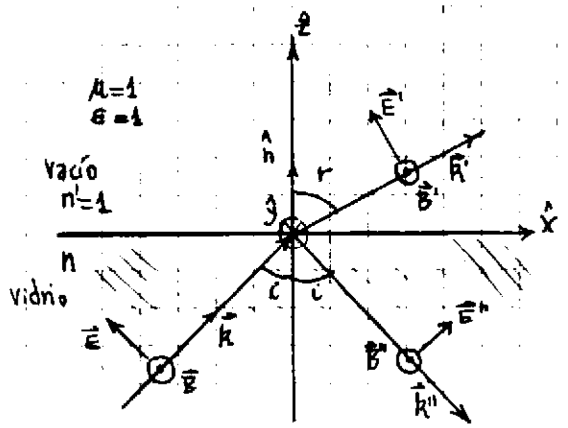
GUÍA 6

I. Problemas

1.



onda l.p. con TM , $n > n' = 1$

$i > i_0 = \text{asen}\left(\frac{n'}{n}\right)$ ← ángulo límite $\qquad \frac{n'}{n} < 1$

$\text{sen}\, i > \frac{n'}{n}$

$n . \text{sen}\, i = n' . \text{sen}\, r$ (snell law)

$\text{sen}\, i > \frac{\text{sen}\, i}{\text{sen}\, r} \Rightarrow \text{sen}\, r > 1$

$\therefore r \in \mathbb{C} \Rightarrow$

$\text{sen}\, r > 1 \rightarrow \qquad \cos r = \sqrt{-\text{sen}^2 r + 1} = i \sqrt{\text{sen}^2 r - 1}$

$\cos r = i . \sqrt{\text{sen}^2 i . \left(\frac{n}{n'}\right)^2 - 1}$

$\cos r = i . \sqrt{\left(\frac{\text{sen}\, i}{\text{sen}\, i_0}\right)^2 - 1}$

$\therefore \cos r \in \mathbb{I}m$

a.

$\langle \vec{S} \rangle = \frac{c}{8\pi} \mathcal{R}e \left\{ \vec{E} \times \vec{H}^* \right\}$

$\langle \vec{S} \rangle = \frac{c}{8\pi} \mathcal{R}e \left\{ \vec{E}' \times \vec{H}'^* \right\}$

Como el ángulo de i es $> i_0 \Rightarrow$

$\vec{k}' \in \mathbb{C}$

$\vec{B}' = \sqrt{\mu' \varepsilon'} \; \frac{\vec{k}' \times \vec{E}'}{k'}$

pero el medio primado es vacío $\therefore$

$\vec{H}' = \frac{\sqrt{\mu' \varepsilon'}}{\mu'} . \left(\frac{\vec{k}' \times \vec{E}'}{k'} \right)$

$\vec{H}'^* = \frac{\cancel{\sqrt{\mu' \varepsilon'}}}{\mu'} . \frac{\left(\vec{k}' \times \vec{E}'\right)^*}{\cancel{\sqrt{\mu' \varepsilon'}} . \frac{\omega}{c}}$

$\langle \vec{S} . \hat{n} \rangle = \frac{c}{8\pi} \mathcal{R}e \left\{ \frac{c}{\mu' . \omega} \left(\vec{E}' \times \vec{k}'^* \times \vec{E}'^* \right) . \hat{n} \right\}$

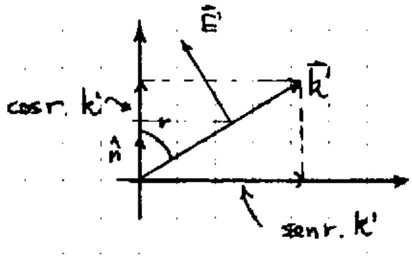
$= \frac{c^2}{8\pi \mu' \omega} \mathcal{R}e \left\{ \left(\vec{k}'^* |\vec{E}'|^2 - \vec{E}'^* \underbrace{\left(\vec{E}' . \vec{k}'^* \right)}_{=0} \right) \hat{n} \right\}$

$\langle \vec{S} . \hat{n} \rangle = \frac{c^2}{8\pi \mu' \omega} \mathcal{R}e \left\{ \vec{k}'^* . |\vec{E}'|^2 . \hat{n} \right\}$

$\frac{c^2 . |\vec{E}'|^2}{8\pi \mu' \omega} \mathcal{R}e \left\{ \vec{k}'^* . \hat{n} \right\}$

$\mathcal{R}e \left\{ \cos r . k' \right\} = 0$

$\Rightarrow \boxed{\langle \vec{S} . \hat{n} \rangle = 0}$



No hay flujo del vector de Poynting en la dirección normal a la superficie

b. El ángulo de Brewster es tal que:

Brewster $i_B = \operatorname{atan}\left(\frac{n'}{n}\right)$

(> límite) → $i > \operatorname{asen}\left(\frac{n'}{n}\right)$

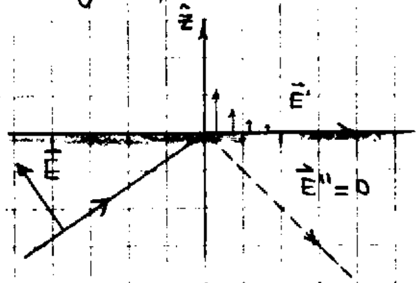
Para incidir además con ángulo de Brewster necesitaré i':

$$\tan i' = \frac{n'}{n} \quad \text{pero} \quad \operatorname{sen} i' > \frac{n'}{n}$$

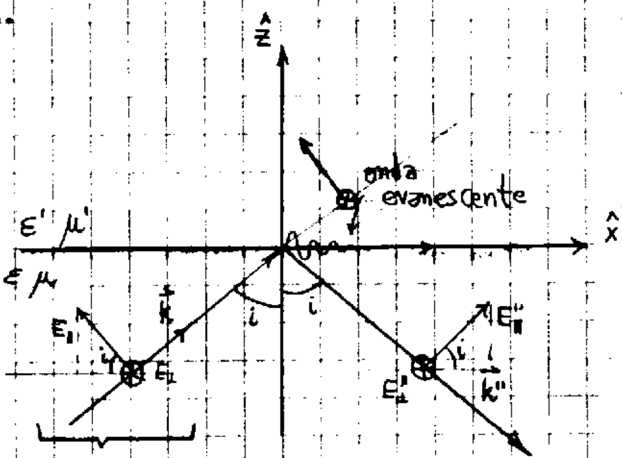
luego quiero i': $\operatorname{sen} i' > \frac{\operatorname{sen} i'}{\cos i'} \Rightarrow \cos i' > 1$ $\boxed{\text{No hay } i' \text{ real que verifique esto}}$

Parece lógico que no puedan darse ambas situaciones en simultáneo, porque eso significaría que la interfase se engulle a la onda.

Esto significa que el medio está tragándose constantemente energía de la onda, lo cual parecería violar la conservación ¿?



2.

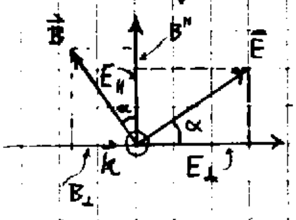


$i > i_0 \Rightarrow \vec{k}' \in \mathbb{C}$

incide luz L.P.

$$\vec{E} = (\hat{e}_\parallel E_\parallel + \hat{e}_\perp E_\perp)\, e^{i(\vec{k}\cdot\vec{x} - \omega t)}$$

$$\vec{E}'' = (\hat{e}_\parallel E''_\parallel + \hat{e}_\perp E''_\perp)\, e^{i(\vec{k}''\cdot\vec{x} - \omega t)}$$



$\boxed{\text{snell} \quad \sqrt{\varepsilon\mu}\,.\operatorname{sen} i = \sqrt{\varepsilon'\mu'}\,.\operatorname{sen} r}$

$E_\perp = E.\cos\alpha$
$E_\parallel = E.\operatorname{sen}\alpha$
$B_\parallel = B.\cos\alpha$
$B_\perp = B.\operatorname{sen}\alpha$

* $D\hat{n}$ continuo en contorno

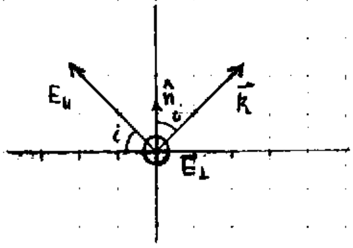
$$\varepsilon\,(E_\parallel + E''_\parallel)\,.\operatorname{sen} i - \varepsilon'.E'_\parallel \operatorname{sen} r = 0$$
$$\varepsilon\,(E_\parallel + E''_\parallel)\,.\sqrt{\frac{\varepsilon'\mu'}{\varepsilon\mu}}\,\operatorname{sen} r - \varepsilon'.E'_\parallel.\operatorname{sen} r = 0$$
$$\sqrt{\varepsilon}\,(E_\parallel + E''_\parallel)\,1/\sqrt{\mu} - \sqrt{\varepsilon'}/\sqrt{\mu'}\,E'_\parallel = 0$$

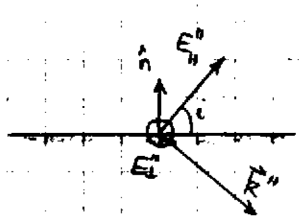
* $E\hat{t}$ continuo en contorno

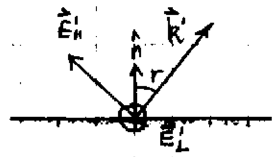
$$E_\perp + E''_\perp - E'_\perp = 0$$
$$E_\perp + E''_\perp = E'_\perp$$
$$\sqrt{\frac{\varepsilon}{\mu}}\,(E.\operatorname{sen}\alpha + E''_\parallel) = \sqrt{\frac{\varepsilon'}{\mu'}}$$

* $B\hat{n}$ continuo

$$\left[\left(\vec{k}\times\vec{E}_0 + \vec{k}''\times\vec{E}''_0\right) - \left(\vec{k}'\times\vec{E}'_0\right)\right].\hat{n} = 0$$
$$\left[(\vec{k}\times\vec{E}_\perp).\hat{n} + (\vec{k}''\times\vec{E}''_\perp).\hat{n}\right] - \left[(\vec{k}'\times\vec{E}'_\perp).\hat{n}\right] = 0$$







$$(\vec{k}\times\vec{E}_{\parallel})\cdot\hat{n}=0 \qquad \vec{k}''\times\vec{E}''_{\parallel}\cdot\hat{n}=0 \qquad \vec{k}'\times\vec{E}'_{\parallel}\cdot\hat{n}=0$$

$$\vec{k}\times\vec{E}_{\perp}\cdot\hat{n}\neq 0 \qquad \vec{k}''\times\vec{E}''_{\perp}\cdot\hat{n}\neq 0 \qquad \vec{k}'\times\vec{E}'_{\perp}\cdot\hat{n}\neq 0$$

$$k\cdot E_{\perp}\cos\left(\frac{\pi}{2}-i\right) \qquad k''\cdot E''_{\perp}\cos\left(\pi/2-i\right) \qquad k'\cdot E'_{\perp}\cos\left(\pi/2-r\right)$$

$$k\cdot E_{\perp}\cdot \operatorname{sen} i \qquad k''\cdot E''_{\perp}\cdot\operatorname{sen} i \qquad k'\, E'_{\perp}\cdot\operatorname{sen} r$$

$$k\cdot E_{\perp}\cdot\operatorname{sen} i + k''\cdot E''_{\perp}\cdot\operatorname{sen} i = k'\,E'_{\perp}\operatorname{sen} r = k'\,E'_{\perp}\cdot\operatorname{sen} i\sqrt{\frac{\varepsilon\mu}{\varepsilon'\mu'}}$$

$$\boxed{k''=k} \qquad k\left(E_{\perp}+E''_{\perp}\right)=k'\cdot E'_{\perp}\cdot\sqrt{\frac{\varepsilon\mu}{\varepsilon'\mu'}}$$

$$\boxed{k'=k\sqrt{\frac{\varepsilon'\mu'}{\varepsilon\mu}}}$$

* H_t continuo

$$\left[\frac{1}{\mu}\left(\vec{k}\times\vec{E}_0+\vec{k}''\times\vec{E}''_0\right)-\frac{1}{\mu'}\left(\vec{k}'\times\vec{E}'_0\right)\right]\times\hat{n}=0$$

$$\left[-\frac{1}{\mu}\,\hat{n}\times\left(\vec{k}\times\vec{E}_0\right)-\frac{1}{\mu}\,\hat{n}\times\left(\vec{k}''\times\vec{E}''_0\right)\right]+\left[\hat{n}\times\frac{1}{\mu'}\,\vec{k}'\times\vec{E}'_0\right]=0$$

$$-\frac{1}{\mu}\left[\vec{k}\left(\hat{n}\cdot\vec{E}_0\right)-\vec{E}_0\left(\hat{n}\cdot\vec{k}\right)\right]-\frac{1}{\mu}\left[\vec{k}''\left(\hat{n}\cdot\vec{E}''_0\right)-\vec{E}''_0\left(\hat{n}\cdot\vec{k}''\right)\right]$$

$$+\frac{1}{\mu'}\left[\vec{k}'\left(\hat{n}\cdot\vec{E}'_0\right)-\vec{E}'_0\left(\hat{n}\cdot\vec{k}'\right)\right]=0$$

$$-\frac{1}{\mu}\left[\vec{k}\,E_{\parallel}\cos\left(\frac{\pi}{2}-i\right)-\vec{E}_0\cdot k\cdot\cos i\right]-\frac{1}{\mu}\left[\vec{k}''\,E''_{\parallel}\cos\left(\frac{\pi}{2}-i\right)-\vec{E}''_0\cdot k''\cos i\right]$$

$$+\frac{1}{\mu'}\left[\vec{k}'\cdot E'_0\cos\left(\frac{\pi}{2}-r\right)-\vec{E}'_0\cdot k'\cdot\cos r\right]=0$$

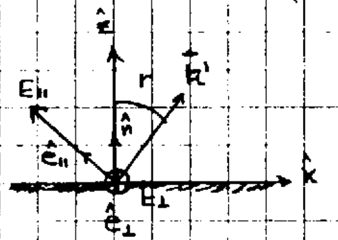
$$-\frac{\vec{k}\cdot E_{\parallel}\cdot\operatorname{sen} i}{\mu}+\frac{\vec{E}_0\cdot k\cdot\cos i}{\mu}-\frac{\vec{k}\cdot E''_{\parallel}\cdot\operatorname{sen} i}{\mu}+\frac{\vec{E}''_0\cdot k\cdot\cos i}{\mu}+\frac{\vec{k}'\operatorname{sen} r\,E'_0}{\mu'}$$

$$-\frac{\vec{E}'_0\cdot k'\cdot\cos r}{\mu'}=0$$

$$\vec{E}' = \left(\hat{e}_\parallel E'_\parallel + \hat{e}_\perp E'_\perp\right) e^{i\left(\vec{k}'\cdot\vec{x} - \omega t\right)}$$

$$e^{i\left[k'(\cos r\, z + \operatorname{sen} r\, x) - \omega t\right]}$$

$$\left(\hat{e}_\parallel E'_\parallel + \hat{e}_\perp E'_\perp\right) e^{-k'|\cos r|\, z} \cdot e^{i\left[k' \operatorname{sen} r\, x - \omega t\right]}$$



* Coeficientes de Fresnel

E paralelo

$$\frac{E'_\parallel}{E_\parallel} = \frac{2nn'\cos i}{\frac{\mu}{\mu'} n'^2 \cos i + nn'\cos r} = \frac{E'_\parallel}{E\,\operatorname{sen}\alpha} = \frac{2\sqrt{\mu\varepsilon\mu'\varepsilon'}\cdot\cos i}{\frac{\mu}{\cancel{\mu'}}\cdot\cancel{\mu'}\varepsilon'\cdot\cos i + \sqrt{\mu\varepsilon\mu'\varepsilon'}\cdot\cos r}$$

$$\frac{\sqrt{\mu\varepsilon'}\, 2\sqrt{\mu'\varepsilon}\cdot\cos i}{\sqrt{\mu\varepsilon'}\left(\sqrt{\mu\varepsilon'}\cdot\cos i + \sqrt{\mu'\varepsilon}\cos r\right)}$$

$$E'_\parallel = \frac{E\cdot\operatorname{sen}\alpha\cdot 2\sqrt{\mu'\varepsilon}\cos i\left(\sqrt{\mu\varepsilon'}\cos i - i\sqrt{\mu'\varepsilon}\cdot|\cos r|\right)}{\mu\varepsilon'\cos^2 i + \mu'\varepsilon|\cos r|^2}$$

$$\frac{E''_\parallel}{E_\parallel} = \frac{\frac{\mu}{\mu'}n'^2\cos i - nn'\cos r}{\frac{\mu}{\mu'}n'^2\cos i + nn'\cos r} = \frac{\left[\frac{\mu}{\mu'}n'^2\cos i - nn'\cos r\right]^2}{\left[\frac{\mu}{\mu'}n'^2\cos i\right]^2 + \left[nn'|\cos r|\right]^2} \ldots$$

$$= \frac{\left[\frac{\mu}{\mu'}n'^2\cos i\right]^2 - 2\left(\frac{\mu}{\mu'}n'^2\cos i\right)\left(nn'|\cos r|\right) i - \left(nn'|\cos r|\right)^2}{\left[\frac{\mu}{\mu'}n'^2\cos i\right]^2 + \left[nn'|\cos r|\right]^2}$$

$$E''_\parallel = \left(\frac{A^2 - B^2}{A^2 + B^2} - \frac{i\cdot 2|A||B|}{A^2 + B^2}\right)\cdot\operatorname{sen}\alpha\cdot E$$

E perpendicular

$$\frac{E'_\perp}{E_\perp} = \frac{2n\cos i}{n\cos i + \frac{\mu}{\mu'}\sqrt{n'^2 - n^2\operatorname{sen}^2 i}} = \frac{E'_\perp}{E\cos\alpha} = \frac{2\sqrt{\mu\varepsilon}\cdot\cos i}{\sqrt{\mu\varepsilon}\cdot\cos i + \frac{\mu}{\cancel{\mu'}\sqrt{\mu'}}\cancel{\sqrt{\mu'}}\sqrt{\varepsilon'}\cdot\cos r}$$

$$E'_\perp = \frac{2\cos i\cdot\cos\alpha\cdot E}{\cos i + \frac{\sqrt{\mu}\sqrt{\varepsilon'}}{\sqrt{\varepsilon}\sqrt{\mu'}}\cdot\cos r}$$

$$E'_\perp = \frac{2\cos i\cdot\cos\alpha\cdot E\left(\cos i - |\cos r|\cdot i\cdot\left[\sqrt{\varepsilon'\mu}/\sqrt{\varepsilon\mu'}\right]\right)}{\cos^2 i + \frac{\mu\varepsilon'}{\varepsilon\mu'}|\cos r|^2}$$

$$\frac{E''_\perp}{E_\perp} = \frac{\sqrt{\mu\varepsilon}\cos i - \mu/\mu'\,\sqrt{\mu'\varepsilon'}\cos r}{\sqrt{\mu\varepsilon}\cdot\cos i + \mu/\mu'\cdot\sqrt{\mu'\varepsilon'}\cos r} = \frac{\sqrt{\mu\varepsilon}\cdot\cos i - \mu\cdot\sqrt{\varepsilon'}/\sqrt{\mu'}\cdot\cos r}{\sqrt{\mu\varepsilon}\cdot\cos i + \mu\cdot\sqrt{\frac{\varepsilon'}{\mu'}}\cdot\cos r}$$

$$= \frac{\cos i - \sqrt{\frac{\mu\varepsilon'}{\varepsilon\mu'}}\cdot\cos r}{\cos i + \sqrt{\frac{\mu\varepsilon'}{\varepsilon\mu'}}\cdot\cos r}$$

$$\frac{E''_\perp}{E\cdot\cos\alpha} = \frac{\cos^2 i - 2\cos i\,|\cos r|\cdot i\sqrt{\frac{\mu\varepsilon'}{\mu'\varepsilon}} - \frac{\mu\varepsilon'}{\varepsilon\mu'}\cdot|\cos r|^2}{\cos^2 i + |\cos r|^2\cdot\frac{\mu\varepsilon'}{\varepsilon\mu'}}$$

$$E''_\perp = \frac{E\cos\alpha\,\left(\cos^2 i - \frac{\mu\varepsilon'}{\varepsilon\mu'}|\cos r|^2\right)}{\cos^2 i + |\cos r|^2\frac{\mu\varepsilon'}{\varepsilon\mu'}} - \frac{i\,E\cos\alpha\cdot 2\cos i\,|\cos r|\sqrt{\frac{\mu\varepsilon'}{\mu'\varepsilon}}}{\cos^2 i + |\cos r|^2\frac{\mu\varepsilon'}{\mu'\varepsilon}}$$

$$|E''_\perp| = \sqrt{\frac{\left[\left(\cos^2 i - \frac{\mu\varepsilon'}{\varepsilon\mu'}|\cos r|^2\right)E\cos\alpha\right]^2 + \left[E\cos\alpha\cdot 2\cos i\,|\cos r|\right]^2\frac{\mu\varepsilon'}{\mu'\varepsilon}}{\left[\cos^2 i + |\cos r|^2\frac{\mu\varepsilon'}{\varepsilon\mu'}\right]^2}}$$

$$= E\cos\alpha\sqrt{\left(\cos^4 i + \left(\frac{\mu\varepsilon'}{\varepsilon\mu'}\right)^2|\cos r|^4 - 2\cos^2 i\,\frac{\mu\varepsilon'}{\varepsilon\mu'}|\cos r|^2 + 4\cos^2 i\,|\cos r|^2\frac{\mu\varepsilon'}{\mu'\varepsilon}\right.}$$

$$|E''_\perp| = \frac{E\cos\alpha\,\left(\cos^2 i + |\cos r|^2\frac{\mu\varepsilon'}{\varepsilon\mu'}\right)}{\cos^2 i + |\cos r|^2\frac{\mu\varepsilon'}{\varepsilon\mu'}} \rightarrow |E''_\perp| = E\cos\alpha$$

$$\varphi_\perp = \operatorname{atan}\left\{\frac{\mathcal{Im}\,E''_\perp}{\mathcal{Re}\,E''_\perp}\right\} \Rightarrow$$

$$\varphi_\perp = \operatorname{atan}\left\{\frac{\cancel{-E\cos\alpha}\,2\cos i\,|\cos r|\sqrt{\frac{\mu\varepsilon'}{\varepsilon\mu'}}}{\cancel{E\cos\alpha}\left(\cos^2 i - |\cos r|^2\frac{\mu\varepsilon'}{\varepsilon\mu'}\right)}\right\}$$

$$|E''_\parallel| = E\,\mathrm{sen}\,\alpha\,\frac{\sqrt{\left(\left[\frac{\mu}{\mu'}n'^2\cos i\right] - \left[n n'|\cos r|^2\right]\right)^2 + \left(2\frac{\mu}{\mu'}n'^2\cos i\cdot n n'|\cos r|\right)^2}}{\sqrt{\left[\left(\frac{\mu}{\mu'}n'^2\cos i\right)^2 + \left(n n'|\cos r|\right)^2\right]^2}}$$

$$|E''_\parallel| = E\,\mathrm{sen}\,\alpha$$

$$\varphi_\parallel = \operatorname{atan}\left\{\frac{\mathcal{Im}\,E''_\parallel}{\mathcal{Re}\,E''_\parallel}\right\} \Rightarrow$$

$$\varphi_\parallel = \operatorname{atan}\left\{-\frac{2\frac{\mu}{\mu'}n'^2\cos i\cdot n n'|\cos r|}{\left[\frac{\mu}{\mu'}n'^2\cos i\right]^2 - \left[n n'|\cos r|\right]^2}\right\}$$

$$\varphi_\parallel = \operatorname{atan}\left\{\frac{-2\frac{\mu}{\cancel{\mu'}}\cancel{\mu'}\varepsilon'\sqrt{\mu\varepsilon\mu'\varepsilon'}\,|\cos r|\cos i}{\left(\frac{\mu}{\cancel{\mu'}}\cancel{\mu'}\varepsilon'\cos i\right)^2 - \left(\sqrt{\mu\varepsilon\mu'\varepsilon'}|\cos r|\right)^2}\right\}$$

*** Ángulo r**

$$\cos r = i|\cos r| =$$

$$\cos^2 r = 1 - \mathrm{sen}^2 r$$

$$\cos r = i\sqrt{\mathrm{sen}^2 r - 1}$$

$$\cos r = i\sqrt{\mathrm{sen}^2 i\,\frac{\varepsilon\mu}{\varepsilon'\mu'} - 1}$$

$$|\cos r|^2 = \left(\mathrm{sen}^2 i\,\frac{\varepsilon\mu}{\varepsilon'\mu'} - 1\right)$$

$$\varphi_\parallel = \operatorname{atan}\left\{\frac{-2\cos i\cdot\mu\varepsilon'\sqrt{\mu\varepsilon\mu'\varepsilon'}\sqrt{\mathrm{sen}^2 i\,\frac{\varepsilon\mu}{\varepsilon'\mu'} - 1}}{(\mu\varepsilon')^2\cos^2 i - \mu\varepsilon\mu'\varepsilon'\left(\mathrm{sen}^2 i\,\frac{\varepsilon\mu}{\varepsilon'\mu'} - 1\right)}\right\}$$

$$\varphi_\parallel = \operatorname{atan}\left\{\frac{-2\cos i\cdot\mu\sqrt{\mu\varepsilon\mu'\varepsilon'}\sqrt{\mathrm{sen}^2 i\,\varepsilon\mu\varepsilon'^{-1}\mu'^{-1} - 1}}{\mu^2\varepsilon'^2\cos^2 i - \mu^2\varepsilon^2\,\mathrm{sen}^2 i + \mu\varepsilon\mu'\varepsilon'}\right\}$$

$$\varphi_\parallel = \operatorname{atan}\left\{\frac{-2\cos i\sqrt{\frac{\varepsilon\mu'}{\mu\varepsilon'}}\sqrt{\mathrm{sen}^2 i\,\frac{\varepsilon\mu}{\varepsilon'\mu'} - 1}}{\cos^2 i - \left(\frac{\varepsilon}{\varepsilon'}\right)^2\mathrm{sen}^2 i + \frac{\varepsilon\mu'}{\mu\varepsilon'}}\right\}$$

$$\vec{E}'' = \left(\hat{e}_{\parallel}\, E \operatorname{sen}\alpha . e^{i\varphi_{\parallel}} + \hat{e}_{\perp}\, E . \cos\alpha\, e^{i\varphi_{\perp}}\right) . e^{i\left(\vec{k}.\vec{x} - \omega t\right)}$$

$$\varphi_{\perp} = \operatorname{atan}\left\{\frac{-2 . \cos i . \sqrt{\operatorname{sen}^2 i . \left(\varepsilon\mu/\varepsilon'\mu'\right) - 1} . \sqrt{\frac{\varepsilon'\mu}{\varepsilon\mu'}}}{\cos^2 i - \frac{\varepsilon'\mu}{\varepsilon\mu'} . \left(\operatorname{sen}^2 i . \frac{\varepsilon\mu}{\varepsilon'\mu'} - 1\right)}\right\}$$

$$\varphi_{\perp} = \operatorname{atan}\left\{\frac{-2 \cos i . \sqrt{\operatorname{sen}^2 i . \left(\mu/\mu'\right)^2 - \varepsilon'\mu/\varepsilon\mu'}}{\cos^2 i + \frac{\varepsilon'\mu}{\varepsilon\mu'} - \operatorname{sen}^2 i \left(\frac{\mu}{\mu'}\right)^2}\right\}$$

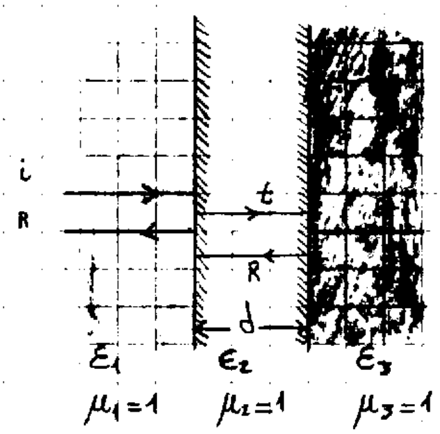
$$\varphi_{\parallel} = \operatorname{atan}\left\{\frac{-2 . \cos i . \sqrt{\operatorname{sen}^2 i \left(\varepsilon/\varepsilon'\right)^2 - \varepsilon\mu'/\varepsilon'\mu}}{\cos^2 i + \frac{\varepsilon\mu'}{\varepsilon'\mu} - \operatorname{sen}^2 i . \left(\frac{\varepsilon}{\varepsilon'}\right)^2}\right\}$$

si suponemos frecuencias ópticas $\Rightarrow$

$$\varphi_{\perp} = \operatorname{atan}\left\{\frac{-2 . \cos i \sqrt{\operatorname{sen}^2 i - \varepsilon'/\varepsilon}}{\cos^2 i + \frac{\varepsilon'}{\varepsilon} - \operatorname{sen}^2 i}\right\}$$

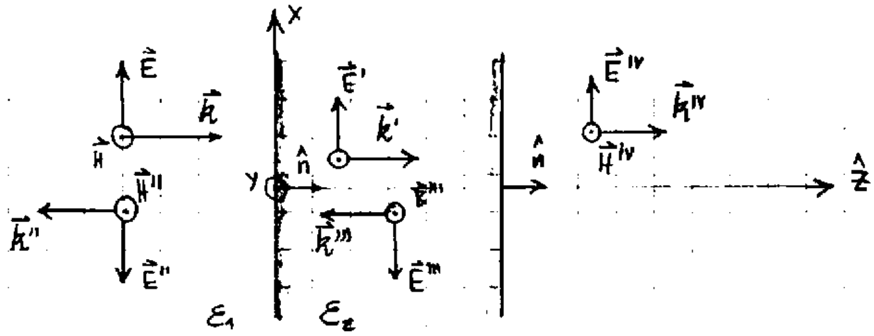
$$\varphi_{\parallel} = \operatorname{atan}\left\{\frac{-2 \cos i . \sqrt{\operatorname{sen}^2 i \left(\varepsilon/\varepsilon'\right)^2 - \varepsilon/\varepsilon'}}{\cos^2 i + \frac{\varepsilon}{\varepsilon'} - \operatorname{sen}^2 i . \left(\frac{\varepsilon}{\varepsilon'}\right)^2}\right\}$$

3.



$$\vec{E}(\vec{x},t) = \vec{E}_0 . e^{i(\vec{k}\cdot\vec{x} - \omega t)}$$

a.



* $\underline{E\hat{t} \text{ continuo}}$ $(z=0)$

$$\left[(\vec{E} + \vec{E}'') - (\vec{E}' + \vec{E}''')\right] \times \hat{n} = 0$$

$$(-E\hat{y} + E''\hat{y}) - (-E'\hat{y} + E'''\hat{y}) = 0$$

$$-E_0 + E_0'' = -E_0' + E_0'''$$

* $\underline{H\hat{t} \text{ continuo}}$ $(z=0)$

$$\left[(\vec{k} \times \vec{E}_0 + \vec{k}'' \times \vec{E}_0'') - (\vec{k}' \times \vec{E}_0' + \vec{k}''' \times \vec{E}_0''')\right] \times \hat{n} = 0$$

$\hat{x}$)
$$k.E_0 + k''.E_0'' - k'.E_0' - k'''.E_0''' = 0$$

$$\sqrt{\varepsilon_1\mu_1}\,\frac{\omega}{c} E_0 + \sqrt{\varepsilon_1\mu_1}\,\frac{\omega}{c} E_0'' - \sqrt{\varepsilon_2\mu_2}\,\frac{\omega}{c} E_0' - \sqrt{\varepsilon_2\mu_2}\,\frac{\omega}{c} E_0''' = 0$$

$$\sqrt{\varepsilon_1}\,E_0 + \sqrt{\varepsilon_1}\,E_0'' - \sqrt{\varepsilon_2}\,E_0' - \sqrt{\varepsilon_2}\,E_0''' = 0 \;\rightarrow\; \sqrt{\varepsilon_1}\,(E_0 + E_0'') = \sqrt{\varepsilon_2}\,(E_0' + E_0''')$$

* $\underline{E\hat{t} \text{ continuo}}$ $(z=d)$

$$\left[(\vec{E}_0'.e^{ik'.d} + \vec{E}_0'''.e^{-ik'''.d}) - \vec{E}_0^{IV}.e^{ik^{IV}.d}\right]_{z=d} \times \hat{n} = 0$$

$-\hat{y}$)
$$-E_0'.e^{ik'.d} + E_0'''.e^{-ik'''.d} - (-E_0^{IV}.e^{ik^{IV}.d}) = 0$$

$$-E_0'.e^{i\sqrt{\varepsilon_2\mu_2}.\frac{\omega}{c}.d} + E_0'''.e^{-i\sqrt{\varepsilon_2\mu_2}.\frac{\omega}{c}.d} = -E_0^{IV}.e^{i\sqrt{\varepsilon_3\mu_3}.\frac{\omega}{c}.d}$$

$$-E_0' + E_0'''.e^{-i\sqrt{\varepsilon_2}\frac{\omega}{c}2.d} = +E_0^{IV}.e^{(\sqrt{\varepsilon_3}-\sqrt{\varepsilon_2})i\frac{\omega}{c}d}$$

$$-E_0' + E_0'''.e^{-i\sqrt{\varepsilon_2}\frac{\omega}{c}2d} = -E_0^{IV}.e^{(\sqrt{\varepsilon_3}-\sqrt{\varepsilon_2})i\frac{\omega}{c}d}$$

* $\underline{H\hat{t} \text{ continuo}}$ $(z=d)$

$$\left[\left(\vec{k}'\times\vec{E}' + \vec{k}'''\times\vec{E}'''\right) - \left(\vec{k}^{IV}\times\vec{E}^{IV}\right)\right]_{z=d}\times\hat{n} = 0$$

$$\hat{x})\quad k'E_0' e^{ik'd} + k'''E_0''' e^{-ik'''d} - k^{IV}E_0^{IV}e^{ik^{IV}d} = 0$$

$$\sqrt{\varepsilon_2\mu_2}\frac{\omega}{c}E_0' e^{id\sqrt{\varepsilon_2\mu_2}\frac{\omega}{c}} + \sqrt{\varepsilon_2\mu_2}\frac{\omega}{c}E_0''' e^{-id\sqrt{\varepsilon_2\mu_2}\frac{\omega}{c}} = \sqrt{\varepsilon_3\mu_3}\frac{\omega}{c}E_0^{IV} e^{id\sqrt{\varepsilon_3\mu_3}\frac{\omega}{c}}$$

$$\sqrt{\varepsilon_2}E_0' + \sqrt{\varepsilon_2}E_0''' e^{-i\sqrt{\varepsilon_2}\frac{d2\omega}{c}} = \sqrt{\varepsilon_3}E_0^{IV} e^{(\sqrt{\varepsilon_3}-\sqrt{\varepsilon_2})\frac{i\omega d}{c}}$$

$$\sqrt{\varepsilon_2}\left(E_0' + E_0''' e^{-i\sqrt{\varepsilon_2}\frac{2\omega d}{c}}\right) = \sqrt{\varepsilon_3}e^{(\sqrt{\varepsilon_3}-\sqrt{\varepsilon_2})\frac{i\omega d}{c}}E_0^{IV}$$

$\boxed{1}$ $\quad -E_0 + E_0'' = -E_0' + E_0'''$

Nadesito $\quad E_0''=0$

$\boxed{2}$ $\quad \sqrt{\varepsilon_1}(E_0 + E_0'') = \sqrt{\varepsilon_2}(E_0''' + E_0')$

$\boxed{3}$ $\quad E_0' - E_0''' e^{i\sqrt{\varepsilon_2}\frac{2\omega d}{c}} = E_0^{IV} e^{(\sqrt{\varepsilon_3}-\sqrt{\varepsilon_2})\frac{i\omega d}{c}}$

$\boxed{4}$ $\quad \sqrt{\varepsilon_2}\left(E_0' + E_0''' e^{-i\frac{2\omega d}{c}\sqrt{\varepsilon_2}}\right) = \sqrt{\varepsilon_3}\, e^{(\sqrt{\varepsilon_3}-\sqrt{\varepsilon_2})\frac{i\omega d}{c}} E_0^{IV}$

de 1,2 $\quad \sqrt{\varepsilon_1}(E_0 + E_0'') = \sqrt{\varepsilon_2}(E_0' + E_0'' - E_0 + E_0')$

$$E_0''(\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}) = \sqrt{\varepsilon_2}\,2E_0' - (\sqrt{\varepsilon_2}+\sqrt{\varepsilon_1})E_0$$

$$E_0'' = \frac{2E_0'\sqrt{\varepsilon_2} - E_0(\sqrt{\varepsilon_2}+\sqrt{\varepsilon_1})}{\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2}}$$

de 3,4 $\quad \sqrt{\varepsilon_2}E_0' + \sqrt{\varepsilon_2}E_0''' e^{-i\frac{2\omega d}{c}\sqrt{\varepsilon_2}} = \sqrt{\varepsilon_3}E_0' - \sqrt{\varepsilon_3}E_0''' e^{i\frac{2\omega d}{c}\sqrt{\varepsilon_2}}$

$$E_0''' e^{-i\frac{2\omega d}{c}\sqrt{\varepsilon_2}}(\sqrt{\varepsilon_2}+\sqrt{\varepsilon_3}) = E_0'(\sqrt{\varepsilon_3}-\sqrt{\varepsilon_2})$$

$$(E_0'' - E_0 + E_0')\,e^{\quad}(\quad) = E_0'(\sqrt{\varepsilon_3}-\sqrt{\varepsilon_2})$$

$$E_0'' = E_0'\frac{(\sqrt{\varepsilon_3}-\sqrt{\varepsilon_2})}{(\sqrt{\varepsilon_2}+\sqrt{\varepsilon_3})}e^{i\frac{2\omega d}{c}\sqrt{\varepsilon_2}} - E_0' + E_0$$

$$\frac{2E_0'\sqrt{\varepsilon_2} - E_0(\sqrt{\varepsilon_2}+\sqrt{\varepsilon_1})}{(\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2})} = E_0'\left(\frac{(\sqrt{\varepsilon_3}-\sqrt{\varepsilon_2})}{(\sqrt{\varepsilon_3}+\sqrt{\varepsilon_2})}e^{i\frac{2\omega d}{c}\sqrt{\varepsilon_2}} - 1\right) + E_0$$

$$E_0'\underbrace{\left(\frac{2\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2}} - \frac{\sqrt{\varepsilon_3}-\sqrt{\varepsilon_2}}{(\sqrt{\varepsilon_3}+\sqrt{\varepsilon_2})}e^{i\frac{2\omega d}{c}\sqrt{\varepsilon_2}} + 1\right)}_{\to\, =\phi} = E_0\left(1 + \frac{\sqrt{\varepsilon_2}+\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2}}\right) = \frac{E_0 2\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1}\sqrt{\varepsilon_2}}$$

$$E_0' = \frac{2E_0\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2}}\cdot\frac{1}{[\quad\phi\quad]}$$

$$E_0'' = \frac{\dfrac{2\sqrt{\varepsilon_2}\,2E_0\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2}}\cdot\dfrac{1}{[\phi]} - E_0(\sqrt{\varepsilon_2}+\sqrt{\varepsilon_1})}{\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2}} = 0$$

$$\frac{4\sqrt{\varepsilon_2}\sqrt{\varepsilon_1}}{(\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2})\,\phi} = \sqrt{\varepsilon_2}+\sqrt{\varepsilon_1} \quad\Rightarrow$$

$$\frac{4\sqrt{\varepsilon_1}\sqrt{\varepsilon_2}}{(\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2})}\frac{1}{(\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2})} = \phi$$

$$\frac{\cancel{4}\sqrt{\varepsilon_1}\sqrt{\varepsilon_2}}{(\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2})}\cdot\frac{1}{(\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2})} = \frac{2\sqrt{\varepsilon_2}}{(\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2})} - \text{———}\, e^{i\frac{2\omega d}{c}\sqrt{\varepsilon_2}} + 1$$

$$\frac{2\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2}}\left(\frac{\sqrt{\varepsilon_1}\,2}{\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2}} - 1\right) = - \text{———}\, e + 1$$

$$\frac{2\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2}}\left(\frac{2\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2}} - 1\right) = 1 - \frac{(\sqrt{\varepsilon_3}-\sqrt{\varepsilon_2})}{(\sqrt{\varepsilon_2}+\sqrt{\varepsilon_3})}\, e^{i\frac{2\omega d}{c}\sqrt{\varepsilon_2}}$$

$$\frac{2\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2}}\left(\frac{2\sqrt{\varepsilon_1}-\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2}}\right) = \frac{2\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2}} =$$

$$\frac{2\sqrt{\varepsilon_2}-\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2}} = \frac{\sqrt{\varepsilon_2}-\sqrt{\varepsilon_3}}{(\sqrt{\varepsilon_2}+\sqrt{\varepsilon_3})}\, e^{i\frac{2\omega d}{c}\sqrt{\varepsilon_2}}$$

$$\frac{(\sqrt{\varepsilon_2}-\sqrt{\varepsilon_1})}{\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2}}\cdot\frac{(\sqrt{\varepsilon_2}+\sqrt{\varepsilon_3})}{(\sqrt{\varepsilon_2}-\sqrt{\varepsilon_3})} = e^{i\frac{2\omega d}{c}\sqrt{\varepsilon_2}}$$

$$= \begin{cases} \cos\left(\frac{2\omega d}{c}\sqrt{\varepsilon_2}\right) \\ i.\,\mathrm{sen}\left(\frac{2\omega d}{c}\sqrt{\varepsilon_2}\right) \end{cases}$$

$$\frac{2\omega d}{c}\sqrt{\varepsilon_2} = m\pi$$

$$\boxed{d = \frac{m\,\pi . c}{2\omega\sqrt{\varepsilon_2}}} \quad m \in \mathbb{Z}$$

$$\frac{\sqrt{\varepsilon_2}-\sqrt{\varepsilon_1}}{(\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2})}\cdot\frac{\sqrt{\varepsilon_2}+\sqrt{\varepsilon_3}}{(\sqrt{\varepsilon_2}-\sqrt{\varepsilon_3})} = 1$$

$*$ 1, m par

$$\cancel{\varepsilon_2} - \sqrt{\varepsilon_1 . \varepsilon_2} + \sqrt{\varepsilon_2\varepsilon_3} - \cancel{\sqrt{\varepsilon_1\varepsilon_3}} =$$

$$\sqrt{\varepsilon_1\varepsilon_2} + \cancel{\varepsilon_2} - \cancel{\sqrt{\varepsilon_1\varepsilon_3}} - \sqrt{\varepsilon_2\varepsilon_3}$$

$$\Rightarrow \quad -\sqrt{\varepsilon_1\varepsilon_2} + \sqrt{\varepsilon_2\varepsilon_3} = -\sqrt{\varepsilon_2\varepsilon_3} + \sqrt{\varepsilon_1\varepsilon_2}$$

$$\cancel{2}\sqrt{\cancel{\varepsilon_2}\,\varepsilon_3} = \cancel{2}\sqrt{\varepsilon_1\cancel{\varepsilon_2}}$$

$$\boxed{\varepsilon_3 = \varepsilon_1}$$

$*$ -1, m impar

$$\varepsilon_2 - \cancel{\sqrt{\varepsilon_1\varepsilon_2}} + \cancel{\sqrt{\varepsilon_2\varepsilon_2}} - \sqrt{\varepsilon_1\varepsilon_3} =$$

$$-\cancel{\sqrt{\varepsilon_1\varepsilon_2}} - \varepsilon_2 + \sqrt{\varepsilon_1\varepsilon_3} + \cancel{\sqrt{\varepsilon_2\varepsilon_3}}$$

$$\cancel{2}\varepsilon_2 = \cancel{2}\sqrt{\varepsilon_1\varepsilon_3}$$

$$\boxed{\varepsilon_2 = \sqrt{\varepsilon_1\varepsilon_3}}$$

$$\cancel{\varepsilon_2\sqrt{\varepsilon_2}} + \varepsilon_1\sqrt{\varepsilon_2} + \varepsilon_2\sqrt{\varepsilon_3} + \cancel{\varepsilon_1\sqrt{\varepsilon_3}} =$$

$$-\varepsilon_2\sqrt{\varepsilon_3} + \cancel{\varepsilon_1\sqrt{\varepsilon_3}} + \cancel{\varepsilon_2\sqrt{\varepsilon_2}} - \varepsilon_1\sqrt{\varepsilon_2}$$

$$2\varepsilon_2\sqrt{\varepsilon_3} = -2\varepsilon_1\sqrt{\varepsilon_2}$$

$$\frac{\sqrt{\varepsilon_3}}{\varepsilon_1} = -\frac{1}{\sqrt{\varepsilon_2}}$$

$$\varepsilon_2 = \frac{\varepsilon_1^2}{\varepsilon_3}$$

4.

$$\vec{E} = \vec{E}_0 \cdot e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$\vec{H} = \frac{\vec{B}}{\mu} = \frac{c}{\mu . \omega}\, \vec{k}\times\vec{E}$$

MLIH (μ, ε) homogéneos

$$\mu_{em} = \frac{1}{8\pi}\left(\vec{E}\cdot\vec{D} + \vec{B}\cdot\vec{H}\right) \Rightarrow$$

$$\langle \mu_e \rangle = \frac{1}{8\pi} \mathrm{Re}\left(\frac{\vec{E}\cdot\vec{D}^*}{2}\right) \qquad \langle \mu_m \rangle = \frac{1}{8\pi} \mathrm{Re}\left(\frac{\vec{B}\cdot\vec{H}^*}{2}\right)$$

Pero, en un conductor $\vec{k} \in \mathbb{C}$ y es

$$\vec{K} = k\sqrt{1 + i\frac{4\pi\sigma}{\omega\varepsilon}} = \sqrt{\mu\varepsilon}\cdot\frac{\omega}{c}\sqrt{1 + i\cdot\frac{4\pi\sigma}{\omega\varepsilon}}$$

$$\vec{K} = \left(\beta + i\frac{\alpha}{2}\right)\hat{k}$$ supomemos propagación en x

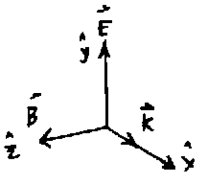
$$\vec{E} = \vec{E}_0 \cdot e^{i\left(\left[\beta + i\frac{\alpha}{2}\right]x - \omega t\right)} = \vec{E}_0 \cdot e^{-\frac{\alpha}{2}x} \cdot e^{i(\beta x - \omega t)}$$

con $$\frac{\alpha}{2} = \sqrt{\mu\varepsilon}\cdot\frac{\omega}{c}\cdot\left(\frac{\sqrt{1 + \left(4\pi\sigma/\omega\varepsilon\right)^2} - 1}{2}\right)^{1/2}$$

$$\langle \mu_e \rangle = \frac{1}{8\pi}\cdot\frac{\vec{E}_0}{2}\cdot e^{-\frac{\alpha}{2}x}\cdot e^{i(\beta x - \omega t)}\cdot\varepsilon\cdot\vec{E}_0\cdot e^{-\frac{\alpha}{2}x}\cdot e^{-i(\beta x - \omega t)}$$

$$\langle \mu_e \rangle = \frac{1}{16\pi}\left|\vec{E}_0\right|^2 \cdot e^{-\alpha x}\cdot\varepsilon$$ ← energía asociada al campo eléctrico

$$\vec{H} = \frac{c}{\mu\omega}\cdot\vec{K}\times\vec{E} = \frac{c}{\mu\omega}\left(\beta + i\frac{\alpha}{2}\right)\cdot\vec{E}_0\cdot e^{-\frac{\alpha}{2}x}\cdot e^{i(\beta x - \omega t)}$$



$$\langle \mu_m \rangle = \frac{1}{8\pi}\cdot\frac{c}{2\omega}\vec{E}_0\cdot e^{-\frac{\alpha x}{2}}\cdot\frac{c}{\mu\omega}\vec{E}_0\cdot e^{-\frac{\alpha x}{2}}\cdot\left(\beta^2 + \frac{\alpha^2}{4}\right)$$

forma de $\vec{H}$

$$\frac{c}{\mu\omega}\vec{E}_0\, e^{-\frac{\alpha x}{2}}\cdot\left(\beta + i\frac{\alpha}{2}\right) e^{i(\beta x - \omega t)}$$

$$\cdot\, e^{i(\beta x - \omega t)}\cdot\left(\beta^2 + \frac{\alpha^2}{4}\right)^{1/2}\cdot e^{i\cdot\mathrm{atan}\left(\frac{\alpha}{2\beta}\right)}$$

$$\frac{c}{\mu\omega}\vec{E}_0\cdot e^{-\frac{\alpha x}{2}}\cdot e^{i[\beta x - \omega t + \mathrm{atan}(\alpha/2\beta)]}\cdot\left(\beta^2 + \frac{\alpha^2}{4}\right)^{1/2}$$

$$\langle \mu_m \rangle = \frac{1}{16\pi}\frac{c^2}{\mu\omega^2}\left|\vec{E}_0\right|^2 e^{-\alpha x}\cdot\left(\beta^2 + \frac{\alpha^2}{4}\right)$$

$$\langle \mu_m \rangle = \frac{c^2}{\mu\omega^2}\cdot\frac{\langle \mu_e \rangle}{\varepsilon}\cdot\left(\beta^2 + \frac{\alpha^2}{4}\right)$$

$$\langle \mu_m \rangle = \frac{c^2}{\omega^2}\cdot\frac{1}{\mu\varepsilon}\langle \mu_e \rangle\left(\beta^2 + \frac{\alpha^2}{4}\right)$$

$$\boxed{\langle \mu_m \rangle = \frac{1}{k^2}\cdot\left(\beta^2 + \frac{\alpha^2}{4}\right)\langle \mu_e \rangle}$$

Relación entre las energías asociadas a los campos

a. Supongamos un mal conductor (dieléctrico)

$$\Rightarrow \quad \frac{4\pi\sigma}{\omega\varepsilon} \ll 1 \quad \rightarrow \quad \left(1+\left(\frac{4\pi\sigma}{\omega\varepsilon}\right)^2\right)^{1/2} = 1 + \frac{1}{2}\left(\frac{4\pi\sigma}{\omega\varepsilon}\right)^2$$

$$\beta = \sqrt{\mu\varepsilon}\cdot\frac{\omega}{c}\cdot\left(\frac{\sqrt{1+0}+1}{2}\right)^{1/2} \qquad \frac{\alpha}{2} = \sqrt{\mu\varepsilon}\cdot\frac{\omega}{c}\left(\frac{\sqrt{1+0}-1}{2}\right)^{1/2}$$

$$\left(\frac{1+1/2\left(4\pi\sigma/\omega\varepsilon\right)^2+1}{2}\right)^{1/2} \qquad \left(\frac{1+1/2\left(4\pi\sigma/\omega\varepsilon\right)^2-1}{2}\right)^{1/2}$$

$$\left[1+\underbrace{\frac{1}{4}\left(\frac{4\pi\sigma}{\omega\varepsilon}\right)^2}_{\sim 0}\right]^{1/2} \qquad \left[\frac{1}{4}\left(\frac{4\pi\sigma}{\omega\varepsilon}\right)^2\right]^{1/2}$$

$$\beta \cong \sqrt{\mu\varepsilon}\cdot\frac{\omega}{c} \qquad \frac{\alpha}{2} \approx \sqrt{\mu\varepsilon}\,\frac{\cancel{\omega}}{c}\cdot\frac{1}{2}\cdot\frac{4\pi\sigma}{\cancel{\omega}\varepsilon}$$

$$\beta^2 \cong \mu\varepsilon\frac{\omega^2}{c^2} \qquad \frac{\alpha}{2} \approx \frac{4\pi\sigma\sqrt{\mu}}{\sqrt{\varepsilon}\,2c}$$

$$\frac{\alpha^2}{4} \approx \frac{\cancel{16}^{4}\pi^2\sigma^2\mu}{\varepsilon\,\cancel{4}_{1}\,c^2}$$

$$\left(\beta^2+\frac{\alpha^2}{4}\right) = \mu\varepsilon\frac{\omega^2}{c^2}+\frac{4\pi^2\sigma^2\mu}{\varepsilon c^2}$$

$$\langle\mu_M\rangle = \frac{1}{\mu\varepsilon\frac{\omega^2}{c^2}}\cdot\left(\mu\varepsilon\frac{\omega^2}{c^2}+\frac{4\pi^2\sigma^2\mu}{\varepsilon . c^2}\right)\langle\mu_e\rangle = \left(1+\frac{4\pi^2\sigma^2}{\omega^2\varepsilon^2}\right)\langle\mu_e\rangle$$

$$\boxed{\langle\mu_M\rangle = \left(1+\frac{1}{4}\left[\frac{4\pi\sigma}{\omega . \varepsilon}\right]^2\right)\langle\mu_e\rangle}$$

b. Supongamos ahora un buen conductor (muy buen conductor)

$$\frac{4\pi\sigma}{\omega\varepsilon} \gg 1 \quad \Rightarrow \quad 1+\left(\frac{4\pi\sigma}{\omega\varepsilon}\right)^2 \cong \left(\frac{4\pi\sigma}{\omega\varepsilon}\right)^2$$

$$\beta \cong \sqrt{\mu\varepsilon}\cdot\frac{\omega}{c}\cdot\left(\frac{\frac{4\pi\sigma}{\omega\varepsilon}}{2}\right)^{1/2} \qquad \frac{\alpha}{2} \cong \sqrt{\mu\varepsilon}\,\frac{\omega}{c}\cdot\left(\frac{\frac{4\pi\sigma}{\omega\varepsilon}}{2}\right)^{1/2}$$

$$\sqrt{\mu\varepsilon}.\ \frac{\omega}{c}\left(\frac{2\pi\sigma}{\omega\varepsilon}\right)^{1/2}$$

$$\beta^2+\frac{\alpha^2}{4} = \left(\mu\varepsilon.\frac{\omega^2}{c^2}\cdot\frac{2\pi\sigma}{\omega\varepsilon}\right)2 \quad \Rightarrow$$

$$\langle\mu_M\rangle = \frac{1}{\cancel{\mu\varepsilon}\cancel{\frac{\omega^2}{c^2}}}\cdot\cancel{\mu\varepsilon}\cancel{\frac{\omega^2}{c^2}}\cdot\frac{2\pi\sigma}{\omega\varepsilon}.2\langle\mu_e\rangle = \frac{4\pi\sigma}{\omega\varepsilon}\langle\mu_e\rangle$$

$$\boxed{\langle\mu_M\rangle = \frac{4\pi\sigma\langle\mu_e\rangle}{\omega\varepsilon}}$$

5.

$\vec{A}(\vec{x},t)$, $\vec{B}(\vec{x},t)$ armónicos $\Rightarrow$

$$\left.\begin{aligned} \vec{A}(\vec{x},t) &= \vec{A}(\vec{x}).e^{i\omega t} \\ \vec{B}(\vec{x},t) &= \vec{B}(\vec{x}).e^{i\omega t} \end{aligned}\right\} \text{con } \vec{A}(\vec{x}), \vec{B}(\vec{x}) \in \mathbb{C}$$

$$\mathcal{R}e\{\vec{A}(\vec{x},t)\}.\mathcal{R}e\{\vec{B}(\vec{x},t)\} =$$

$$|\vec{A}(\vec{x})|.\cos(\omega t+\varphi_A).\cos(\omega t+\varphi_B).|\vec{B}(\vec{x})|$$

Pero esto ya es muy complicado por las fases de cada complejo

$$\mathcal{R}e\{\vec{A}(\vec{x},t)\} = \frac{\vec{A}(\vec{x}).e^{i\omega t} + \vec{A}^*(\vec{x}).e^{-i\omega t}}{2}$$

$$\mathcal{R}e\{\vec{B}(\vec{x},t)\} = \frac{\vec{B}(\vec{x}).e^{i\omega t} + \vec{B}^*(\vec{x}).e^{-i\omega t}}{2}$$

$$\Rightarrow \quad \mathcal{R}e\{\vec{A}(\vec{x},t)\}.\mathcal{R}e\{\vec{B}(\vec{x},t)\} =$$

$$\frac{1}{4}\left(\vec{A}(\vec{x}).\vec{B}(\vec{x}).e^{2i\omega t} + \vec{A}(\vec{x}).\vec{B}^*(\vec{x}) + \vec{A}^*(\vec{x}).\vec{B}(\vec{x}) + \vec{A}^*(\vec{x}).\vec{B}^*(\vec{x}).e^{-2i\omega t}\right) =$$

$$\frac{1}{4}\left(2\,\mathcal{R}e\{\vec{A}(\vec{x}).\vec{B}(\vec{x}).e^{2i\omega t}\} + 2\,\mathcal{R}e\{\vec{A}(\vec{x}).\vec{B}^*(\vec{x})\}\right) =$$

$$\frac{1}{2}\,\mathcal{R}e\{\vec{A}(\vec{x}).\vec{B}^*(\vec{x}) + \vec{A}(\vec{x}).\vec{B}(\vec{x}).e^{2i\omega t}\} \Rightarrow$$

$$\langle \mathcal{R}e\,\vec{A}\,.\,\mathcal{R}e\,\vec{B}\rangle = \frac{1}{2}\mathcal{R}e\left\{\vec{A}(\vec{x}).\vec{B}^*(\vec{x}) + \vec{A}(\vec{x}).\vec{B}(\vec{x}).\int_0^{-\frac{\pi n}{\omega}} e^{2i\omega t}\,dt\,.\frac{1}{n}\right\}$$

$$\cancel{2}i\omega t = \cancel{2}\pi n$$
$$t = -\frac{\pi n}{\omega}$$

integral

$2i\omega t = u$

$2i\omega dt = du$

$dt = -\frac{du}{2\omega}$

$$-\int_0^{2i\pi n} \frac{e^u.du}{2\omega}.\frac{1}{n} =$$

$$-\left(e^{2i\pi n} - e^0\right).\frac{1}{n} =$$

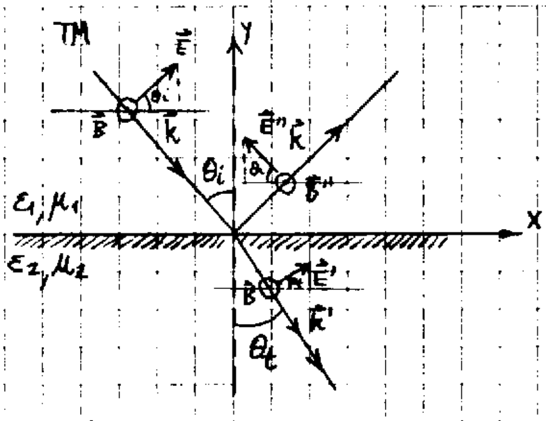
$$\left[1 - \left(\cos(2\pi n) + i.\text{sen}\,2\pi n\right)\right].\frac{1}{n} =$$

$$[1-1].\frac{1}{n} = 0$$

$$\boxed{\langle \mathcal{R}e(\vec{A}).\mathcal{R}e(\vec{B})\rangle = \frac{1}{2}\,\mathcal{R}e\left(\vec{A}(\vec{x}).\vec{B}^*(\vec{x})\right)}$$

$$\mathbb{P}_{rad} = \frac{dF_{mec}}{dS}$$

$$\frac{d\vec{P}}{dt} = \oint_S \overleftrightarrow{T} \cdot d\vec{S} \qquad \text{con } \vec{P} = \vec{P}_{Mec} + \vec{P}_{campo}$$



$$\vec{F}_{mec} = \oint_S \overleftrightarrow{T} \cdot d\vec{S} - \frac{d}{dt} \int_V \frac{\vec{E} \times \vec{B}}{4\pi c}\, dV$$

La superficie S debe usar la normal $-\hat{y}$; pero dado nuestro sistema coordenado invertiremos el signo y será $\frac{d\vec{P}}{dt} = -\oint_S \overleftrightarrow{T} \cdot d\vec{S}$ con $\hat{n} \equiv \hat{y}$

Para la presión me interesará la fuerza en $\hat{y}$

* contornos

$\vec{D}_n$ $\quad \left[\varepsilon_1 (\vec{E}_0 + \vec{E}_0'') - \varepsilon_2 \vec{E}_0' \right] \cdot \hat{n} = 0$

$$\varepsilon_1 \operatorname{sen}(\theta_i) (E_0 + E_0'') = \varepsilon_2 E_0' \operatorname{sen}(\theta_t)$$

$\vec{B}_t$ $\quad \frac{1}{\mu_1} (k E_0 + k E_0'') - \frac{1}{\mu_2} k' E_0' = 0$

$$\sqrt{\frac{\varepsilon_1}{\mu_1}} (E_0 + E_0'') = \sqrt{\frac{\varepsilon_2}{\mu_2}} E_0'$$

$\vec{E}_t$ $\quad \left[(\vec{E}_0 + \vec{E}_0'') - \vec{E}_0' \right] \times \hat{n} = 0$

$$\cos(\theta_i) E_0 - \cos(\theta_i) E_0'' = \cos(\theta_t) E_0'$$

$$\vec{B} = \sqrt{\mu \varepsilon}\, \frac{\vec{k} \times \vec{E}}{k}$$

$$B_0 = \sqrt{\mu \varepsilon}\, E_0$$

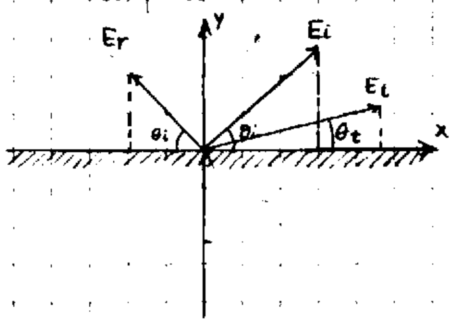
definimos

$E_0 \equiv E_i$
$E_0'' \equiv E_r$
$E_0' \equiv E_t$

$$\begin{cases} \varepsilon_1 \operatorname{sen} \theta_i (E_i + E_r) = \varepsilon_2 E_t \operatorname{sen} \theta_t \\ \cos \theta_i (E_i - E_r) = \cos \theta_t E_t \end{cases}$$

$B_i = \sqrt{\mu_1 \varepsilon_1}\, E_i$
$B_r = \sqrt{\mu_1 \varepsilon_1}\, E_r$
$B_t = \sqrt{\mu_2 \varepsilon_2}\, E_t$

En el plano ZX



$\vec{E}_r \cdot \hat{y} = |E_r| \operatorname{sen} \theta_i$

$\vec{E}_i \cdot \hat{y} = |E_i| \operatorname{sen} \theta_i$

$\vec{E}_t \cdot \hat{y} = |E_t| \operatorname{sen} \theta_t$

$\therefore$ En el plano ZX es:

$\vec{E} = E\hat{y} + E\hat{x}$
$\vec{B} = B\hat{z}$

$$\overleftrightarrow{T} = \begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix}$$

$$d\vec{S} = \begin{pmatrix} 0 \\ dx\, dz\ \hat{y} \\ 0 \end{pmatrix}$$

$$\overleftrightarrow{T} \cdot d\vec{S} \cdot \hat{y} = \frac{1}{4\pi} \left(\varepsilon E_y^2 - \frac{1}{2} \varepsilon E_x^2 - \frac{1}{2} \varepsilon E_y^2 - \frac{1}{2} \frac{B_z^2}{\mu} \right) d\vec{S} \cdot \hat{y}$$

$$d\vec{F} \cdot \hat{y} = T_{yy}\, dS$$

$$d\vec{F} \cdot \hat{y} = \frac{1}{4\pi} \left(\varepsilon \frac{E_y^2}{2} - \varepsilon \frac{E_x^2}{2} - \frac{\mu^{-1}}{2} B_z^2 \right)$$

$$B_z^2 = \left(\sqrt{\mu_1 \varepsilon_1} (E_i + E_r) \right)^2$$

la normal es en $-\hat{y}$

$$-\frac{1}{4\pi} \left[\frac{\varepsilon_1}{2} \operatorname{sen}^2 \theta_i (E_i + E_r)^2 - \frac{\varepsilon_1}{2} (E_i - E_r)^2 \cos^2 \theta_i - \frac{1}{2 \cancel{\mu_1}} (E_i + E_r)^2 \cancel{\mu_1} \varepsilon_1 \right]$$

$$+ \frac{1}{4\pi} \left[\frac{\varepsilon_2}{2} \operatorname{sen}^2 \theta_t E_t^2 - \frac{\varepsilon_2}{2} E_t^2 \cos^2 \theta_t - \frac{\cancel{\mu_2^{-1}}}{2} \cancel{\mu_2} \varepsilon_2 E_t^2 \right]$$

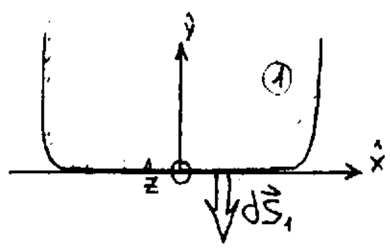
$$= \frac{1}{8\pi}\Big(-\varepsilon_1 \operatorname{sen}^2\theta_i \,(E_i+E_r)^2 + \varepsilon_1 (E_i-E_r)^2 \cos^2\theta_i + \varepsilon_1 (E_i+E_r)^2 - \varepsilon_2 \operatorname{sen}^2\theta_t E_t^2$$
$$+\varepsilon_2 E_t^2 \cos^2\theta_t + \varepsilon_2 E_t^2 \Big)$$

$$= \frac{1}{8\pi}\Big[(E_i+E_r)^2 \varepsilon_1 \left[1-\operatorname{sen}^2\theta_i\right] + \varepsilon_1 (E_i-E_r)^2 \cos^2\theta_i - \varepsilon_2 E_t^2 \left(1+\cos^2\theta_t - \operatorname{sen}^2\theta_t\right)\Big]$$

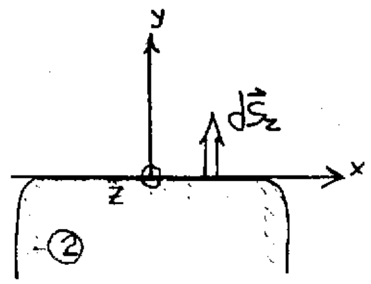
$$d\vec{F}\cdot\hat{y} = \frac{1}{8\pi}\left[2\varepsilon_1 \cos^2\theta_i \left(E_i^2+E_r^2\right) + \varepsilon_2 E_t^2 \cdot 2\cos^2\theta_t \right]$$

$$\underset{\text{Radiación}}{\text{Presión}} = \frac{1}{4\pi}\left(\left[|E_i|^2 + |E_r|^2\right] \varepsilon_1 \cos^2\theta_i + |E_t|^2 \varepsilon_2 \cos^2\theta_t \right)$$

* Signos [CONVENCIÓN]



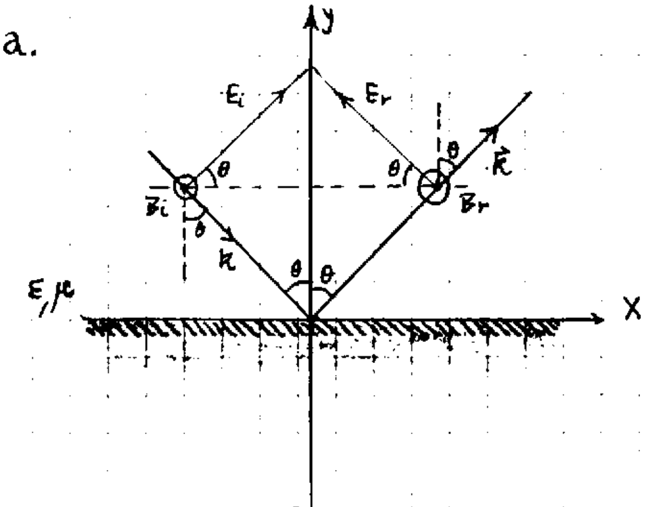
$d\vec{S}_1 = dx\,dz(-\hat{y})$



$d\vec{S}_2 = dx\,dz(\hat{y})$

6.

a.



Onda plana incide sobre espejo perfecto con ángulo θ $\Rightarrow$
No hay onda transmitida

$k_{inc} = k_{refl.}$

$$\vec{E}_i = E_i \, e^{i[\vec{k}\vec{x} - \omega t]} = E_i . e^{i[-k.y.\cos\theta + k.x.\operatorname{sen}\theta - \omega t]} . (\cos\theta\,\hat{x} + \operatorname{sen}\theta\,\hat{y})$$

$$\vec{E}_r = E_r . e^{i[k.y.\cos\theta + k.x.\operatorname{sen}\theta - \omega t]} . (-\cos\theta\,\hat{x} + \operatorname{sen}\theta\,\hat{y})$$

$$\vec{B}_i = \sqrt{\mu\varepsilon}\ \hat{k} \times \vec{E}_i = \sqrt{\mu\varepsilon} . E_i . e^{i(-k.y.\cos\theta + k.x.\operatorname{sen}\theta - \omega t)} . \hat{z}$$

$$\vec{B}_r = \sqrt{\mu\varepsilon} . E_r . e^{i(k.y.\cos\theta + k.x.\operatorname{sen}\theta - \omega t)} . \hat{z}$$

* Contornos

$E_{\hat{t}}$, $B_{\hat{n}}$ continuos $\Rightarrow$

• $D_{\hat{n}}$, $H_{\hat{t}}$ no pueden usarse porque hay $\vec{j}$, σ superficiales

$$E_i . \cos\theta - E_r . \cos\theta = 0$$
$$E_i = E_r$$

* Tensor de Maxwell

$$dS\,\hat{y} = d\vec{S} = \begin{pmatrix} 0 \\ -dx.dz \\ 0 \end{pmatrix} \Rightarrow \overleftrightarrow{T} . d\vec{S} = \begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix} . \begin{pmatrix} 0 \\ -dS \\ 0 \end{pmatrix} = \begin{pmatrix} -T_{xy}.dS \\ -T_{yy}.dS \\ -T_{zy}.dS \end{pmatrix}$$

$$-T_{yy} . dS = -\frac{1}{4\pi}\left(\varepsilon E_y^2 + \mu \underbrace{H_y^2}_{=0} - \frac{\varepsilon}{2}\left(E_x^2 + E_y^2 + \underbrace{E_z^2}_{=0}\right) - \frac{\mu}{2}\left(\underbrace{H_x^2}_{=0} + \underbrace{H_y^2}_{=0} + H_z^2\right)\right) . dS$$

$$-T_{yy} . dS = -\frac{1}{4\pi}\left(\frac{\varepsilon}{2} E_y^2 - \frac{\varepsilon}{2} E_x^2 - \frac{\mu}{2} H_z^2\right) dS \Rightarrow$$

$$\langle -T_{yy} \rangle = \left(\frac{\varepsilon}{4}\left[\mathcal{Re}(E_y \cdot E_y^*) - \mathcal{Re}(E_x \cdot E_x^*)\right] - \frac{\mu}{4}\left[\mathcal{Re}(H_z \cdot H_z^*)\right]\right)\left(-\frac{1}{4\pi}\right)$$

$$E_y . E_y^* = \left[E_i . e^{i(-ky\cos\theta + kx\operatorname{sen}\theta - \omega t)} . \operatorname{sen}\theta + E_i . e^{i(ky\cos\theta + kx\operatorname{sen}\theta - \omega t)} . \operatorname{sen}\theta\right] .$$
$$\left[E_i . e^{i(ky\cos\theta - kx\operatorname{sen}\theta + \omega t)} . \operatorname{sen}\theta + E_i . e^{i(-ky\cos\theta - kx\operatorname{sen}\theta + \omega t)} . \operatorname{sen}\theta\right]$$

$$= E_i . \operatorname{sen}\theta . e^{i(kx\operatorname{sen}\theta - \omega t)} \left(e^{-iky\cos\theta} + e^{iky\cos\theta}\right) .$$
$$E_i . \operatorname{sen}\theta . e^{-i(kx\operatorname{sen}\theta - \omega t)} \cdot \left(e^{iky\cos\theta} + e^{-iky\cos\theta}\right)$$

$$= E_i^2 . \operatorname{sen}^2\theta . \left(\cos[k.y.\cos\theta]\right) . \left(\cos[k.y.\cos\theta]\right) . 4 \Rightarrow E_y E_y^*\big|_{y=0} = 4E_i^2 \operatorname{sen}^2\theta$$

$$E_x \cdot E_x^* = E_i \cos\theta \cdot e^{i(kx\,\text{sen}\theta - \omega t)} \left[e^{-iky\cos\theta} - e^{iky\cos\theta} \right]$$

$$E_i \cos\theta \cdot e^{-i(kx\,\text{sen}\theta - \omega t)} \left[e^{iky\cos\theta} - e^{-iky\cos\theta} \right] = 0 = E_x E_x^* \big|_{y=0}$$

$$H_z \cdot H_z^* = \frac{B_z B_z^*}{\mu^2} = \frac{\cancel{4}\,\varepsilon\, E_i^2}{\mu^2}$$

$$\Rightarrow$$

$$\langle -T_{yy} \rangle = \left(\frac{\varepsilon}{4} 4 E_i^2 \,\text{sen}^2\theta - \frac{\cancel{4}}{4} \frac{\varepsilon}{\cancel{4}} E_i^2 \right) \frac{1}{4\pi}$$

$$= \frac{\varepsilon E_i^2}{4} \left[4\,\text{sen}^2\theta - 1 \right] \frac{1}{4\pi} = \frac{\varepsilon}{16\pi} E_i^2 \left(1 - 4\,\text{sen}^2\theta \right)$$

$$\boxed{\text{Presión Radiación} = \langle -T_{yy} \rangle = \frac{\varepsilon}{16\pi} E_i^2 \left(1 - 4\,\text{sen}^2\theta \right)}$$

b. Si $\theta = 0$ (incidencia normal) $\Rightarrow$

$$\boxed{\text{Presión Radiación} = \frac{\varepsilon}{16\pi} E_i^2}$$

$$\langle \mu_{eM} \rangle = \frac{1}{8\pi} \cdot \frac{1}{2} \,\mathrm{Re} \left(E \cdot D^* + B \cdot H^* \right)$$

$$\frac{1}{16\pi} \,\mathrm{Re} \Big\{ \varepsilon E_i^2 \,\text{sen}^2\theta \, e^{i(kx\,\text{sen}\theta - \omega t)} \, 2^2 \cos^2(ky\cos\theta) \, e^{-i(kx\,\text{sen}\theta - \omega t)}$$

$$\varepsilon E_i^2 \cos^2\theta \, e^{i(kx\,\text{sen}\theta - \omega t)} \, 2^2 i^2 \,\text{sen}^2(ky\cos\theta) \, e^{-i(kx\,\text{sen}\theta - \omega t)}$$

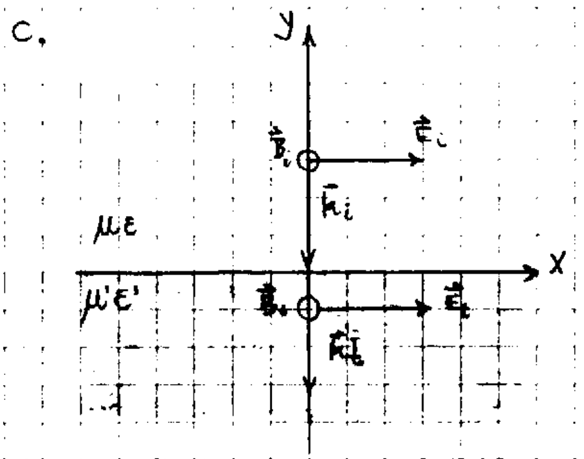
$$+ \frac{\cancel{4}\,\varepsilon E_i^2}{\cancel{4}} e^{i(\ldots)} e^{-i(\ldots)} \Big\}$$

$$\langle \mu_{eM} \rangle = \frac{1}{16\pi} \Big(\varepsilon E_i^2 \,\text{sen}^2\theta \, 4 \cos^2(ky\cos\theta) - \varepsilon E_i^2 \cos^2\theta \, 4 \,\text{sen}^2(ky\cos\theta) + \varepsilon E_i^2 \Big)$$

Pero si $\theta = 0$ $\Rightarrow$

$$\boxed{\mu_{eM} = \frac{1}{16\pi} \varepsilon \cdot E_i^2}$$

c.

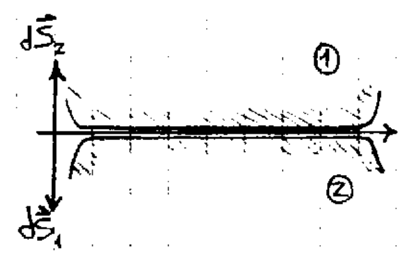


$$\vec{E}_i = E_i \, e^{i(ky - \omega t)} \,\hat{x}$$

$$\vec{B}_i = \sqrt{\mu\varepsilon}\, E_i \, e^{i(ky - \omega t)} \,\hat{z}$$

$$\vec{E}_t = E_t \, e^{i(-k'y - \omega t)} \,\hat{x}$$

$$\vec{B}_t = \sqrt{\mu'\varepsilon'}\, E_t \, e^{i(-k'y - \omega t)} \,\hat{z}$$



$$d\vec{F} = \overleftrightarrow{T}\cdot d\vec{S} = \begin{pmatrix} T_{xx} & & \\ T_{yx} & T_{yy} & T_{yz} \\ & & T_{zz}\end{pmatrix}\begin{pmatrix}0\\ -dx\,dz\\ 0\end{pmatrix} = \begin{pmatrix}-T_{xy}\cdot dx\,dz\\ -T_{yy}\cdot dx\,dz\\ -T_{zy}\cdot dx\,dz\end{pmatrix}$$

me interesa $\hat{y}$

$$\text{Presión}_{\text{rad}} = \langle -T_{yy}\rangle$$

* medio ①

$$(4\pi).-T_{yy} = -\left[\varepsilon E_y^2 + \mu H_y^2 - \frac{\varepsilon}{2}\left(E_y^2+E_z^2+E_x^2\right) - \frac{\mu}{2}\left(H_x^2+H_y^2+H_z^2\right)\right]$$

$$(4\pi).-T_{yy} = \left[-\varepsilon\frac{E_y^2}{2} + \varepsilon\frac{E_x^2}{2} + \varepsilon\frac{E_z^2}{2} - \mu\frac{H_y^2}{2} + \mu\frac{H_x^2}{2} + \mu\frac{H_z^2}{2}\right]$$

Pero $\vec{E} = E_x$, $\vec{B} = B_z \rightarrow$ $\quad -T_{yy} = \left(\varepsilon\frac{E_x^2}{2} + \mu\frac{H_z^2}{2}\right)\frac{1}{4\pi}$ ∴

$$\langle -T_{yy}\rangle = \frac{1}{8\pi}\cdot\frac{1}{2}\,\mathrm{Re}\left\{\varepsilon E_x\cdot E_x^* + \mu H_z\cdot H_z^*\right\}$$

$$= \frac{1}{16\pi}\,\mathrm{Re}\left\{\varepsilon.\,E_i.\,e^{-i\cancel{(ky+\omega t)}}.\,E_i.\,e^{i(ky+\omega t)} + \frac{\cancel{\mu}\sqrt{\mu\varepsilon}\,E_i\,e^{-i(ky+\omega t)}\;\sqrt{\mu\varepsilon}.\,E_i.\,e^{i(ky+\omega t)}}{\cancel{\mu^2}}\right\}$$

$$P_{\text{resión rad}} = \frac{1}{16\pi}\,\mathrm{Re}\left\{\varepsilon E_i^2 + \varepsilon E_i^2\right\} = \frac{1}{16\pi}\,2.E_i^2.\varepsilon$$

* medio ②

$$d\vec{F} = \overleftrightarrow{T}\cdot d\vec{S} = \begin{pmatrix} T_{xx} & & \\ T_{yx} & T_{yy} & T_{yz} \\ & & T_{zz}\end{pmatrix}\begin{pmatrix}0\\ dx\,dz\\ 0\end{pmatrix} = \left(T_{yy}.dx\,dz\right)$$

me interesa $\hat{y}$

$\text{Presión}_{\text{rad}} = \langle T_{yy}\rangle$ $\Rightarrow$ análogamente a lo considerado para el medio 1 es:

$$T_{yy} = \frac{1}{4\pi}\left(-\varepsilon'\frac{E_x^2}{2} - \mu'\frac{H_z^2}{2}\right) \quad ∴$$

$$\langle T_{yy}\rangle = \frac{1}{16\pi}\,\mathrm{Re}\left\{-\varepsilon' E_x\cdot E_x^* - \mu' H_z\cdot H_z^*\right\}$$

$$= \frac{1}{16\pi}\,\mathrm{Re}\left\{-\varepsilon'.\,E_t^2 - \cancel{\mu'}.\,\frac{\mu'\varepsilon}{\cancel{\mu'^2}}\,E_t^2\right\} = \frac{1}{16\pi}\left(-2\varepsilon' E_t^2\right)$$

* Contorno

$B\hat{n}$ continua

$$\hookrightarrow \sqrt{\mu\varepsilon}\,E_i - \sqrt{\mu'\varepsilon'}\,E_t = 0 \quad\rightarrow\quad E_t = E_i\left(\frac{\mu\varepsilon}{\mu'\varepsilon'}\right)^{1/2}$$

$$P_{\text{rad}} = \frac{1}{16\pi}\,\varepsilon\,2E_i^2 - \left(\frac{-1}{16\pi}\,\cancel{\varepsilon'}\,2\,E_i^2\,\frac{\mu\varepsilon}{\mu'\cancel{\varepsilon'}}\right) = \boxed{\frac{1}{16\pi}\,E_i^2.2.\varepsilon\left(1+\frac{\mu}{\mu'}\right)}$$

$$\langle \mu_{EM} \rangle = \frac{1}{16\pi} \, \mathrm{Re}\left\{ E \cdot D^* + B \cdot H^* \right\}$$

* medio 1

$$= \frac{1}{16\pi} \, \mathrm{Re}\left\{ \varepsilon \, E_i^2 + \frac{\cancel{\mu}\varepsilon}{\cancel{\mu}} E_i^2 \right\}$$

* medio 2

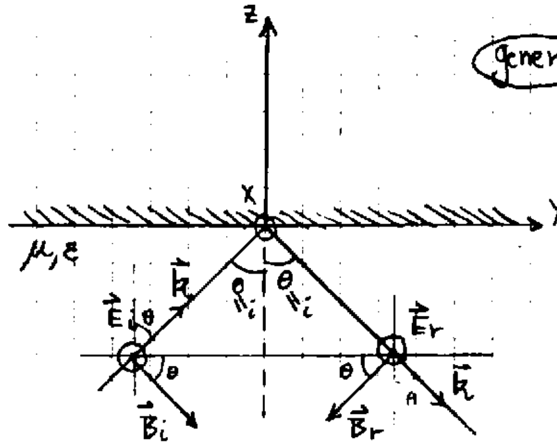
$$= \frac{1}{16\pi} \, \mathrm{Re}\left\{ \varepsilon' E_t^2 + \frac{\cancel{\mu}\varepsilon'}{\cancel{\mu}} E_t^2 \right\}$$

$$\langle \mu_{EM} \rangle = \frac{1}{16\pi} 2 \varepsilon E_i^2 + \frac{1}{16\pi} 2 \cancel{\varepsilon} \left(E_i^2 \frac{\mu \varepsilon}{\mu' \cancel{\varepsilon}} \right)$$

$$\boxed{\langle \mu_{em} \rangle = \frac{1}{16\pi} 2 \varepsilon E_i^2 \left(1 + \frac{\mu}{\mu'} \right)}$$

▲ La densidad de energía es igual a la preción de radiación

7. Análisis de las experiencias de Wiener



Generic a.

$$\vec{E}_i = E_i . e^{i(\vec{k}.\vec{x} - \omega t)} . \hat{x}$$
$$\vec{E}_i = E_i . e^{i(k_y . y + k_z . z - \omega t)} . \hat{x}$$
$$\vec{B}_i = \sqrt{\mu\varepsilon} . E_i . e^{i(k_y . y + k_z . z - \omega t)} . (\cos\theta \, \hat{y} - \operatorname{sen}\theta \, \hat{z})$$
$$\vec{E}_r = E_r . e^{i(k_y . y - k_z . z - \omega t)} . \hat{x}$$
$$\vec{B}_r = \sqrt{\mu\varepsilon} . E_r . e^{i(k_y . y - k_z . z - \omega t)} . (-\cos\theta \, \hat{y} - \operatorname{sen}\theta \, \hat{z})$$

* Contornos Usamos $B\hat{n} = 0 \quad E\hat{t} = 0 \rightarrow$

$$-\sqrt{\mu\varepsilon} . E_i . \operatorname{sen}\theta - \sqrt{\mu\varepsilon} \, E_r . \operatorname{sen}\theta = 0 \Rightarrow E_i = -E_r$$

$$k_y = |\vec{k}| . \operatorname{sen}\theta \, , \quad k_z = |\vec{k}| . \cos\theta$$ (k es la misma en cada cuadrante)

* Campos

$$\boxed{\vec{E} = \vec{E}_i + \vec{E}_r = E_i . e^{i(k \operatorname{sen}\theta \, y - \omega t)} \left(e^{ik . \cos\theta \, z} - e^{-ik\cos\theta \, z} \right) \hat{x}}$$

$$\vec{B} = B_i + B_r = \sqrt{\mu\varepsilon} . E_i \, e^{i(k \operatorname{sen}\theta y - \omega t)} \left[e^{ik\cos\theta z} (\cos\theta \, \hat{y} - \operatorname{sen}\theta \, \hat{z}) - e^{-ik\cos\theta z} (-\cos\theta \, \hat{y} - \operatorname{sen}\theta \, \hat{z}) \right]$$

$$\boxed{\begin{aligned} \vec{B} = \; & \sqrt{\mu\varepsilon} . E_i . e^{i(k\operatorname{sen}\theta y - \omega t)} \left(e^{ik\cos\theta z} + e^{-ik\cos\theta z} \right) . \cos\theta \, \hat{y} \\ & \sqrt{\mu\varepsilon} . E_i . e^{i(k\operatorname{sen}\theta y - \omega t)} \left(-e^{ik\cos\theta z} + e^{-ik\cos\theta z} \right) . \operatorname{sen}\theta \, \hat{z} \end{aligned}}$$

b.

$$\vec{S} = \frac{c}{4\pi} (\vec{E} \times \vec{H}) = \frac{c}{4\pi\mu} (\vec{E} \times \vec{B})$$

$$\vec{E} = E_i . e^{i(k\operatorname{sen}\theta y - \omega t)} . 2i . \operatorname{sen}(k\cos\theta z) \, \hat{x}$$

$$\vec{B} = \sqrt{\mu\varepsilon} . E_i . e^{i(k\operatorname{sen}\theta y - \omega t)} . 2 . \cos(k\cos\theta z) . \cos\theta \, \hat{y}$$
$$- \sqrt{\mu\varepsilon} . E_i \, e^{i(k . \operatorname{sen}\theta y - \omega t)} . 2i . \operatorname{sen}(k . \cos\theta z) . \operatorname{sen}\theta \, \hat{z}$$

$$\vec{S} = \frac{c}{4\pi\mu} . \left(2^2 i \, E_i^2 . e^{2i(k\operatorname{sen}\theta y - \omega t)} . \operatorname{sen}(k\cos\theta z) \sqrt{\mu\varepsilon} . \cos(k\cos\theta z) . \cos\theta \, \hat{z} + 2^2 i^2 E_i^2 . e^{2i(k . \operatorname{sen}\theta y - \omega t)} . \operatorname{sen}^2(k\cos\theta z) \sqrt{\mu\varepsilon} . \operatorname{sen}\theta \, \hat{y} \right)$$

$$\vec{S} = \frac{c}{4\pi\mu} \cdot 4 E_i^2 \sqrt{\mu\varepsilon} \cdot e^{i\,2(ky\,\text{sen}\,\theta - \omega t)} \Big[\text{sen}(kz\cos\theta)\cos(kz\cos\theta)\cdot\cos\theta \cdot i\,\hat{z} - \text{sen}^2(kz\cos\theta)\,\text{sen}\,\theta\,\hat{y} \Big]$$

$$\langle\vec{S}\rangle = \frac{c}{4\pi\mu}\cdot\frac{1}{2}\,\text{Re}\left(\vec{E}\times\vec{B}^*\right) = \frac{c}{4\pi\mu}\cdot\frac{1}{2}\,\text{Re}\left(\vec{E}^*\times\vec{B}\right)$$

$$\vec{E}^* = E_i\, e^{-i(k\,\text{sen}\,\theta\, y - \omega t + \pi/2)}\, 2\,\text{sen}(kz\cos\theta)\,\hat{x}$$

$$i = e^{i\pi/2}$$

$$\vec{B} = \sqrt{\mu\varepsilon}\cdot E_i\, 2\cdot e^{+i(k\,\text{sen}\,\theta\, y - \omega t)}\cos(k\cos\theta\, z)\cos\theta\;\hat{y}$$
$$- \sqrt{\mu\varepsilon}\cdot E_i\, 2\cdot e^{i(k\,\text{sen}\,\theta\, y - \omega t + \pi/2)}\cdot\text{sen}(k\cos\theta\, z)\,\text{sen}\,\theta\;\hat{z}$$

$$\vec{E}^*\times\vec{B} = 2^2 E_i^2\cdot\text{sen}(kz\cos\theta)\cdot\cos(k\cos\theta\, z)\cdot\cos\theta\cdot e^{-i\pi/2}\sqrt{\mu\varepsilon}\;\hat{z}$$
$$2^2 E_i^2\,\sqrt{\mu\varepsilon}\cdot\text{sen}^2(kz\cos\theta)\cdot\text{sen}\,\theta\;\hat{y}$$

$$\boxed{\langle\vec{S}\rangle = \frac{c}{8\pi\mu}\cdot 4E_i^2\sqrt{\mu\varepsilon}\;\text{sen}^2(k\,z\cos\theta)\,\text{sen}\,\theta\;\hat{y}}$$

c.

$$\mu_E = \frac{1}{8\pi}\left(\vec{E}\cdot\vec{D}\right)$$

$$\mu_E = \frac{1}{8\pi}\,\varepsilon|\vec{E}|^2 = \frac{1}{8\pi}\,\varepsilon\, 2^2 E_i^2\cdot\text{sen}^2(k\cos\theta\, z)$$

$$\langle\mu_E\rangle = \frac{1}{16\pi}\,\text{Re}\left(\vec{E}\cdot\vec{D}^*\right) = \frac{1}{16\pi}\,\varepsilon\cdot 4E_i^2\,\text{sen}^2(k\cos\theta\, z)$$

d.

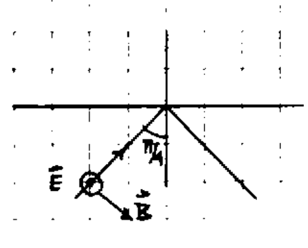
$$\mu_M = \frac{1}{8\pi}\left(\vec{B}\cdot\vec{H}\right)$$

$$\mu_M = \frac{1}{8\pi}\,\frac{1}{\mu}|\vec{B}|^2 = \frac{1}{8\pi\mu}\left(\mu\varepsilon\, E_i^2\, 4\cos^2(k\cos\theta\, z)\cdot\cos^2\theta + \mu\varepsilon\, E_i^2\, 4\cdot\text{sen}^2(k\cos\theta\, z)\,\text{sen}^2\theta\right)$$

$$\mu_M = \frac{1}{8\pi}\,\varepsilon\, E_i^2\, 4\left[\cos^2(k\cos\theta\, z)\cdot\cos^2\theta + \text{sen}^2(k\cos\theta\, z)\cdot\text{sen}^2\theta\right]$$

$$\langle\mu_M\rangle = \frac{1}{16\pi}\,\text{Re}\left(\vec{B}\cdot\vec{H}^*\right) = \frac{1}{16\pi}\cdot\frac{1}{\cancel{\mu}}\cdot\varepsilon\cancel{\mu}\, E_i^2\, 4\left[\cos^2(k\cos\theta\, z)\cos^2\theta + \text{sen}^2(k\cos\theta\, z)\,\text{sen}^2\theta\right]$$

2



AFortunadamente este es un caso particular del punto genenc y tan solo necesitamos especializar en $\theta = \frac{\pi}{4}$

$\cos\pi/4 = \operatorname{sen}\pi/4 = a \quad a = \frac{1}{\sqrt{2}}$

a. * Campos

$$\boxed{\vec{E} = E_i \cdot e^{i(kay + \omega t + \pi/2)} \, 2 \operatorname{sen}(kaz) \, \hat{x}}$$

$$\boxed{\begin{aligned} \vec{B} &= \sqrt{\mu\varepsilon} \cdot E_i \cdot e^{i(kay - \omega t)} \cdot 2 \cos(kaz) \, a \, \hat{y} \\ &- \sqrt{\mu\varepsilon} \cdot E_i \, e^{i(kay - \omega t + \pi/2)} \cdot 2 \operatorname{sen}(kaz) \, a \, \hat{z} \end{aligned}}$$

b. * Vector de Poynting

$$\vec{S} = \frac{c}{4\pi\mu} \cdot 4 E_i^2 \sqrt{\mu\varepsilon} \, e^{i2(kya - \omega t)} \left(\operatorname{sen}(kaz) \cos(kaz) \cdot a \, i \, \hat{z} - \operatorname{sen}^2(kaz) \, a \, \hat{y} \right)$$

$$\boxed{\langle \vec{S} \rangle = \frac{c}{8\pi\mu} 4 E_i^2 \sqrt{\mu\varepsilon} \, a \operatorname{sen}^2(kaz) \, \hat{y}}$$

c. * energía E

$$\mu_E = \frac{1}{8\pi} \varepsilon \, 4 E_i^2 \cdot \operatorname{sen}^2(kaz) \qquad \boxed{\langle \mu_E \rangle = \frac{1}{16\pi} \varepsilon \, 4 E_i^2 \operatorname{sen}^2(kaz)}$$

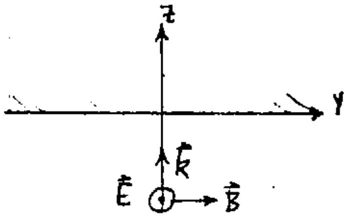
d. * energía M

$$\mu_M = \frac{1}{8\pi} \varepsilon \, 4 E_i^2 \left(\cos^2(kaz) \cdot a^2 + \operatorname{sen}^2(kaz) \cdot a^2 \right)$$

$$\mu_M = \frac{1}{8\pi} \varepsilon \, 4 E_i^2 \, a^2$$

$$\boxed{\langle \mu_M \rangle = \frac{1}{16\pi} \varepsilon \, 4 E_i^2 \, a^2}$$

1



Incidencia normal es $\theta = 0 \Rightarrow$

$\cos\theta = 1 \qquad \operatorname{sen}\theta = 0$

$$\boxed{\vec{E} = E_i \cdot e^{-i(\omega t + \pi/2)} \, 2 \cdot \operatorname{sen}(k \cdot z) \, \hat{x}}$$

$$\boxed{\vec{B} = \sqrt{\mu\varepsilon} \cdot E_i \, e^{-i(\omega t)} \, 2 \cdot \cos(kz) \, \hat{y}}$$

$$\vec{S} = \frac{c}{4\pi\mu} 4 E_i^2 \cdot \sqrt{\mu\varepsilon} \cdot e^{-2i\omega t + \pi/2 i} \cdot \operatorname{sen}(kz) \cos(kz) \, \hat{z}$$

$$\boxed{\langle \vec{S} \rangle = 0}$$

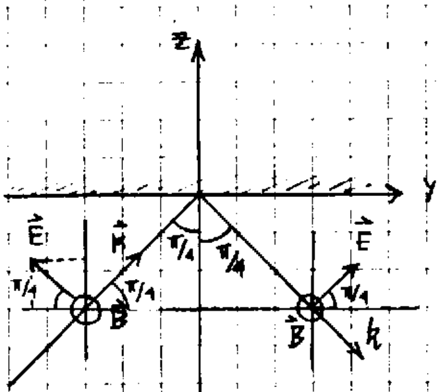
no hay flujo de energía en el tiempo

$$\langle \mu_E \rangle = \frac{1}{4\pi} \varepsilon E_i^2 \cdot \operatorname{sen}^2(kz)$$

$$\langle \mu_M \rangle = \frac{1}{4\pi} \varepsilon E_i^2 \cdot \cos^2(kz)$$

$$\boxed{\langle \mu_{EM} \rangle = \frac{1}{4\pi} \varepsilon E_i^2}$$

③



$$\vec{E}_i = E_i \, e^{i(\vec{k}\cdot\vec{x}-\omega t)} \, a \, (\hat{z}-\hat{y})$$
$$\vec{E}_r = E_r \, e^{i(\vec{k}\cdot\vec{x}-\omega t)} \, a \, (\hat{z}+\hat{y})$$
$$\vec{B}_i = \sqrt{\mu\varepsilon} \, . E_i \, e^{i(\vec{k}\cdot\vec{x}-\omega t)} \, . a \, \hat{x}$$
$$\vec{B}_r = \sqrt{\mu\varepsilon} \, . E_r \, e^{i(\vec{k}\cdot\vec{x}-\omega t)} \, a \, \hat{x}$$

k es el mismo para incidente y reflejado

$*\underline{i}$ $\quad$ $\vec{k}\cdot\vec{x} = k a y + k a z$ $\qquad$ $*\underline{r}$ $\quad$ $\vec{k}\cdot\vec{x} = k a y - k a z$

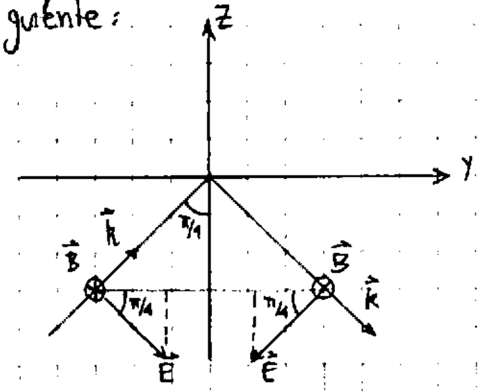
* Contornos

$B_{\hat{n}}$, $E_{\hat{t}}$ continuos

$$-E_i . a + E_r . a = 0 \Rightarrow E_i = E_r$$

$$\vec{E} = \vec{E}_i + \vec{E}_r = E_i . a . e^{i(kay-\omega t)} \left[e^{ikaz} (\hat{z}-\hat{y}) + e^{-ikaz} (\hat{z}+\hat{y}) \right]$$
$$E_i . a . e^{i(kay-\omega t)} \left[(e^{ikaz} + e^{-ikaz}) \hat{z} \right.$$
$$\left. (-e^{ikaz} + e^{-ikaz}) \hat{y} \right]$$
$$\vec{E} = E_i a \, e^{i(kay-\omega t)} \, 2 . \left[(\cos(kaz) \hat{z} + i . \operatorname{sen}(kaz) \hat{y} \right]$$
$$\vec{B} = \vec{B}_i + \vec{B}_r = \sqrt{\mu\varepsilon} \, E_i . a \, e^{i(kay-\omega t)} . 2 . \cos(kaz) \, \hat{x}$$

Podemos, llegados a este paso, evitarnos todo el trabajo de recalcular. Comparando con el caso anterior vemos que podríamos elegir ejes en el modo siguiente:



$$\vec{E}_i = E_i . e^{i(kay+kaz-\omega t)} \, a \, (-\hat{z}+\hat{y})$$
$$\vec{E}_r = E_r . e^{i(kay-kaz-\omega t)} \, a \, (-\hat{z}-\hat{y})$$
$$\vec{B}_i = \sqrt{\mu\varepsilon} . E_i . e^{i(kay+kaz-\omega t)} \, a \, (-\hat{x})$$
$$\vec{B}_r = \sqrt{\mu\varepsilon} . E_r . e^{i(kay-kaz-\omega t)} \, a \, (-\hat{x})$$

* Contornos

$B_{\hat{n}}$, $E_{\hat{t}}$ continuos.

a. * Campos $\qquad$ $E_i . a - E_r . a = 0 \Rightarrow E_i = E_r$

$$\vec{E} = \vec{E}_i + \vec{E}_r = E_i . a . e^{i(kay-\omega t)} \left[-e^{ikaz} \hat{z} + e^{ikaz} \hat{y} - e^{-ikaz} \hat{z} - e^{-ikaz} \hat{y} \right]$$

$$\boxed{\vec{E} = E_i . a . e^{i(kay-\omega t)} \left[-\cos(kaz) 2 \hat{z} + 2i \operatorname{sen}(kaz) \hat{y} \right]}$$

$$\vec{B} = \vec{B}_i + \vec{B}_r = -\sqrt{\mu\varepsilon}\,.\,E_i\,.\,e^{i(kay-\omega t)}\,.\,a\left[e^{ikaz} + e^{-ikaz}\right]\hat{x}$$

$$\boxed{\vec{B} = -\sqrt{\mu\varepsilon}\,.\,E_i\,.\,a\,.\,e^{i(kay-\omega t)}\,.\,\cos(kaz)\,.\,2\,\hat{x}}$$

Fue en vano; $\vec{B}_{TM}$ es parecido en forma a $\vec{E}$ del caso TE pero no igual y lo mismo con $\vec{E}_{TM}$ que se parece a $\vec{B}_{TE}$. Hay que recalcular

b. * Vector de Poynting

$$<\vec{S}> = \frac{c}{4\pi\mu}\,.\,\frac{1}{2}\,\mathcal{R}e\left(\vec{E}\times\vec{B}^*\right)$$

$$\vec{E}\times\vec{B}^* = -E_i^2 a^2.\sqrt{\mu\varepsilon}\,.\,4\,.\,\cos^2(kaz)\,\hat{y} + E_i^2 a^2\sqrt{\mu\varepsilon}\,.\,4.e^{i\pi/2}.\,\mathrm{sen}\,(kaz).\cos(kaz)\,\hat{z}$$

$$\boxed{<\vec{S}> = \frac{c}{8\pi\mu}\,.\,E_i^2.a^2.4\sqrt{\mu\varepsilon}\,.\cos^2(kaz)\,\hat{y}}$$

c., d. * Energía eléctrica y magnética

$$<\mu_E> = \frac{1}{16\pi}\,.\,\mathcal{R}e\left(\varepsilon|\vec{E}|^2\right) = \frac{\varepsilon}{16\pi}\,.\left(E_i^2 a^2.4.\left[\cos^2(kaz) + \mathrm{sen}^2(kaz)\right]\right)$$

$$\boxed{<\mu_E> = \frac{\varepsilon}{16\pi}\,.\,a^2 E_i^2 4}$$

$$<\mu_M> = \frac{1}{16\pi}\,.\,\mathcal{R}e\left(\frac{1}{\mu}|\vec{B}|^2\right) = \frac{1}{16\pi}\,\frac{\cancel{\mu}\varepsilon}{\cancel{\mu}}\,E_i^2 a^2.4.\cos^2(kaz)$$

$$\boxed{<\mu_M> = \frac{\varepsilon}{16\pi}\,a^2 E_i^2 4\,.\cos^2(kaz)}$$

* Conclusiones *

e. El vector ópticos es $\vec{E}$

En la experiencia ③ no se observan franjas; es decir que la película no se impresionó con datos lumínicos. En dicha experiencia se tiene:

$<\mu_E>$: constante
$<\mu_M>$: depende de la coordenada z

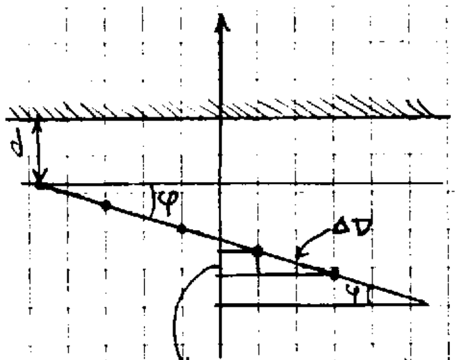
En las experiencias ① y ② tenemos.

①	②
$<\mu_E>$: depende de z	$<\mu_E>$: depende de z
$<\mu_M>$: depende de z	$<\mu_M>$: constante

Luego como en ① y ② hubo franjas y en ③ no $<\mu_E>$ es el causante de las franjas sobre la película; $\Rightarrow$ $\vec{E}$ es el vector óptico

Asimismo en $z=0$ (pegado al espejo) $<\mu_E> = 0$ lo cual sustenta aún más el argumento.

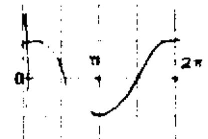
f.



$\frac{m\pi}{ak} = (\text{sen}\,\varphi).\Delta D$

$\Delta D = \frac{1}{\text{sen}\,\varphi}\cdot\frac{m\pi}{a.k}$

interfranja ▶ $\boxed{\Delta D = \frac{1}{\text{sen}\,\varphi}.m.d}$



$\cos^2(kaz) = 1 \Leftrightarrow$ /

$kaz = m\pi$

$z = \frac{m\pi}{ak} = \frac{\sqrt{2}.\pi.m}{\sqrt{\mu\varepsilon}.\omega}c = \frac{m}{\sqrt{2}}\lambda$

$k = \frac{\omega}{c}.\sqrt{\mu\varepsilon}$

$z =$

$k = \frac{2\pi}{\lambda}$

$\nu = \frac{\omega}{2\pi}$

$z = -d$ y $m = 1$

$-d = \frac{\pi}{ak}$