega}}{2\hbar} = -\frac{i}{\hbar}\left(px + xp\right)$$

$$xp = \frac{i\hbar}{2}\left(1 - \frac{i}{\hbar}px - \frac{i}{\hbar}xp\right)$$

$$[x, p] = i\hbar$$

$$xp = \frac{i\hbar}{2} - \frac{i^2}{2}px - \frac{i^2}{2}xp = \frac{i\hbar}{2} + \frac{px}{2} + \frac{xp}{2}$$

$$xp - px = i\hbar$$

$$px = xp - i\hbar$$

$$\frac{xp}{\cancel{2}} = \frac{i\hbar}{\cancel{2}} + \frac{px}{\cancel{2}}$$

Esta data la obtengo mucho más fácil del commutador $[x,p] = i\hbar$ → $xp = i\hbar + px$

$$\langle m|2xp - i\hbar|n\rangle = \langle m|2\cdot\frac{i\hbar}{2}\left(1 + a^{+}a^{+} - aa\right) - i\hbar|n\rangle$$

$$\langle m|a^{+}a^{+} - aa|n\rangle$$

$$= \langle m|a^{+}a^{+}|n\rangle - \langle m|aa|n\rangle$$

$$= \langle m|a^{+}\sqrt{n+1}\,|n+1\rangle - \langle m|\sqrt{n}\,a\,|n-1\rangle$$

$$\langle m|xp+px|n\rangle = \langle m|\sqrt{n+1}\,\sqrt{n+2}\,|n+2\rangle - \langle m|\sqrt{n}\sqrt{n-1}\,|n-2\rangle$$

$$\boxed{\langle m|xp+px|n\rangle = \sqrt{n+1}\,\sqrt{n+2}\,.\,\delta_{m,n+2} - \sqrt{n}\,\sqrt{n-1}\,\delta_{m,n-2}}$$

$*\ \langle m|x^2|n\rangle$

$$x^2 = \frac{\hbar}{2m\omega}(a+a^{+})(a+a^{+}) = \left(aa + a^{+}a^{+} + a^{+}a + aa^{+}\right)\frac{\hbar}{2m\omega}$$

$$\langle m|aa|n\rangle + \langle m|a^{+}a^{+}|n\rangle + \langle m|a^{+}a|n\rangle + \langle m|aa^{+}|n\rangle$$

$$\langle m|a\sqrt{n}\,|n-1\rangle + \langle m|a^{+}\sqrt{n+1}\,|n+1\rangle + \langle m|a^{+}\sqrt{n}\,|n-1\rangle + \langle m|a\sqrt{n+1}\,|n+1\rangle$$

$$\sqrt{n}\,\sqrt{n-1}\,\langle m|n-2\rangle + \sqrt{n+1}\,\sqrt{n+2}\,\langle m|n+2\rangle + \sqrt{n}\sqrt{n}\,.\,\langle m|n\rangle + \sqrt{n+1}\,\sqrt{n+1}\,\langle m|n\rangle$$

$$\sqrt{n}\,\sqrt{n-1}\,\delta_{m,n-2} + \sqrt{n+1}\,\sqrt{n+2}\,\delta_{m,n+2} + \underbrace{\left(\sqrt{n}\sqrt{n} + \sqrt{n+1}\sqrt{n+1}\right)}_{(n\ \ +\ n+1)}\delta_{mn}$$

$$\boxed{\langle m|x^2|n\rangle = \left[\sqrt{n(n-1)}\,\delta_{m,n-2} + \sqrt{(n+1)(n+2)}\,\delta_{m,n+2} + (2n+1)\,\delta_{mn}\right]\frac{\hbar}{2m\omega}}$$

$*\ \langle m|p^2|n\rangle$ $$\left(i\sqrt{\frac{m\omega\hbar}{2}}\right)^2(a^{+}-a)(a^{+}-a) = \left(-\frac{m\omega\hbar}{2}\right)\left(a^{+}a^{+} - aa^{+} - a^{+}a + aa\right)$$

$$\langle m|a^{+}a^{+}|n\rangle - \langle m|aa^{+}|n\rangle - \langle m|a^{+}a|n\rangle + \langle m|aa|n\rangle$$

$$\sqrt{n+1}\,\sqrt{n+2}\,\delta_{m,n+2} \quad -(n+1)\,\delta_{mn} \quad - n\,\delta_{mn} \quad + \sqrt{n}\,\sqrt{n-1}\,\delta_{m,n-2}$$

$$\boxed{\langle m|p^2|n\rangle = \left[\sqrt{n(n-1)}\,\delta_{m,n-2} + \sqrt{(n+1)(n+2)}\,\delta_{m,n+2} - (2n+1)\,\delta_{mn}\right]\frac{-m\omega\hbar}{2}}$$

b)

$$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} \Rightarrow$$

$$\left.\begin{aligned} \langle n|\frac{p^2}{2m}|n\rangle &= -\frac{\cancel{m}\omega\hbar}{2}\left(-2n+1\right)\frac{1}{2\cancel{m}} = (2n+1)\,\frac{\omega\hbar}{4} \\ \langle n|\frac{m\omega^2}{2}x^2|n\rangle &= \frac{\cancel{m}\omega^{\cancel{2}}}{2}\,\frac{\hbar}{2\cancel{m\omega}}(2n+1) = (2n+1)\,.\,\frac{\omega\hbar}{4} \end{aligned}\right\} \Rightarrow \boxed{\left\langle \frac{p^2}{2m}\right\rangle = \left\langle \frac{m\omega^2 x^2}{2}\right\rangle}$$

$\Rightarrow$ Se cumple el teorema del virial con respecto a un autoestado de la energía $|n\rangle$

9.

$$\left.\begin{array}{l}\langle(\Delta x)^2\rangle = \langle x^2\rangle - \langle x\rangle^2 \\ \langle(\Delta p)^2\rangle = \langle p^2\rangle - \langle p\rangle^2\end{array}\right\} \Rightarrow$$ Tomado en un autoestado $|n\rangle$ es

$$\langle(\Delta x)^2\rangle_{|n\rangle} = \langle n|x^2|n\rangle - (\langle n|x|n\rangle)^2 = (2n+1)\frac{\hbar}{2m\omega} - 0$$

$$\langle(\Delta p)^2\rangle_{|n\rangle} = \langle n|p^2|n\rangle - (\langle n|p|n\rangle)^2 = (2n+1)\frac{m\omega\hbar}{2} - 0$$

$$(2n+1)^2\frac{\hbar}{2\cancel{m\omega}}\cdot\frac{\cancel{m\omega}\hbar}{2} = \cancel{4}\left(n+\frac{1}{2}\right)^2\frac{\hbar^2}{\cancel{4}}$$

$$\Rightarrow \qquad \langle(\Delta x)^2\rangle_{|n\rangle}\langle(\Delta p)^2\rangle_{|n\rangle} = \left(n+\frac{1}{2}\right)^2\cdot\hbar^2$$

Con $n=0$ es $\langle(\Delta x)^2\rangle\langle(\Delta p)^2\rangle = \dfrac{\hbar^2}{4}$ incerteza mínima

$\therefore$ La Función de onda del estado fundamental es un paquete gaussiano

11.

$$a|0\rangle = 0 \quad \equiv |0\rangle$$

$$a^{+}|0\rangle = |1\rangle$$

$$a^{+}|1\rangle = a^{+}(a^{+}|0\rangle)$$

$$\sqrt{2}\,|2\rangle = (a^{+})^{2}|0\rangle$$

$$a^{+}\sqrt{2}\,|2\rangle = (a^{+})^{3}|0\rangle$$

$$\sqrt{3}\sqrt{2}\,|3\rangle = (a^{+})^{3}|0\rangle$$

general $\rightarrow$

$$|3\rangle = \frac{(a^{+})^{3}}{\sqrt{3!}}|0\rangle \rightarrow \quad |n\rangle = \frac{(a^{+})^{n}}{\sqrt{n!}}|0\rangle$$

$$a^{+} = \sqrt{\frac{m\omega}{2\hbar}}\left(x - \frac{ip}{m\omega}\right)$$

$$a = \sqrt{\frac{m\omega}{2\hbar}}\left(x + \frac{ip}{m\omega}\right)$$

$$x = \sqrt{\frac{\hbar}{2m\omega}}\left(a + a^{+}\right)$$

$\langle x|0\rangle$ y $\langle x|1\rangle$

$$\langle x|0\rangle = \langle x|.(|0\rangle) = \langle x|.a|0\rangle = \sqrt{\frac{m\omega}{2\hbar}}\langle x|\hat{x} + \frac{i.\hat{p}}{m\omega}|0\rangle$$

$$\langle x|0\rangle = \sqrt{\frac{m\omega}{2\hbar}}\left(\langle x|0\rangle x + \frac{i}{m\omega}\langle x| -i\hbar\frac{\partial}{\partial x}|0\rangle\right)$$

$$0 = \sqrt{\frac{m\omega}{2\hbar}}.x\langle x|0\rangle + \frac{\hbar}{m\omega}\frac{\partial}{\partial x}\left(\langle x|0\rangle\right)\sqrt{\frac{m\omega}{2\hbar}}$$

$$0 = +\sqrt{\frac{m\omega}{2\hbar}}\,x\langle x|0\rangle + \sqrt{\frac{\hbar}{2m\omega}}\,\frac{\partial}{\partial x}\langle x|0\rangle$$

$$\frac{\partial}{\partial x}\langle x|0\rangle = -\sqrt{\frac{2m\omega}{\hbar}}\sqrt{\frac{m\omega}{2\hbar}}\,x\langle x|0\rangle$$

$$\frac{\partial}{\partial x}\langle x|0\rangle = -\frac{m\omega}{\hbar}.x\langle x|0\rangle \rightarrow$$

$$\int\frac{1}{\langle x|0\rangle}d\langle x|0\rangle = -\int_{x_0}^{x}\frac{m\omega}{\hbar}x.dx$$

$$\ln\left(\langle x|0\rangle\right) = -\frac{m\omega}{\hbar}\left(\frac{x^{2}}{2} - \frac{x_0^{2}}{2}\right)$$

$$\langle x|0\rangle = e^{-\frac{m\omega}{\hbar}\frac{(x^{2})}{2} + \frac{m\omega x_0^{2}}{\hbar}}$$

$$\langle x|0\rangle = K.e^{-\left(\frac{x}{\text{Ж}}\right)^{2}\frac{1}{2}}$$

constante de normalización

$$\text{Ж} = \sqrt{\frac{\hbar}{m\omega}}$$

$$\int_{-\infty}^{+\infty}|\langle x|0\rangle|^{2}.dx = 1$$

$$\int_{-\infty}^{+\infty}|K|^{2}.e^{-\left(\frac{x}{\text{Ж}}\right)^{2}}.dx = 1$$

$$|K|^{2}\sqrt{\frac{\pi}{1/\text{Ж}^{2}}} = 1$$

$$|K|^{2} = \frac{1}{(\pi.\text{Ж}^{2})^{1/2}}$$

$$|K| = \frac{1}{(\pi\,\text{Ж}^{2})^{1/4}} = \frac{1}{(\pi)^{1/4}\sqrt{\frac{\hbar}{m\omega}}}$$

$$\boxed{\langle x|0\rangle = \frac{1}{(\pi)^{1/4}\sqrt[4]{\frac{\hbar}{m\omega}}}.e^{-\frac{1}{2}\frac{x^{2}}{\hbar}.m\omega}}$$

$$\langle x|1\rangle = \langle x|a^{+}|0\rangle$$

$$\langle x|1\rangle = \sqrt{\frac{m\omega}{2\hbar}}\,\langle x|\hat{x} - \frac{i\,\hat{p}}{m\omega}|0\rangle = \sqrt{\frac{m\omega}{2\hbar}}\left(\langle x|x + \frac{i^2\hbar}{m\omega}\frac{\partial}{\partial x}|0\rangle\right)$$

$$\langle x|1\rangle = \sqrt{\frac{m\omega}{2\hbar}}\left(\langle x|0\rangle x - \frac{\hbar}{m\omega}\frac{\partial}{\partial x}\langle x|0\rangle\right)$$

$$\langle x|1\rangle = \sqrt{\frac{m\omega}{2\hbar}}\cdot x\langle x|0\rangle - \frac{\hbar}{m\omega}\sqrt{\frac{m\omega}{2\hbar}}\cdot\frac{\partial}{\partial x}\langle x|0\rangle$$

$$\sqrt{\frac{m\omega}{2\hbar}}\,x\langle x|0\rangle - \sqrt{\frac{\hbar}{2m\omega}}\,\frac{\partial}{\partial x}\langle x|0\rangle$$

$$= \sqrt{\frac{m\omega}{2\hbar}}\,x\,\frac{1}{(\pi)^{1/4}\sqrt{\frac{\hbar}{m\omega}}}\,e^{-\frac{1}{2}x^2\frac{m\omega}{\hbar}} - \sqrt{\frac{\cancel{\hbar}}{2\cancel{m\omega}}}\cdot\frac{1}{(\pi)^{1/4}\sqrt{\frac{\cancel{\hbar}}{\cancel{m\omega}}}}\,e^{-\frac{1}{2}x^2\frac{m\omega}{\hbar}}\cdot\left(-\frac{m\omega}{\hbar}\right)x$$

$$= \frac{1}{(\pi)^{1/4}}\,x\cdot e^{-\frac{1}{2}x^2\cdot\frac{m\omega}{\hbar}}\left(\frac{m\omega}{\sqrt{2}\,\hbar} - -\frac{m\omega}{\sqrt{2}\,\hbar}\right)$$

$$\boxed{\langle x|1\rangle = \sqrt{2}\,\frac{m\omega}{\pi^{1/4}\cdot\hbar}\cdot x\cdot e^{-\frac{x^2}{2}\frac{m\omega}{\hbar}}}$$

12.

a)

$$\langle x'|p'\rangle = \frac{1}{\sqrt{2\pi\hbar}}\cdot e^{+i\frac{x'p'}{\hbar}}$$

$$\langle p'|\hat{x}|\alpha\rangle = \int dx'\,\langle p'|x'\rangle\langle x'|\hat{x}|\alpha\rangle$$

$$\int dx'\left(\frac{e^{-i\frac{x'p'}{\hbar}}}{\sqrt{2\pi\hbar}}\cdot x'\right)\langle x'|\alpha\rangle$$

$$\int dx'\left\{-\frac{\hbar}{i}\frac{\partial}{\partial p'}\left(\frac{e^{-\frac{ix'p'}{\hbar}}}{\sqrt{2\pi\hbar}}\right)\right\}\langle x'|\alpha\rangle$$

$$= \underbrace{\int dx'}\left(i\hbar\frac{\partial}{\partial p'}\langle p'|\underbrace{x'\rangle\langle x'|}_{\mathbb{1}}\alpha\rangle\right)$$

$$\boxed{\langle p'|\hat{x}|\alpha\rangle = i\hbar\frac{\partial}{\partial p'}\langle p'|\alpha\rangle}$$

b) oscilador armónico 1D

13. $C(t) = \langle\ x(t)\ x(0)\ \rangle$ oscilador armónico en 1D

$$C(t) = \langle 0|\, \mathcal{U}^{+}(t,0)\, x(0)\, \mathcal{U}(t,0)\, x(0)\, |0\rangle \qquad H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2}$$

$x(0) = \sqrt{\frac{\hbar}{2m\omega}}\,(a+a^{+})$ → en $t=0$ coinciden Schrödinger y Heisenberg

con $a|0\rangle \equiv 0$ (el Fundamental es 0)
$a|0\rangle \equiv |0\rangle$ ←

$$\hat{H} = \hbar\omega\left(\hat{N} + \frac{1}{2}\right)$$

$$\hat{H}|0\rangle = \hbar\omega\Big(\underbrace{\hat{N}|0\rangle}_{0} + \frac{1}{2}|0\rangle\Big)$$

$$C(t) = \frac{\hbar}{2m\omega}\langle 0|\, e^{\frac{i\hat{H}t}{\hbar}}(a+a^{+})\, e^{-\frac{i\hat{H}t}{\hbar}}(a+a^{+})\,|0\rangle$$

$$\hat{H}|0\rangle = \frac{\hbar\omega}{2}|0\rangle$$

$$\hat{H}^2|0\rangle = \frac{\hbar\omega}{2}\left(\frac{\hbar\omega}{2}\right)|0\rangle$$

$$C(t) = \frac{\hbar}{2m\omega}\langle 0|\, e^{i\frac{\hbar\omega t}{2\hbar}}(a+a^{+})\Big(\overset{(|1\rangle)}{} + e^{-\frac{i\hat{H}t}{\hbar}}|1\rangle\Big)$$

$$C(t) = \frac{\hbar}{2m\omega}\langle 0|\, e^{i\frac{\hbar\omega t}{2\hbar}}(a+a^{+})\Big(\ |1\rangle\ \ e^{-i\frac{\hbar\omega}{\hbar}\frac{3}{2}t}\Big)$$

$$\hat{H}|1\rangle = \hbar\omega\Big(|1\rangle + \frac{1}{2}|1\rangle\Big)$$

$$\hat{H}|1\rangle = \hbar\omega\,\frac{3}{2}|1\rangle$$

$$C(t) = \frac{\hbar}{2m\omega}\langle 0|\,(a+a^{+})\ \ \Big(e^{-\frac{i\hbar\omega t}{\hbar}}|1\rangle\Big)$$

$$\hat{H}^2|1\rangle = \hbar\omega\frac{3}{2}\left(\hbar\omega\frac{3}{2}\right)|1\rangle$$

$$C(t) = \frac{\hbar}{2m\omega}\langle 0|(a+a^{+})|1\rangle\, e^{-i\omega t}$$

$$C(t) = \frac{\hbar}{2m\omega}\Big(\langle 0|a|1\rangle + \langle 0|a^{+}|1\rangle\Big)\, e^{-i\omega t}$$

$$C(t) = \frac{\hbar}{2m\omega}\Big(\langle 0|0\rangle + \langle 0|2\rangle\Big)\, e^{-i\omega t}$$

$$C(t) = \frac{\hbar}{2m\omega}\Big(\langle 0|0\rangle\Big)\, e^{-i\omega t}$$

$$C(t) = \frac{\hbar}{2m\omega}\, e^{-i\omega t}$$

$$\boxed{C(t) = \frac{\hbar}{2m\omega}\, e^{-i\omega t}}$$

* Cálculos Auxiliares

$\hat{N} \equiv a^{+}a$

$a^{+}a\,|0\rangle = a^{+}|0\rangle = |1\rangle$

$\downarrow_{DC} \qquad \downarrow_{DC}$

$\langle 0|aa^{+} = \langle 0|a^{+} = \langle 1|$

$\langle 0|N^{+}$

$\langle 0|e^{\frac{iHt}{\hbar}} \Rightarrow \langle 0|\,\hbar\omega\left(\hat{N}^{+} + \frac{1}{2}\right)$

$\hbar\omega\left(\langle 0|\hat{N}^{+} + \frac{1}{2}\langle 0|\right)$

$\underbrace{\langle 0|a}_{=0}a^{+}$

$\hbar\omega\left(\hat{N} + \frac{1}{2}\right)|0\rangle$

$\hbar\omega$

$\hbar\omega\left(\langle 0|\cdot\frac{1}{2}\right) = \langle 0|\,\frac{\hbar\omega}{2}$

14.

Oscilador armónico en 1D $\qquad H = \hbar\omega\left(N + \frac{1}{2}\right)$

a) $|\alpha\rangle$ CL $\{|0\rangle, |1\rangle\}$ con $\langle x\rangle_{|\alpha\rangle}$ máximo

$$|\alpha\rangle = c_1|0\rangle + c_2|1\rangle \quad \text{con} \quad c_1, c_2 \in \mathbb{C} \quad y$$

$$|c_1|^2 + |c_2|^2 = 1$$

Como la fase global puede ser arbitraria es conveniente escribir:

$$|\alpha\rangle = c_1 e^{i\phi_1}|0\rangle + c_2 e^{i\phi_2}|1\rangle \qquad \text{con} \quad c_1, c_2 \in \mathbb{R} \quad y$$

$$|\alpha\rangle = c_1 e^{i\phi}|0\rangle + c_2|1\rangle \qquad c_1^2 + c_2^2 = 1$$

$$\phi = \phi_1 - \phi_2$$

$$\langle x\rangle_{|\alpha\rangle} = \langle\alpha|\, x\, |\alpha\rangle \qquad c_2^2 = 1 - c_1^2$$

$$c_2 = (1 - c_1^2)^{1/2}$$

Usamos

$$x = \sqrt{\frac{\hbar}{2m\omega}}\,(a + a^+) \quad \Rightarrow \text{es } x(t=0) \text{ el } x \text{ inicial}$$

$$\langle x\rangle = \left(\langle 0|c_1 e^{-i\phi} + \langle 1|c_2\right) e^{+\frac{iHt}{\hbar}} \sqrt{\frac{\hbar}{2m\omega}}\,(a + a^+)\, e^{-\frac{iHt}{\hbar}} \left(c_1 e^{i\phi}|0\rangle + c_2|1\rangle\right)$$

$$\langle x\rangle = \sqrt{\frac{\hbar}{2m\omega}} \left(\langle 0|c_1 e^{-i\phi} . e^{\frac{i\hbar\omega t}{2\cancel{\hbar}}} + \langle 1|c_2 . e^{\frac{i\frac{3}{2}\hbar\omega t}{\cancel{\hbar}}}\right)(a + a^+) . \left(c_1 . e^{i\phi} . e^{-\frac{i\hbar\omega t}{2\cancel{\hbar}}}|0\rangle\right.$$

$$\left. + c_2 . e^{-\frac{i\hbar\omega 3t}{2\cancel{\hbar}}}|1\rangle\right)$$

$$\langle x\rangle = \left(\frac{\hbar}{2m\omega}\right)^{1/2}\left[\left(\langle 0|c_1 . e^{-i\phi} . e^{i\frac{\omega t}{2}} + \langle 1|c_2 e^{i\frac{3}{2}\omega t}\right)\left(c_1 . e^{i\phi} . e^{-i\frac{\omega t}{2}}|1\rangle + c_2 . e^{-i\frac{\omega t}{2}z}|0\rangle\right.\right.$$

$$\left.\left. + c_2 . e^{-\frac{i\omega t 3}{2}}\sqrt{2}\,|2\rangle\right)\right]$$

$$\langle x\rangle = \left(\frac{\hbar}{2m\omega}\right)^{1/2}\left[\langle 1|1\rangle c_1 c_2 e^{i(\phi + \omega t)} + \langle 0|0\rangle c_1 e^{-i\phi} e^{-i\omega t} c_2\right]$$

AUXILIAR

$$(a + a^+)|0\rangle = a|0\rangle + a^+|0\rangle =$$

$$= 0 + |1\rangle$$

$$(a + a^+)|1\rangle = a|1\rangle + a^+|1\rangle$$

$$|0\rangle + \sqrt{2}\,|2\rangle$$

$$\langle x\rangle = \left(\frac{\hbar}{2m\omega}\right)^{1/2} c_1 c_2 \left[e^{i(\phi + \omega t)} + e^{-i(\phi + \omega t)}\right]$$

$$\langle x\rangle_{|\alpha\rangle} = \sqrt{\frac{\hbar}{2m\omega}}\; \underbrace{c_1(1 - c_1^2)^{1/2}}_{\text{variables}} . \underbrace{2\cos(\phi + \omega t)}_{\text{variables}}$$

$$\frac{\partial\langle x\rangle}{\partial c_1} = 0 \quad \Rightarrow \quad \frac{2c_1(1 - c_1^2) + c_1^2(-2c_1)}{2\sqrt{c_1^2(1 - c_1^2)}} = 0 = \frac{\cancel{2}\overset{1}{\cancel{c_1}} - \cancel{2}c_1^{\overset{2}{\cancel{3}}} - \cancel{2}c_1^{\overset{2}{\cancel{3}}}}{\cancel{2}\cancel{c_1}(1 - c_1^2)} \Rightarrow \quad 1 - 2c_1^2 = 0$$

$$\boxed{c_1 = \frac{1}{\sqrt{2}}}$$

$$\Rightarrow \overset{\text{con } c_1 \neq 1}{\boxed{c_2 = 1/\sqrt{2}}}$$

$$\frac{\partial\langle x\rangle}{\partial\phi}=0 \quad\Rightarrow\quad \frac{d}{d\phi}\cos(\phi+\omega t)=0$$

$$-\text{sen}(\phi+\omega t)=0 \quad\rightarrow\quad \phi+\omega t=n\pi \qquad n\in\mathbb{N}$$

$$\phi=n.\pi+\omega t$$

en $t=0$ es $\phi=n\pi$

$$\boxed{|\alpha\rangle=\frac{1}{\sqrt{2}}(e^{i\pi n}|0\rangle+|1\rangle}$$

b) $|\alpha\rangle=e^{in\pi}\frac{|0\rangle}{\sqrt{2}}+\frac{|1\rangle}{\sqrt{2}}$ → elijo $n=2$ → $|\alpha\rangle=\frac{|0\rangle}{\sqrt{2}}+\frac{|1\rangle}{\sqrt{2}}$

$$\mathcal{U}(t,0)|\alpha,0\rangle=|\alpha,0;t\rangle$$

$$|\alpha,0;t\rangle=e^{-iHt/\hbar}\left(\frac{|0\rangle}{\sqrt{2}}+\frac{|1\rangle}{\sqrt{2}}\right)$$

$$=\frac{1}{\sqrt{2}}\left(e^{-i\frac{\cancel{\hbar}\omega}{2}t/\cancel{\hbar}}|0\rangle+e^{-i\frac{3}{2}\cancel{\hbar}\omega t/\cancel{\hbar}}|1\rangle\right)$$

$$\boxed{|\alpha,0;t\rangle=\frac{1}{\sqrt{2}}\cdot e^{-\frac{i\omega t}{2}}\left(|0\rangle+e^{-i\omega t}|1\rangle\right)}$$

i)

$$\langle x\rangle_s=\langle\alpha,0;t|x|\alpha,0;t\rangle\Rightarrow$$

$$=\sqrt{\frac{\hbar}{2m\omega}}\cdot\frac{1}{2}\left(\langle0|+\langle1|e^{i\omega t}\right)(a+a^{\dagger})\left(|0\rangle+e^{-i\omega t}|1\rangle\right)$$

$$=\frac{1}{2}\left(\frac{\hbar}{2m\omega}\right)^{1/2}\left(\langle0|+\langle1|e^{i\omega t}\right)\left(e^{-i\omega t}|0\rangle+|1\rangle+e^{-i\omega t}|2\rangle\right)$$

$$=\left(e^{-i\omega t}\langle0|0\rangle+e^{i\omega t}\langle1|1\rangle\right)$$

$$\langle x\rangle_s=\frac{1}{\cancel{2}}\left(\frac{\hbar}{2m\omega}\right)^{1/2}\cancel{2}\cos(\omega t)=\boxed{\sqrt{\frac{\hbar}{2m\omega}}\cos(\omega t)=\langle x\rangle_s}$$

ii)

$\langle x\rangle_H$; aquí hay que evolucionar el operador

$$=\langle\alpha,0|\mathcal{U}^{\dagger}(t,0)\,x(0)\,\mathcal{U}(t,0)|\alpha,0\rangle$$

$$=\left(\frac{\langle0|+\langle1|}{\sqrt{2}}\right)\underbrace{\left(e^{\frac{iHt}{\hbar}}x(0)\,e^{-\frac{iHt}{\hbar}}\right)}_{x(t)}\left(\frac{|1\rangle+|0\rangle}{\sqrt{2}}\right)$$

Ecuaciones de Heisenberg

$$\left[\begin{aligned}\frac{dx}{dt}&=\frac{1}{i\hbar}[x,H]=\frac{1}{i\hbar\,2m}[x,p^2]=\frac{1}{\cancel{2}\cancel{i\hbar}m}\cancel{i\hbar}\cancel{2}p=\frac{p}{m}\\ \frac{dp}{dt}&=\frac{1}{i\hbar}[p,H]=\frac{m\omega^2}{2i\hbar}[p,x^2]=\frac{m\omega^2}{\cancel{2}\cancel{i\hbar}}\cdot-\cancel{i\hbar}\cancel{2}x=-m\omega^2x\end{aligned}\right.$$

Están acopladas, pero podemos desacoplarlas metiendo una en la otra:

$$\frac{d^2x}{dt^2} = \frac{1}{m}\frac{dp}{dt} = -\omega^2 x \rightarrow \qquad x = x(0).e^{i\omega t}$$

$$p = m.\frac{dx}{dt} = m.\,x(0).\,\omega i.\,e^{i\omega t}$$

$$p(0) = m\omega i.x(0) \rightarrow \qquad p(t) = p(0).e^{i\omega t}$$

$$x(t) = x(0).\cos[\omega t] + \frac{p(0)}{m\omega}.\operatorname{sen}[\omega t]$$

$$\langle x\rangle_H = \left(\frac{\langle 0| + \langle 1|}{\sqrt{2}}\right)\left(x(0).\cos[\omega t]\right)\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) +$$

$$\left(\frac{\langle 0| + \langle 1|}{\sqrt{2}}\right)\left(\frac{p(0)}{m\omega}.\operatorname{sen}[\omega t]\right)\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)$$

$$= \frac{1}{2}\cos[\omega t]\sqrt{\frac{\hbar}{2m\omega}}\underbrace{\left(\langle 0| + \langle 1|\right)\left(a + a^{+}\right)\left(|1\rangle + |0\rangle\right)}_{\left(\langle 0| + \langle 1|\right)\left(|0\rangle + |2\rangle + |1\rangle\right) \;=\; (1+1)} +$$

$$\frac{1}{2}.\operatorname{sen}[\omega t]\sqrt{\frac{m\hbar\omega}{2}}.i.\underbrace{\left(\langle 0| + \langle 1|\right)\left(a^{+} - a\right)\left(|1\rangle + |0\rangle\right)}_{\substack{\left(\langle 0| + \langle 1|\right)\left(|2\rangle - |0\rangle + |1\rangle\right) \\ (-1+1)}}$$

$$\boxed{\langle x\rangle_H = \sqrt{\frac{\hbar}{2m\omega}}.\cos[\omega t]}$$

c) $\langle (\Delta x)^2\rangle$

$\langle(\Delta x)^2\rangle = \langle x^2\rangle - \langle x\rangle^2 \rightarrow$ $*$ schrödinger $\begin{cases}\text{estados evolucionados} \\ \text{operadores fijos}\end{cases}$

$\langle (\Delta x)^2\rangle_S \Rightarrow$

$$\langle \alpha, 0; t|\, x^2\, |\alpha, 0; t\rangle = \frac{1}{2}.\frac{\hbar}{2m\omega}\left(\langle 0| + \langle 1|.e^{i\omega t}\right)(a+a^{+})(a+a^{+})\left(|0\rangle + e^{-i\omega t}|1\rangle\right)$$

$$= \left(aa + a^{+}a + aa^{+} + a^{+}a^{+}\right)\left(|0\rangle + e^{-i\omega t}|1\rangle\right)$$

$$\left(\langle 0| + \langle 1|.e^{i\omega t}\right)\left(|0\rangle + \sqrt{2}|2\rangle + e^{-i\omega t}\left[|1\rangle + 2|1\rangle + \sqrt{2}\sqrt{3}|3\rangle\right]\right)$$

$$= \frac{1}{2}\cdot\frac{\hbar}{2m\omega}\left(1 + 1 + 2\right) = \frac{\hbar}{m\omega}$$

$$aa^{+}|1\rangle = a\sqrt{2}|2\rangle = 2|1\rangle$$

$$a^{+}a|1\rangle = a^{+}|0\rangle = |1\rangle$$

$$\langle x\rangle^2 = \frac{\hbar}{2m\omega}\cos^2(\omega t)$$

↖ ya fue calculado

$$\langle(\Delta x)^2\rangle_S = \frac{\hbar}{m\omega}\left(1 - \frac{\cos^2(\omega t)}{2}\right)$$

$$\langle(\Delta x)^2\rangle_S = \frac{\hbar}{m\omega}\left(\frac{1}{2}+\frac{1}{2}-\frac{\cos^2(\omega t)}{2}\right) = \frac{\hbar}{m\omega}\left(\frac{1}{2}+\frac{\text{sen}^2\omega t}{2}\right)$$

$$\boxed{\langle(\Delta x)^2\rangle_S = \frac{\hbar}{2m\omega}\left[1+\text{sen}^2(\omega t)\right]}$$

* Heisenberg

$$x^2(t) = \left[x(0)\cdot\cos[\omega t] + \frac{p(0)}{m\omega}\cdot\text{sen}[\omega t]\right]^2$$

$$x^2(t) = x^2(0)\cdot\cos^2(\omega t) + \frac{x(0)\,p(0)}{m\omega}\cos(\omega t)\,\text{sen}(\omega t) + \frac{p(0)\cdot x(0)}{m\omega}\cdot\text{sen}(\omega t)\cos(\omega t)$$
$$+\frac{p^2(0)}{m^2\omega^2}\,\text{sen}^2(\omega t)$$

$$\langle x^2\rangle = \left(\langle\alpha,0|\right)\left[x^2(0)\cdot\cos^2(\omega t)\right]\left(|\alpha,0\rangle\right) + \left(\langle\alpha,0,t|\right)\left[\frac{p^2(0)}{m^2\omega^2}\text{sen}^2(\omega t)\right]\left(|\alpha,t\rangle\right)$$
$$+\left(\langle\alpha,t|\right)\left[\frac{x(0)(p(0)}{m\omega}\cos(\omega t)\,\text{sen}(\omega t)\right]\left(|\alpha,t\rangle\right) + \left(\langle\alpha,t|\right)\left[\frac{p(0)x(0)}{m\omega}\cos(\omega t)\,\text{sen}(\omega t)\right]\left(|\alpha,t\rangle\right)$$

$$\langle x^2\rangle = \frac{\cos^2(\omega t)}{2}\left(\langle 0|+\langle 1|\right)x^2(0)\left(|0\rangle+|1\rangle\right) + \frac{\text{sen}^2(\omega t)}{m^2 2\omega^2}\left(\langle 0|+\langle 1|\right)p^2(0)\left(|0\rangle+|1\rangle\right)$$
$$+\frac{\cos(\omega t)\cdot\text{sen}(\omega t)}{2\,m\cdot\omega}\left\{\left(\langle 0|+\langle 1|\right)x(0)p(0)\left(|0\rangle+|1\rangle\right)+\left(\langle 0|+\langle 1|\right)p(0)\,x(0)\left(|0\rangle+|1\rangle\right)\right\}$$

$$= \frac{\cos^2(\omega t)}{\cancel{2}}\,\frac{\hbar}{\cancel{2}m\omega}\cancel{4} + \frac{\text{sen}^2(\omega t)}{m^2 2\omega^2}\cdot\frac{m\hbar\omega}{2}\left(-1+2+3\right) =$$

$$= \frac{\hbar}{m\omega}\cdot\left[\cos^2(\omega t)\right] + \frac{\cancel{4}}{\cancel{4}}\frac{m\hbar\omega}{m^2\omega^2}\cdot\text{sen}^2(\omega t)$$

$$\langle x^2\rangle_H = \frac{\hbar}{m\omega}\left(\cos^2(\omega t) + \text{sen}^2(\omega t)\right)$$

$$\langle(\Delta x)^2\rangle_H = \langle x^2\rangle_H - \langle x\rangle_H^2 = \frac{\hbar}{m\omega}\cos^2(\omega t) + \frac{\hbar}{m\omega}\text{sen}^2(\omega t) - \frac{\hbar}{m\omega}\cdot\frac{\cos^2(\omega t)}{2}$$

$$\langle x\rangle_H^2 = \frac{\hbar}{2m\omega}\cdot\cos^2(\omega t)$$

$$\frac{\hbar}{2m\omega}\left[\cos^2(\omega t) + 2\cdot\text{sen}^2(\omega t)\right]$$

$$\boxed{\langle(\Delta x)^2\rangle_H = \frac{\hbar}{2m\omega}\left[1+\text{sen}^2(\omega t)\right]}$$

* Cálculos Auxiliares

$$p^2(0) = -\frac{m\hbar\omega}{2}(a^\dagger-a)(a^\dagger-a) = -\frac{m\hbar\omega}{2}\left(a^\dagger a^\dagger - aa^\dagger - a^\dagger a + aa\right) = \frac{m\hbar\omega}{2}\left(-a^\dagger a^\dagger + aa^\dagger + a^\dagger a - aa\right)$$

$$p^2(0)\left(|0\rangle + e^{-i\omega t}|1\rangle\right) = \frac{m\hbar\omega}{2}\left(-\sqrt{2}|2\rangle - |0\rangle + e^{-i\omega t}\left[-\sqrt{2}\sqrt{3}\,|3\rangle + 2|1\rangle + |1\rangle\right]\right) \Rightarrow$$ al hacer producto interno se anula

$$x(0)\,p(0) = \sqrt{\ }\sqrt{\ }\,i\,(a+a^\dagger)(a^\dagger-a) = \left(\cancel{a}a^\dagger + a^\dagger a^\dagger - aa - a^\dagger a\right)i\sqrt{\ }\sqrt{\ }$$
$$= (a^\dagger-a)(a+a^\dagger) = \left(a^\dagger a - aa + a^\dagger a^\dagger - \cancel{a}a^\dagger\right)i\sqrt{\ }\sqrt{\ }$$

$$x(0)\,p(0) + p(0)\,x(0) = 2\left(a^\dagger a^\dagger - aa\right)i\sqrt{\ }\sqrt{\ } \Rightarrow \{x,p\}\left(|0\rangle + e^{-i\omega t}|1\rangle\right) =$$

$$2i\left(\ \right)^{1/2}\left(\ \right)^{1/2}\left(\sqrt{2}\,|2\rangle + \sqrt{2}\sqrt{3}\,|3\rangle e^{-i\omega t}\right) \Rightarrow$$ al hacer producto interno se anula

15. $\langle 0|e^{ik\hat{x}}|0\rangle = e^{-\frac{k^2}{2}\langle 0|\hat{x}^2|0\rangle}$

$$|n\rangle = \frac{(a^+)^n}{\sqrt{n!}}|0\rangle$$

$$e^{ik\hat{x}} = \sum_{n=0}^{\infty} \frac{1}{n!}(ik\hat{x})^n$$

$$\hat{x}|0\rangle = \sqrt{\frac{\hbar}{2m\omega}}(a+a^+)|0\rangle = \sqrt{\frac{\hbar}{2m\omega}}|1\rangle$$

$$\hat{x}^2|0\rangle = \sqrt{\frac{\hbar}{2m\omega}}(a+a^+)\left[\sqrt{\frac{\hbar}{2m\omega}}\right]|1\rangle = \sqrt{\;}\sqrt{\;}\left(|0\rangle+\sqrt{2}|2\rangle\right) =$$

$$\hat{x}^3|0\rangle = \left[\sqrt{\;}\sqrt{\;}\right]\left(|0\rangle+|2\rangle\right)$$

$$= \sqrt{\;}\sqrt{\;}\sqrt{\;}\left(|1\rangle+|1\rangle+|3\rangle\right)$$

No se ve de aquí alguna relación de recurrencia sencilla ⇒ dejamos esto

$$\langle 0|e^{ik\hat{x}}|0\rangle = \int dx'\langle 0|\underbrace{e^{ik\hat{x}}|x'\rangle}\langle x'|0\rangle = \int dx'\langle 0|e^{ikx'}|x'\rangle\underbrace{\langle x'|0\rangle}_{\text{Función de onda para el ground state [ej. 11]}}$$

$$\hat{x}|x'\rangle = x'|x'\rangle$$
$$\hat{x}^n|x'\rangle = (x')^n|x'\rangle$$

$$= \int dx'\langle 0|e^{ikx'}|x'\rangle\frac{e^{-\frac{x'^2}{2}\frac{m\omega}{\hbar}}}{(\pi)^{1/4}\sqrt{\frac{\hbar}{m\omega}}}$$

$$\langle 0|e^{ik\hat{x}}|0\rangle = \int dx'\underbrace{\langle 0|x'\rangle}_{(\langle x'|0\rangle)^*}e^{ikx'}\cdot\frac{e^{-\frac{x'^2m\omega}{2\hbar}}}{(\pi)^{1/4}\sqrt[4]{\frac{\hbar}{m\omega}}} = \int dx'\, e^{ikx'}\cdot\frac{e^{-\frac{x'^2m\omega}{\hbar}}}{(\pi)^{1/2}\left(\frac{\hbar}{m\omega}\right)^{1/2}}$$

* NOTA
[ej. 8]
$$\langle 0|\hat{x}^2|0\rangle = (2n+1)\frac{\hbar}{2m\omega} = \frac{\hbar}{2m\omega}$$
$$-\frac{k^2}{2}\cdot\frac{\hbar}{2m\omega} = -\frac{k^2\hbar}{4m\omega}$$

$$= \frac{\sqrt{m\omega}}{\sqrt{\hbar}\sqrt{\pi}}\int dx'\cdot e^{ikx' - x'^2\frac{m\omega}{\hbar}}$$

$$-\left[-ikx' - i^2x'^2\frac{m\omega}{\hbar}\right] = \Omega$$

* completamos el cuadrado

$$A = ix'\sqrt{\frac{m\omega}{\hbar}}$$
$$2AB = 2ix'\sqrt{\frac{m\omega}{\hbar}}\quad B = ikx'$$
$$= \qquad B = \frac{k}{2}\sqrt{\frac{\hbar}{m\omega}}$$
$$B^2 = \frac{k^2\hbar}{4m\omega}$$

$$\Omega = -\left(-ikx' - i^2x'^2\frac{m\omega}{\hbar}\right) + \frac{k^2\hbar}{4m\omega} - \frac{k^2\hbar}{4m\omega}$$
$$-\left(ix'\sqrt{\frac{m\omega}{\hbar}} + \frac{k}{2}\sqrt{\frac{\hbar}{m\omega}}\right)^2 - \frac{k^2\hbar}{4m\omega}$$

$$= \frac{\sqrt{m\omega}}{\sqrt{\hbar}\sqrt{\pi}}\int_{-\infty}^{+\infty} dx'\cdot e^{-(\quad+\quad)^2}\cdot e^{-\frac{k^2\hbar}{4m\omega}}$$

$$\left.\begin{aligned} ix'\sqrt{\frac{m\omega}{\hbar}} + \frac{k}{2}\sqrt{\frac{\hbar}{m\omega}} &= z \\ dx' &= \frac{dz}{i}\sqrt{\frac{\hbar}{m\omega}}\end{aligned}\right\}\text{ cambio de variables}$$

$$= \frac{\cancel{\sqrt{m\omega}}}{\cancel{\sqrt{\hbar}}\sqrt{\pi}}\cancel{\sqrt{\frac{\hbar}{m\omega}}}\cdot\frac{1}{i}\underbrace{\int_{-i\infty}^{i\infty} dz\, e^{-z^2}}_{\sqrt{\pi}\,i}\cdot e^{-\frac{k^2\hbar}{4m\omega}}$$

$$\langle 0|e^{ik\hat{x}}|0\rangle = e^{-\frac{k^2\hbar}{4m\omega}} = e^{-\frac{k^2}{2}\left(\frac{\hbar}{2m\omega}\right)} \Rightarrow$$ Usando resultado [ej. 8]

$$\boxed{\langle 0|e^{ik\hat{x}}|0\rangle = e^{-\frac{k^2}{2}\langle 0|\hat{x}^2|0\rangle}}$$

16.

$$|\varphi,0\rangle = e^{-\frac{i\hat{p}.d}{\hbar}}|0\rangle$$

Usando Heisenberg evaluar $\langle x\rangle_{|\varphi,0\rangle}$ para $t>0$, es decir $\langle x(t)\rangle_{|\varphi,0\rangle}$

En Heisenberg los estados generales se mantienen estacionarios, pero los operadores evolucionan

$$\langle x\rangle = \langle\varphi,0|\,x(t)\,|\varphi,0\rangle$$

$$\begin{cases} \frac{dx}{dt} = \frac{1}{i\hbar}[x,H] = \frac{1}{i\hbar}\left[x,\frac{p_x^2}{2m}\right] = \frac{1}{i\hbar}\cdot\frac{1}{2m}[x,p_x^2] = \frac{1}{\cancel{2}mi\cancel{\hbar}}(+i\cancel{\hbar}\cancel{2}p) = \frac{p}{m} \\ \frac{dp}{dt} = \frac{1}{i\hbar}[p,H] = \frac{1}{i\hbar}\left[p,\frac{m\omega^2x^2}{2}\right] = \frac{1}{i\hbar}\frac{m\omega^2}{2}[p,x^2] = \frac{m\omega^2}{\cancel{2i\hbar}}\cdot(\cancel{-i\hbar 2}x) = -m\omega^2x \end{cases}$$

↑ Ecuaciones de Schrödinger (acopladas)

$$\frac{dx}{dt} = \frac{p}{m} \qquad \frac{dp}{dt} = -m\omega^2x$$

$$\frac{d^2x}{dt^2} = \frac{dp}{dt}\cdot\frac{1}{m} = -\frac{\cancel{m}\omega^2x}{\cancel{m}} = -\omega^2x \rightarrow$$

$$\frac{d^2x}{dt^2} + \omega^2x = 0$$

$$x = A.e^{\lambda t}$$

$$x'' = A.\lambda^2.e^{\lambda t}$$

$$\lambda^2+\omega^2 = \rightarrow$$

$$x(t) = X_0.e^{i\omega t} =$$

$$\frac{dx}{dt} = x_0.e^{i\omega t}\,\omega i = \frac{p}{m}$$

$$m\omega.i\,x_0.e^{i\omega t} = p(t)$$

$$x(t) = x_0(\cos(\omega t) + i.\text{sen}\,\omega t)$$

$$p(t) = m\omega x_0[-\text{sen}(\omega t) + i.\cos(\omega t)]$$

hago desaparecer la unidad i $\quad p(0) = m\omega x(0)\,i$,

$$x(t) = x_0.\cos(\omega t) + \frac{p_0}{m\omega}\text{sen}(\omega t)$$

$$p(t) = -m\omega x_0.\text{sen}(\omega t) + p_0.\cos(\omega t)$$

$$\langle\varphi,0|\,x_0.e^{i\omega t}\,|\varphi,0\rangle$$

$$\langle x\rangle = \langle 0|e^{i\hat{p}.d/\hbar}\,x(0).e^{i\omega t}\,e^{-i\hat{p}.d/\hbar}|0\rangle$$

$$\langle 0|e^{i\hat{p}\frac{d}{\hbar}}\left\{x_0.\cos(\omega t) + \frac{p_0}{m\omega}\text{sen}(\omega t)\right\}e^{-i\frac{\hat{p}d}{\hbar}}|0\rangle$$

$$\iint dx'dx''\langle 0|x''\rangle\left(\langle x''|e^{i\hat{p}d/\hbar}\right)x(t)\left(e^{-i\frac{\hat{p}d}{\hbar}}|x'\rangle\right)\langle x'|0\rangle$$

$$\iint dx'dx''\langle 0|x''\rangle\langle x''-d|\,x(t)\,|x'-d\rangle\langle x'|0\rangle$$

$$\iint dx'dx''\langle 0|x''\rangle x(t).\,\delta(x''\cancel{-d}-x'\cancel{+d})\langle x'|0\rangle$$

$$\langle x\rangle = \int dx'\,|\langle 0|x'\rangle|^2\,x(t)$$

$$= x(t)\int dx'\,\pi^{-1/2}.x_0^{-1}.e^{-\left(\frac{x'}{x_0}\right)^2}$$

$$\langle x\rangle = x(t)\,\pi^{-1/2}.x_0^{-1}.\frac{\sqrt{\pi}}{\sqrt{\frac{1}{x_0^2}}}$$

$$\langle x\rangle = \frac{x(t)}{\cancel{\sqrt{\pi}}\cancel{x_0}}\cancel{\sqrt{\pi}}\cancel{x_0} = x(t)$$

$$\boxed{\langle x\rangle = x(t)}$$

$$e^{-i\hat{p}d/\hbar}|x'\rangle = |x'-d\rangle$$

↑ [Ver ej. 18]

$$a = \left(x + \frac{ip}{m\omega}\right)\sqrt{\frac{m\omega}{2\hbar}} \quad \Rightarrow \quad a|0\rangle = 0 = \hat{x}\sqrt{\frac{m\omega}{2\hbar}}\,|0\rangle + \frac{i\hat{p}}{m\omega}\sqrt{\frac{m\omega}{2\hbar}}\,|0\rangle = 0$$

$$\hat{x}|0\rangle + \frac{i}{m\omega}\hat{p}|0\rangle = 0$$

$$\frac{i\hat{p}}{m\omega}|0\rangle = -\hat{x}|0\rangle$$

$$a|\varphi\rangle = \left(x + \frac{ip}{m\omega}\right)\sqrt{} \cdot e^{-\frac{ipd}{\hbar}}\,|0\rangle$$

$$a|\varphi\rangle = \left(x.\,e^{-\frac{ipd}{\hbar}}\,|0\rangle + \frac{ip}{m\omega}\,e^{-\frac{ipd}{\hbar}}\,|0\rangle\right)\sqrt{\frac{m\omega}{2\hbar}}$$

$$\left[x, e^{-\frac{ipd}{\hbar}}\right] = d.\,e^{-\frac{ipd}{\hbar}} \quad \wedge \quad \left[p, e^{-\frac{ipd}{\hbar}}\right] = 0$$

$$\frac{ipd}{\hbar} = \square$$

$$x.e^{-\square} - e^{-\square}.x$$

$$= \left[\left(d.e^{-\square} + e^{-\square}x\right)|0\rangle + e^{-\square}.\frac{ip}{m\omega}\,|0\rangle\right]\sqrt{\frac{m\omega}{2\hbar}}$$

$$a|\varphi\rangle = \left[d.e^{-\square}|0\rangle + \cancel{e^{-\square}x\,|0\rangle} - \cancel{e^{-\square}.x\,|0\rangle}\right]\cdot\sqrt{\frac{m\omega}{2\hbar}}$$

$$a|\varphi\rangle = a\left(e^{-\frac{ipd}{\hbar}}\,|0\rangle\right) = d.\left(e^{-\frac{ipd}{\hbar}}\,|0\rangle\right) \Rightarrow$$

$|\varphi\rangle = e^{-\frac{i\hat{p}d}{\hbar}}\,|0\rangle$ es autoestado del operador de destrucción a con autovalor d

$|\varphi\rangle$ describe un estado de posición ; pues $[d]$ = longitud

17.

$$a|\lambda\rangle = \lambda|\lambda\rangle \longleftarrow$$ Estados coherentes

a) $$|\lambda\rangle = e^{-|\lambda|^2/2} \cdot e^{\lambda a^+}|0\rangle \rightsquigarrow$$ quiero ver que es coherente y está normalizado

$$a|\lambda\rangle = e^{-|\lambda|^2/2} \; a \cdot e^{\lambda a^+}|0\rangle$$

$$[a, e^{\lambda a^+}] = [a, \sum_{n=0}^{\infty} \frac{1}{n!} \lambda^n (a^+)^n] = \sum \frac{1}{n!} \lambda^n [a, a^{+n}] = a \cdot e^{\lambda a^+} - e^{\lambda a^+} a$$

$$[a, a^{+n}] = a a^{+n} - \underbrace{a^{+n} a}_{=0} = a^{+n-1}[a \cdot a^+] + [a, a^{+n-1}] a^+ \nearrow$$ Esto no es útil $a^+a - a a^+ = -1$

será Sobre el $|0\rangle$

$$|n\rangle = \frac{(a^+)^n}{\sqrt{n!}}|0\rangle$$

$$a^{+n-1}(a a^+) + \left(a^{+n-2}[a, a^+] + [a, a^{+n-2}] a^+\right) a^+$$

$$a^{+n-1}(a a^+) + \left(a^{+n-2}(a a^+) + [a, a^{+n-2}] a^+\right) a^+ \Rightarrow$$

$$a^{+n-1}(a a^+) + \left(a^{+n-2}(a a^+) + \{a^{+n-3}[a, a^+] + [a, a^{+n-3}] a^+\} a^+\right) a^+$$

$$a^{+n-1}(a a^+) + a^{+n-1}(a a^+) + a^{+n-1}(a a^+) + (\ldots\ )$$

$$\sum_{n=1}^{\infty} n \cdot a^{+n-1}(a a^+) \Rightarrow \sum$$

$$a|\lambda\rangle = e^{-\frac{|\lambda|^2}{2}} \cdot a\, e^{\lambda a^+}|0\rangle = e^{-\frac{|\lambda|^2}{2}} \sum_{n=1}^{\infty} \frac{1}{n!} \lambda^n n \cdot a^{+n-1} \underbrace{a a^+ |0\rangle}_{|0\rangle}$$

$$\searrow \sum_{n=1}^{\infty} \frac{\lambda^n \cancel{n} a^{+n-1}}{\cancel{n}(n-1)!}$$

$n-1=N$

$$\searrow \sum_{N=0}^{\infty} \frac{\lambda^{N+1}}{N!} a^{+N} = \lambda \sum_{N=0}^{\infty} \frac{(\lambda a^+)^N}{N!} \swarrow$$

$$a|\lambda\rangle = e^{-\frac{|\lambda|^2}{2}} \cdot \lambda \cdot e^{\lambda a^+}|0\rangle$$

$$= \lambda\left(e^{-\frac{|\lambda|^2}{2}} \cdot e^{\lambda a^+}|0\rangle\right)$$

$$\Rightarrow \boxed{a|\lambda\rangle = \lambda|\lambda\rangle}$$

Veamos la normalización ahora;

$$\langle\lambda|\lambda\rangle = \langle 0| e^{-\frac{|\lambda|^2}{2}} e^{\lambda^* a} e^{-\frac{|\lambda|^2}{2}} e^{\lambda a^+}|0\rangle$$

$$\langle 0| e^{-|\lambda|^2} e^{\lambda^* a + \lambda a^+}|0\rangle = e^{-|\lambda|^2}\langle 0|e^{\lambda^* a + \lambda a^+}|0\rangle$$

$$= e^{-|\lambda|^2}\langle 0|\sum_n \frac{1}{n!}(\lambda^*)^n (a)^n \sum_m \frac{1}{m!}(\lambda)^m (a^+)^m |0\rangle$$

$$e^{-|\lambda|^2} \sum_n \sum_m \langle 0| \frac{1}{n!}(\lambda^*)^n (a)^n \frac{1}{m!}(\lambda)^m \sqrt{m!}\,|m\rangle$$

$$\langle n|\sqrt{n!}\,(\lambda^*)^n \frac{1}{n!} \frac{1}{m!}(\lambda)^m \sqrt{m!}\,|m\rangle$$

$$a^+|0\rangle \searrow \langle 0| a$$

$$\langle\lambda|\lambda\rangle = e^{-|\lambda|^2} \sum_n \sum_m \frac{(\lambda^*)^n (\lambda)^m}{\sqrt{n!m!}} \underbrace{\langle n|m\rangle}_{=\delta_{nm}} = e^{-|\lambda|^2} \sum_n^{\infty} \frac{(|\lambda|^2)^n}{n!} = e^{-|\lambda|^2} \cdot e^{|\lambda|^2} \Rightarrow$$

$$\boxed{\langle\lambda|\lambda\rangle = 1}$$

18.

$$|\varphi\rangle = e^{-i\frac{\hat{p}\cdot d}{\hbar}}|0\rangle$$

$$\langle x'|\varphi\rangle = \langle x'|e^{-i\frac{\hat{p}\cdot d}{\hbar}}|0\rangle$$

* Este es el método de fuerza bruta

$$= \int dp''dp'\langle x'|p'\rangle\langle p'|e^{-i\frac{\hat{p}\cdot d}{\hbar}}|p''\rangle\langle p''|0\rangle$$

$$= \int dp'dp'' e^{-i\frac{p'd}{\hbar}}\langle x'|p'\rangle\underbrace{\langle p'|p''\rangle}_{\delta(p'-p'')}\langle p''|0\rangle$$

$\delta(p'-p'') \leftarrow$ delta de Dirac

$$\langle x'|\varphi\rangle = \int dp'\cdot e^{-i\frac{p'd}{\hbar}}\langle x'|p'\rangle\langle p'|0\rangle$$

$$= \int dx''\cdot dp'\cdot e^{-i\frac{p'd}{\hbar}}\langle x'|p'\rangle\langle p'|x''\rangle\langle x''|0\rangle$$

$p = i\frac{m\omega\hbar}{2}(a^{+}-a)$

$$= \iint dx''\cdot dp'\cdot e^{-i\frac{p'd}{\hbar}}\frac{e^{i\frac{x'p'}{\hbar}}}{\sqrt{2\pi\hbar}}\cdot\frac{e^{-i\frac{x''p'}{\hbar}}}{\sqrt{2\pi\hbar}}\cdot\frac{1}{(\pi)^{1/4}x_0^{1/2}}\cdot e^{-\frac{1}{2}\left(\frac{x''}{x_0}\right)^2}$$

$$\langle x'|\varphi\rangle = \frac{1}{2\pi\hbar\cdot\pi^{1/4}\cdot x_0^{1/2}}\int dp'\cdot e^{-i\frac{p'd}{\hbar}}\cdot e^{\frac{ix'p'}{\hbar}}\underbrace{\int dx''\cdot e^{-\left[\frac{ix''p'}{\hbar}+\left(\frac{x''}{\sqrt{2}x_0}\right)^2\right]}}_{①}$$

$\nearrow \psi_{|\alpha\rangle}(x',\overset{t}{\underset{0}{\shortparallel}})$

Completar cuadrado

$$\frac{x''}{\sqrt{2}x_0} = A$$

$$\frac{i\cancel{x''}p'}{\hbar} = 2AB = \frac{2\cancel{x''}}{\sqrt{2}x_0}B$$

$$B = \frac{\sqrt{2}x_0 ip'}{2\hbar}$$

$$B = \frac{x_0 ip'}{\sqrt{2}\hbar}$$

$$\left(\frac{x''}{\sqrt{2}x_0}\right)^2 + \frac{ix''p'}{\hbar} + \frac{i^2x_0^2p'^2}{2\hbar^2} - \frac{i^2x_0^2p'^2}{2\hbar^2}$$

$$\frac{\sqrt{2}\cdot\cancel{\sqrt{\pi}}\cdot x_0}{2\underset{\sqrt{\pi}}{\cancel{\pi}}\hbar\ \pi^{1/4}x_0^{1/2}}\int dp'\cdot e^{-i\frac{p'd}{\hbar}+\frac{ix'p'}{\hbar}+\frac{x_0^2p'^2}{2\hbar^2}}$$

$$\langle x'|\varphi\rangle = \frac{x_0^{1/2}}{\sqrt{2}\hbar\ \pi^{1/4}\ \pi^{1/2}}\int dp'\ e^{\frac{ip'}{\hbar}(x'-d)}\cdot e^{\frac{x_0^2\cdot p'^2}{2\hbar^2}}$$

$$= \frac{\sqrt{x_0}}{\sqrt{2}\hbar\ \pi^{1/4}\pi^{1/2}}\int dp'\ e^{\left(\frac{x_0p'}{\sqrt{2}\hbar}+\frac{i(x'-d)}{2x_0}\right)^2}\cdot e^{\frac{(x'-d)^2}{2x_0^2}}$$

$$\frac{\cancel{\sqrt{x_0}}\cancel{\sqrt{2}}\cancel{\hbar}}{\underset{\sqrt{x_0}}{\cancel{x_0}}\cancel{\sqrt{2}}\sqrt{\pi}\ \pi^{1/4}\cancel{\hbar}}\int dz\ e^{-z^2}\cdot e^{-\frac{(x'-d)^2}{2x_0^2}}$$

$$\langle x'|\varphi\rangle = \frac{1}{x_0^{1/2}\cancel{\sqrt{\pi}}\ \pi^{1/4}}\cancel{\sqrt{\pi}}\ e^{-\frac{(x'-d)^2}{2x_0^2}}$$

$$\boxed{\langle x'|\varphi\rangle = x_0^{-1/2}\cdot\pi^{-1/4}\cdot e^{-\frac{[x'-d]^2}{2x_0^2}}}$$

completar cuadrados

$$\frac{x_0p'}{\sqrt{2}\hbar} = A$$

$$\frac{i\cancel{p'}(x'-d)}{\cancel{\hbar}} = 2AB = \frac{2x_0\cancel{p'}}{\sqrt{2}\cancel{\hbar}}B$$

$$\frac{\sqrt{2}i(x'-d)}{2x_0} = B$$

$dz' = dp'\frac{x_0}{\sqrt{2}\hbar}$

$$\frac{i(x'-d)}{\sqrt{2}x_0} = B$$

$$\frac{-(x'-d)^2}{2x_0^2} = B^2$$

$$-A\mp 2AB + B^2 - B^2$$

$$-(A+B)^2 + B^2$$

①

$$\int dx''\ e^{-\left[\frac{x''}{\sqrt{2}x_0}+\frac{ix_0p'}{\sqrt{2}\hbar}\right]^2}\cdot e^{\frac{x_0^2p'^2}{2\hbar^2}}$$

change of variable

$$\boxed{\begin{array}{l}\frac{x''}{\sqrt{2}x_0}+\frac{ix_0p'}{\sqrt{2}\hbar} = z\\ dx'' = dz\sqrt{2}x_0\end{array}}$$

$$\sqrt{2}x_0\int_{-\infty}^{+\infty}dz\cdot e^{-z^2}\cdot e^{\frac{x_0^2p'^2}{2\hbar^2}} =$$

$$\downarrow\ \sqrt{2}x_0\sqrt{\pi}\cdot e^{\frac{x_0^2p'^2}{2\hbar^2}}$$

* Hay un modo muy directo, que es como sigue:

$$\langle x'|\varphi\rangle = \left(\langle x'| e^{-\frac{i\hat{p}\cdot d}{\hbar}}\right)|0\rangle = \langle 0|\left(e^{\frac{i\hat{p}d}{\hbar}}|x'\rangle\right)$$

$$[x', e^{+\frac{i\hat{p}d}{\hbar}}] = \hat{x}' e^{\frac{i\hat{p}d}{\hbar}} - e^{\frac{i\hat{p}d}{\hbar}}\hat{x} = i\hbar\frac{\partial}{\partial p}\left(e^{\frac{i\hat{p}d}{\hbar}}\right) = i\hbar\, e^{\frac{i\hat{p}d}{\hbar}}\cdot\frac{id}{\hbar}$$

$$= -d\cdot e^{\frac{i\hat{p}d}{\hbar}}$$

$$\hat{x}\left(e^{\frac{i\hat{p}d}{\hbar}}|x'\rangle\right) = [x'-d]\left(\underbrace{e^{\frac{i\hat{p}d}{\hbar}}|x'\rangle}_{\equiv}\right)$$

$$\hat{x}|\psi\rangle = (x'-d)|\psi\rangle \Rightarrow$$

$|\psi\rangle \equiv |x'-d\rangle$

$$\langle x'|\varphi\rangle = \langle 0|e^{\frac{i\hat{p}d}{\hbar}}|x'\rangle = \langle 0|\psi\rangle = \langle 0|x'-d\rangle$$

$$\boxed{\langle x'|\varphi\rangle = \pi^{-1/4}\cdot x_0^{-1/2}\cdot e^{-\frac{1}{2}\left(\frac{x'-d}{x_0}\right)^2}}$$

$|\varphi\rangle \equiv |x'-d\rangle$ autoket de $\hat{x}'$

Probabilidad $\rightsquigarrow |\langle 0|\varphi\rangle|^2 \Rightarrow$

$$\langle 0|\varphi\rangle = \langle 0|e^{\frac{i\hat{p}d}{\hbar}}|0\rangle$$

$$= \int dx'\,\langle 0|x'\rangle\langle x'|e^{\frac{i\hat{p}d}{\hbar}}|0\rangle$$

$$= \int dx'\left(\pi^{-1/4}x_0^{-1/2}e^{-\frac{1}{2}\left(\frac{x'}{x_0}\right)^2}\right)\langle 0|\left(e^{-\frac{i\hat{p}d}{\hbar}}|x'\rangle\right)$$

$$= \qquad\qquad \langle 0|x'-d\rangle$$

$$\langle 0|\varphi\rangle = \int dx'\,\pi^{-1/4}x_0^{-1/2}e^{-\frac{1}{2}\left(\frac{x'}{x_0}\right)^2}\pi^{-1/4}x_0^{-1/2}e^{-\frac{1}{2}\frac{(x'-d)^2}{x_0^2}}$$

$$\langle 0|\varphi\rangle = \int \pi^{-1/2}\cdot x_0^{-1}\cdot e^{-\frac{1}{2}\cdot\frac{1}{x_0^2}\cdot[x'^2+(x'-d)^2]}\cdot dx'$$

$$\langle 0|\varphi\rangle = \pi^{-1/2}x_0^{-1}\int e^{-\frac{1}{2x_0^2}\left[\sqrt{2}x'-\frac{d}{\sqrt{2}}\right]^2}e^{-\frac{d^2}{2x_0^2 2}}\cdot dx'$$

$$\pi^{-1/2}x_0^{-1}\cdot e^{-\frac{d^2}{4x_0^2}}\underbrace{\int e^{-\frac{z^2}{2x_0^2}}\frac{dz'}{\sqrt{2}}}$$

cambio de variables
$\sqrt{2}x'-\frac{d}{\sqrt{2}} = z'$
$\sqrt{2}dx' = dz'$

completar cuadrados

$x'^2 + x'^2 - 2x'd + d^2$

$\underline{2x'^2 - 2x'd + d^2}$

$\sqrt{2}x' = A$

$-2x'd = -2AB$

$-\cancel{2}x'd = \cancel{2}\sqrt{2}\cancel{x'}B$

$+\frac{d}{\sqrt{2}} = B$

$\frac{d^2}{2} = B^2$

$[\quad] = \left(\sqrt{2}x'-\frac{d}{\sqrt{2}}\right)^2 + d^2 - \underbrace{\frac{d^2}{2}}_{+\frac{d^2}{2}}$

$$= \pi^{-1/2}\cdot x_0^{-1}\cdot e^{-\frac{d^2}{4x_0^2}}\frac{\sqrt{\pi}}{\sqrt{2}}\cdot\frac{1}{\sqrt{\frac{1}{2x_0^2}}}$$

$$= \frac{1}{\cancel{\sqrt{\pi}}\cdot\cancel{x_0}}\, e^{-\frac{d^2}{4x_0^2}}\,\cancel{\sqrt{\pi}}\,\cancel{x_0}$$

$$\langle 0|\varphi\rangle = e^{-d^2/4x_0^2} \Rightarrow \boxed{|\langle 0|\varphi\rangle|^2 = e^{-\frac{d^2}{2x_0^2}}}$$

Veamos que pasa en el tiempo

$$\langle 0|\mathcal{U}(t)|\varphi\rangle = \left(\langle 0|e^{-\frac{i\hat{H}t}{\hbar}}\right)e^{-\frac{i\hat{p}\cdot d}{\hbar}}|0\rangle =$$

$$= \langle 0|e^{\frac{i\hat{p}d}{\hbar}}\left(e^{\frac{i\hat{H}t}{\hbar}}|0\rangle\right) = \langle 0|e^{\frac{i\hat{p}d}{\hbar}}\, e^{i\omega t(1/2)}|0\rangle$$

$$= \langle 0|e^{\frac{i\hat{p}d}{\hbar}}|0\rangle\, e^{\frac{i\omega t}{2}} \rightarrow |\langle 0|\varphi,t\rangle|^2 = |\langle 0|\varphi\rangle|^2 = e^{-\frac{d^2}{2x_0^2}}$$

$\boxed{\text{No cambia para } t>0}$

c) $T(\ell) = e^{-i\hat{p}\frac{\ell}{\hbar}}$; $|0\rangle$ fundamental $\Rightarrow$

$|\psi\rangle = e^{-i\hat{p}\frac{\ell}{\hbar}}|0\rangle$ ← Este debería ser un estado coherente

$$a|\psi\rangle = a.e^{-i\hat{p}\ell/\hbar}|0\rangle$$

$$[a, e^{-i\hat{p}\ell/\hbar}] = \left[\sqrt{\frac{m\omega}{2\hbar}}\left(x + \frac{ip}{m\omega}\right), e^{-i\hat{p}\ell/\hbar}\right] = \sqrt{\frac{m\omega}{2\hbar}}[x, f(p)]$$

$$= \sqrt{\frac{m\omega}{2\hbar}} . i\hbar\frac{\partial}{\partial p}\left(e^{-i\hat{p}\ell/\hbar}\right) = \cancel{i\hbar}\sqrt{\frac{m\omega}{2\hbar}}e^{-i\hat{p}\ell/\hbar}\left(-\frac{\cancel{i}\ell}{\cancel{\hbar}}\right)$$

$$= \sqrt{\frac{m\omega}{2\hbar}}e^{-ip\ell/\hbar}.\ell$$

$$\Rightarrow$$

$$a|\psi\rangle = \left(\sqrt{\frac{m\omega}{2\hbar}}\ell.e^{-ip\ell/\hbar} - e^{-ip\ell/\hbar}.a\right)|0\rangle$$

$$a|\psi\rangle = \sqrt{\frac{m\omega}{2\hbar}}.\ell.e^{-i\hat{p}\ell/\hbar}|0\rangle = \underbrace{\left(\sqrt{\frac{m\omega}{2\hbar}}.\ell\right)}_{\equiv\lambda}|\psi\rangle$$

$\therefore$ $\boxed{\text{Sí; se obtiene un estado coherente}}$

b) Que verifiquen incerteza mínima significa que :

$$\langle\lambda|(\Delta x)^2|\lambda\rangle\langle\lambda|(\Delta p)^2|\lambda\rangle \geq \frac{\hbar^2}{4}$$

$\langle(\Delta x)^2\rangle = \langle x^2\rangle - \langle x\rangle^2$

$\Delta x = x^2 - 2\langle x\rangle x + \langle x\rangle^2 \mathbb{1}$

Sea que $\langle 0|(\Delta x)^2|0\rangle = x_0^2 \rightarrow \langle 0|e^{ip\ell/\hbar}(\Delta x)^2 e^{-ip\ell/\hbar}|0\rangle$

$$\langle(\Delta x)^2\rangle_\psi = \iint dp'\,dp''\,\langle 0|p''\rangle\langle p''|e^{i\hat{p}\ell/\hbar}(\Delta x)^2 e^{-i\hat{p}\ell/\hbar}|p'\rangle\langle p'|0\rangle$$

$$= \iint \partial p'\,\partial p''\,\langle 0|p''\rangle\langle p''|(\Delta x)^2|p'\rangle\langle p'|0\rangle\, e^{\frac{i\ell}{\hbar}(p''-p')}$$

$$\iint$$

19. $a|\alpha\rangle = \alpha|\alpha\rangle$ $\qquad \alpha \in \mathbb{C}$

a) $$\langle H\rangle_{|\alpha\rangle} = \langle\alpha|H|\alpha\rangle = \langle\alpha|\left(\hat N + \frac{1}{2}\right)\hbar\omega|\alpha\rangle$$

$$= \langle\alpha|\left(a^+a + \frac{1}{2}\right)\hbar\omega|\alpha\rangle = \hbar\omega.\left[\langle\alpha|a^+a|\alpha\rangle + \frac{1}{2}\langle\alpha|\alpha\rangle\right]$$

$$= \hbar\omega\left[\alpha\langle\alpha|a^+|\alpha\rangle + \frac{1}{2}\right]$$

*<u>DC</u>

$a|\alpha\rangle = \alpha|\alpha\rangle \longrightarrow \langle\alpha|a^+ = \langle\alpha|\alpha^* \nearrow$

$$= \hbar\omega\left[|\alpha|^2\langle\alpha|\alpha\rangle + \frac{1}{2}\right]$$

$$\boxed{\langle H\rangle_{|\alpha\rangle} = \hbar\omega\left(|\alpha|^2 + \frac{1}{2}\right)}$$

$$\langle p\rangle_{|\alpha\rangle} = \langle\alpha|p|\alpha\rangle = i\sqrt{\frac{m\omega\hbar}{2}}\langle\alpha|a^+ - a|\alpha\rangle$$

$$= \text{''}\left(\langle\alpha|a^+|\alpha\rangle - \langle\alpha|a|\alpha\rangle\right)$$

$$\langle p\rangle_{|\alpha\rangle} = i\left(\frac{m\omega\hbar}{2}\right)^{1/2}\left(\alpha^* - \alpha\right) = i\left(\frac{m\omega\hbar}{2}\right)^{1/2}(-2i)\Im m(\alpha)$$

$$= \boxed{\sqrt{\frac{m\omega\hbar}{2}}\, 2\Im m(\alpha)}$$

$$\langle x\rangle_{|\alpha\rangle} = \langle\alpha|x|\alpha\rangle = \sqrt{\frac{\hbar}{2m\omega}}\langle\alpha|a + a^+|\alpha\rangle$$

$$= \text{''}\left(\langle\alpha|a|\alpha\rangle + \langle\alpha|a^+|\alpha\rangle\right)$$

$$\langle x\rangle_{|\alpha\rangle} = \sqrt{\frac{\hbar}{2m\omega}}\left(\alpha + \alpha^*\right) = \boxed{\sqrt{\frac{\hbar}{2m\omega}}\, 2\Re e(\alpha)}$$

$$E = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2}$$

$$E = \frac{m\omega\hbar}{\cancel{2}}\cancel{2}^2\Im m^2\{\alpha\}\frac{1}{\cancel{2}\cancel{m}} + \frac{\cancel{m}\omega^{\cancel{2}}}{\cancel{2}}\frac{\hbar}{\cancel{2m\omega}}\cancel{2}^2\Re e^2\{\alpha\}$$

$$E = \omega\hbar\left[\Im m^2\{\alpha\} + \Re e^2\{\alpha\}\right]$$

si $E \gg \hbar\omega \rightarrow 1 \gg \frac{\hbar\omega}{E}$ $\qquad E = \hbar\omega|\alpha|^2$

$$E = \hbar\omega\left(n + \frac{1}{2}\right)$$

$$1 = \frac{\hbar\omega}{E}n + \underbrace{\frac{\hbar\omega}{E}.\frac{1}{2}}_{\sim 0}$$

$E \approx \hbar\omega.n \longrightarrow$ $\boxed{|\alpha|^2 = n}$, con $n \in \mathbb{N}$

$$\Re e^2\{\alpha\} + \Im m^2\{\alpha\} = n$$

$\boxed{\text{la norma al cuadrado del complejo } \overset{\alpha}{} \text{ debe ser } \mathbb{N}}$

b) $|\alpha\rangle$ $\qquad \{|n\rangle\}$ base de autoestados de H

$$\hat H|n\rangle = \hbar\omega\left(\hat N + \frac{1}{2}\right) = \left[\hbar\omega\left(n + \frac{1}{2}\right)\right]|n\rangle$$

↓ autovalor $\qquad n \in \mathbb{N}$

$$a|\alpha\rangle = \alpha|\alpha\rangle$$

$$|\alpha\rangle = \sum_{n'}^{N}|n'\rangle\langle n'|\alpha\rangle = \sum_{n'}^{N}\underbrace{\left(\langle n'|\alpha\rangle\right)}_{\hookrightarrow \text{ coeficientes}}|n'\rangle$$

$$\langle n' | \alpha \rangle = C_{n'} \quad \rightarrow \quad \langle n' | \overset{\#}{\alpha} | \alpha \rangle = \alpha C_{n'}$$

$$\langle n' | a | \alpha \rangle = \langle n'+1 | \sqrt{n'+1} | \alpha \rangle$$

$$a^{+} | n' \rangle = \sqrt{n'+1} \, | n'+1 \rangle$$

$$\alpha C_{n'} = \sqrt{n'+1} \langle n'+1 | \alpha \rangle$$

$$\boxed{C_{n'} = \frac{\sqrt{n'+1}}{\alpha} \langle n'+1 | \alpha \rangle} \qquad (\langle n' | a)$$

$$\mathcal{U}_{(t,0)} | \alpha \rangle = e^{-iHt/\hbar} \sum_{n'=0}^{\infty} \frac{\sqrt{n'+1}}{\alpha} \langle n'+1 | \alpha \rangle | n' \rangle$$

$$| \alpha, t \rangle = \sum_{n'=0}^{\infty} \sqrt{\frac{n'+1}{\alpha}} \langle n'+1 | \alpha \rangle \left(\sum_{k}^{\infty} \frac{1}{k!} \left(\frac{-it}{\hbar} \right)^{k} H^{k} \right) | n' \rangle \xrightarrow{} \text{el evolucionador temporal}$$

$$\sum_{k}^{\infty} \frac{1}{k!} \left(\frac{it}{\hbar} \right)^{k} \left(\hbar\omega \left[n' + \frac{1}{2} \right] \right)^{k} | n' \rangle$$

$$| \alpha, t \rangle = \sum_{n'=0}^{\infty} \frac{(n'+1)^{1/2}}{\alpha} \langle n'+1 | \alpha \rangle e^{-it\omega(n'+1/2)} | n' \rangle$$

Habría que ver que sigue siendo autoestado de a ; pero que su autovalor varía en el tiempo. ⇒

$$a | \alpha, t \rangle = a \left(\sum_{n'=0}^{\infty} \frac{(n'+1)^{1/2}}{\alpha} \langle n'+1 | \alpha \rangle e^{-it\omega(n'+1/2)} | n' \rangle \right) \qquad n' = N' - 1$$

$$a | \alpha, t \rangle = a \left(\sum_{N'=1}^{\infty} \frac{(N')^{1/2}}{\alpha} \langle N' | \alpha \rangle e^{-it\omega(N'-1/2)} | N'-1 \rangle \right)$$

$$= \sum_{n'=0}^{\infty} \frac{(n'+1)^{1/2}}{\alpha} \langle n'+1 | \alpha \rangle . e^{-i\omega t(n'+1/2)} (n')^{1/2} | n'-1 \rangle \qquad n' = N' + 1$$

pero con $N' = -1$ da cero ⇒

$$= \sum_{n'=1}^{\infty} \frac{(n'+1)^{1/2}}{\alpha} \langle n'+1 | \alpha \rangle e^{-i\omega t(n'+1/2)} (n')^{1/2} | n'-1 \rangle$$

$$\sum_{n''=0}^{\infty} \frac{(n''+2)^{1/2}}{\alpha} \langle n''+2 | \alpha \rangle e^{-i\omega t(n''+3/2)} (n''+1)^{1/2} | n'' \rangle \qquad n'-1 = n''$$