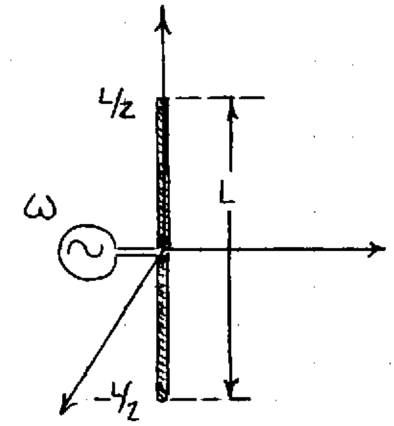
2.

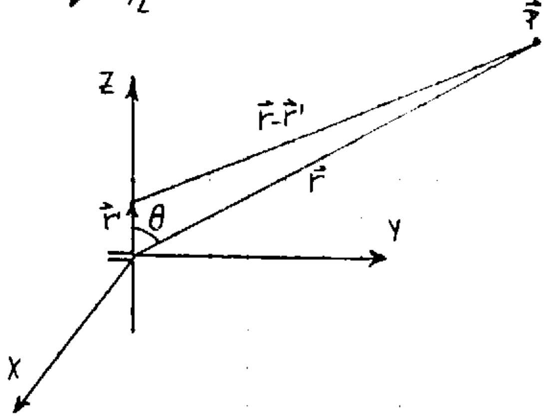


Antena con fuente armónica

$$\vec{A}(\vec{x},t) = \underbrace{\vec{A}(\vec{x})}_{} \cdot e^{-i\omega t}$$

↘ la parte espacial satisface

$$\vec{A}(\vec{x}) = \frac{1}{c} \int_{V'} \vec{J}(\vec{x}') \cdot \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \cdot dV'$$



Considero corriente del tipo

$$I = I_0(z) \cdot e^{-i\omega t'}$$

con nodos en los extremos

$$I = I_0 \operatorname{sen}\left(\frac{kL}{2} - k|z'|\right) \cdot e^{-i\omega t + i\omega \frac{R}{c}}$$

$$\vec{J}(x',y',z') = I_0 \operatorname{sen}\left(\frac{kL}{2} - k|z'|\right) \cdot \delta(x') \cdot \delta(y') \; \hat{z}'$$

$$\vec{A}(\vec{x}) = \frac{1}{c} \iiint_{-L/2}^{+L/2} I_0 \operatorname{sen}\left(kL/2 - k|z'|\right) \cdot \delta(x') \cdot \delta(y') \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \cdot dx' \, dy' \cdot dz' \; \hat{z}'$$

* Aproximamos campo lejano $r' \ll r \rightarrow$

$$|\vec{r}-\vec{r}'| = \sqrt{r^2 + r'^2 - 2rr'\cos\theta} = r\sqrt{1 + \underbrace{\frac{r'^2}{r^2}}_{\to 0} - \frac{2r'}{r}\cos\theta} \cong r \cdot \left(1 + \frac{1}{\cancel{2}}\left(-\frac{\cancel{2}r'}{r}\cos\theta\right)\right)$$

Taylor mediante

$|\vec{r}-r'\hat{z}|$

$$\cong r - r' \cdot \cos\theta$$
$$r - z' \cdot \cos\theta$$

$$\frac{1}{|\vec{r}-\vec{r}'|} \cdot e^{ik|\vec{r}-\vec{r}'|} = \frac{1}{r} \cdot e^{ikr} \cdot e^{-ik\underbrace{r'}_{z'}\cos\theta} \Rightarrow$$

$$\vec{A}(\vec{x}) = \frac{1}{c} \cdot I_0 \cdot \frac{1}{r} \cdot e^{ikr} \int_{-L/2}^{+L/2} \operatorname{sen}\left[k\left(\frac{L}{2} - |z'|\right)\right] \cdot e^{ikz'\cos\theta} \cdot dz' \cdot \hat{z}$$

$$= \frac{I_0 e^{ikr}}{cr} \int_{-L/2}^{+L/2} \operatorname{sen}\left[k\left(\frac{L}{2} - |z'|\right)\right] \cdot \cos(kz'\cos\theta) \cdot dz' \cdot \hat{z}$$

$$= \frac{I_0 e^{ikr}}{c \cdot r} \cdot 2 \int_0^{L/2} \operatorname{sen}\left[k\frac{L}{2} - z'\right] \cos(kz'\cos\theta) \cdot dz' \cdot \hat{z}$$

$$\vec{A}(\vec{x}) = \frac{I_0 e^{ikr} 2}{c \cdot r} \left(\frac{\cos\left(\frac{kL\cos\theta}{2}\right) - \cos\left(\frac{kL}{2}\right)}{k \; \operatorname{sen}^2\theta}\right) \cdot \hat{z}$$

$$\vec{A}(\vec{x},t) = \frac{I_0\, e^{i(kr-\omega t)}}{c.r}\,.\,2\,\underbrace{\left[\frac{\cos\left(kL/2\,.\cos\theta\right) - \cos\left(kL/2\right)}{k.\,sen^2\theta}\right]}_{\equiv\, \Psi(kL,\theta)}\hat{z}$$

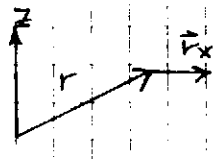
le pegué el $e^{-i\omega t}$ para no olvidar el tiempo

$$\vec{B}_{rad} = -\frac{1}{c}\,\hat{r}\times\frac{I_0}{c.r}\,\Psi\left(\frac{kL}{2},\theta\right).e^{ikr}.e^{-i\omega t}\,(-i\omega)\,\hat{z}$$

$$\vec{B}_{rad} = -\frac{1}{c}\,\frac{I_0}{c.r}\,\Psi.e^{i(kr-\omega t)}\,i.\omega.\,sen\,\theta\,\hat{\varphi}$$

$$\vec{E}_{rad} = \vec{B}_{rad}\times\hat{n} = -\frac{i\,I_0\,\omega}{c^2 r}\,\Psi\, e^{i(kr-\omega t)}\,sen.\theta.\,\hat{\theta}$$

* CÁLCULO AUX



$\vec{r}\times\hat{z} = \hat{\varphi}$

$\hat{r}\,\hat{\theta}\,\hat{\varphi}$

$\hat{r}\times\hat{z} =$
$\hat{r}\times(\cos\theta\,\hat{r}\,,-sen\theta\,\hat{\theta}) =$
$-sen\theta\,\hat{\varphi}$

$$\langle\vec{S}\rangle = \frac{c}{4\pi}\,Re\left\{\frac{1}{2}\left(\vec{E}\times\vec{B}^*\right)\right\}$$

$$= \frac{c}{8\pi}\,Re\left\{\frac{I_0\omega}{c^2 r}\,\Psi.\,e^{i(kr-\omega t+\pi/2)}\,sen\,\theta\,\hat{\theta}\times\frac{I_0\omega}{c^2 r}\,\Psi\, e^{-i(kr-\omega t+\pi/2)}\,sen\,\theta\,\hat{\varphi}\right\}$$

$$\langle\vec{S}\rangle = \frac{\cancel{c}}{8\pi}.\frac{I_0^2\,\omega^2\,\Psi^2}{c^{\cancel{4}3}\,r^2}\,sen^2\theta\;\hat{r} = \frac{I_0^2\,k^2\,\Psi^2\,sen^2\theta}{8\pi.\,c.\,r^2}\,\hat{r}$$

$$\langle dP\rangle = \langle\vec{S}\rangle.\,d\vec{S} = \langle\vec{S}\rangle.\hat{n}\,dS = \langle\vec{S}\rangle.\hat{n}\,r^2\,d\Omega$$

Potencia por ángulo sólido $\rightarrow$ $\boxed{\left\langle\frac{dP}{d\Omega}\right\rangle = \frac{I_0^2\,k^2\,\Psi^2.\cancel{r^2}.\,sen^2\theta}{8\pi.c.\cancel{r^2}}}$

$$\langle P\rangle = \int_0^{2\pi}\!\!\int_0^{\pi}\frac{I_0^2\,\cancel{k^2}}{8\pi.c}.\,\cancel{sen^2\theta}.\,2^2\,\frac{\left[\cos\left(kL/2\,\cos\theta\right) - \cos\left(kL/2\right)\right]^2}{\cancel{k^2}\,sen^{\cancel{4}}\theta}\,sen\theta\;d\theta\,d\varphi$$

$sen\,\theta$

$$\langle P\rangle = \frac{I_0^2\,\cancel{4}.\cancel{2}\pi}{\cancel{8\pi}.c}\int_0^{\pi}\frac{\left[\cos\left(kL/2.\cos\theta\right) - \cos\left(kL/2\right)\right]^2 d\theta}{sen\,\theta}$$

$$\langle P\rangle = \frac{I_0^2}{c}\int_0^{\pi}\left[\cos\left(\frac{kL}{2}.\cos\theta\right) - \cos\left(\frac{kL}{2}\right)\right]^2\frac{d\theta}{sen\theta}$$

Pero las $\lambda = m\frac{\lambda}{2}$

$kL = m\pi$

$\frac{2\cancel{\pi}}{\lambda}L = m\cancel{\pi} \rightarrow L = \frac{\lambda}{2}$

Para que la antena funcione

$kL = \pi$

antena de media onda

$$\Rightarrow \langle P\rangle = \frac{I_0^2}{c}\int_0^{\pi}\frac{\cos^2\left(m\frac{\pi}{2}.\cos\theta\right)}{sen\theta}.d\theta$$

para $m=1$ es:

$$\boxed{\langle P\rangle = \frac{I_0^2}{c}.\,1{,}852}$$

Potencia total irradiada

para $m=2$

$$\langle P\rangle = \frac{I_0^2}{C}$$

$$\left\langle \frac{dP}{d\Omega}\right\rangle = \frac{I_0^2 \cancel{k^2}}{\cancel{8}_2\pi c}\cdot\frac{\cancel{\operatorname{sen}^2\theta}}{\operatorname{sen}^2\theta}\cdot\cancel{4}\left[\frac{\cos\left(kL/2\cdot\cos\theta\right)-\cos\left(kL/2\right)}{\cancel{k}\cdot\cancel{\operatorname{sen}^2\theta}}\right]^2$$

$$\left\langle \frac{dP}{d\Omega}\right\rangle = \frac{I_0^2}{2\pi\cdot c}\cdot\frac{1}{\operatorname{sen}^2\theta}\left(\cos\left\{\frac{kL}{2}\cos\theta\right\}-\cos\left\{\frac{kL}{2}\right\}\right)^2$$

$$\text{con } L=\frac{\lambda}{2} \Rightarrow \quad \frac{kL}{2}=\frac{\cancel{2}\pi}{\cancel{\lambda}}\cdot\frac{\cancel{\lambda}}{\cancel{2}}\cdot\frac{1}{2} \Rightarrow$$

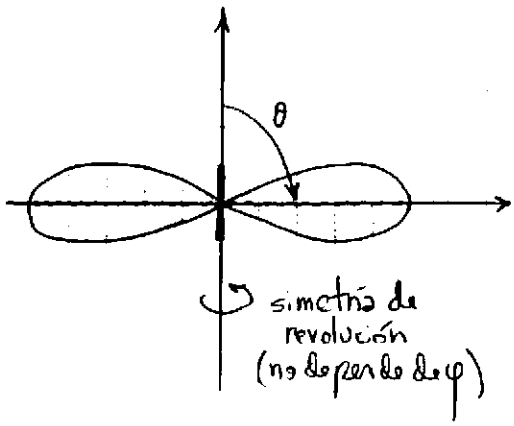
$$\left\langle \frac{dP}{d\Omega}\right\rangle = \frac{I_0^2}{2\pi c}\cdot\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\operatorname{sen}^2\theta} \qquad \text{con } m=1$$

$$\frac{d}{d\theta}\left(\frac{\cos^2\left(\pi/2\cdot\cos\theta\right)}{\operatorname{sen}^2\theta}\right)=0$$

$$-\cancel{2}\cdot\cancel{\cos\left(\frac{\pi}{2}\cos\theta\right)}\cdot\operatorname{sen}\left(\frac{\pi}{2}\cdot\cos\theta\right)\cdot\frac{\pi}{2}\cdot\operatorname{sen}^{\cancel{2}}\theta-\cancel{2}\,\cancel{\operatorname{sen}\theta}\cos\theta\cdot\cos^{\cancel{2}}\left(\pi/2\cdot\cos\theta\right)=0$$

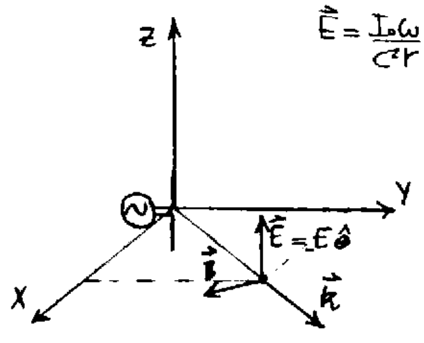
$$-\operatorname{sen}\left(\frac{\pi}{2}\cdot\cos\theta\right)\frac{\pi}{2}\operatorname{sen}^2\theta-\cos\theta\cdot\cos\left(\frac{\pi}{2}\cdot\cos\theta\right)=0$$

$\theta=0 \rightarrow$ verifica extremo (mínimo)
$\theta=\pi/2 \rightarrow$ verifica extremo (máximo)
$\theta=\pi/4 \rightarrow$ no es extremo
$\theta=\pi \rightarrow$ verifica extremo (mínimo)



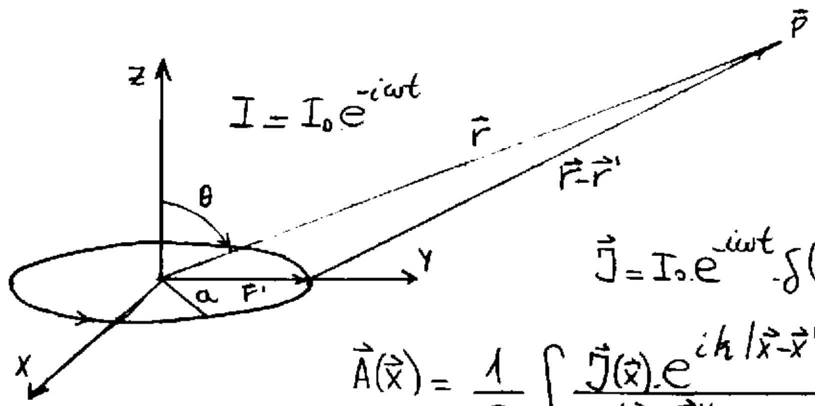
En $\theta=\pi/2$ es

$$\vec{E}=\frac{I_0\omega}{c^2 r}\cdot\frac{1}{k}\,2\,e^{i(kr-\omega t+\pi/2)}\cdot\hat{\theta}$$



La polarización en la dirección de máxima radiación ($\theta=\pi/2$) es lineal

3.



$$I=I_0 e^{-i\omega t}$$

$$\vec{J}=I_0 e^{-i\omega t}\,\delta(r-a)\,\delta\left(\cos\theta-\cos\frac{\pi}{2}\right)\hat{\varphi}$$

EL $\vec{A}(\vec{x},t)$ tendrá la misma ω que la fuente

$$\vec{A}(\vec{x})=\frac{1}{c}\int_{V'}\frac{\vec{J}(\vec{x})\cdot e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}\,dV'$$

(pongo sólo la parte espacial)

* Campo lejano $\dfrac{1}{|\vec{x}-\vec{x}'|}=\dfrac{1}{r}$; $ik|\vec{x}-\vec{x}'|\equiv ik|\vec{r}-\vec{r}'|=$ ($\theta'=\pi/2$)

$ik\cdot\sqrt{r^2+r'^2-2rr'(\operatorname{sen}\theta\cdot\cos(\varphi-\varphi')}\cdot$

$ik\cdot r\cdot\left(1-r'/r\left[\operatorname{sen}\theta\cdot\cos(\varphi-\varphi')\right]\right)$ con $r'=a$

son dos vectores esféricos $\vec{r}'$ se mueve

$$\vec{A}(\vec{x},t) = \frac{1}{c}\int_0^a\int_0^{2\pi}\int_0^{\pi} \frac{I_0 e^{-i\omega t}}{r'} \cdot \frac{\delta(r'-a) \cdot \delta(\cos\theta)}{r} \cdot e^{ik(r - a\,\text{sen}\theta\,\cos(\varphi-\varphi'))} \cdot \hat{\varphi} \cdot r'^2\,\text{sen}\theta' \, d\theta' \, d\varphi' \, dr'$$

$$\vec{A}(\vec{x},t) = \frac{I_0}{c\,r} e^{i(kr-\omega t)} \iiint \delta(r'-a) \cdot \delta(\cos\theta') \cdot e^{-ika\,\text{sen}\theta\,\cos(\varphi-\varphi')} \, r' \, \text{sen}\theta' \cdot d\theta' \, d\varphi' \, dr' \, \hat{\varphi}$$

$$= \frac{I_0 \cdot e^{i(kr-\omega t)} \cdot a}{c\,r} \iint \delta(\cos\theta') \, e^{-ika\,\text{sen}\theta\,\cos(\varphi-\varphi')} \, \text{sen}\theta' \, d\theta' \, d\varphi' \, \hat{\varphi}$$

$$\vec{A}(\vec{x},t) = \frac{a \cdot I_0 e^{i(kr-\omega t)}}{c\,r} \int_0^{2\pi} e^{-ika\,\text{sen}\theta\,\cos(\varphi-\varphi')} \, d\varphi' \, \hat{\varphi}'$$

$$\vec{A} = \frac{a \, I_0 e^{i(kr-\omega t)}}{c\,r} \int_0^{2\pi} e^{-ika\,\text{sen}\theta \cdot \cos(\varphi-\varphi')} \cdot (-\text{sen}\varphi' \hat{x} + \cos\varphi' \hat{y}) \cdot d\varphi'$$

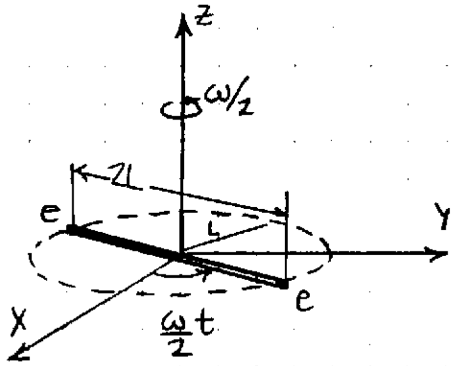
usando

$$\frac{\delta(r'-a)}{r'} \delta(\cos\theta)$$

evito

$$\frac{\delta(r'-a)\,\delta(\theta'-\pi/2)}{r'^2\,\text{sen}\theta}$$

4.



Acá no podemos usar en general, las fórmulas que presuponen dependencia temporal $e^{-i\omega t}$ aunque se puede llevar a una forma parecida

(a)

$$\vec{p} = e.\vec{r}(t) = e.\left(\frac{L}{2}\right)\left[\cos\left(\frac{\omega t}{2}\right)\hat{x} + \operatorname{sen}\left(\frac{\omega t}{2}\right)\hat{y}\right]$$

$$+ e\left(\frac{L}{2}\right)\left[\underbrace{\cos\left(\frac{\omega t}{2}+\pi\right)\hat{x}}_{-\cos\left(\frac{\omega t}{2}\right)\hat{x}} + \underbrace{\operatorname{sen}\left(\frac{\omega t}{2}+\pi\right)\hat{y}}_{-\operatorname{sen}\left(\frac{\omega t}{2}\right)\hat{y}}\right]$$

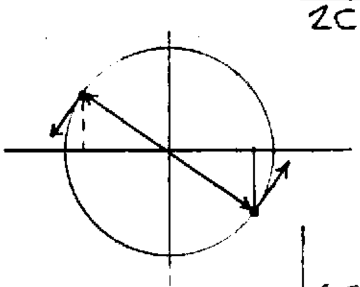
$$\boxed{\vec{p} = 0}$$ no hay desbalance de carga neta respecto del origen

(b) $$\vec{m} = \left[\frac{1}{2c}.L\hat{r}\right)\times e\left(\frac{\omega}{2}.L\right)\hat{\varphi}\right] + \left(\frac{1}{2c}L\hat{r}'\times e\left(\frac{\omega}{2}L\right)\hat{\varphi}'\right) \qquad \varphi'=\varphi+\pi$$

$$\vec{m} = \frac{1}{2c}\left[L.\cos\left(\frac{\omega t}{2}\right)\hat{x} + L\operatorname{sen}\left(\frac{\omega t}{2}\right)\hat{y}\right]\times e\left(\frac{\omega}{2}.L\right)\left[-\operatorname{sen}\left(\frac{\omega t}{2}\right)\hat{x} + \cos\left(\frac{\omega t}{2}\right)\hat{y}\right]$$

$$+ \frac{1}{2c}\left[-L\cos\left(\frac{\omega t}{2}\right)\hat{x} - L.\operatorname{sen}\left(\frac{\omega t}{2}\right)\hat{y}\right]\times e\left(\frac{\omega}{2}L\right)\left[\operatorname{sen}\left(\frac{\omega t}{2}\right)\hat{x} - \cos\left(\frac{\omega t}{2}\right)\hat{y}\right]$$

$$= \frac{1}{2c}e\frac{\omega}{2}L^2\Big([\cos\hat{x}, \operatorname{sen}\hat{y}]\times[-\operatorname{sen}\hat{x}, +\cos\hat{y}] + $$
$$-[\cos\hat{x}, \operatorname{sen}\hat{y}]\times[\operatorname{sen}\hat{x}, -\cos\hat{y}]\Big)$$



$$\boxed{\vec{m} = \frac{e\omega L^2}{4c}.2\hat{z}}$$ ← Como sospechábamos no depende del tiempo

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \cos\varphi & \operatorname{sen}\varphi & 0 \\ -\operatorname{sen}\varphi & \cos\varphi & 0 \end{vmatrix} = \cos^2\varphi\,\hat{z} + \operatorname{sen}^2\varphi\,\hat{z} = \hat{z}$$

(c) $$Q_{ij} = \sum_\ell q_\ell\left(3x_ix_j - \delta_{ij}r_\ell^2\right)$$

	x	y	z	r^2
e_1	$L\cos\left(\frac{\omega t}{2}\right)$	$L.\operatorname{sen}\left(\frac{\omega t}{2}\right)$	0	L^2
e_2	$-L\cos\left(\frac{\omega t}{2}\right)$	$-L\operatorname{sen}\left(\frac{\omega t}{2}\right)$	0	L^2

$$\overleftrightarrow{Q}_{(1)} = e\begin{pmatrix} 3L^2\cos^2 - L^2 & 3L^2\cos\operatorname{sen} & 0 \\ 3L^2\cos\operatorname{sen} & 3L^2\operatorname{sen}^2 - L^2 & 0 \\ 0 & 0 & -L^2 \end{pmatrix}$$

$$\overleftrightarrow{Q}_{(2)} = e\begin{pmatrix} 3L^2\cos^2 - L^2 & 3L^2\cos\operatorname{sen} & 0 \\ 3L^2\cos\operatorname{sen} & 3L^2\operatorname{sen}^2 - L^2 & 0 \\ 0 & 0 & -L^2 \end{pmatrix} \searrow \overleftrightarrow{Q} = 2eL^2\begin{pmatrix} 3\cos^2-1 & 3\cos.\operatorname{sen} & 0 \\ 3\cos.\operatorname{sen} & 3\operatorname{sen}^2-1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\boxed{\overleftrightarrow{Q} = 2eL^2 \begin{pmatrix} 3\cos^2\left(\frac{\omega t}{2}\right) - 1 & 3\cos\left(\frac{\omega t}{2}\right)\operatorname{sen}\left(\frac{\omega t}{2}\right) & 0 \\ 3\cos\left(\frac{\omega t}{2}\right)\operatorname{sen}\left(\frac{\omega t}{2}\right) & 3\operatorname{sen}^2\left(\frac{\omega t}{2}\right) - 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}}$$

(d)

$$\overleftrightarrow{Q}\cdot\hat{n} = 2eL^2 \begin{pmatrix} 3\cos^2-1 & 3\cos\operatorname{sen} & 0 \\ 3\cos\operatorname{sen} & 3\operatorname{sen}^2-1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} \operatorname{sen}\theta\cos\varphi \\ \operatorname{sen}\theta\operatorname{sen}\varphi \\ \cos\theta \end{pmatrix}$$

lo vectorizo

$$\vec{Q} = 2eL^2 \begin{pmatrix} (3\cos^2-1)\operatorname{sen}\theta\cos\varphi + 3\cos\operatorname{sen}\operatorname{sen}\theta\operatorname{sen}\varphi \\ 3\cos\operatorname{sen}\operatorname{sen}\theta\cos\varphi + (3\operatorname{sen}^2-1)\operatorname{sen}\theta\operatorname{sen}\varphi \\ -\cos\theta \end{pmatrix}$$

* Aproximo onda larga

$k.L \ll 1$

$2\pi L \ll \lambda$

$$\vec{Q} \sim 2eL^2 \begin{pmatrix} 2\operatorname{sen}\theta.\cos\varphi + 0 \\ 0 - \operatorname{sen}\theta.\operatorname{sen}\varphi \\ \cos\theta \end{pmatrix}$$

$\frac{\omega}{c}.L \ll 1 \to \omega \sim 0 \to \operatorname{sen}\left(\frac{\omega t}{2}\right) \sim \frac{\omega t}{2}$

$\cos\left(\frac{\omega t}{2}\right) \sim 1$

Conviene simplificar a C el tensor pues sería suicidio derivarlo dos veces así

$$\overleftrightarrow{Q} = 2eL^2 \begin{pmatrix} 3\cos^2 & 3\cos\operatorname{sen} & 0 \\ 3\cos\operatorname{sen} & 3\operatorname{sen}^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} + 2eL^2 \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\overleftrightarrow{Q} = 2eL^2 \begin{pmatrix} \frac{3}{2}(1+\cos\omega t) & \frac{3}{2}\operatorname{sen}(\omega t) & 0 \\ \frac{3}{2}\operatorname{sen}(\omega t) & \frac{3}{2}(1-\cos\omega t) & 0 \\ 0 & 0 & 0 \end{pmatrix} + \ldots$$

$\cos^2 A - \frac{1}{2} = \frac{\cos 2A}{2}$

$\cos^2 A = \frac{1+\cos 2A}{2}$

$$\overleftrightarrow{Q} = 3eL^2 \begin{pmatrix} \cos\omega t & \operatorname{sen}\omega t & 0 \\ \operatorname{sen}\omega t & -\cos\omega t & 0 \\ 0 & 0 & 0 \end{pmatrix} + 2eL^2 \begin{pmatrix} \frac{3}{2}-1 & 0 & 0 \\ 0 & \frac{3}{2}-1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\overleftrightarrow{Q} = \operatorname{Re}\left\{ 3eL^2 \begin{pmatrix} e^{i\omega t} & -i\,e^{+i\omega t} & 0 \\ -i\,e^{+i\omega t} & -e^{i\omega t} & 0 \\ 0 & 0 & 0 \end{pmatrix} + 2eL^2 \begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & -1 \end{pmatrix} \right\}$$

$$\overleftrightarrow{Q} = \operatorname{Re}\left\{ 3eL^2 . e^{i\omega t} \begin{pmatrix} 1 & -i & 0 \\ -i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + eL^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \right\}$$

$$\vec{A}_{\substack{\text{CUADRUPOLAR} \\ \text{ELÉCTRICO}}} = \frac{1}{6c^2 r} \dddot{\overleftrightarrow{Q}}.\hat{n}$$

$$\dot{\overleftrightarrow{Q}} = 3eL^2 . e^{i\omega t}\, i\omega \begin{pmatrix} 1 & -i & 0 \\ -i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad \ddot{\overleftrightarrow{Q}} = \underbrace{(i\omega)^2}_{"-\omega^2"} . 3eL^2 . e^{i\omega t} \begin{pmatrix} 1 & -i & 0 \\ -i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\vec{A}_{\substack{CUAD\\ELE}} = \frac{1}{\cancel{6}c^2 r}\cdot(-\omega^2)\cdot\cancel{3}eL^2 e^{i\omega t}\begin{pmatrix}1 & -i & 0\\ -i & -1 & 0\\ 0 & 0 & 0\end{pmatrix}\cdot\begin{pmatrix}\operatorname{sen}\theta\cdot\cos\varphi\\ \operatorname{sen}\theta\cdot\operatorname{sen}\varphi\\ \cos\theta\end{pmatrix}$$

$$= -\frac{\omega^2}{c^2}\cdot\frac{eL^2}{r}\cdot e^{i\omega t}\cdot\begin{pmatrix}\operatorname{sen}\theta\cdot\cos\varphi - i\cdot\operatorname{sen}\theta\cdot\operatorname{sen}\varphi & \hat{x}\\ -i\cdot\operatorname{sen}\theta\cdot\cos\varphi - \operatorname{sen}\theta\cdot\operatorname{sen}\varphi & \hat{y}\\ 0 & \end{pmatrix}$$

$$= +\frac{\omega^2}{c^2}\cdot\frac{eL^2}{r}\cdot e^{i\omega t}\cdot\operatorname{sen}\theta\begin{pmatrix}-\cos\varphi + i\cdot\operatorname{sen}\varphi & \hat{x}\\ +\operatorname{sen}\varphi + i\cdot\cos\varphi & \hat{y}\end{pmatrix}$$

$-i\operatorname{sen}\varphi + \cos\varphi$

$$= +\frac{\omega^2 eL^2}{c^2 r}e^{i\omega t}\cdot\operatorname{sen}\theta\left(-e^{-i\varphi}\hat{x} + i\cdot e^{-i\varphi}\hat{y}\right)$$

$$\vec{A}_{\substack{CUAD\\elec}} = \frac{+\omega^2 eL^2}{c^2 r}\cdot e^{i(\omega t-\varphi)}\operatorname{sen}\theta\cdot\left(-\hat{x}+i\hat{y}\right)$$

$$\vec{B}_{rad} = -\frac{1}{c}\hat{n}\times\dot{\vec{A}} = -\frac{1}{c}\hat{r}\times -\frac{i\omega^3 eL^2}{c^2 r}\cdot e^{i(\omega t'-\varphi)}\operatorname{sen}\theta(\hat{x}-i\hat{y})$$

$$\hat{n}\times(\hat{x}-i\hat{y}) = \begin{vmatrix}\hat{r} & \hat{\theta} & \hat{\varphi}\\ 1 & 0 & 0\\ \begin{pmatrix}\operatorname{sen}\theta\cdot\cos\varphi\\ -i\cdot\operatorname{sen}\theta\cdot\operatorname{sen}\varphi\end{pmatrix} & \begin{pmatrix}\cos\theta\cdot\cos\varphi\\ -i\cdot\cos\theta\cdot\operatorname{sen}\varphi\end{pmatrix} & \begin{pmatrix}-\operatorname{sen}\varphi\\ -i\cdot\cos\varphi\end{pmatrix}\end{vmatrix} = (\cos\theta\cdot\cos\varphi - i\cos\theta\cdot\operatorname{sen}\varphi)\,\hat{\varphi} + (\operatorname{sen}\varphi + i\cdot\cos\varphi)\,\hat{\theta}$$

$i\,e^{-i\varphi}$

$i\cdot\cos\varphi - i^2\operatorname{sen}\varphi$

$$= \left(e^{-i\varphi}\cdot\cos\theta\,\hat{\varphi} + i\cdot e^{-i\varphi}\cdot\hat{\theta}\right)$$

$$= e^{-i\varphi}\cdot[\cos\theta\,\hat{\varphi} + i\cdot\hat{\theta}]$$

$$\vec{B}_{rad} = +\frac{1}{c^3}\cdot\frac{\omega^3 eL^2}{r}\,e^{i(\omega t'-\varphi)}\cdot e^{-i\varphi}\cdot\operatorname{sen}\theta\,[\cos\theta\,\hat{\varphi} + i\cdot\hat{\theta}]$$

$$\vec{E}_{rad} = -\hat{n}\times\vec{B}_{rad} = -\begin{vmatrix}\hat{r} & \hat{\theta} & \hat{\varphi}\\ 1 & 0 & 0\\ 0 & i & \cos\theta\end{vmatrix}\,\frac{\omega^3 eL^2}{c^3 r}\cdot e^{i(\omega t'-2\varphi)}\cdot\operatorname{sen}\theta$$

$$\vec{E}_{rad} = \left(-i\hat{\varphi} + \cos\theta\cdot\hat{\theta}\right)\cdot\operatorname{sen}\theta\cdot\frac{\omega^3 eL^2}{c^3 r}\cdot e^{i(\omega t - \frac{r}{c}\omega - 2\varphi)}$$

$$\boxed{\left.\begin{matrix}\vec{B}_{rad}\\ \\ \vec{E}_{rad}\end{matrix}\right\} = \underbrace{\frac{\omega^3 eL^2}{c^3 r}\cdot\operatorname{sen}\theta}_{\equiv\Omega}\cdot e^{i\underbrace{(\omega t - kr - 2\varphi)}_{\equiv\Phi}}\cdot\left\{\begin{matrix}i\hat{\theta} + \cos\theta\,\hat{\varphi}\\ \\ \cos\theta\cdot\hat{\theta} - i\hat{\varphi}\end{matrix}\right\}}$$

$$\vec{E}\times\vec{B}^* = \Omega^2 e^{i\Phi}\begin{vmatrix}\hat{r} & \hat{\theta} & \hat{\varphi}\\ 0 & \cos\theta & -i\\ 0 & -i & \cos\theta\end{vmatrix} e^{-i\Phi}$$

$$\mathrm{Re}\left\{\frac{1}{2}\vec{E}\times\vec{B}^*\right\} = \frac{\Omega^2}{2}\left(\cos^2\theta\,\hat{r} - 1\,\hat{r}\right)$$

$$\langle \vec{S} \rangle = \frac{c}{4\pi}\,\frac{\Omega}{2}\,(\cos^2\theta - 1)\,\hat{r}$$

$$\langle \vec{S} \rangle . \hat{n} = \frac{c}{8\pi}\,.\,\Omega\,.\,\mathrm{sen}^2\theta$$

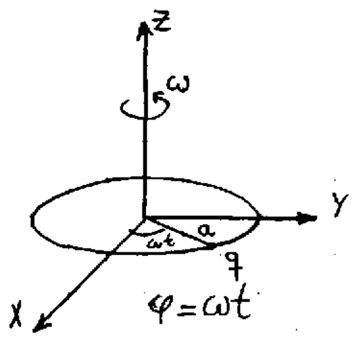
$$\left\langle \frac{dP}{d\Omega} \right\rangle = \langle \vec{S} \rangle . \hat{n}\; r^2 \quad \rightarrow \quad \left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi}\left(\frac{\omega^3}{c^3}\,\frac{e L^2}{\cancel{r}}\right)^2 \cancel{r^2}\,.\,\mathrm{sen}^2\theta$$

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{1}{8\pi}\,\frac{\omega^6}{c^5}\,e^2 L^4\,\mathrm{sen}^2\theta$$

$$\langle P \rangle = \int_0^\pi \frac{\omega^6 e^2 L^4}{8\pi c^5}\,.\,\mathrm{sen}^3\theta\,.\,d\theta$$

$$\langle P \rangle = \frac{\omega^6 e^2 L^4}{\underset{2}{\cancel{8}}\pi c^5}\,.\,\frac{\overset{1}{\cancel{4}}}{3} = \boxed{\frac{\omega^6 e^2 L^4}{6\pi c^5} = \langle P \rangle}$$

7.



$$\vec{p} = q.\vec{x}$$
$$\vec{m} = \frac{1}{2c}\left(\vec{x} \times q.\vec{v}\right)$$
$$Q_{ij} = q\left(3x_ix_j - \delta_{ij} r^2\right)$$

$$\vec{p} = q.\ a\left(\cos(\omega t)\hat{x} + \operatorname{sen}(\omega t)\hat{y}\right)$$
$$\vec{P} = q.a\left(e^{-i\omega t}.\hat{x} + i.e^{-i\omega t}\hat{y}\right) \Rightarrow \vec{P} = q.a.e^{-i\omega t}\left(\hat{x} + i.\hat{y}\right)$$

donde $\vec{p} = Re\{\vec{P}\}$

$$\vec{m} = \frac{1}{2c}q(a^2).\left[\cos\omega t\,\hat{x} + \operatorname{sen}\omega t\,\hat{y}\right] \times \omega.\left[-\operatorname{sen}(\omega t).\hat{x} + \cos(\omega t)\hat{y}\right]$$

$$\vec{m} = \frac{q.a^2\omega}{2c}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \cos\omega t & \operatorname{sen}\omega t & 0 \\ -\operatorname{sen}\omega t & \cos\omega t & 0 \end{vmatrix} = \frac{q.a^2\omega}{2c}\left[\cos^2(\omega t)\hat{z} + \operatorname{sen}^2(\omega t)\hat{z}\right]$$

$$\vec{x} = \left(a\cos(\omega t), a\operatorname{sen}(\omega t), 0\right)$$

$$\vec{m} = q\frac{a^2.\omega}{2c}\hat{z}$$

$$\Rightarrow \boxed{\dot{\vec{m}} = 0}$$

$$\overleftrightarrow{Q} = q\begin{pmatrix} 3\cos^2(\omega t)a^2 - a^2 & 3\cos.\operatorname{sen}.a^2 & 0 \\ 3a^2\cos.\operatorname{sen}. & 3\operatorname{sen}^2(\omega t)a^2 - a^2 & 0 \\ 0 & 0 & -a^2 \end{pmatrix}$$

$$\overleftrightarrow{Q} = q.a^2\begin{pmatrix} 3\cos^2 - 1 & 3\cos\operatorname{sen} & 0 \\ 3\cos\operatorname{sen} & 3\operatorname{sen}^2 - 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$\cos(\omega t).\operatorname{sen}(\omega t) = \frac{1}{2}\operatorname{sen}2\omega$

$\cos^2 A = \frac{1 + \cos(2A)}{2}$

$3\cos^2 - 1 = \frac{3}{2} + \frac{3}{2}\cos(2A$

$$\overleftrightarrow{Q} = q.a^2\begin{pmatrix} 1/2 + 3/2\cos(2\omega t) & 3/2\operatorname{sen}(2\omega t) & 0 \\ 3/2\operatorname{sen}(2\omega t) & 1/2 - 3/2\cos(2\omega t) & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\overleftrightarrow{Q} = 3/2\,qa^2\begin{pmatrix} \cos(2\omega t) & \operatorname{sen}(2\omega t) & 0 \\ \operatorname{sen}(2\omega t) & -\cos(2\omega t) & 0 \\ 0 & 0 & 0 \end{pmatrix} + qa^2\begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\overleftrightarrow{Q} = 3/2\,qa^2\begin{pmatrix} e^{-i2\omega t} & i e^{-i2\omega t} & 0 \\ i.e^{-i2\omega t} & -e^{-i2\omega t} & 0 \\ 0 & 0 & 0 \end{pmatrix} + \text{''}$$

$$\overleftrightarrow{Q} = \frac{3}{2}q.a^2.e^{-i2\omega t}.\begin{pmatrix} 1 & i & 0 \\ i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \text{''}$$

$$\vec{A}_{dip} = \frac{\dot{\vec{P}}}{c.r} \rightarrow \vec{A}_{dip} = \frac{1}{c.r}\, q.a\,(\hat{x}+i\hat{y}).e^{-i\omega t}.(-i\omega)$$

$$\vec{A}_{dip} = -\frac{i\omega}{cr}\, q a\, e^{-i\omega t}(\hat{x}+i\hat{y})$$

$$\dot{\vec{A}}_{dip} = -\frac{\omega^2}{cr}\, q a\, e^{-i\omega t}(\hat{x}+i\hat{y})$$

$$\vec{A}_{cuad} = \frac{1}{6c^2 r}\,\dddot{\overleftrightarrow{Q}}.\hat{n} \rightarrow \vec{A}_{cuad} = \frac{1}{\cancel{6}_2 c^2 r}\,\frac{\cancel{3}}{2}\, q a^2\, e^{-i2\omega t}(-i2\omega)^2.\begin{pmatrix}1 & i & 0\\ i & -1 & 0\\ 0 & 0 & 0\end{pmatrix}.\hat{r}$$

$$\vec{A}_{cuad} = -\frac{\omega^2}{c^2 r}.q.a^2.e^{-i2\omega t'}\begin{pmatrix}1 & i & 0\\ i & -1 & 0\\ 0 & 0 & 0\end{pmatrix}.\hat{r}$$

$$\vec{B}_{dip} = -\frac{1}{c}\,\hat{n}\times\dot{\vec{A}}(t')$$

$$\vec{B}_{dip} = -\frac{1}{c}\,\frac{1}{cr}\, q.a.e^{-i\omega t'}(-i\omega)^2.\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ sen\theta.\cos\varphi & sen\theta.sen\varphi & \cos\theta\\ 1 & i & 0\end{vmatrix}$$

conviene pasar a un sistema donde los versores sean tales que el producto $\times$ tenga un renglón con dos ceros

$$\vec{B}_{dip} = \frac{\omega^2}{c^2.r}\, q.a.e^{-i\omega t'}.\begin{vmatrix}\hat{r} & \hat{\theta} & \hat{\varphi}\\ 1 & 0 & 0\\ \begin{pmatrix}sen\theta.\cos\varphi\\ +i.sen\theta.sen\varphi\end{pmatrix} & \begin{pmatrix}\cos\theta.\cos\varphi\\ +i.\cos\theta.sen\varphi\end{pmatrix} & \begin{pmatrix}-sen\varphi\\ +i.\cos\varphi\end{pmatrix}\end{vmatrix}$$

$$\vec{B}_{dip} = \frac{\omega^2 q.a.e^{-i\omega(t-r/c)}}{r.c^2}\left[\cos\theta.(\cos\varphi+i.sen\varphi)\hat{\varphi} - (-sen\varphi+i.\cos\varphi)\hat{\theta}\right]$$

$-ie^{i\varphi}$ $+sen\varphi - i\cos\varphi$

$$\left[\cos\theta.e^{i\varphi}\hat{\varphi} + -i.e^{i\varphi}\hat{\theta}\right]$$

$$\boxed{\vec{B}_{dip} = \frac{\omega^2 q.a}{r.c^2}\, e^{-i(\omega t-kr-\varphi)}.\left[\cos\theta\,\hat{\varphi} - i.\hat{\theta}\right]}$$

$$\vec{E}_{dip} = -\hat{n}\times\vec{B}_{dip} = -\begin{vmatrix}\hat{r} & \hat{\theta} & \hat{\varphi}\\ 1 & 0 & 0\\ 0 & -i & \cos\theta\end{vmatrix}\frac{\omega^2 q a}{r.c^2}\, e^{i(kr-\omega t+\varphi)}$$

$$\boxed{\vec{E}_{dip} = -\left(-i\hat{\varphi} - \cos\theta.\hat{\theta}\right)\frac{\omega^2 q a}{r.c^2}\, e^{i(kr-\omega t+\varphi)}}$$

$$\vec{A}_{cuad} = -\frac{\omega^2}{c^2 r}\, q a^2 e^{-i2\omega(t-\frac{r}{c})}.\begin{pmatrix}1 & i & 0\\ i & -1 & 0\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}sen\theta.\cos\varphi\\ sen\theta.sen\varphi\\ \cos\theta\end{pmatrix}$$

$i(\cos\varphi - i\,sen\varphi)$

$$\vec{A}_{cuad} = \text{''}\ \begin{pmatrix}[sen\theta.\cos\varphi + i.sen\theta.sen\varphi]\,\hat{x}\\ [sen\theta.\cos\varphi.i - sen\theta\ sen\varphi]\,\hat{y}\\ 0\ \hat{z}\end{pmatrix} = \text{''}\ sen\theta\begin{pmatrix}e^{i\varphi}\ \hat{x}\\ ie^{i\varphi}\ \hat{y}\\ 0\ \hat{z}\end{pmatrix}$$

$$\vec{B}_{cuad} = -\frac{1}{c}\hat{r}\times\left(-\frac{\omega^2}{c^2 r}qa^2 e^{-i2\omega t}\, e^{i2kr}\right).(-2i\omega)\ sen\theta.e^{i\varphi}$$

mismo vectorial que antes

$$\left(-\frac{1}{c}\right)\left(\frac{-\omega^2 q a^2 e^{-i2\omega t}\, e^{i2kr}}{c^2 r}\right)(-2i\omega)\begin{vmatrix}\hat{r} & \hat{\theta} & \hat{\varphi}\\ 1 & 0 & 0\\ \begin{matrix}sen\theta.\cos\varphi\\ +i.sen\theta.sen\varphi\end{matrix} & \begin{matrix}\cos\theta.\cos\varphi\\ +i.sen\varphi\cos\theta\end{matrix} & \begin{matrix}-sen\varphi\\ +i.\cos\varphi\end{matrix}\end{vmatrix}.sen\theta.e^{i\varphi}$$

$$\vec{B}_{cuad} = -\frac{\omega^3 2}{c^3 r} q a^2 e^{-i(2\omega t - 2kr - \pi/2 - \varphi)} . \operatorname{sen}\theta . e^{i\varphi}\left(\cos\theta . \hat{\varphi} - i . \hat{\theta}\right)$$

$$\boxed{\vec{B}_{cuad} = -\frac{2\omega^3}{c^3 r} q a^2 e^{2i(kr - \omega t + \pi/4 + \varphi)} . \operatorname{sen}\theta . \left(\cos\theta . \hat{\varphi} - i . \hat{\theta}\right)}$$

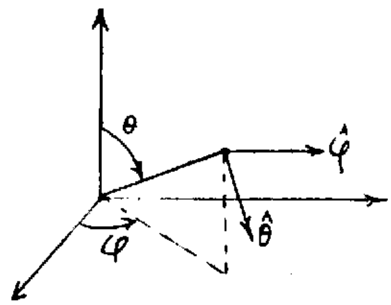
$$\vec{E}_{cuad} = -\hat{r} \times \vec{B}_{cuad} \propto \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\varphi} \\ 1 & 0 & 0 \\ 0 & -i & \cos\theta \end{vmatrix}$$

$$\boxed{\vec{E}_{cuad} = \frac{2\omega^3}{c^3 r} q a^2 . e^{2i(kr - \omega t + \pi/4 + \varphi)} \operatorname{sen}\theta . \left(\cos\theta . \hat{\theta} + i . \hat{\varphi}\right)}$$

* Análisis de la polarización

tomando la parte real resulta en:

$$\vec{E}_{dip} = \left[\frac{\omega^2 q a}{c^2 . r} . \cos(kr - \omega t + \varphi) \cos\theta \, \hat{\theta} \right.$$
$$\left. \frac{\omega^2 q a}{c^2 r} \cos(kr - \omega t + \varphi + \pi/2) \; \hat{\varphi} \right]$$



con $\begin{cases} \theta = 0 \rightarrow \text{polarización circular} \\ \theta = \pi/2 \rightarrow \text{polarización lineal en } \hat{\varphi} \\ \theta \neq 0, \pi/2 \rightarrow \text{polarización elíptica} \end{cases}$

Para el $\vec{E}_{cuad}$ el análisis es exactamente el mismo

* Cálculo de Potencia

$$\langle \vec{S} \rangle = \frac{c}{4\pi} \operatorname{Re}\left\{ \frac{1}{2} \vec{E} \times \vec{B}^* \right\} \qquad \vec{E} \perp \vec{B} \rightarrow \vec{E} \times \vec{B} = |E||B| . \hat{r}$$

$$\Rightarrow \quad \frac{1}{2} \left(\frac{\omega^2 q a}{r . c^2} \right)^2 \left[e^{+i\phi} \left(+\cos\theta \hat{\theta} + e^{i\pi/2} \hat{\varphi} \right) \times e^{-i\phi} \left(\cos\theta \hat{\varphi} + e^{i\pi/2} \hat{\theta} \right) \right]$$

$$\begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\varphi} \\ 0 & +\cos\theta & +e^{i\pi/2} \\ 0 & e^{i\pi/2} & \cos\theta \end{vmatrix}$$

$$\langle \vec{S} \rangle_{dip} = \frac{c}{4\pi} \frac{1}{2} \left(\frac{\omega^2 q a}{r . c^2} \right)^2 \left(\cos^2\theta + 1 \right) \hat{r}$$

$$\langle \vec{S} \rangle_{cuad} = \frac{c}{4\pi} \frac{1}{2} \left(\frac{2\omega^3 q a^2}{c^3 r} \right)^2 \operatorname{sen}^2\theta \left(\cos^2\theta + 1 \right) \hat{r}$$

en forma análoga resulta

$$\langle \vec{S} \rangle_{tot} . \hat{r} = \frac{c}{8\pi} \left(\cos^2\theta + 1 \right) \left[\operatorname{sen}^2\theta . \frac{4\omega^6 q^2 a^4}{c^6 r^2} + \frac{\omega^4 q^2 a^2}{r^2 c^4} \right]$$

$$\langle \vec{S} \rangle_{tot} . \hat{r} = \frac{c}{8\pi} \left(\cos^2\theta + 1 \right) . \frac{\omega^4 q^2 a^2}{c^4 r^2} \left(\operatorname{sen}^2\theta . \frac{4\omega^2 a^2}{c^2} + 1 \right)$$

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \langle \vec{S} \rangle . \hat{n}\, r^2 \Rightarrow$$

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{\omega^4 q^2 a^2}{c^3 . 8\pi} . (\cos^2\theta + 1) . \left[\operatorname{sen}^2\theta . \frac{4\omega^2 a^2}{c^2} + 1 \right]$$

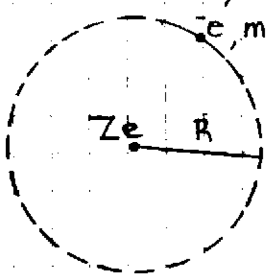
$$\langle P \rangle = \iint \left\langle \frac{dP}{d\Omega} \right\rangle . \operatorname{sen}\theta . d\theta . d\varphi$$

$$\langle P \rangle = \frac{\omega^4 q^2 a^2}{c^3 \cancel{8\pi}_{4}} . \cancel{2\pi} \left[\int_0^\pi (\cos^2\theta + 1) \overset{\operatorname{sen}\theta}{} d\theta + \int_0^\pi (\cos^2\theta + 1) \left(\operatorname{sen}^3\theta . \frac{4\omega^2 a^2}{c^2} \right) d\theta \right]$$

$$\boxed{\langle P \rangle = \frac{\omega^4 q^2 a^2}{4 c^3} . \left(\frac{8}{3} + \frac{4\omega^2 a^2}{c^2} . \frac{8}{5} \right)}$$

ésto es la potencia irradiada por la parte cuadrupolar; es mucho más chica que la irradiada por la parte dipolar

8.



suponemos que gire a $\omega = \dot{\varphi}$ constante

$P = \vec{J} . \vec{E}$ ← La potencia tiene esta expresión general

$$P = \frac{dU}{dt} = \vec{J} . \vec{E}$$

$$\frac{dU}{dt} = q . \vec{v} . \vec{E}$$ ← esta fórmula no nos es útil aquí porque despreciaremos $\dot{r}$

(a) $F_{\text{entre partículas}} = -\frac{e . e Z}{r^2} \hat{r}$ $\qquad \vec{v} = \omega . r \hat{\varphi}$

$$F) \quad -\frac{e^2 Z}{r^2} = m . (-r . \omega^2)$$

$$\frac{e^2 Z}{r^2} = m . \frac{v^2}{r} \rightarrow v^2 = \frac{e^2 Z}{m r}$$

Potencia radiada

$$P \cong -\frac{d}{dt} \left(\frac{1}{2} \cancel{m} \frac{e^2 Z}{\cancel{m} r} - \frac{Z e^2}{r} \right) = -\frac{d}{dt} \left(-\frac{Z e^2}{2r} \right)$$

$$P = \frac{Z e^2}{2 r^2} . \frac{dr}{dt} \qquad P_{(t=0)} = -\frac{Z e^2}{2 R^2} . \dot{r} \Big|_{t=0}$$

Fórmula Larmor

$$\frac{2 q^2}{3 c^3} \left(\frac{d\vec{v}}{dt} \right)^2 = \frac{2 \cancel{e^2}}{3 c^3} \left(\frac{e^2 Z}{m r^2} \right)^2 = -\frac{\cancel{Z} \cancel{e^2}}{2 \cancel{R^2}} . \frac{dr}{dt}$$

$$\int_0^{\Delta t} \frac{4}{3} \frac{e^4 Z}{c^3 m^2} dt = -\int_R^0 r^2 . dr$$

$$r_0 = \frac{e^2}{m c^2}$$

$$\frac{4 e^4 Z}{\cancel{3} c^3 m^2} \Delta t = -\frac{1}{\cancel{3}} \left(-R^3 \right)$$

$$\Delta t = \frac{R^3 m^2 c^3}{4 e^4 Z} = \frac{R^3 \cancel{m^2} c^3}{r_0^2 \cancel{m^2} c^4 4 Z}$$

$$\boxed{\Delta t \approx \left(\frac{R^2}{r_0^2} \right) . \frac{R}{4 Z c}}$$

(b)

$$\frac{2e^2}{3c^3} r^2\omega^4 = -\frac{Ze^2}{2r^2}\cdot\dot{r}$$

$$\omega^2 r = \frac{e^2 Z}{m r^2}$$

$$\int r^2 dr = -\int \frac{4e^4 Z}{3m^2c^3}\,dt$$

$$\left(\frac{d\varphi}{dt}\right)^2 = \frac{e^2 Z}{m r^3}$$

$$\frac{r^3}{\cancel{3}} = -\frac{4e^4 Z}{\cancel{3} m^2 c^3}\cdot t$$

$$\frac{d\varphi}{dt} = e\left(\frac{Z}{m}\right)^{1/2}\frac{1}{r^{3/2}}$$

$$\frac{r^3 - R^3}{\cancel{3}} = -\frac{4e^4 Z}{\cancel{3} m^2 c^3}\cdot t$$

$$r^3 = R^3 - \frac{4e^4 Z}{m^2c^3}\cdot t$$

$$\frac{d\varphi}{dt} = e\cdot\sqrt{\frac{Z}{m}}\cdot\frac{1}{\left(R^3 - \underbrace{\frac{t}{\Delta t}}_{}\right)^{1/2}}$$

Δt → tiempo de caída

$$\sqrt{\frac{m}{Z}}\cdot 2\pi = \int_0^{t\text{ de una vuelta}} \frac{dt}{\left(R^3 - \frac{1}{\Delta t}\cdot t\right)^{1/2}}$$

$$= -\int_{R^3}^{-t_0/\Delta t + R^3} \frac{\Delta t\cdot dU}{U^{1/2}}$$

$$R^3 - \frac{t}{\Delta t} = U \qquad -\frac{dt}{\Delta t} = dU$$

$$\frac{\text{tiempo en caer}}{\text{tiempo en un giro}} = \frac{R^3 m^2 c^3}{4e^4 Z}$$

$$\sqrt{\frac{m\cdot 4\pi^2}{Z}} = -\frac{1}{1/2}\cdot\left(\sqrt{-\frac{t_0}{\Delta t} + R^3} - \sqrt{R^3}\right)\Delta t$$

$$\frac{1}{\Delta t}\cdot\frac{1}{2}\sqrt{\frac{m\cdot 4\pi^2}{Z}} = R^{3/2} - \left(R^3 - \frac{t_0}{\Delta t}\right)^{1/2}$$

$$R^3 - \frac{t_0}{\Delta t} = \left(R^{3/2} - \frac{1}{Z\cdot\Delta t}\sqrt{\frac{m\cdot 4\pi^2}{Z}}\right)^2$$

$$\frac{t_0}{\Delta t} = -\left(R^{3/2} - \frac{1}{\quad}\sqrt{\quad}\right)^2 + R^3$$

$$\frac{t_0}{\Delta t} = -\cancel{R^3} + \frac{1}{Z\cdot\Delta t}\cdot R^{3/2} Z\sqrt{\frac{m4\pi^2}{Z}} + \frac{1}{Z^2\Delta t^2}\frac{m4\pi^2}{Z} + \cancel{R^3}$$

$$\frac{t_0}{\Delta t} = \frac{R^{3/2}\cdot 2}{\cancel{Z}\cdot\frac{R^3 m^2 c^3}{4e^4\cancel{Z}}}\sqrt{\frac{m4\pi^2}{Z}} + \frac{1}{\cancel{Z^2}\cdot\frac{R^6 \cancel{m^4} c^6}{16e^8\cancel{Z^2}}}\frac{\cancel{m}4\pi^2}{Z}$$

$$\frac{t_0}{\Delta t} = \frac{4e^4\cdot 2}{R^{3/2} m^2 c^3}\left(\frac{m}{Z}\right)^{1/2} 2\pi + \underbrace{\frac{16e^8}{R^6 m^3 c^6}\frac{4\pi^2}{Z}}_{\sim 0}$$

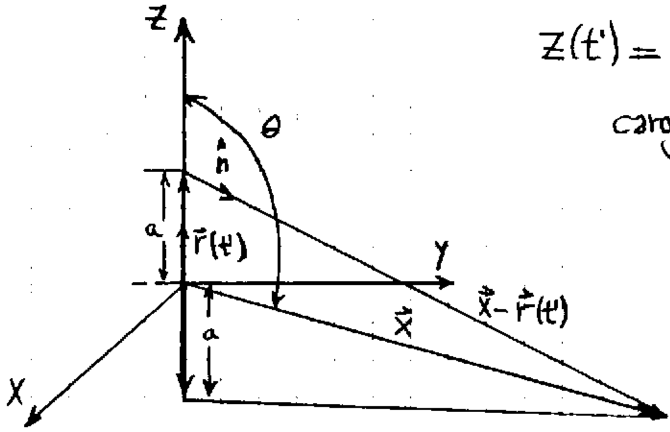
$$\frac{t_0}{\Delta t} = \frac{8e^4}{\cancel{e^4}\,\cancel{m^{3/2}c^3}}\frac{2\pi}{\sqrt{Z}}$$

$\sqrt{Z} = 1$

$$\#\text{vueltas} = \frac{t_0}{\Delta t} = \frac{16\pi e}{1}$$

no llega a dar ni una vuelta

10.



$$z(t') = a.\cos(\omega_0 t')$$

carga q

$x \gg 2a$ $\left\{ \begin{array}{l} \text{aprox. de} \\ \text{campo lejano} \end{array} \right.$

$\hat{n} \approx \hat{r}$.

(a) Como se quiere ver potencia irradiada puédese usar $\vec{E}_{acel.}$ (proveniente de potenciales Liénard-Wiechert)

$$\vec{E} = \left(\frac{q}{c}\right) \cdot \frac{\hat{r} \times \{(\hat{r} - \beta\hat{z}) \times \dot{\beta}\hat{z}\}}{(1-\beta.\cos\theta)^3 . r} \qquad \hat{r} \times \{\hat{r} \times \dot{\beta}\hat{z} - \underbrace{\beta\hat{z} \times \dot{\beta}\hat{z}}_{=0}\}$$

$$\vec{\beta} = \frac{\vec{V}}{c} = -\frac{1}{c}\, a.\,\text{sen}(\omega_0 t').\,\omega_0 \hat{z} = -\frac{\omega_0 a}{c}.\text{sen}(\omega_0 t')\,\hat{z}$$

$$\dot{\vec{\beta}} = -\frac{\omega_0^2 a}{c}.\cos(\omega_0 t')\,\hat{z}$$

$$K = 1 - \hat{r}.\vec{\beta} = 1 - \hat{r}.\hat{z}.\beta = 1 - \beta.\cos\theta = 1 + \frac{\omega_0 a}{c}\,\text{sen}(\omega_0 t').\cos\theta$$

$$\vec{E} = \frac{q}{c} \cdot \frac{+\frac{\omega_0^2 a}{c}.\cos(\omega_0 t').\,\text{sen}\,\theta\;\hat{\theta}}{r.\left(1 + \frac{\omega_0 a}{c}\,\text{sen}(\omega_0 t').\cos\theta\right)^3}$$

* numerador

$$\hat{r} \times \dot{\beta}\hat{z} = \dot{\beta} \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\varphi} \\ 1 & 0 & 0 \\ \cos\theta & -\text{sen}\theta & 0 \end{vmatrix}$$

$$= \dot{\beta}\,(-\text{sen}\,\theta\,\hat{\varphi}) = -\dot{\beta}.\,\text{sen}\,\theta\,\hat{\varphi}$$

$$\hat{r} \times (-\dot{\beta}.\text{sen}\,\theta\,\hat{\varphi}) = -\dot{\beta}.\text{sen}\,\theta \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\varphi} \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= -\dot{\beta}.\,\text{sen}\,\theta(-\hat{\theta}) = \dot{\beta}.\,\text{sen}\,\theta\;\hat{\theta}$$

$$dP = \left|\frac{q}{c} \cdot \frac{\dot{\beta}.\text{sen}\,\theta}{r.K^3}\right|^2 . \frac{c}{4\pi}.r^2.k\; d\Omega$$

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c} \cdot \frac{\dot{\beta}^2.\,\text{sen}^2\theta}{K^5}$$

$$\left.\frac{dP}{d\Omega}\right|_{t'} = \frac{q^2}{4\pi c} \cdot \frac{\omega_0^4.a^2.\cos^2(\omega_0 t').\,\text{sen}^2\theta}{c^2\left(1 + \frac{\omega_0 a}{c}\,\text{sen}(\omega_0 t').\cos\theta\right)^5}$$

si defino $\beta = \frac{a.\omega_0}{c}$ →

$$\boxed{\left.\frac{dP}{d\Omega}\right|_{t'} = \frac{q^2\, c\, \beta^4 . \cos^2(\omega_0 t').\,\text{sen}^2\theta}{4\pi\, a^2 \left[1 + \beta.\,\text{sen}(\omega_0 t')\cos\theta\right]^5}}$$

(b)

$$\left\langle \frac{dP}{d\Omega} \right\rangle \propto \int_0^{\frac{2\pi}{\omega_0}} \frac{\cos^2(\omega_0 t')}{[1+\beta\cdot\cos\theta\cdot \text{sen}(\omega_0 t')]^5}\, dt'$$

$\omega_0 t' =$

$\omega_0 \left(t \frac{2\pi}{\omega_0}\right)$

$1+\beta\cdot\cos\theta\cdot \text{sen}(\omega_0 t') = u$

$\beta\cdot\cos\theta\cdot\cos(\omega_0 t')\,\omega_0\, dt' = du$

$$= \frac{1/2 + 1/2\cdot\cos(2\omega_0 t')}{\quad}$$

(c) El caso no relativista emplea: $\beta \ll 1 \rightarrow v \ll c \Rightarrow$

$$k = 1 - \hat{n}\cdot\hat{\beta} \cong 1 \Rightarrow$$

$$\vec{E} = \frac{q}{c}\cdot\frac{\hat{r}\times[\hat{r}\times\dot{\beta}\hat{z}]}{r} = \frac{q}{c\cdot r}\left(\hat{r}\times -\dot{\beta}\cdot\text{sen}\theta\,\hat{\varphi}\right) = \frac{q}{c\cdot r}\,\dot{\beta}\cdot\text{sen}\theta\,\hat{\theta}$$

$$\left.\frac{dP}{d\Omega}\right|_{t'\sim t} = \frac{c}{4\pi}\,\frac{q^2}{c^2}\,\dot{\beta}^2\,\text{sen}^2\theta$$

$$\left.\frac{dP}{d\Omega}\right|_t = \frac{q^2}{4\pi c}\cdot\frac{\omega_0^4 a^2}{c^2}\cos^2(\omega_0 t')\,\text{sen}^2\theta$$

$$\boxed{\frac{dP}{d\Omega} = \frac{q^2 c}{4\pi a^2}\,\beta^4\cdot\cos^2(\omega_0 t')\,\text{sen}^2\theta}$$

Expresión no relativista

La expresión difiere de la relativista en que se ha despreciado el término

$$\frac{1}{1+\beta\cdot\text{sen}(\omega_0 t')\cdot\cos\theta} \sim 1$$

$\frac{\omega_0 a}{c} \rightarrow \frac{V_{lineal}}{c} \ll 1 \Rightarrow \omega_0 a \ll c$

$\therefore$ Es consistente la aproximación hecha