GUÍA 1

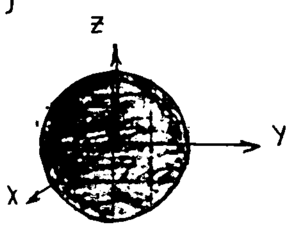
I. Transformaciones de Simetría. Ley de Gauss. Ley de Ampere

1.

* a.

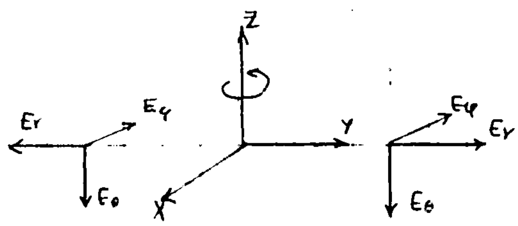
i) $\rho(\vec{r}) = \begin{cases} 0 & r > R \\ \rho_0 & r < R \end{cases}$ $\rho_0 = \dfrac{Q}{\frac{4}{3}\pi R^3}$ $Q \to$ carga total, $R \to$ radio esfera

ii)

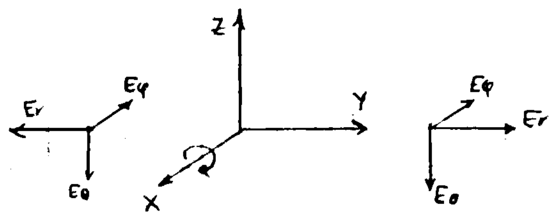


simetría de rotación en φ
simetría de rotación en θ
simetría de reflexión en XZ, YZ, XY

(Valen también dentro de la esfera pues ρ es uniforme)



$$\underbrace{\begin{aligned} E_r' &= E_r \\ E_\varphi' &= -E_\varphi \\ E_\theta &= E_\theta' \end{aligned}}_{\text{reflexión en XZ}} \; \underbrace{\begin{aligned} &= E_r \\ &= E_\varphi \\ &= E_\theta \end{aligned}}_{\text{rotación en } \varphi\,(\pi)} \quad \to E_\varphi = 0$$

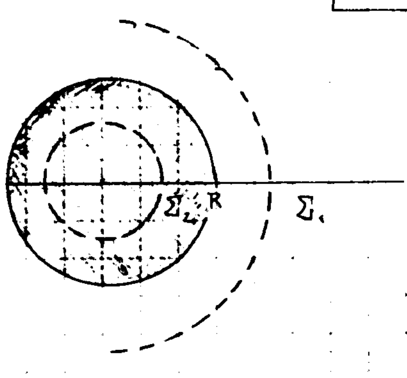


$$\underbrace{\begin{aligned} E_r &= E_r \\ E_\varphi &= -E_\varphi \\ E_\theta &= E_\theta \end{aligned}}_{\text{reflexión en XZ}} \; \underbrace{\begin{aligned} &= E_r \\ &= E_\varphi \\ &= -E_\theta \end{aligned}}_{\text{rotación en } \theta\,(\pi)} \quad \to E_\theta = 0$$

$\vec{E} \neq \vec{E}(\varphi, \theta)$ pues rotando en φ, θ tengo situaciones físicas indistinguibles

$$\Rightarrow \boxed{\vec{E} = E(r)\,\hat{r}}$$

iii)

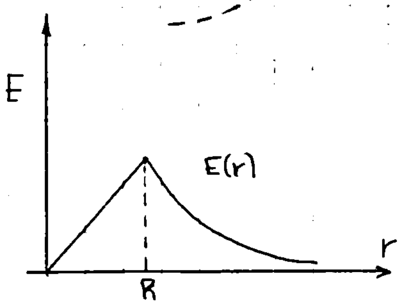


$r > R$

$$\oint_{S_1} \vec{E} \cdot d\vec{S} = 4\pi . Q$$

$$E . 4\pi r^2 = 4\pi\, Q$$

$$\boxed{\vec{E} = \frac{Q}{r^2}\,\hat{r}}$$



$r < R$

$$\oint_{S_2} \vec{E} \cdot d\vec{S} = 4\pi . \left(\frac{4}{3}\pi r^3\right) . \rho_0$$

$$E . \cancel{4\pi} r^{\cancel{2}} = \cancel{4\pi} . \frac{4\pi}{3} r^{\cancel{3}} \rho_0$$

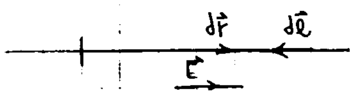
$$E = \frac{4}{3}\pi r . \rho_0\, \hat{r}$$

$$\boxed{\vec{E} = \frac{Q . r}{R^3}\,\hat{r}}$$

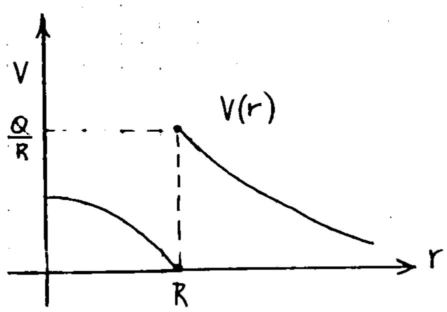
iv) $V = -\int_{\infty \to r} \vec{E} \cdot d\vec{\ell} \quad \rightarrow \quad -\int_r^{\infty} \left(\frac{Q}{r^2}\right)(-dr) - \int_r^{R} \frac{Q}{R^3} r \cdot (-dr)$

$$-\frac{Q}{r}\Big|_r^{\infty} + \frac{Q}{R^3}\frac{r^2}{2}\Big|_r^{R} \ldots$$

$$-Q\left(\frac{1}{\infty} - \frac{1}{r}\right) + \frac{Q}{2R^3}(R^2 - r^2)$$

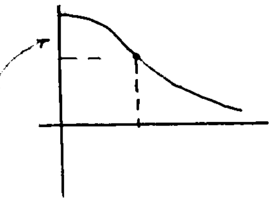


$$\boxed{V(r>R) = \frac{Q}{r}} \qquad V(r<R) = \frac{Q}{2R} - \frac{Qr^2}{2R^3}$$

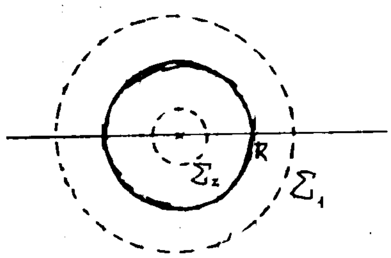


$$\boxed{V(r<R) = \frac{3Q}{2R} - \frac{Qr^2}{2R^3}}$$

este sí empalma



* b.



i) $\rho(\vec{r}) = \begin{cases} 0 & r>R \\ \delta(r-R)\cdot\sigma & r=R \\ 0 & r<R \end{cases}$

ii)

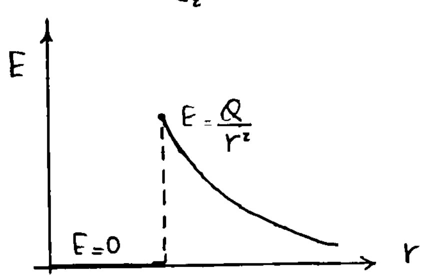
con $r>R$ ó $r<R$ $\begin{cases} \text{Simetría de rotación en } \varphi, \theta \\ \text{Simetría de reflexión en } XY, XZ, YZ \end{cases}$

[Ya sea dentro o fuera del casquete] $\rightarrow \vec{E} = E(r)\hat{r} \leftarrow$ (por los mismos argumentos que la esfera maciza) $\rightarrow$ cumple las mismas simetrías

iii) $\oint_{\Sigma_1} \vec{E} \cdot d\vec{S} = 4\pi . Q \quad \rightarrow \quad \vec{E} = \frac{Q}{r^2}\hat{r} \qquad (r>R)$

$\oint_{\Sigma_2} \vec{E} \cdot d\vec{S} = 0 \quad \rightarrow \quad \vec{E} = 0 \qquad (r<R)$

Campo



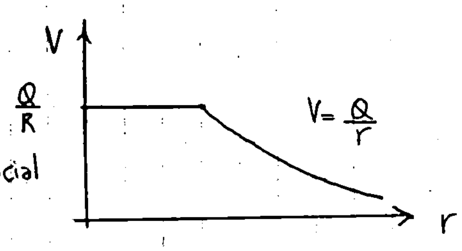
iv)

$V(r) = \frac{Q}{r} \qquad r>R$

$V(r) = (\text{cte}) \qquad r<R$

tomo $\quad V(r=R) = \text{cte} = \frac{Q}{R}$

Potencial



* C

i) $\rho = \begin{cases} 0 & r>R \\ f(r) & r<R \end{cases}$

ii) Como la ρ es radial a un r_0 fijo cualquiera, se tiene una configuración de carga totalmente simétrica

Simetría de rotación en φ, θ
Simetría de reflexión en XY, YZ, XZ

$$\vec{E} = E(r)\,\hat{r}$$

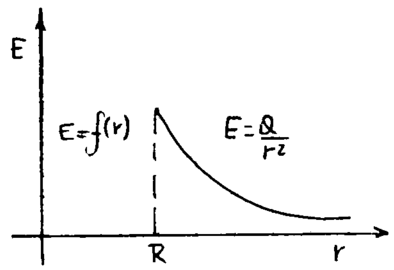
iii)

$$\oint \vec{E}\cdot d\vec{S} = 4\pi . Q \rightarrow \boxed{\vec{E} = \frac{Q}{r^2}\hat{r}} \quad r>R$$

$$f(R) = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$\oint \vec{E}\cdot d\vec{S} = 4\pi . f(r).\frac{4\pi}{3} r^3$$

$$E = f(r).\frac{4\pi}{3}.r \Rightarrow \boxed{\vec{E} = \frac{f(r)}{f(R)}.\frac{Q}{R^3}.r\,\hat{r}} \quad r<R$$



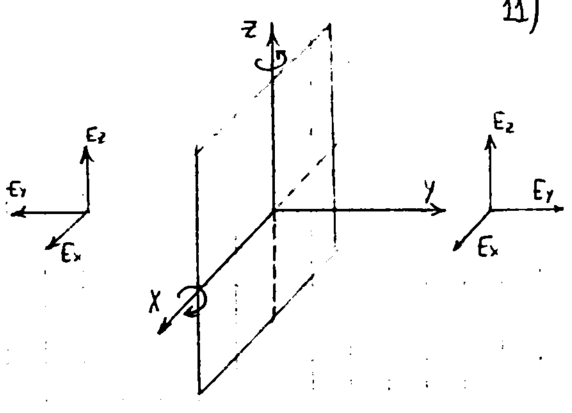
$$V(r) = \frac{Q}{r} \quad r>R$$

* d

i)

$$\rho = \begin{cases} \delta(y-0).\sigma & y=0 \\ 0 & y\neq 0 \end{cases}$$

ii)



Simetría reflexión en ZX, XY, ZY
Simetría de traslación en x, z

Simetría de rotación en π

reflexión del vector $\vec{E}$

$$E_x' = E_x = -E_x \rightarrow E_x = 0$$
$$E_y' = -E_y = -E_y$$
$$E_z' = E_z = E_z$$

rotación en π entorno a $\hat{z}$

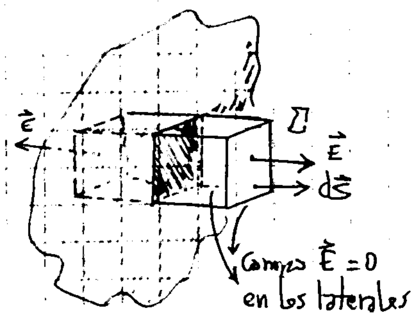
$$E_x' = E_x = E_x$$
$$E_y' = -E_y = -E_y \rightarrow E_z = 0$$
$$E_z' = E_z = -E_z$$

rotación en π entorno a $\hat{x}$

A y_0 fijo no depende de x,z por la infinitud; en cualquier x,z que me sitúe tengo ∞ plano a cada lado $\Rightarrow$ la situación física es la misma $\Rightarrow$ No puede depender de x,z

$$\boxed{\vec{E} = E(y)\,\hat{y}}$$

iii)

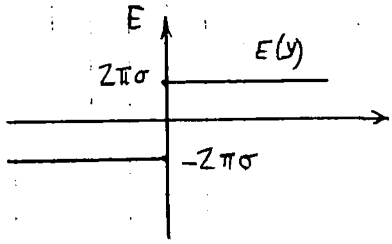


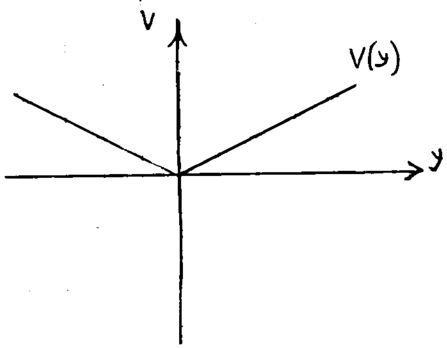
$$\oint_{\Sigma} \vec{E}\cdot d\vec{S} = 4\pi . Q$$

$$2E.A = 4\pi.\sigma.A \quad \therefore$$

$$\boxed{\vec{E} = 2\pi.\sigma\,\hat{y}} \qquad y>0$$

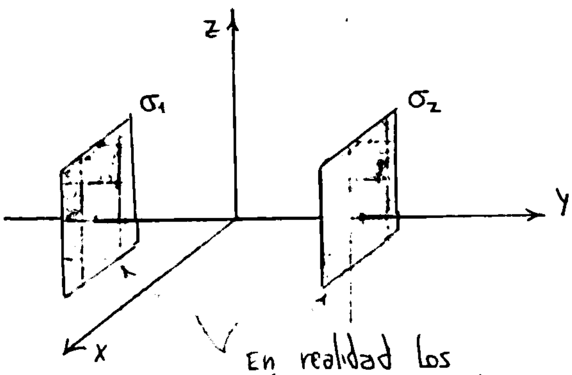
$$\boxed{\vec{E} = -2\pi\sigma\,\hat{y}} \qquad y<0$$





$$V = \begin{cases} 2\pi\sigma.y & y>0 \\ -2\pi\sigma.y & y<0 \end{cases}$$

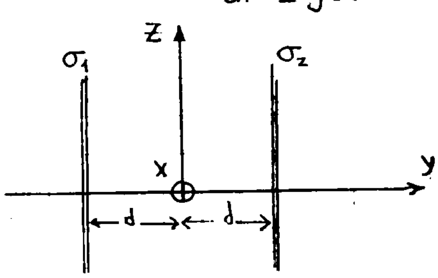
* e



En realidad los planos son infinitos en Z y en X

i)

$$\rho = \begin{cases} 0 & |y| \neq d \\ \delta(y-d).\sigma & y=d \\ \delta(y+d).\sigma & y=-d \end{cases}$$



ii)

Simetría de traslación en X, Z
Simetría de reflexión en XY, ZY

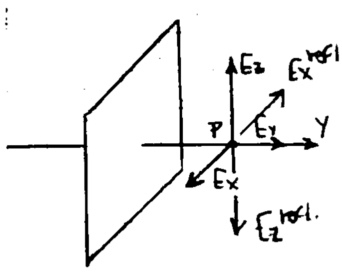
$\vec{E} = \vec{E}(y)$ (no depende de x, z) por la infinitud de los planos en x, z

reflexión en XY

$$E_x' = E_x = E_x$$
$$E_y' = E_y = E_y \qquad \rightarrow E_z = 0$$
$$E_z' = \underbrace{-E_z = E_z}_{\text{punto P ubicado en el plano de reflexión}}$$

reflexión en ZY

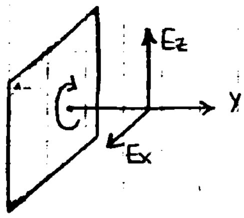
$$E_x' = -E_x = E_x \qquad \rightarrow E_x = 0$$
$$E_y' = E_y = E_y$$
$$E_z' = \underbrace{E_z = E_z}_{\text{Punto P ubicado en el plano de reflexión}}$$



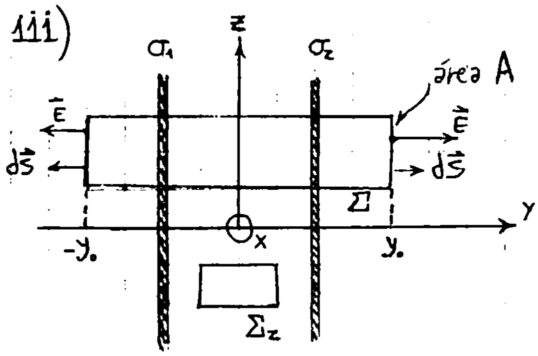
$$\boxed{\vec{E} = E(y)\,\hat{y}}$$

NOTA

En realidad, dada la infinitud en X y en Z podemos girar en cualquier ángulo, según la flecha circular. Luego no puede haber componentes en z, x $\Rightarrow$
$E_z = 0$, $E_x = 0$



iii)



Considero: $\sigma_1 = \sigma_2$

Por reflexión de $\vec{E}$ se que E_y apunta hacia afuera de los planos $\Rightarrow$ $E(y_0)$ y $E(-y_0)$ apuntan hacia afuera

$$\int_\Sigma \vec{E}\cdot d\vec{S} = 4\pi . Q$$

$$2.E.A = 4\pi . \sigma . 2.A$$

$$E = 4\pi . \sigma \rightarrow \boxed{\vec{E} = 4\pi . \sigma \, \hat{y}} \; y>d$$

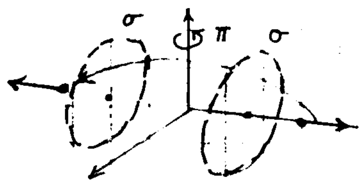
$$\boxed{\vec{E} = -4\pi\sigma\hat{y}} \; y<-d$$

$$\int \vec{E}\cdot d\vec{S} = 0$$

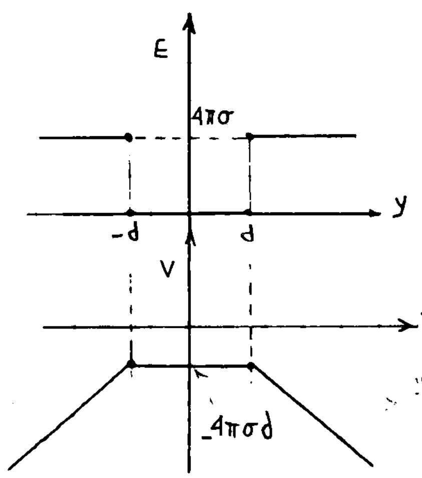
$$E . 2A = 0 \rightarrow \boxed{\vec{E} = 0}$$

$$|y|<d$$

$\sigma_1 = \sigma_2$ rotación en π deja todo igual

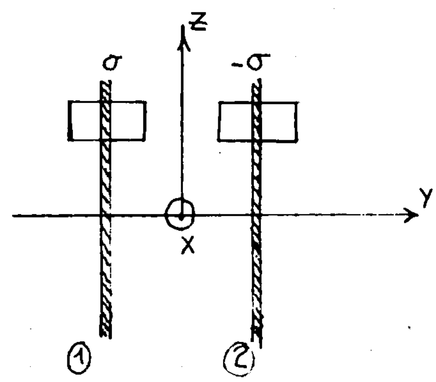


$\sigma_1 = \sigma_2$



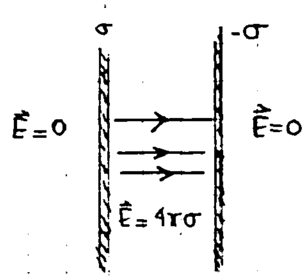
$V = -4\pi\sigma y$	$y > d$
$V = 4\pi\sigma y$	$y < -d$
$V = -4\pi\sigma d$	$\lvert y\rvert \leq d$

Considero $\sigma_1 = -\sigma_2$ $\rightarrow$ $\begin{cases} \sigma_1 = \sigma \\ \sigma_2 = -\sigma \end{cases}$

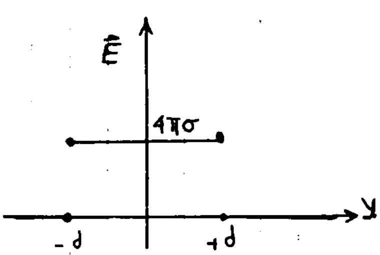


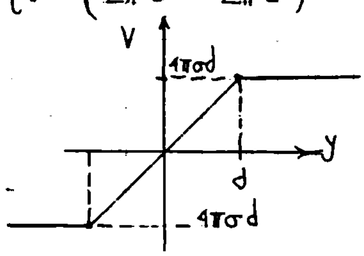
Superpongo dos planos infinitos como en el caso *d

$$\vec{E}^{①} = \begin{cases} 2\pi\sigma & y > -d \\ -2\pi\sigma & y < -d \end{cases} \qquad \vec{E}^{②} = \begin{cases} -2\pi\sigma & y > d \\ +2\pi\sigma & y < d \end{cases}$$

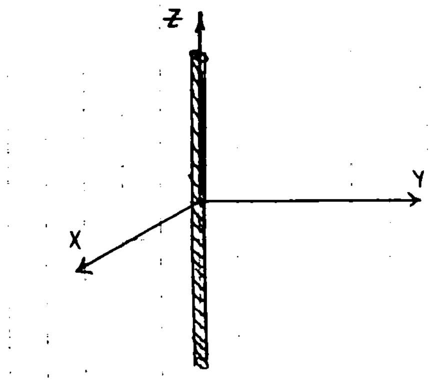


$$\vec{E} = \vec{E}^1 + \vec{E}^2 \rightarrow \vec{E} = \begin{cases} 0 & (-2\pi\sigma + 2\pi\sigma) & y < -d \\ 4\pi\sigma & (2\pi\sigma + 2\pi\sigma) & -d < y < d \\ 0 & (2\pi\sigma - 2\pi\sigma) & y > d \end{cases}$$





* f

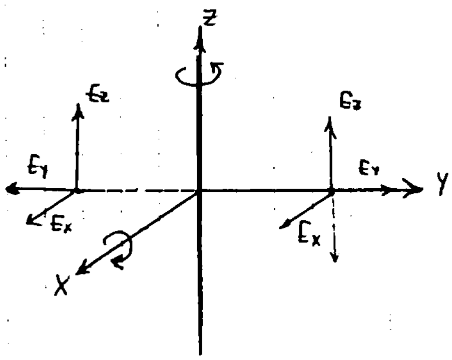


i) $\rho = \begin{cases} 0 & z \neq 0 \\ \delta(x-0).\,\delta(y-0).\,\lambda \end{cases}$

ii) Simetría de rotación en φ

Simetría de traslación en z

Simetría de reflexión en XY, XZ, YZ



$$\overbrace{E'_r = E_r}^{\text{reflexión en } ZX} = E_r$$
$$E'_\varphi = -E_\varphi = E_\varphi$$
$$E'_z = \underbrace{E_z = E_z}_{\text{rotación de } \pi \text{ en } \hat{z}}$$

$$E_\varphi = 0$$

reflexión en XY

$$E_r = E_r$$
$$-E_z = E_z$$
$$E_\varphi = E_\varphi$$

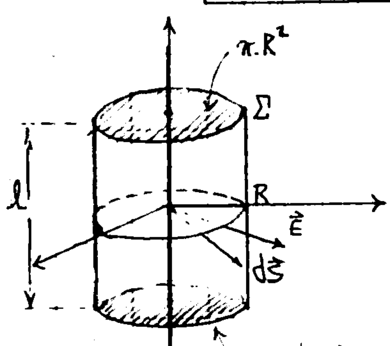
rotación de π en $\hat{z}$

$$E_z = 0$$

Por la traslación en $\hat{z}$ no puede depender de z

Por la rotación en $\hat{\varphi}$ no puede depender de φ $\longrightarrow$ $\boxed{\vec{E} = E(r)\,\hat{r}}$

iii)



por las tapas $\vec{E} \perp d\vec{S} \to \vec{E}\,d\vec{S} = 0$

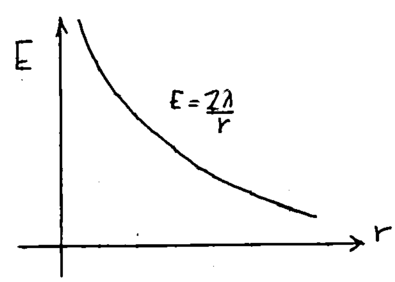
$$\oint_\Sigma \vec{E}\cdot d\vec{S} = 4\pi.\,l.\,\lambda$$

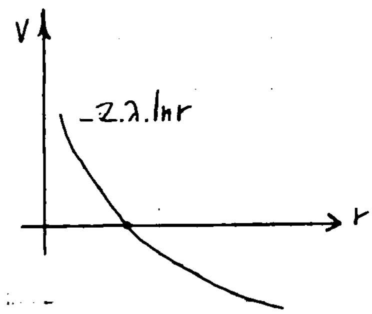
$$E.\,\cancel{2}\cancel{\pi}R.\,l = \overset{2}{\cancel{4}}\cancel{\pi}.\,l.\,\lambda$$

$$E = \frac{2\lambda}{R} \quad \to \quad \boxed{\vec{E} = \frac{2\lambda}{r}\,\hat{r}}$$

iv)

$$\boxed{V = -2\lambda.\ln r}$$





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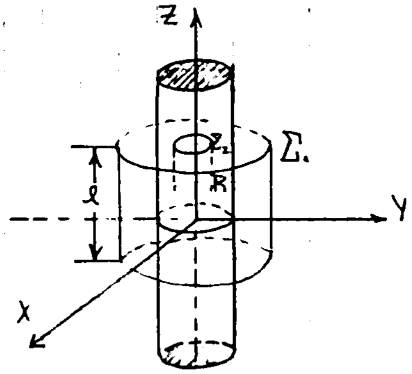
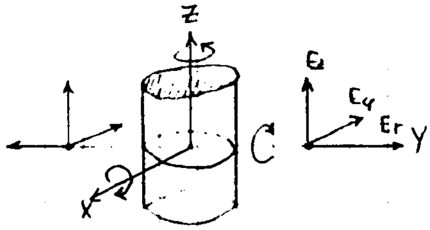


i) $\rho = \begin{cases} 0 & r > R \\ \rho_0 & r \leq R \end{cases}$

ii) Simetría de rotación en φ
Simetría de traslación en z
Simetría de reflexión en XY, YZ, ZX

Σ_1 : sup. gaussiana exterior
Σ_2 : sup. gaussiana interior



La rotación en φ garantiza $\vec{E} \neq \vec{E}(\varphi)$
La traslación en z garantiza $\vec{E} \neq \vec{E}(z)$

reflexión en ZX

$$\overbrace{E'_r = E_r}\ = E_r$$
$$E'_\varphi = -E_\varphi = E_\varphi$$
$$\underbrace{E_z = E_z}_{\text{rotación de } \pi \text{ en } z} = E_z$$

$E_\varphi = 0$

reflexión en ZX

$$E_r = E_r$$
$$-E_\varphi = E_\varphi$$
$$E_z = -E_z$$

rotación de π en $\hat{x}$

$E_z = 0$

$$\boxed{\vec{E} = E(r)\,\hat{r}} \leftarrow (\hat{r} \text{ de cilíndricas})$$

iii)

$$\oint_{\Sigma_1} \vec{E} \cdot d\vec{S} = 4\pi . Q$$

solo sup. lateral

$$E . \cancel{2}\cancel{\pi}.\cancel{l}.r = \overset{2}{\cancel{4}}\cancel{\pi}.\rho_0 . \pi . R^2 . \cancel{l}$$

$$E = \frac{2\rho_0 . \pi . R^2}{r} \quad \rightarrow \quad \boxed{\vec{E} = \frac{2\pi R^2}{r}\rho_0\,\hat{r}} \qquad r > R$$

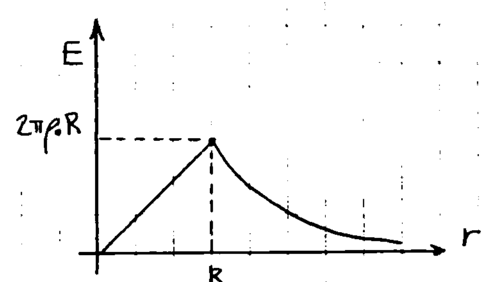
$$\oint_{\Sigma_2} \vec{E} \cdot d\vec{S} = 4\pi Q$$

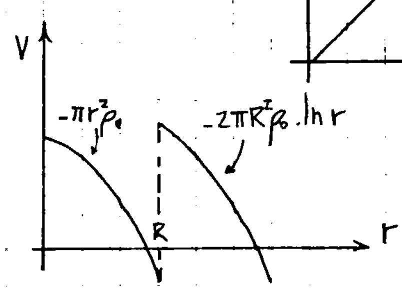
$$E \cdot \cancel{2}\cancel{\pi}\cancel{l}\cancel{r} = \overset{2}{\cancel{4}}\cancel{\pi}\rho_0\, \pi\, r^{\cancel{2}} \cancel{l}$$

$$E = 2\pi\rho_0 r \quad \rightarrow \quad \boxed{\vec{E} = 2\pi . r . \rho_0\,\hat{r}} \qquad r < R$$

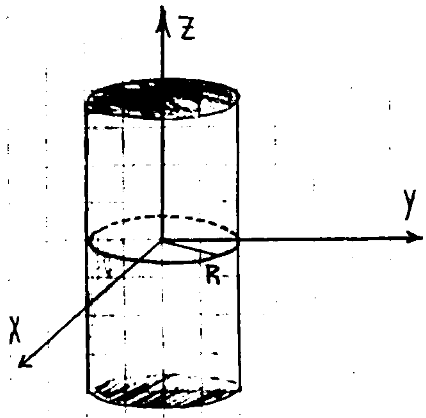
iv)

$$V = \begin{cases} -\ln r . 2\pi R^2 \rho_0 & r > R \\ -\pi r^2 \rho_0 & r < R \end{cases}$$





* h



i) $\rho = \begin{cases} 0 & \text{si } r \neq R \\ \delta(r-R).\sigma & \text{si } r = R \end{cases}$

(r de coordenadas cilíndricas)

ii) Simetría de traslación en z
Simetría de rotación en φ
Simetría de reflexion en XY, ZX, ZY

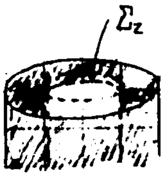
Procediendo en forma análoga al *g se tiene: $\vec{E} = E(r)\,\hat{r}$

iii) $\oint_{\Sigma_1} \vec{E}.d\vec{S} = 4\pi.\sigma.2\pi.R.\ell$

$E.\cancel{2\pi}.\cancel{\ell}r = \cancel{8}4\pi^{\cancel{2}}\sigma R\cancel{\ell}$

$E = \dfrac{4\pi\sigma R}{r}$

$\boxed{\vec{E} = 4\pi\sigma \dfrac{R}{r}\,\hat{r}}$ $r>R$



$\oint_{\Sigma_2} \vec{E}.d\vec{S} = 0$

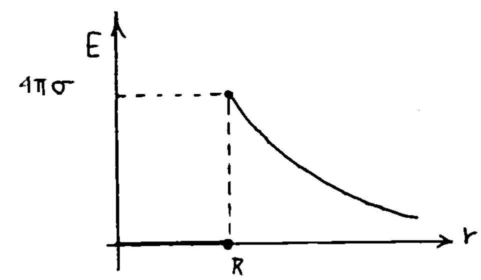
$E.2\pi\ell r = 0$

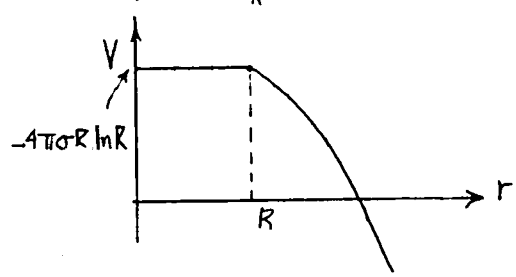
$\boxed{\vec{E} = 0}$ $r<R$

iv)

$$\boxed{\begin{array}{ll} V = -4\pi\sigma R.\ln r & r>R \\ V = cte & \text{si } r<R \end{array}}$$

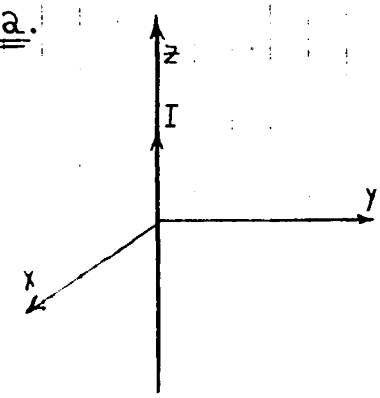
podemos tomar $-4\pi\sigma R.\ln R$





2.

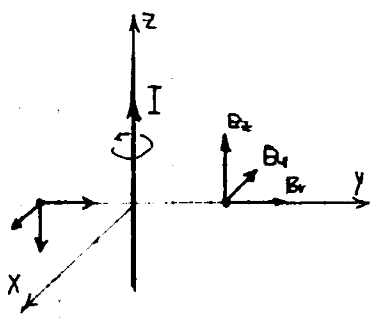
***a.**



i) $\boxed{\vec{j}(\vec{r}) = I \cdot \delta(x) \cdot \delta(y) \cdot \hat{z}}$

$$I = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g \cdot \underbrace{\delta(x')\delta(y') \cdot dx' \, dy'}_{\equiv J} = g$$

ii) Simetría de rotación en φ
Simetría de traslación en z
Simetría de reflexión en ZX, ZY

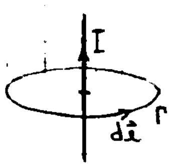


$$\overbrace{B'_r = -B_r}^{\text{reflexión en } ZY} = B_r$$
$$B'_\varphi = B_\varphi = B_\varphi$$
$$B'_z = \underbrace{-B_z = B_z}_{\text{Rotación en } \hat{z} \text{ de } \pi}$$

$\rightarrow$ $B_r = 0$
$B_z = 0$

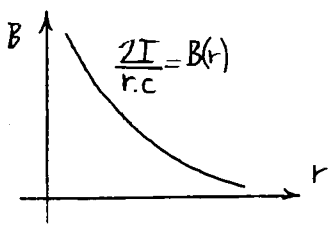
Como hay simetría de traslación y rotación $\Rightarrow$ $\boxed{\vec{B} = B(r)\hat{\varphi}}$ (r de coordenadas cilíndricas)

iii)

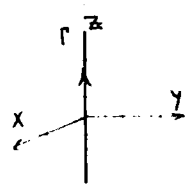


$$\oint_\Gamma \vec{B} \cdot d\vec{l} = \frac{4\pi I}{c}$$

$$B \cdot 2\pi r = \frac{4\pi I}{c} \quad \rightarrow \quad \boxed{\vec{B} = \frac{2I}{r \cdot c}\hat{\varphi}}$$



iv) $$\vec{A} = \frac{1}{c}\int_\Gamma \frac{I \, d\vec{l}'}{|\vec{r} - \vec{r}'|}$$ $|(x,y,z) - (0,0,z')|$



$$\vec{A} = \frac{1}{c}\int_\Gamma \frac{I \, dz' \, \hat{z}'}{\sqrt{x^2+y^2+(z-z')^2}}$$

$$\vec{A} = \frac{1}{c} \cdot I \hat{z}' \cdot \int_\Gamma \frac{dz'}{\sqrt{x^2+y^2+(z-z')^2}}$$

$$\vec{A} = \frac{I\hat{z}}{c}\int_{z+A}^{z-A} \frac{-du}{\sqrt{\underbrace{x^2+y^2}_{\equiv b^2}+u^2}}$$

$z - z' = u$
$-dz' = du$

$$\ln\left|u + \sqrt{x^2+y^2+u^2}\right| \Big|_{z+A}^{z-A}$$

$$\lim_{A\to\infty} \ln \frac{\left|(z-A) + \sqrt{x^2+y^2+(z-A)^2}\right|}{\left|(z+A) + \sqrt{x^2+y^2+(z+A)^2}\right|}$$

$$\lim_{A\to\infty} \ln \left| \frac{(z-A)\left(1 + \sqrt{\frac{x^2}{(z-A)^2} + \frac{y^2}{(z-A)^2} + 1}\right)}{(z+A)\left(+1 + \sqrt{\frac{x^2}{(z+A)^2} + \frac{y^2}{(z+A)^2} + 1}\right)} \right|$$

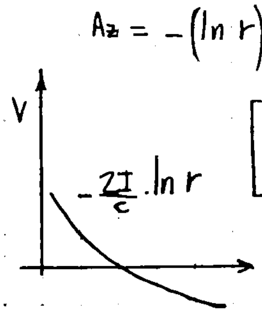
$\vec{B} = \vec{\nabla} \times \vec{A}$ $\Rightarrow$ en cilíndricas es:

$$\frac{2I}{c\,r} = \underbrace{\frac{\partial (A_r)}{\partial z}}_{=0 \text{ pues } A \neq A(z)} - \frac{\partial A_z}{\partial r} \Rightarrow$$

$$A_z = \int -\frac{2I}{cr} \, dr$$

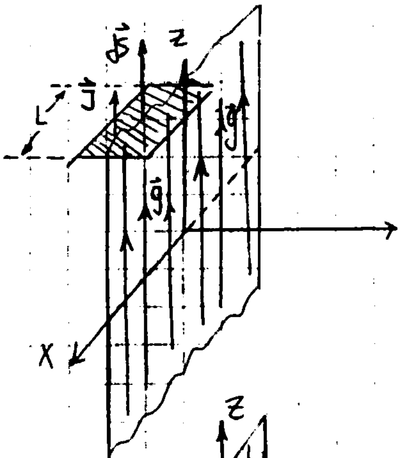
$$A_z = -(\ln r)\frac{2I}{c} + \underbrace{Cte.}_{\text{la tomo } 0}$$

$$\boxed{\vec{A} = -(\ln r)\frac{2I}{c}\hat{\varphi}}$$



En potenciales debidos a distribuciones no finitas la integral diverge [en realidad no hay distribuciones ∞] $\Rightarrow$ hay que usar otro método

* b.

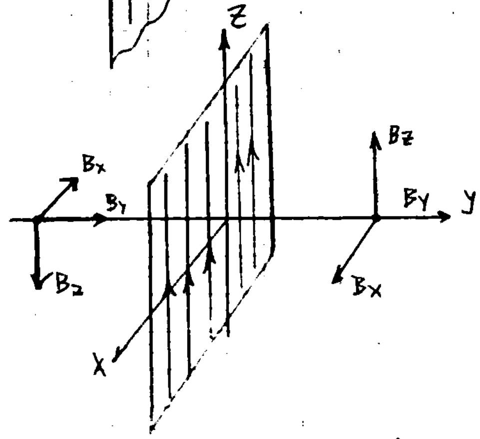


1)

$$\boxed{\vec{J}(\vec{r}) = \frac{I}{L}.\delta(y)\,\hat{z}}$$

$$I = \int_0^L \int_{-\infty}^{\infty} \underbrace{g.\delta(y').\,dy'.dx'}_{=J} = g.L \longrightarrow$$ g es una densidad de corriente

ii) Simetría de traslación en $\hat{z}$
Simetría de traslación en $\hat{x}$
Simetría de reflexión en ZX, ZY



Reflexión en ZX

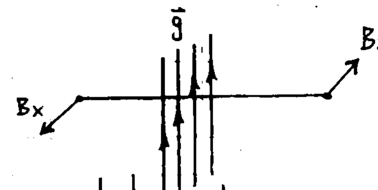
$$\begin{aligned} B'_x &= -B_x &&= -B_x \\ B'_y &= B_y &&= -B_y \\ B'_z &= -B_z &&= B_z \end{aligned} \quad \Rightarrow \quad \begin{aligned} B_y &= 0 \\ B_z &= 0 \end{aligned}$$

giro en π

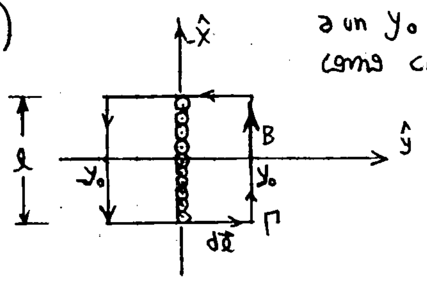
$$\boxed{\vec{B} = B(y)\hat{x}}$$

Por la traslación en $\hat{z}$ y en $\hat{x}$ $\Rightarrow$ $\vec{B} \neq \vec{B}(z,x)$

Por regla de la mano derecha será con los sentidos aquí mostrados



iii) a un y_0 fijo el campo $\vec{B}$ es constante entonces puedo tomar Γ como circuito para la Ley de Ampere.



$$\oint_\Gamma \vec{B}\cdot d\vec{l} = \frac{4\pi}{c} I_c$$

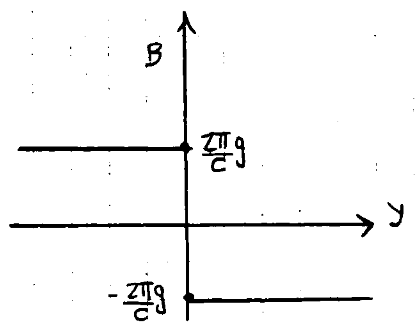
$$2B.l = \frac{4\pi}{c}.g.l \quad \rightarrow \quad B = \frac{2\pi g}{c}$$

$$I_c = \int \vec{J}.d\vec{S} \qquad g\hat{z}\,\underbrace{\delta(y)\hat{y}\,dxdy}_{d\vec{S}}$$

$$i_c = \int g.\delta(y).dx\,dy$$

$$i_c = g.l$$

$$\boxed{\begin{aligned} \vec{B} &= -\frac{2\pi}{c} g\,\hat{x} && y>0 \\ \vec{B} &= \frac{2\pi}{c} g\,\hat{x} && y<0 \end{aligned}}$$



iv)
$$\bar{A} = \frac{1}{c}\int \frac{I.d\vec{l}}{|\vec{r}-\vec{r}'|} \quad , \quad \bar{A} = \frac{1}{c}\int_\Omega \frac{\vec{J}}{|\vec{r}-\vec{r}'|}\,d\Omega$$

$$\frac{1}{c}\iiint \frac{g.\delta(y).\,dx.dy\,dz}{\sqrt{(x-x')^2+(y)^2+(z-z')^2}}$$

▲ Otra vez la integral que diverge

$$\vec{B} = \vec{\nabla}\times\vec{A} \qquad \text{sea } y>0 \rightarrow$$

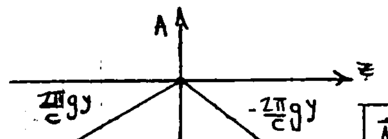
$$\hat{x})\quad -\frac{2\pi}{c}g = \frac{\partial}{\partial y}A_z - \underbrace{\frac{\partial}{\partial z}A_y}_{=0}$$

$$A_z = -\frac{2\pi}{c}.g\,y$$

las otras componentes

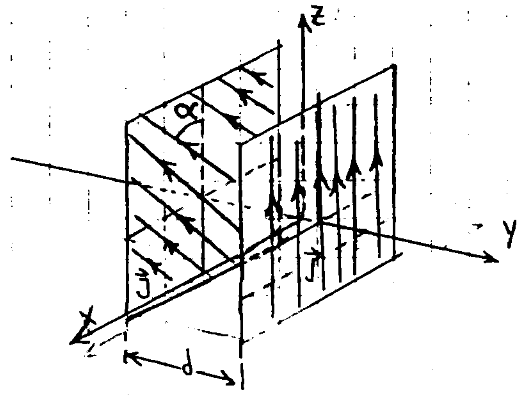
$$0 = \underbrace{\frac{\partial A_x}{\partial z}}_{=0} - \underbrace{\frac{\partial A_z}{\partial x}}_{=0}$$

$$0 = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$$



$$\boxed{\bar{A} = -\frac{2\pi}{c}.g.y\,\hat{z}} \quad y>0$$

*C



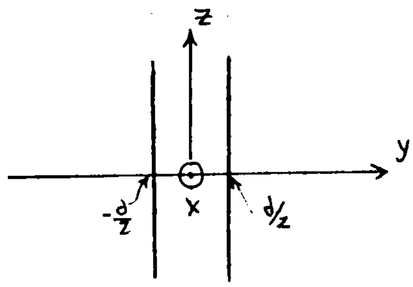
1)

$$\vec{j}(\vec{r}) = \begin{cases} \vec{g}\cdot\delta\left(y-\frac{d}{2}\right) & \text{si } y>0 \quad (g\hat{z}) \\ \vec{g}\cdot\delta\left(y+\frac{d}{2}\right) & \text{si } y<0 \end{cases}$$

$\vec{g} = g\cdot\cos\alpha\,\hat{z} + g\cdot\operatorname{sen}\alpha\,\hat{x}$

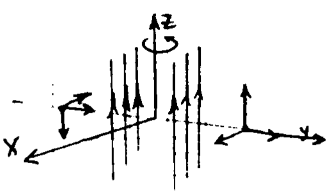
ii) Simetría de traslación en $\hat{z}$, $\hat{x}$

$\vec{B} \neq \vec{B}(x,z) \Rightarrow \vec{B} = \vec{B}(y)$



• CASO $\alpha = 0$

simetría reflexión en ZX, ZY



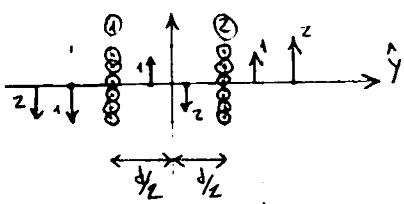
reflejo ZX

$$\begin{aligned} B_x' &= -B_x = -B_x \\ B_y' &= B_y = -B_y \\ B_z' &= -B_z = B_z \end{aligned}$$

Giro π en $\hat{z}$

$B_y = 0$, $B_z = 0$

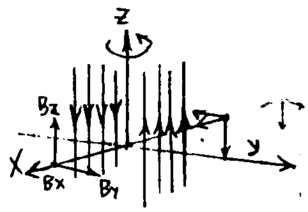
$\boxed{\vec{B} = B(y)\,\hat{x}}$ y por mano derecha será:



Adentro se cancela por superposición

• CASO $\alpha = \pi$

simetría de reflexión en ZY



Giro π en $\hat{z}$ y pongo $(-)$

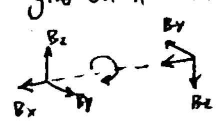
$$\begin{aligned} B_x' &= B_x = -(-B_x) \\ B_y' &= -B_y = -(-B_y) \\ B_z' &= -B_z = -(B_z) \end{aligned}$$

Reflexión en ZY

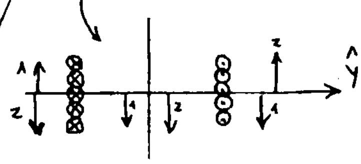
$B_y = 0$,

$\vec{B} = B(y)\,\hat{x}$

giro en π en $\hat{x}$



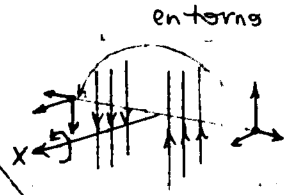
$\rightarrow B_y = 0$
$B_z = 0$



Afuera se cancela por superposición

▲ Pero esto solo vale para $y=0$ (en el eje)

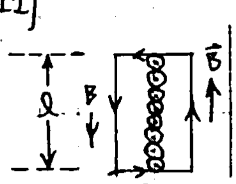
simetría de rotación en π entorno a $\hat{x}$



$\rightarrow B_z = 0$

$\boxed{\vec{B} = B(y)\,\hat{x}}$

iii)



$\oint \vec{B}\cdot d\vec{\ell} = \frac{4\pi}{c}\iint g\cdot\delta\left(y+\frac{d}{2}\right)\cdot dx\cdot dy$

$B\cdot\cancel{2\ell} = \frac{4\pi}{\cancel{2}\,c}\cdot g\cdot\cancel{\ell}$

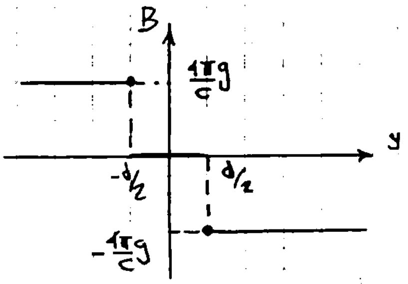
Para la situación de chapa ② el cálculo es simil

$B = \frac{2\pi g}{c} \rightarrow \vec{B}_1 = \frac{2\pi}{c}g\,\hat{x} \quad y<-d/2$

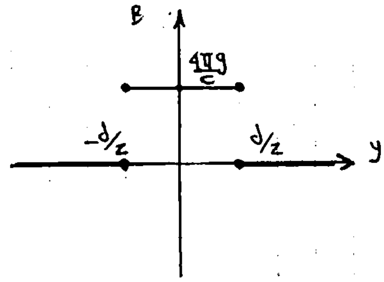
$\vec{B}_1 = -\frac{2\pi}{c}g\,\hat{x} \quad y>-d/2$

$$\Rightarrow \boxed{\vec{B} = \begin{cases} \frac{4\pi}{c}g\,\hat{x}, & y<-d/2 \\ 0, & -d/2<y<d/2 \\ -\frac{4\pi}{c}g\,\hat{x}, & y>d/2 \end{cases} \quad (\text{caso } \alpha=0)}$$

• Caso $\alpha = 0$



• Caso $\alpha = \pi$

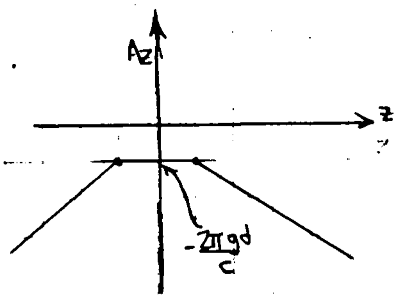


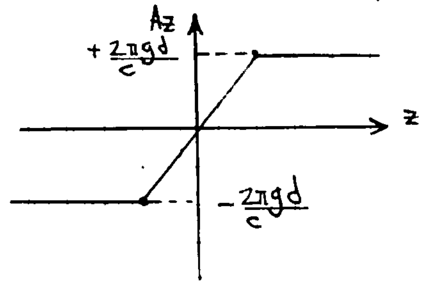
Para $\alpha = \pi$

$$\vec{B} = \begin{cases} 0 & y > d/2 \\ & y < -d/2 \\ \frac{4\pi}{c} g\,\hat{x} & -d/2 < y < d/2 \end{cases}$$

↑ Esto sale usando mismo circuito de Ampere que ya utilizamos

iv) el cálculo del potencial $\vec{A}$ puede hacerse en forma similar al punto anterior





$$\boxed{\begin{array}{ll} \vec{A} = -\frac{4\pi g}{c} y\,\hat{z} & y > d/2 \\ \vec{A} = \frac{4\pi g}{c} y\,\hat{z} & y < -d/2 \end{array}}$$

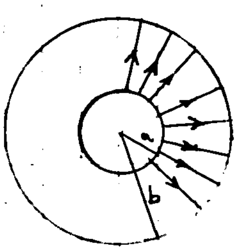
$\hat{x})\quad \frac{4\pi g}{c} = \frac{\partial}{\partial y}(A_z) - \underbrace{\frac{\partial}{\partial z}(A_y)}_{=0}$

$A_z = \frac{4\pi g}{c} y$

$\vec{A} = -\frac{2\pi g d}{c}\hat{z} \quad -d/2 < y < d/2$

$$\boxed{\begin{array}{ll} \vec{A} = 0 & y > d/2,\ y < -d/2 \\ \vec{A} = \frac{2\pi g d}{c}\hat{z} & |y| < d/2 \end{array}}$$

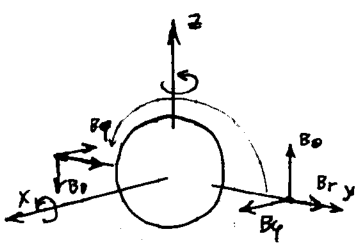
∗ d.



1) $\vec{J}(\vec{r}) = \begin{cases} 0 & r > b,\ r < a \\ J\hat{r} & a < r < b \end{cases}$ [$\hat{r}$ de esféricas]

ii) Simetría de revolución (rotación en θ, φ de esféricas)
Reflexión XY, ZX, ZY

$\vec{B}$ no puede depender de θ, φ
$\rightarrow \vec{B} = \vec{B}(r)$



reflejo en ZX y giro en π torno a $\hat{z}$

$B'_r = -B_r = B_r$
$B'_\theta = -B_\theta = B_\theta$
$B'_\varphi = \underbrace{-B_\varphi = -B_\varphi}$

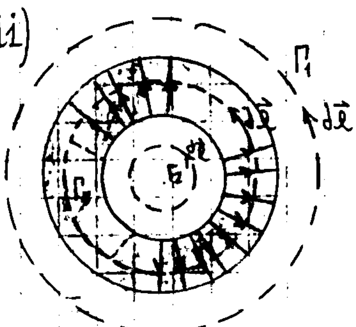
$B_r = 0$, $B_\theta = 0$

si giro en π entorno a $\hat{x}$ →

$B'_r = -B_r = B_r$
$B'_\theta = -B_\theta = -B_\theta$
$B'_\varphi = \underbrace{-B_\varphi = B_\varphi}$

$B_\varphi = 0$

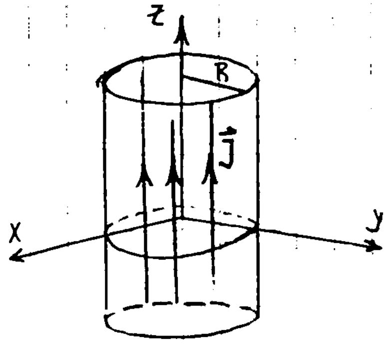
iii)



$\vec{B} = 0$ fuera
$\vec{B} = 0$ dentro
porque no hay dirección privilegiada dentro de la zona esférica de las corrientes salvo $\hat{r}$ y no puede ir en $\hat{r}$ el $\vec{B}$

$\therefore \boxed{\vec{B} = 0}$ fuera de la esfera "b" o en el interior de la esfera "a"

* e

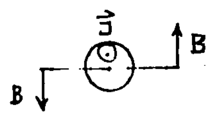


1) $\vec{J}(\vec{r}) = \begin{cases} 0 & r>R \\ \vec{J} = J\hat{z} & r<R \end{cases}$ [r de cilíndricas]

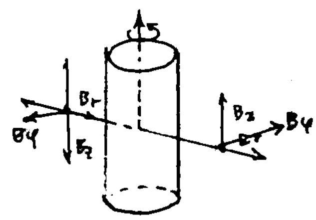
ii) Simetría de rotación en $\hat{z}$ (φ)
Simetría de traslación en $\hat{z}$
Simetría de reflexión en ZX, ZY

traslación en $\hat{z}$
rotación en $\hat{\varphi}$ $\rightarrow$ $\vec{B} = \vec{B}(r)$

$$\boxed{\vec{B} = B(r)\,\hat{\varphi}}$$



← dirección de acuerdo a la mano derecha

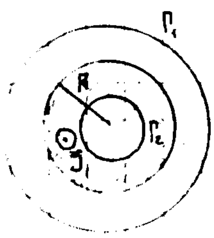


reflexión ZX

$$\begin{aligned} B'_r &= -B_r = B_r \\ B'_\varphi &= B_\varphi = B_\varphi \\ B'_z &= \underbrace{-B_z = B_z}_{\text{rotación en } \pi \text{ en } \hat{z}} \end{aligned}$$

$$\Rightarrow \begin{cases} B_z = 0 \\ B_r = 0 \end{cases}$$

iii)

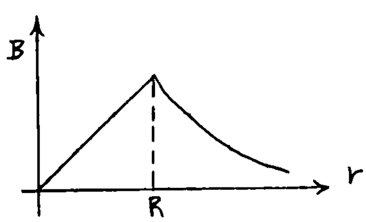


$$\int_{\Gamma_1} \vec{B}\cdot d\vec{l} = \frac{4\pi}{c}\overbrace{(J\,\pi R^2)}^{I_c}$$

$$B\cdot 2\pi r = \frac{4\pi^2 R^2}{c} J \rightarrow \boxed{\vec{B} = \frac{2\pi R^2}{c} J \frac{1}{r}\hat{\varphi}} \quad r>R$$

$$\oint_{\Gamma_2} \vec{B}\cdot d\vec{l} = \frac{4\pi}{c} J\cdot\pi r^2$$

$$B\cdot 2\pi r = \frac{4\pi^2 r^2}{c} j \rightarrow \boxed{\vec{B} = \frac{2\pi r j}{c}\hat{\varphi}} \quad r<R$$



iv) • r>R

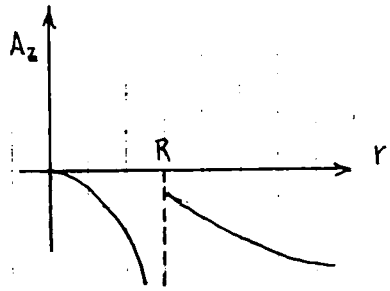
$$\hat{\varphi}]\ \frac{2\pi R^2}{c} J \frac{1}{r} = \underbrace{\frac{\partial A_r}{\partial z}}_{=0} - \frac{\partial A_z}{\partial r}$$

$$A_z = -\frac{2\pi R^2}{c} J \ln r$$

• r<R

$$\hat{\varphi}]\ \frac{2\pi r j}{c} = \underbrace{\frac{\partial A_r}{\partial z}}_{=0} - \frac{\partial A_z}{\partial r}$$

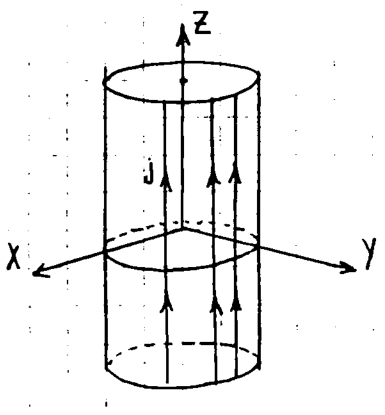
$$A_z = -\frac{2\pi}{c} j \frac{r^2}{2}$$



$\vec{A}$ se podría empalmar con una constante dada

$$\boxed{\begin{aligned} r>R &\rightarrow \vec{A} = -\frac{2\pi R^2 J}{c}\ln r\ \hat{z} \\ r<R &\rightarrow \vec{A} = -\frac{\pi J}{c} r^2\ \hat{z} \end{aligned}}$$

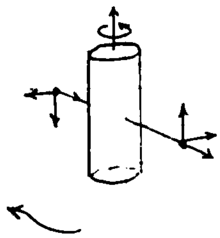
* f.



i) $J(\vec{r}) = \vec{g}\cdot\delta(r-R) = g\cdot\delta(r-R)\,\hat{z}$

ii) Simetría rotación (φ)
Simetría traslación en $\hat{z}$
Reflexión en ZX, YZ

Por rotación y traslación es
$\vec{B} = \vec{B}(r) \rightarrow$

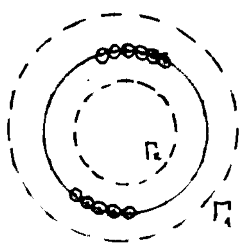


	reflexión	rotación en π
$B_r' =$	$-B_r =$	B_r
$B_\varphi' =$	$B_\varphi =$	B_φ
$B_z' =$	$-B_z =$	B_z

$\therefore \quad B_z = 0 \,,\; B_r = 0$

$$\boxed{\vec{B} = B(r)\,\hat{\varphi}}$$

iii)

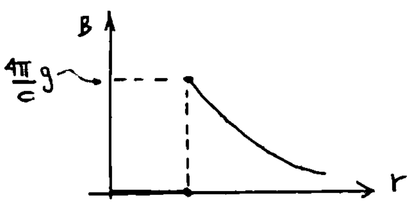


$\oint_{\Gamma_2} \vec{B}\cdot d\vec{l} = 0 \rightarrow \boxed{\vec{B} = 0} \qquad r<R$

$$\oint_{\Gamma_1} \vec{B}\cdot d\vec{l} = \frac{4\pi}{c}\cdot I_c = \frac{4\pi}{c}\cdot\int_0^{2\pi}\!\!\int_0^{R} g\cdot\delta(r-R)\,dr\,d\varphi\,R$$

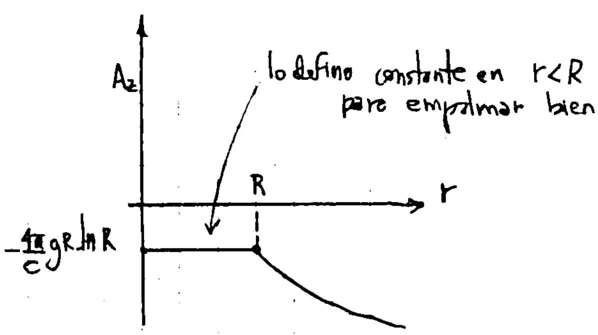
$B\cdot\cancel{2\pi}\cdot r = \frac{4\pi}{c}\cdot\cancel{2\pi}\cdot g\cdot R \qquad r>R$

$B = \frac{4\pi}{c}\,g\,\frac{R}{r} \rightarrow \boxed{\vec{B} = \frac{4\pi g R}{c}\cdot\frac{1}{r}\,\hat{\varphi}}$



iv) Partiendo de $\vec{B} = \vec{\nabla}\times\vec{A}$

$$\hat{\varphi} \quad \frac{4\pi}{c}\,g\,\frac{R}{r} = \underbrace{\frac{\partial A_r}{\partial z}}_{=0} - \frac{\partial A_z}{\partial r}$$

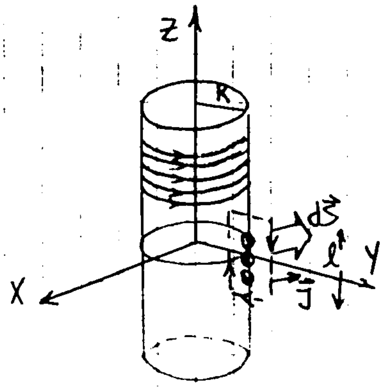


$A_z = -\frac{4\pi g R}{c}\int \frac{1}{r}\,dr$

$A_z = -\frac{4\pi g R}{c}\cdot\ln|r|$

$$\boxed{\vec{A} = -\frac{4\pi}{c}\,g\,R\cdot\ln r\;\hat{z}}$$

* 8.



n vueltas por longitud

$$I.n.\ell = \int_{-\ell/2}^{+\ell/2}\int_{-\infty}^{+\infty} g.\delta(y-R).dy.dz$$

$$I n \ell = g.\ell$$

$$\boxed{\vec{B} = B(r)\,\hat{z}}$$

i) $\vec{J}(\vec{r}) = I.n.\delta(r-R)\,\hat{\varphi}$

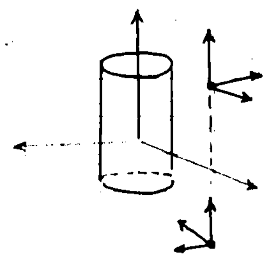
ii)

Simetría de rotación (φ)

Simetría de traslación en $\hat{z}$

Simetría de reflexión en XY

$\vec{B} = \vec{B}(r)$



Reflexión

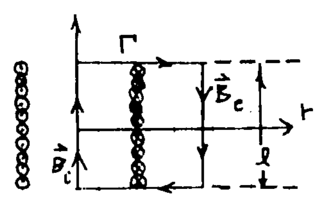
$B_r' = -B_r = B_r$

$B_\varphi' = -B_\varphi = B_\varphi$

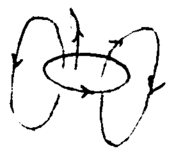
$B_z' = B_z = B_z$ — traslación en $\hat{z}$

$B_r = 0$, $B_\varphi = 0$

iii)

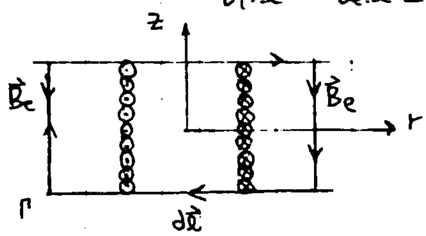


$n = \frac{N}{L}$



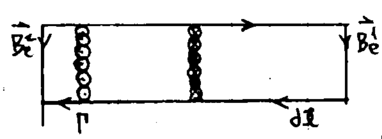
$$\oint_\Gamma \vec{B}.d\vec{\ell} = \frac{4\pi}{c}.i.n.\ell$$

$$B_i.\ell - B_e.\ell = \frac{4\pi}{c}\, i n \ell$$



$\oint_\Gamma \vec{B}.d\vec{\ell} = 0$ La corriente entrante es igual a la saliente $\Rightarrow$ Ic neta es nula

$B_e.\ell - B_e.\ell = 0 \rightarrow$ Este circuito no aporta nada



$B_e^1.\ell = B_e^2.\ell$

$\rightarrow \vec{B}$ afuera es constante (no depende de r)

Hay que calcular B_i lo cual hacemos por integración directa:

$$B_i = \frac{1}{c}\int \frac{\vec{J}\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}.dV$$

$$B_i(\vec{r}=z) = \frac{1}{c}\int \quad \cdot\,\hat{r}$$

$\vec{r}-\vec{r}' = (0,0,z) - (R,0,z') = (-R,0,z-z')$

$$g\hat{\varphi}\times(-R,0,z-z') = \begin{vmatrix} \hat{r} & \hat{\varphi} & \hat{z} \\ 0 & g & 0 \\ -R & 0 & z-z' \end{vmatrix}$$

$$= g.(z-z')\,\hat{r} + Rg\,\hat{z}$$

$$\frac{1}{c}\iint \frac{I n.(z-z')\,\hat{r}'}{(R^2+(z-z')^2)^{3/2}}\,R\,d\varphi'.dz'$$

$$+\frac{1}{c}\int_{-\infty}^{+\infty}\int_0^{2\pi} \frac{R I n\,\hat{z}'.R\,d\varphi'.dz'}{[R^2+(z-z')^2]^{3/2}}$$

$$R^2 I.n\,\hat{z}\;2\pi\int_{-\infty}^{+\infty}\frac{dz'}{[R^2+(z-z')^2]^{3/2}}$$

$$\int_{-\infty}^{+\infty} \frac{dz'}{\left(R^2+(z-z')^2\right)^{3/2}} = -\int_{+\infty}^{-\infty} \frac{du}{(R^2+U^2)^{3/2}} = \int_{-\infty}^{+\infty} \frac{du}{(R^2+U^2)^{3/2}} = \frac{U}{R^2.(R^2+U^2)^{1/2}}\Bigg|_{-\infty}^{+\infty}$$

$z-z'=U$
$-dz' = du$

$$= \frac{U}{R^2(R^2+U^2)^{1/2}} - \frac{-U}{R^2.(R^2+U)^{1/2}}$$

$U \gg R$
$1 \gg \frac{R}{U}$

$$= \lim_{U\to\infty} \frac{2U}{R^2(R^2+U^2)^{1/2}}$$

$$= \lim_{U\to\infty} \frac{2\cancel{U}}{R^2 \cancel{|U|}\left(1+\frac{R^2}{U^2}\right)^{1/2}} = \frac{2}{R^2}$$

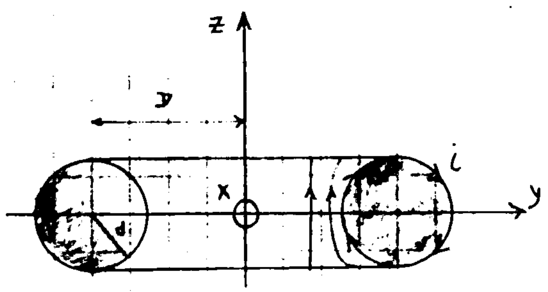
$$\Rightarrow \quad \vec{B_i} = \cancel{R^2}\, \frac{2\pi . I . n}{c} \frac{2}{\cancel{R^2}}\,\hat{z} \quad \to \quad B_i = \frac{4\pi I n}{c}\hat{z} \quad \therefore$$

$$B_i . \ell - B_e . \ell = \frac{4\pi}{c} I n \ell \quad \Rightarrow \quad B_e = \frac{4\pi}{c} I n . \ell - \frac{4\pi}{c} I n \ell = 0 \quad \Rightarrow$$

$$\boxed{B_e = 0}$$

$\boxed{\vec{B} = \frac{4\pi}{c} I n \quad \hat{z}}$ dentro del solenoide $\quad \boxed{\vec{B}=0}$ fuera del solenoide

$*$ h.

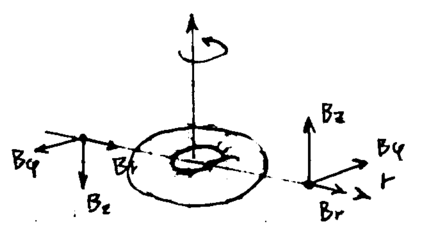


N vueltas

i) $\vec{J} = g$

ii) Simetría de rotación en $\hat{z}$ (en φ)

Simetría de reflexión en ZY, ZX



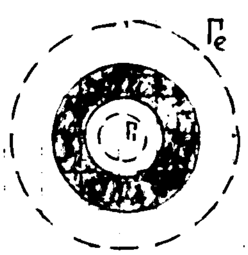
$\vec{B}$ No depende de $\varphi \Rightarrow \vec{B} = \vec{B}(r,z)$

reflexión

$B_r' = -B_r \quad = \quad B_r$
$B_\varphi' = B_\varphi \quad = \quad B_\varphi$
$B_z' = -B_z \quad = \quad B_z$

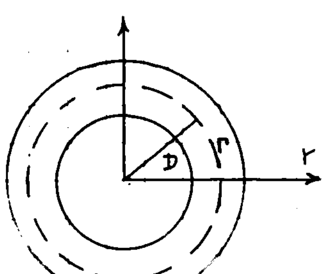
giro de π entorno a $\hat{z}$

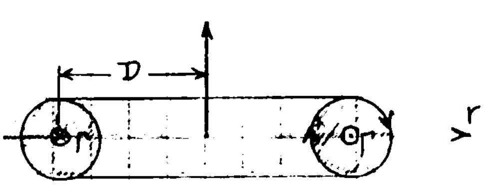
$\therefore \quad B_r = 0, \quad B_z = 0 \quad \Rightarrow \quad \boxed{\vec{B} = B(r,z)\,\hat{\varphi}}$



Γ_e, Γ_i no concatenan corrientes $\Rightarrow$ como $\vec{B}$ es constante sobre $\Gamma_e, \Gamma_i \Rightarrow \vec{B}=0$ fuera del toro

iii)





Hago un Ampere en r, z=0 $\to$ $\vec{B}$ es constante sobre ella

$$\oint_\Gamma \vec{B}\, d\vec{\ell} = \frac{4\pi}{c} . i . N$$

$$B \cdot 2\pi r = \frac{4\pi}{c} i N$$

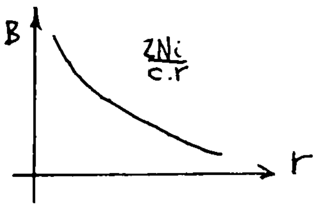
$$B = \frac{2}{c}\frac{iN}{r}$$

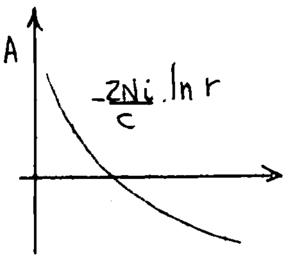
$$\boxed{\vec{B} = \frac{2Ni}{c \cdot r}\,\hat{\varphi}}$$

dentro del toro y a $z=0$
(pero para otro z es igual) $\rightarrow$ $\vec{B} \neq \vec{B}(z)$

$$\boxed{\vec{B} = 0}$$

fuera del toro





iv)

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$\hat{\varphi}$) $\quad \frac{2Ni}{c\,r} = \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}$

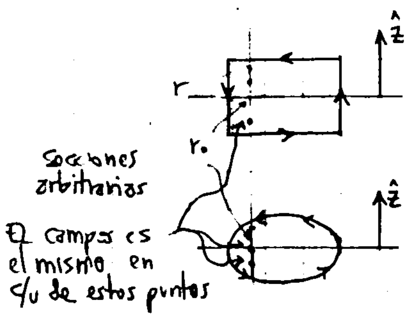
$$-\frac{2Ni}{c.r} = \frac{\partial A_z}{\partial r}$$

$$A_z = -\frac{2Ni}{c\,r}\,dr$$

$$A_z = -\frac{2Ni}{c}.\ln r$$

$$\boxed{\vec{A} = -\frac{2Ni}{c}.\ln r\ \hat{z}}$$

- Toro de sección arbitraria

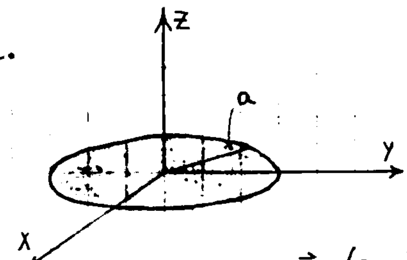


El campo $\vec{B}$ solo depende de la distancia radial; luego a un mismo radio r el $|\vec{B}|$ es el mismo sin importar la altura (obviamente tenemos que estar situados en el interior del toro)

II Integración directa: solución de Poisson

3.

a.



$$\varphi = \int_{\Omega'} \frac{\rho(\vec{r}')\, d\Omega'}{|\vec{r}-\vec{r}'|}$$

$$\rho(\vec{r}') = \sigma \cdot \delta(z-0) \qquad r<a$$

cilíndricas

se calcula sobre el eje z

$$\vec{r} = (0, z) \qquad \vec{r}' = (r', 0) \quad \rightarrow \quad |\vec{r}-\vec{r}'| = \sqrt{r'^2 + z^2}$$

$$\varphi(0,z) = \int_{-\infty}^{+\infty}\int_0^{2\pi}\int_0^a \frac{\sigma\, \delta(z)\, dr'\, r'\, d\varphi'\, dz'}{(r'^2+z^2)^{1/2}} = \int_{-\infty}^{+\infty}\int_0^a \frac{2\pi \cdot \sigma \cdot \delta(z) \cdot r' \cdot dr' \cdot dz'}{(r'^2+z^2)^{1/2}}$$

$$\varphi(0,z) = \pi \cdot \sigma \int_{-\infty}^{+\infty} \delta(z') \cdot dz' \int_{z^2}^{a^2+z^2} \frac{dU}{U^{1/2}} = \pi \cdot \sigma \cdot \left.\frac{U^{1/2}}{1/2}\right|_{z^2}^{a^2+z^2}$$

$r'^2 + z^2 = U$
$2r'dr' = dU$

$$\boxed{\varphi(0,z) = \pi \cdot \sigma \cdot 2 \cdot \left(\sqrt{a^2+z^2} - \sqrt{z^2}\right)}$$

* Algunos casos límites

Sea $z \gg a \rightarrow 1 \gg \frac{a}{z}$

$$\varphi(0, z\gg a) = 2\pi\sigma |z| \left[\left(1 + \left(\frac{a}{z}\right)^2\right)^{1/2} - 1\right]$$

$$\approx 2\pi\sigma|z| \cdot \left(\cancel{1} + \frac{1}{2}\frac{a^2}{z^2} - \cancel{1}\right)$$

$$\varphi(0, z\gg a) \approx \frac{\cancel{2}\pi\sigma}{\cancel{2}} \cdot \frac{a^2}{|z|} = \frac{Q}{|z|}$$ ← Potencial de una carga puntual

Sea $z \ll a \rightarrow \frac{z}{a} \ll 1$

$$\varphi(0, z\ll a) = 2\pi\sigma \cdot \left[a\left(1+\frac{z^2}{a^2}\right)^{1/2} - |z|\right]$$

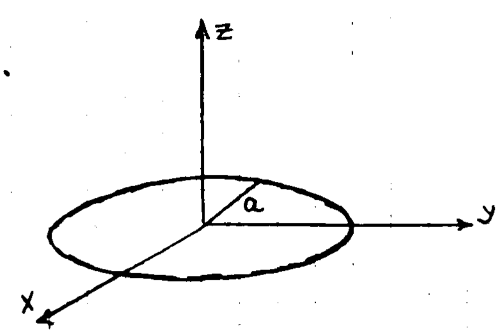
$$2\pi\sigma a \left[\left(1+\left[\frac{z}{a}\right]^2\right)^{1/2} - \frac{|z|}{a}\right]$$

$$\approx 2\pi\sigma a \left(1 + \frac{1}{2}\frac{z^2}{a^2} - \frac{z}{a}\right)$$

$$\varphi(0, z\ll a) \approx 2\pi\sigma a - 2\pi\sigma z$$ ← potencial de un plano infinito (si olvidamos la constante $2\pi\sigma a$)

$$\vec{E}(0,z) = 2\sigma\pi \left(\frac{z}{\sqrt{a^2+z^2}} - \frac{z}{\sqrt{z^2}}\right)$$

b.



$$\rho(\vec{r}) = \lambda \cdot \delta(r'-a) \cdot \delta(z')$$

$$\vec{r} = (0, z)$$
$$\vec{r}' = (a, 0)$$

$$\vec{r}-\vec{r}' = (-a, z) \rightarrow |\vec{r}-\vec{r}'| = (a^2+z^2)$$

$$\varphi(z) = \int_0^{+\infty}\int_{-\infty}^{+\infty}\int_0^{2\pi} \frac{\lambda.\delta(r'-a).\delta(z').d\varphi'.dr'dz'.a}{\sqrt{a^2+z^2}}$$

$$\varphi(z) = \int_0^{+\infty}\delta(r-a).dr' \int_{-\infty}^{+\infty}\delta(z).dz' \int_0^{2\pi}\frac{a.\lambda.d\varphi'}{\sqrt{a^2+z^2}} = \frac{\lambda.a.2\pi}{\sqrt{a^2+z^2}}$$

$$\boxed{\varphi(0,z) = \frac{2\pi a.\lambda}{\sqrt{a^2+z^2}}}$$

* Algunos casos límites

Sea $z \gg a \rightarrow 1 \gg \frac{a}{z}$

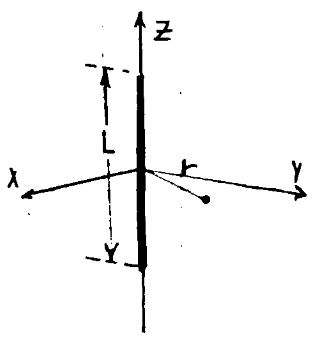
$$\varphi(z\gg a) = 2\pi a\lambda . \frac{1}{|z|\sqrt{\frac{a^2}{z^2}+1}}$$

$$\varphi(z\gg a) \cong \frac{2\pi a\lambda}{|z|}\left(1-\frac{1}{2}\frac{a^2}{z^2}\right)$$

si $z \ggg a \rightarrow \quad \varphi \cong \frac{2\pi a.\lambda}{|z|} = \frac{Q}{|z|}$ ← potencial de una carga puntual

$\varphi(0,0) = 2\pi\lambda$ ← potencial constante en el origen $\therefore \vec{E}(\vec{0}) = 0$

c.



Usamos cilíndricas ⇒

$\vec{r} = (r,0)$ $\qquad \vec{r}-\vec{r}\,' = (r,z')$

$\vec{r}\,' = (0,z')$ $\qquad |\vec{r}-\vec{r}\,'| = \sqrt{r^2+z'^2}$

$$\lambda\,\delta(r).\underbrace{\delta(\varphi-\varphi')}_{\delta(\cos\varphi-\cos\varphi')} = \rho(r',\varphi')$$

$$\varphi(r,0) = \int_{-L/2}^{L/2}\frac{\lambda\, dz'}{\sqrt{r^2+z'^2}} = \lambda.\ \ln\left|z'+\sqrt{r^2+z'^2}\right|\Big|_{-L/2}^{L/2}$$

$$\boxed{\varphi(r,0) = \lambda.\ \ln\left|\frac{L/2+\sqrt{r^2+L^2/4}}{-L/2+\sqrt{r^2+L^2/4}}\right|}$$

$r \ll L \rightarrow \frac{r}{L} \ll 1 \rightarrow$

$$\varphi(r\ll L) = \lambda.\ln\left|\frac{\frac{L}{2}+L.\left(\frac{1}{4}+\frac{r^2}{L^2}\right)^{1/2}}{-\frac{L}{2}+L.\left(\frac{1}{4}+\frac{r^2}{L^2}\right)^{1/2}}\right|$$

$$= \lambda.\ln\left|\frac{\frac{1}{2}+\left(\frac{1}{4}+\frac{r^2}{L^2}\right)^{1/2}}{-\frac{1}{2}+\left(\frac{1}{4}+\frac{r^2}{L^2}\right)^{1/2}}\right| = \lambda.\ln\left|\frac{1+\left(1+\left(\frac{2r}{L}\right)^2\right)^{1/2}}{-1+\left(1+\left(\frac{2r}{L}\right)^2\right)^{1/2}}\right|$$

$$= \lambda.\ln\left|\frac{2+\frac{2r^2}{L^2}}{\frac{2r^2}{L^2}}\right|$$

$$\frac{1+\frac{r^2}{L^2}}{\frac{r^2}{L^2}} = \frac{L^2}{r^2}+1$$

$$\varphi(r\ll L) \approx \lambda.\ln\left|\frac{L^2}{r^2}+1\right| \cong \lambda.\frac{L^2}{r^2}$$

sea $r \gg L \rightarrow$

$1 \gg \frac{L}{r}$

$$\varphi(r \gg L) = \lambda . \ln \left(\frac{L}{2} + r \sqrt{1 + \frac{L^2}{r^2 4}} \Big/ -\frac{L}{2} + r \sqrt{1 + \frac{L^2}{r^2 4}} \right) .$$

$$\varphi(r \gg L) = \lambda . \ln \left(\frac{L}{r} + \left(1 + \frac{L^2}{4r^2}\right)^{1/2} \Big/ -\frac{L}{r} + \left(1 + \frac{L^2}{r^2 4}\right)^{1/2} \right)$$

$$\varphi(r \gg L) = \lambda . \ln \left(1 + L/r \,/\, 1 - L/r\right)$$

$$\lambda . \ln \left(\frac{1 + L/r}{1 - L/r} \right)$$

$$\approx \lambda . \left(\frac{1 + L/r}{1 - L/r} - 1 \right) = \lambda \left(\frac{2L/r}{1 - L/r} \right)$$

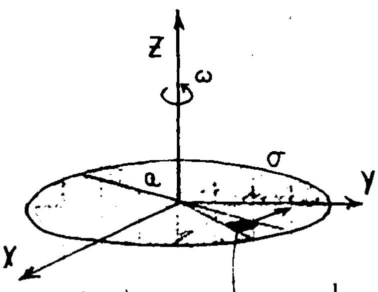
$$\varphi(r \gg L) \cong \frac{\lambda 2L}{r} \left(1 + \frac{L}{r}\right)$$

$$\varphi(r \gg L) \cong \frac{2\lambda L}{r} + 2\lambda \frac{L^2}{r^2}$$

potencial de una carga $Q = 2\lambda L$; parte dipolar

4.

a)



$r . \omega = v$

Se puede pensar que es como una distribución de corrientes

$$r . d\varphi . dr . \sigma = dq$$

$$\sigma . r . d\varphi . dr = dq$$

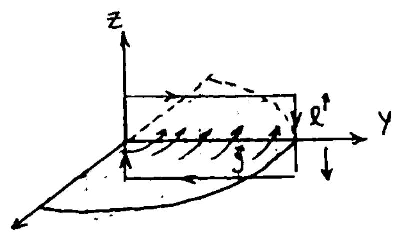
$$\sigma . r . \frac{d\varphi}{dt} dr = \frac{dq}{dt}$$

$$\sigma r . \omega . dr = di$$

$$\sigma . v . dr = di$$

$$\int di = \int_0^a \sigma r . \omega . dr$$

corriente total $\rightarrow$ $I = \frac{\sigma \omega}{2} a^2$



$$I = \int \vec{J} . d\vec{S}$$

$$I = \int_{-l/2}^{+l/2} \int_0^a g . \delta(z) . dz \, dy$$

hago la integración en y pero vale x radio

$$\frac{\sigma \omega a^2}{2} = g . a$$

$$\frac{\sigma \omega a}{2} = g . \rightarrow \vec{J} = \frac{\sigma \omega . a}{2} . \delta(z) \hat{\varphi}$$

$$\vec{B}(\vec{x}) = \frac{1}{c} \int_{V'} \frac{\vec{J} \times (\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3} . dV'$$ $\leftarrow$ Biot-Savart

$\vec{x} = (0, z) \quad \rightarrow \quad \vec{x} - \vec{x}' = (-r', z)$
$\vec{x}' = (r', 0)$

$$\vec{B}(\vec{x}) = \int_{-\infty}^{+\infty}\int_0^a\int_0^{2\pi} \frac{\frac{\sigma.\omega.a}{2}.\delta(z')\,\hat{\varphi}' \times (-r'\hat{r}', z\hat{z})}{(r'^2 + z^2)^{3/2}} . r'.d\varphi'.dr'.dz'$$

$$\vec{B}(z) = \frac{\sigma\omega a}{2}\int_0^a\int_0^{2\pi} \frac{(r'\hat{z}', z\,\hat{r})\,.r'}{(r'^2+z^2)^{3/2}} . d\varphi'.dr'$$

$\hat{r}\ \hat{\varphi}\ \hat{z}\ \hat{r}$
$\hat{\varphi}' \times \hat{z} = \hat{\varphi}' \times \hat{z}' = \hat{r}'$
$\hat{z}' = \hat{z}$

estamos parados en
$z' = 0$
$r = 0$

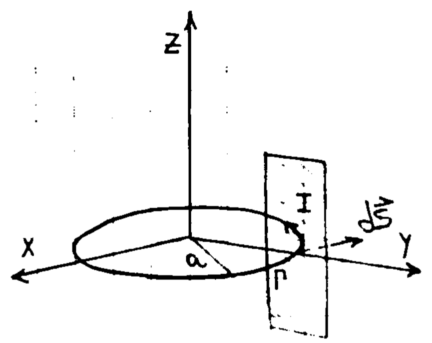
$$\vec{B}(z) = \sigma\omega a . \left[\int_0^a \frac{r'^2.dr'}{(r'^2+z^2)^{3/2}}\hat{z} + \int_0^a \frac{z.r'dr'}{(r'^2+z^2)^{3/2}}\hat{r}\right]$$

a^2+z^2

$\rightarrow$ esta no me interesa pero igualmente su resolución no es fácil porque hay que convertir el $\hat{r}$ en cartesianas para integrar

$\sigma\omega a$

b.



$$\vec{J}(\vec{r}) = I.\delta(z).\delta(r-a)\,\hat{\varphi}$$

$\vec{J} \,//\, d\vec{S}$

$I = \int \vec{J}\, d\vec{S}$

$I = \int_{-\infty}^{\infty}\int_0^{\infty} g\ \delta(z).\delta(r-a)\,dr\,dz$

$I = g$

$\vec{r} - \vec{r}' = (0,0,z) - (x', y', 0)$
$\vec{r} - \vec{r}' = (0, z) - (a, 0)$

i)
$$\vec{B} = \frac{1}{c}\int_V \frac{\vec{J}\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\,dV$$

$\hat{r}\hat{\varphi}\hat{z}\hat{r}\hat{\varphi}$

$$\vec{B} = \frac{1}{c}\int_{-\infty}^{+\infty}\int_0^{2\pi}\int_0^{+\infty} g\hat{\varphi}.\delta(z).\delta(r-a)\times\frac{(-a\hat{r} + z\hat{z})}{(z^2+a^2)^{3/2}}a\,dr'.d\varphi'.dz'$$

$$\vec{B} = \frac{1}{c}\int_{-\infty}^{+\infty}\int_0^{2\pi}\int_0^{+\infty} \frac{g.\delta(z)\,\delta(r-a)\,a\,dr'\,d\varphi'\,dz'}{(z^2+a^2)^{3/2}}\left[+a\hat{z} + z\hat{r}\right]$$

$$\boxed{\vec{B}(r=0) = +\frac{1}{c}\cdot\frac{I.a^2.2\pi}{(z^2+a^2)^{3/2}}\hat{z}}$$

La parte en $\hat{r}$ la tiro (igual da cero por simetría)

ii) Si $z \gg a \rightarrow \quad 1 \gg \frac{a}{z}$

$$\vec{B} = \frac{1}{c}\frac{a^2 I.2\pi}{z^3}\frac{1}{\left(1+\underbrace{\frac{a^2}{z^2}}_{\text{lo tiro}}\right)^{3/2}} \approx \boxed{\frac{2\pi.a^2.I}{c\ z^3}}$$

$\leftarrow$ Para puntos muy alejados

iii)

5.

$$\phi(r) = \frac{e}{r}\left(1+\frac{r}{a}\right)\cdot e^{-\frac{2r}{a}}$$

a.
$$\frac{e}{r}\cdot e^{-\frac{2r}{a}} + \frac{e}{a}\cdot e^{-\frac{2r}{a}} = \phi(r)$$

$$\nabla^2\phi = 4\pi\rho$$

este diverge en $r=0$

esto ahora puede converger

$$\underbrace{\left(\frac{e}{r}\right)\left(e^{-\frac{2r}{a}}-1\right)} + \frac{e}{r} + \underbrace{\frac{e}{a}\cdot e^{-\frac{2r}{a}}}$$

$\rho = q\cdot\delta(\vec{x}) = e\cdot\delta(r)$ → r de esféricas

proviene de una densidad de carga

$$\equiv \phi'(r) \quad \text{con} \quad \phi(r) = \frac{e}{r} + \phi'(r)$$

$$\nabla^2\phi'(r) = \frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(r^2\cdot\left\{\frac{e}{a}\cdot\left(-\frac{2}{a}\right)e^{-\frac{2r}{a}} + \frac{e}{r}\cdot\left[-\frac{2}{a}\cdot e^{-\frac{2r}{a}}\right] - \frac{e}{r^2}\left[e^{-\frac{2r}{a}} - 1\right]\right\}\right)$$

(en esféricas)

$$\nabla^2\phi' = \frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left[-\frac{2}{a^2}r^2\cdot e\cdot e^{-\frac{2r}{a}} - 2\frac{r}{a}\cdot e\cdot e^{-\frac{2r}{a}} - e\cdot e^{-\frac{2r}{a}} + e\right]$$

$$-4\pi\rho = \frac{1}{r^2}\left[-e^{-\frac{2r}{a}}\cdot\frac{2}{a}\left(-\frac{2r^2e}{a^2} - \frac{2re}{a} - e + e\cdot e^{\frac{2r}{a}}\right) + e^{-\frac{2r}{a}}\left(-\frac{4re}{a^2} - \frac{2e}{a} + \frac{2}{a}e^{\frac{2r}{a}}\cdot e\right)\right]$$

$$-4\pi\rho = e^{-\frac{2r}{a}}\frac{4e}{a^3} + \cancel{e^{-\frac{2r}{a}}\frac{4e}{a^2 r}} + \cancel{e^{-\frac{2r}{a}}\frac{e2}{ar^2}} - \cancel{\frac{2e}{ar^2}}$$
$$- \cancel{\frac{e^{-\frac{2r}{a}}4e}{a^2 r}} - \cancel{\frac{e^{-\frac{2r}{a}}2e}{r^2 a}} + \cancel{\frac{2e}{ar^2}}$$

$$\rho(r) = -\frac{1}{\cancel{4}\pi}\cdot\frac{\cancel{4}e}{a^3}\cdot e^{-\frac{2r}{a}}$$

$$\boxed{\rho(r) = -\frac{e}{\pi a^3}\cdot e^{-\frac{2r}{a}} + e.\delta(r)}$$

densidad de carga para todo r

c.

$$Q = \int_V \left[-\frac{e}{\pi a^3} e^{-\frac{2r}{a}} + e.\delta(r)\right].dV$$

$$Q = \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty}\left[-\frac{e}{\pi a^3} e^{-\frac{2r}{a}}\right] r^2.\mathrm{sen}\,\theta.\,dr.\,d\theta.\,d\varphi \quad + \quad \iiint e.\delta(\vec{r}-0).dV$$

$$\iiint e.\delta(\vec{r}-0).dV \to e$$

En rigor habría que poner $\frac{e.\delta(r)}{r^2.\,\mathrm{sen}\,\theta}$

$$-\frac{e}{\pi a^3}.2\pi.2.\int_0^{\infty} e^{-\frac{2r}{a}}.r^2.dr$$

$r^2 = u$

$2r\,dr = du$

$dv = e^{-\frac{2r}{a}}$

$v = e^{-\frac{2r}{a}}.-\frac{a}{2}$

$$\underbrace{-\frac{a}{2} r^2 e^{-\frac{2r}{a}}\Big|_0^{\infty}}_{0} - \int_0^{\infty} -\frac{a}{\cancel{2}}.e^{-\frac{2r}{a}}\cancel{2}r.dr$$

$$+ a\underbrace{\int_0^{\infty} e^{-\frac{2r}{a}}.r.dr}$$

$r = u$

$dr = du$

$$\underbrace{r.\frac{a}{2}e^{-\frac{2r}{a}}\Big|_0^{\infty}}_{=0} + a.\int_0^{\infty} -\frac{a}{2}.e^{-\frac{2r}{a}}.dr$$

$$\underbrace{-\frac{a^2}{2}\cdot\left(-\frac{a}{2}\right)e^{-\frac{2r}{a}}\Big|_0^{\infty}}_{+\frac{a^3}{4}}$$

$$\frac{\cancel{4\pi}.e}{\cancel{\pi}.\cancel{a^3}}.\frac{\cancel{a^3}}{\cancel{4}} \Longrightarrow \quad Q = -e \quad + e \rightarrow \boxed{Q = 0}$$

átomo neutro

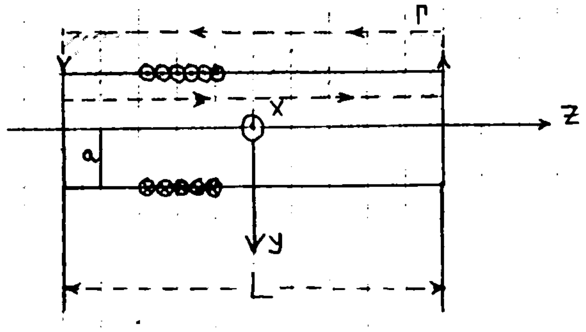
b. La interpretación es que se puede desmenuzar el potencial en una parte correspondiente a una partícula puntual (que sería el núcleo) y otra correspondiente a la nube electrónica

$$\rho(r) = e.\delta(r) - \frac{e}{\pi.a^3}.e^{-\frac{2r}{a}} \qquad \text{[en esféricas]}$$

núcleo   nube electrónica

d.

6.



a.

$$\vec{B} = \frac{1}{c}\int_V \frac{\vec{J}\times(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\, dV$$

campo en el eje $\Rightarrow$

$\vec{r} = (0, z)$ $\qquad \vec{r}\,' = (a, z')$

$\vec{r}-\vec{r}\,' = (-a, z-z')$

puedo utilizar cilíndricas cómodamente porque hay un solo $\hat{r}$

$n = \dfrac{N}{L}$ $\qquad I$

$$I_T = \oint_\Gamma \vec{J}\cdot d\vec{S}$$

$$I.N = \int_{-\frac{L}{2}}^{L/2}\int_{-\infty}^{+\infty} g\cdot\delta(y'+a)\,.\,dy'.dz'$$

$$I n L = g.L$$

$$In = g$$

rφzrφ

$$\vec{J} = In.\,\delta(y'+a)\,\hat{\varphi}$$

$$\vec{B} = \frac{1}{c}\int_{-L/2}^{L/2}\int_0^{2\pi}\int_{-\infty}^{+\infty} \frac{I.n.\,\delta(r'-a)\,\hat{\varphi}\times[-a\hat{r}+(z-z')\hat{z}]}{[a^2+(z-z')^2]^{3/2}}\,.\,a.\,dr'.\,d\varphi'.\,dz'$$

$$\vec{B} = \frac{1}{c}\int_{-L/2}^{L/2}\int_0^{2\pi}\int_{-\infty}^{+\infty} In\,\frac{\delta(r'-a)\,a^2}{[a^2+(z-z')^2]^{3/2}}\,dr'.\,d\varphi'.\,dz'\,\hat{z} \;+\; \#\,\hat{r}$$

Por ahora no me interesa la componente radial.

$$B_z = \frac{1}{c}\, I.n.a^2.\,2\pi \int_{-L/2}^{L/2} \frac{dz'}{[a^2+(z-z')^2]^{3/2}}\,\hat{z}$$

$z-z' = u$

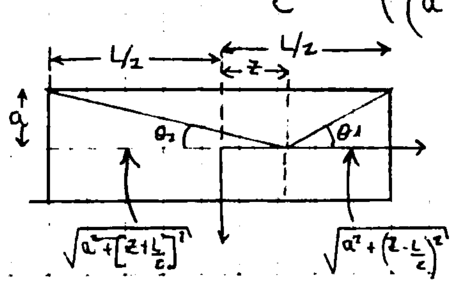
$-dz' = du$

$$\int_{z-L/2}^{z+L/2} \frac{du}{(a^2+u^2)^{3/2}}\,\hat{z}$$

$$\left.\frac{u}{a^2.\sqrt{a^2+u^2}}\right|_{z-L/2}^{z+L/2} = \frac{z+L/2}{a^2.\left(a^2+\left[z+\frac{L}{2}\right]^2\right)^{1/2}} - \frac{z-L/2}{a^2.\left(a^2+\left[z-\frac{L}{2}\right]^2\right)^{1/2}}$$

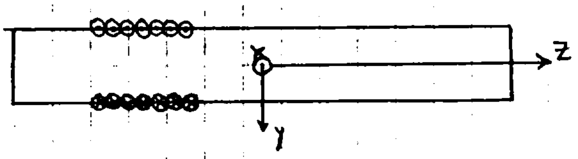
$$B_z = \frac{2\pi n I}{c}\cdot\left(\frac{z+L/2}{\left(a^2+\left[z+\frac{L}{2}\right]^2\right)^{1/2}} - \frac{z-L/2}{\left(a^2+\left[z-\frac{L}{2}\right]^2\right)^{1/2}}\right)$$

usando la figura puede verse que



$$\boxed{B_z = \frac{2\pi n I}{c}\cdot\left[\cos\theta_2 + \cos\theta_1\right]}$$

b. $a \ll L \rightarrow 1 \gg \frac{a}{L}$



Queremos ver en puntos $\vec{r}$ tales que $r \ll a$, $z \ll L$ (es decir cercanos al centro del solenoide)

$$\vec{B} = \frac{1}{c}\int_V \frac{\vec{J}(\vec{r}\,') \times (\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\, dV$$

$\vec{r}-\vec{r}\,' = (r\cos\varphi, r\,\text{sen}\,\varphi, z) - (a\cos\varphi', a\,\text{sen}\,\varphi', z')$

$|\vec{r}-\vec{r}\,'| = r^2\cos^2\varphi + a^2\cos^2\varphi' - 2ra\cos\varphi\cos\varphi' + (z-z')^2$
$+ r^2\text{sen}^2\varphi + a^2\text{sen}^2\varphi' - 2ra\,\text{sen}\,\varphi\,\text{sen}\,\varphi' = (z-z')^2 + r^2 + a^2 - 2ar\cos(\varphi-\varphi')$

$$\vec{B} = \frac{1}{c}\int_{-L/2}^{+L/2}\int_0^{2\pi}\int_0^{+\infty} \frac{I.n.\delta(r'-a)(-\text{sen}\varphi'\hat{x} + \cos\varphi'\hat{y}) \times [(r\cos\varphi - a\cos\varphi')\hat{x} + (r\,\text{sen}\varphi - a\,\text{sen}\varphi')\hat{y} + (z-z')\hat{z}]\, a\,dr\,d\varphi\,dz}{[r^2+a^2+(z-z')^2 - 2ar\cos(\varphi-\varphi')]^{3/2}}$$

Esta es la expresión general; como me interesa B_r y es $B(r_0)=$ cte puedo tomar $\varphi=0$ (es decir $\hat{r}=\hat{x}$) $\rightarrow$ quiero ver la componente $\hat{x}$

$$\vec{B}(x,0,z) = \frac{1}{c}\int_{-L/2}^{L/2}\int_0^{2\pi}\int_{-\infty}^{+\infty} In\,\delta(r'-a).\hat{\varphi} \times ([r - a\cos\varphi']\hat{x} + [-a\,\text{sen}\varphi']\hat{y} + [z-z']\hat{z}).\,a.dr'.d\varphi'\,dz'$$

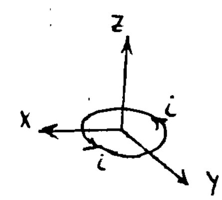
$$B_x = \frac{1}{c}\int_{-L/2}^{+L/2}\int_0^{2\pi}\int_{-\infty}^{+\infty} \frac{In\,\delta(r'-a)(z-z')\,a.\cos\varphi'.dr'\,d\varphi'.dz'}{[r^2+a^2 - 2ar\cos\varphi' + (z-z')^2]^{3/2}}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -In\delta\,\text{sen}\varphi' & In\delta\cos\varphi' & 0 \\ r - a\cos\varphi' & -a.\text{sen}\varphi' & z-z' \end{vmatrix} = \begin{array}{l} In\,\delta(r'-a)(z-z').\cos\varphi'\,\hat{x} \\ \left.\begin{array}{l}\#_1\ \hat{y} \\ \#_2\ \hat{z}\end{array}\right\} \text{NO ME importan}\end{array}$$

$$B_x = \frac{In}{c}\int_{-L/2}^{+L/2}\int_0^{2\pi} \frac{(z-z').a\,dr'\,d\varphi'.dz'.\cos\varphi'}{[\underbrace{r^2+a^2 - 2ar\cos\varphi'}_{\equiv b^2} + (z-z')^2]^{3/2}}$$

$b^2 + (z-z')^2 = u$
$-2(z-z')dz' = du$

$b^2 + u^2 = w$
$2u\,du = dw$



$$B_x = \frac{I.n}{c}\int_0^{2\pi}\int_{b^2+(z+L/2)^2} \frac{(-1/2).du.a\,d\varphi'.\cos\varphi'}{u^{3/2}}$$

$$= \frac{I.n}{c}\int_0^{2\pi} \frac{1}{\cancel{2}}\,a.d\varphi'.\frac{1}{(-1/\cancel{2})}.\frac{1}{u^{1/2}}\Bigg|_{b^2+(z-\frac{L}{2})^2}^{b^2+(z+\frac{L}{2})^2} .\cos\varphi'$$

$$= \frac{I.n.a}{c}\int_0^{2\pi} d\varphi'.\left(\frac{1}{\sqrt{b^2+(z-\frac{L}{2})^2}} - \frac{1}{\sqrt{b^2+(z+\frac{L}{2})^2}}\right).\cos\varphi'$$

$$= \frac{Ina}{c}\int_0^{2\pi}\cos\varphi'.d\varphi'\left[\frac{2}{L}\left(1 - \frac{1}{2}[\alpha-\beta] + \frac{3}{8}[\alpha-\beta]^2\right) - \frac{2}{L}\left(1 - \frac{1}{2}[\alpha+\beta] + \frac{3}{8}[\alpha+\beta]^2\right)\right]$$

$$= \frac{Ina}{c}.\frac{2}{L}\int_0^{2\pi}\cos\varphi'.d\varphi'.\left(\cancel{1} - \cancel{\frac{\alpha}{2}} + \frac{\beta}{2} + \cancel{\frac{3}{8}\alpha^2} + \cancel{\frac{3}{8}\beta^2} - \frac{3}{8}2\alpha\beta - \cancel{1} + \cancel{\frac{\alpha}{2}} + \frac{\beta}{2} - \cancel{\frac{3}{8}\alpha^2} - \cancel{\frac{3}{8}\beta^2} - \frac{3}{8}2\alpha\beta\right)$$

* continúa en la hoja siguiente

detalle

$$\sqrt{r^2+a^2 - 2ar\cos\varphi' + \left(z-\frac{L}{2}\right)^2}$$

$a \ll L$, $r \ll a$, $z \ll L$
$r \ll L$

$1 \gg \frac{r}{L}$ $\quad 1 \gg \frac{a}{L}$ $\quad 1 \gg \frac{z}{L}$

$$\sqrt{L^2\left(\frac{r^2}{L^2} + \frac{a^2}{L^2} - 2\frac{ar}{L^2}\cos\varphi'\right) + L^2\left(\frac{z^2}{L^2} + \frac{1}{4} - \frac{z}{L}\right)}$$

$$L.\left(\frac{r^2}{L^2} + \frac{a^2}{L^2} - 2\cos\varphi'\frac{a}{L}.\frac{r}{L} + \frac{z^2}{L^2} + \frac{1}{4} - \frac{z}{L}\right)^{1/2}$$

$$\frac{L}{2}\Bigg(\underbrace{\left(\frac{2r}{L}\right)^2 + \left(\frac{2a}{L}\right)^2 + \left(\frac{2z}{L}\right)^2 - 8\cos\varphi'\left(\frac{a}{L}\right)\left(\frac{r}{L}\right)}_{\equiv\alpha} - \underbrace{\frac{4z}{L}}_{\equiv\beta} + 1\Bigg)^{1/2}$$

$$\frac{L}{2}\left(1 + [\alpha-\beta]\right)^{1/2} \Rightarrow \sqrt{r^2+a^2-2ar\cos\varphi' + \left(z+\frac{L}{2}\right)^2} \cong \frac{L}{2}\left(1 + [\alpha+\beta]\right)^{1/2}$$

c. Si $L \to \infty \Rightarrow \theta_1, \theta_2 \to 0 \Rightarrow$

$$B_z(L\to\infty) = \frac{2\pi n I}{c}(1+1) = \frac{4\pi n I}{c} \rightarrow \boxed{\text{Coincide con resultado ej. 2.b}}$$

$*$ Viene de la parte b.

$$= \frac{Ina}{c}\cdot\frac{z}{L}\int_0^{2\pi}\cos\varphi'.d\varphi'\left(\beta - \frac{3}{2}\alpha\beta\right)$$

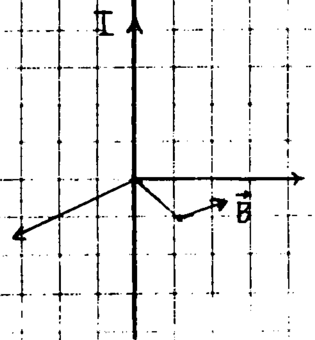
$$= \frac{Ina}{c}\cdot\frac{z}{L}\cdot\frac{4z}{L}\int_0^{2\pi}\cos\varphi'.d\varphi'\left(1 - \frac{3}{2}\left[\frac{4r^2}{L^2}+\frac{4a^2}{L^2}+\frac{4z^2}{L^2}-\frac{8}{L^2}ar\cos\varphi'\right]\right)$$

$$= \frac{4z}{L}\cdot\frac{4}{L^2}\int_0^{2\pi}\cos\varphi'.d\varphi'\left[\frac{L^2}{4}-\frac{3}{2}r^2-\frac{3}{2}a^2+\frac{3}{2}z^2+3ar\cos\varphi'\right]$$

$$= \cdot\int_0^{2\pi}3.ar.\cos^2\varphi'\,d\varphi'$$

$$B_r = \frac{Ina}{c}\cdot\frac{z}{L}\cdot\frac{4z}{L}\,\frac{4}{L^2}\;3.a.r.\pi = \boxed{\frac{96\pi.n.I}{c}\left(\frac{a^2.z.r}{L^4}\right)}$$

9. Se quiere calcular la fuerza que el campo $\vec{B}$ del hilo hace sobre la espira

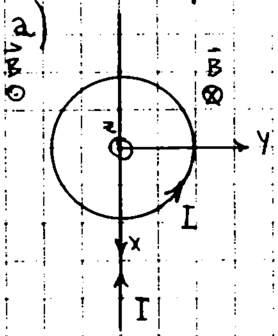


$$\vec{B} = \frac{2I}{r\,c}\hat{\varphi}$$

$$\vec{F} = \frac{1}{c}\oint_{\Gamma} I\,d\vec{l} \times \vec{B}$$

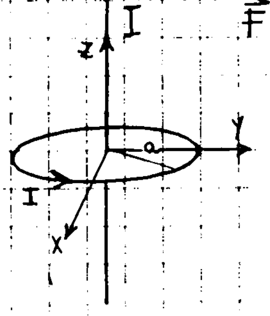
$$\vec{\tau} = \frac{1}{c}\int \vec{x} \times \left(I\,d\vec{l} \times \vec{B}\right)$$

a)



coordenadas cilíndricas en el plano de la espira

b) coordenadas cilíndricas en el plano de la espira



$$\vec{F} = \frac{1}{c}\int I\,a\,d\varphi\,\hat{\varphi} \times \frac{2I}{r\,c}\hat{\varphi}$$

$$\boxed{\vec{F} = 0}$$

la fuerza es nula y por ende el torque también lo es.

No produce efecto de giro tampoco sobre la espira

$$\boxed{\vec{\tau} = 0}$$

$$\vec{F} = \frac{1}{c}\left(\int_0^{\pi} I a\,d\varphi\left(-\operatorname{sen}\varphi\,\hat{x} + \cos\varphi\,\hat{y}\right) \times -\frac{2I}{c}\cdot\frac{1}{\sqrt{x^2+y^2}}\hat{z} + \right.$$

$$\left. + \int_{\pi}^{2\pi} I a\,d\varphi\left(-\operatorname{sen}\varphi\,\hat{x} + \cos\varphi\,\hat{y}\right) \times \frac{2I}{c}\cdot\frac{1}{\sqrt{x^2+y^2}}\hat{z}\right)$$

$$\vec{F} = \frac{1}{c}\left[\int_0^{\pi} -\frac{2I^2 a}{r\,c}\,d\varphi\left(\operatorname{sen}\varphi\,\hat{y} + \cos\varphi\,\hat{x}\right) + \int_{\pi}^{2\pi}\frac{2I^2 a}{r\,c}\,d\varphi\left(\operatorname{sen}\varphi\,\hat{y} + \cos\varphi\,\hat{x}\right)\right]$$

$$\frac{2I^2 a}{r\,c}\left[-\int_0^{\pi}\operatorname{sen}\varphi\,d\varphi\,\hat{y} - \int_0^{\pi}\cos\varphi\,d\varphi\,\hat{x} + \int_{\pi}^{2\pi}\operatorname{sen}\varphi\,d\varphi\,\hat{y} + \int_{\pi}^{2\pi}\cos\varphi\,d\varphi\,\hat{x}\right]$$

$$\cos\varphi\Big|_0^{\pi}\hat{y} - \operatorname{sen}\varphi\Big|_0^{\pi}\hat{x} - \cos\varphi\Big|_{\pi}^{2\pi}\hat{y} + \operatorname{sen}\varphi\Big|_{\pi}^{2\pi}\hat{x}$$

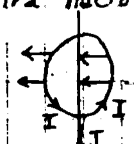
$$(-1-1)\hat{y} - 0\,\hat{x} - (1+1)\hat{y} + 0\,\hat{x}$$

$$\vec{F} = \frac{2I^2 a}{(x^2+y^2)^{1/2}\,c^2}(-4)\,\hat{y}$$

$$\boxed{\vec{F} = -\frac{8\,I^2\,a}{(x^2+y^2)^{1/2}\,c^2}\hat{y}}$$

obs.

Esta fuerza tiende a deformar la espira hacia la izquierda



Para hacerla descentrarse

$$[\vec{F}] = \frac{C^2}{m^2}$$

unidades de la fuerza

$$\vec{\tau} = \frac{1}{c}\int \vec{x} \times \left(I\,d\vec{l} \times \vec{B}\right)$$

$$\vec{\tau} = \frac{1}{c}\,\frac{2I^2 a}{r\,c}\int_0^{\pi}\left(x\hat{x} + y\hat{y}\right) \times \left(\operatorname{sen}\varphi\,\hat{y} + \cos\varphi\,\hat{x}\right)d\varphi + \int_{\pi}^{2\pi}\left(x\hat{x} + y\hat{y}\right) \times \left(\operatorname{sen}\varphi\,\hat{y} + \cos\varphi\,\hat{x}\right)d\varphi$$

$$\vec{\tau} = \frac{2I^2 a}{[x^2+y^2]^{1/2}\,c^2}\hat{z}\left(\int_0^{\pi}\left(-\overset{r\cos\varphi}{x}\operatorname{sen}\varphi + \overset{r\,\operatorname{sen}\varphi}{y}\cos\varphi\right)d\varphi + \int_{\pi}^{2\pi}\left(x\operatorname{sen}\varphi - y\cos\varphi\right)d\varphi\right)$$

$$\int_0^{\pi}\left(-r\cos\varphi\operatorname{sen}\varphi + r\operatorname{sen}\varphi\cos\varphi\right)d\varphi + \int_{\pi}^{2\pi}\left(r\cos\varphi\operatorname{sen}\varphi - r\operatorname{sen}\varphi\cos\varphi\right)d\varphi$$

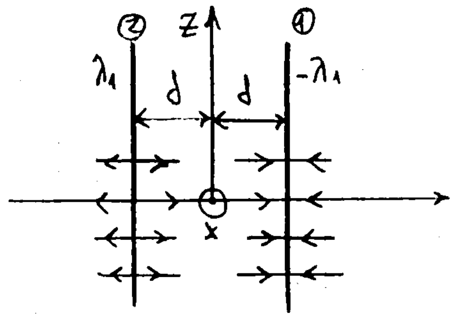
$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ x & y & 0 \\ \cos\varphi & \operatorname{sen}\varphi & 0 \end{vmatrix}$$

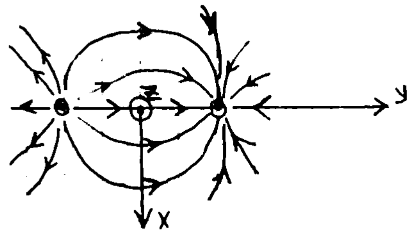
$$x\operatorname{sen}\varphi\,\hat{z} - y\cos\varphi\,\hat{z}$$

$$\Rightarrow \boxed{\vec{\tau} = 0}$$ No hay efecto de giro

IV. Principio de Superposición

10. a.





El campo de cada hilo es:

$$\vec{E} = \frac{2\lambda}{r}\hat{r}$$

$$\vec{E} = \frac{2\lambda}{r}(\cos\varphi\,\hat{x} + \operatorname{sen}\varphi\,\hat{y})$$

$$\vec{E} = \frac{2\lambda . x}{r^2}\hat{x} + \frac{2\lambda . y}{r^2}\hat{y}$$

Habría que desplazar ambos campos en el punto fuente

$$\vec{E} = \frac{2\lambda\, x}{x^2+(y-d)^2}\hat{x} + \frac{2\lambda . (y-d)}{x^2+(y-d)^2}\hat{y}$$

$$\vec{E}_1 = -\frac{2\lambda_1 x}{x^2+(y-d)^2}\hat{x} - \frac{2\lambda_1 (y-d)\hat{y}}{x^2+(y-d)^2}$$

$$\vec{E}_2 = \frac{2\lambda_1 x}{x^2+(y+d)^2}\hat{x} + \frac{2\lambda_1 (y+d)}{x^2+(y+d)^2}\hat{y}$$

$$\boxed{\vec{E} = 2\lambda_1 x . \left(\frac{1}{x^2+(y+d)^2} - \frac{1}{x^2+(y-d)^2}\right)\hat{x} + 2\lambda_1 . \left(\frac{(y+d)}{x^2+(y+d)^2} - \frac{(y-d)}{x^2+(y-d)^2}\right)\hat{y}}$$

$$\vec{E}(0,0,0) = 2\lambda_1\left(\frac{d}{d^2} - \frac{-d}{d^2}\right)\hat{y} = \frac{4\lambda_1}{d}\hat{y}$$

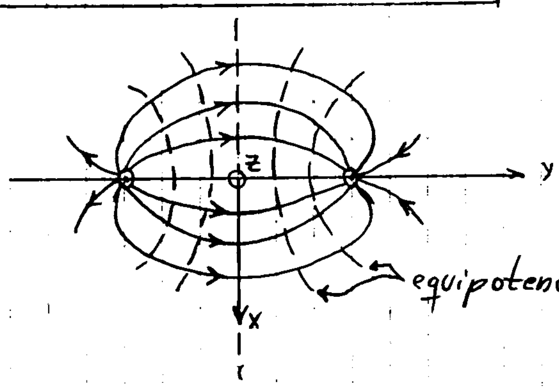
$$\varphi_1 = +2\lambda_1 . \ln\sqrt{x^2+(y-d)^2}$$

$$-\frac{\partial\varphi_1}{\partial x} = -2\lambda_1 . \frac{1}{\sqrt{}} . \frac{1\,\cancel{2}x}{\cancel{2}\sqrt{}} = \vec{E}_1 . \hat{x}$$

$$\varphi_2 = -2\lambda_1 . \ln\sqrt{x^2+(y+d)^2}$$

$$\boxed{\varphi(x,y,z) = 2\lambda_1 . \ln\left(\sqrt{\frac{x^2+(y-d)^2}{x^2+(y+d)^2}}\right)}$$

b.



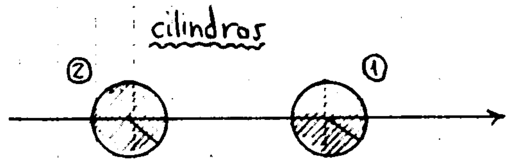
Las equipotenciales son cilindros curvos de sección igual a la sección transversal del equipotencial de dos cargas puntuales $+q$ y $-q$

$$\vec{E}(x,0,z) = 2\lambda_1 x . \underbrace{\left(\frac{1}{x^2+d^2} - \frac{1}{x^2+d^2}\right)}_{=0}\hat{x} + 2\lambda_1\left(\frac{d}{x^2+d^2} - \frac{-d}{x^2+d^2}\right)\hat{y}$$

$$\vec{E}(x,0,z) = 2\lambda_1 . \frac{2d}{x^2+d^2}\hat{y}$$

El campo $\vec{E}$ en el plano equidistante entre los hilos es en $\hat{y}$ (es decir $\perp$ a dicho plano).

c.

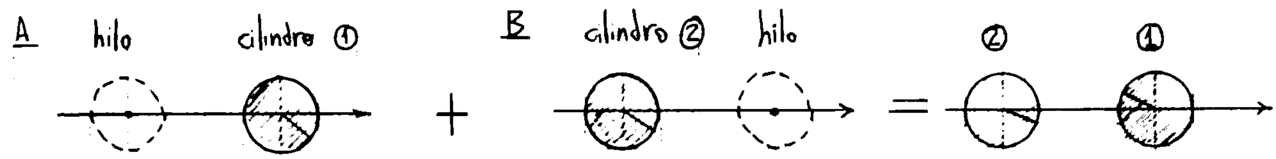


Aquí no puede usarse superposición porque el campo de ① no es el mismo si está presente ② que si no lo está. Hay reacomodamiento de cargas para que ambos cilindros mantengan sus potenciales ~~diferentes.~~ constantes.

Estoy pensando, quien sabe porqué, en cilindros conductores

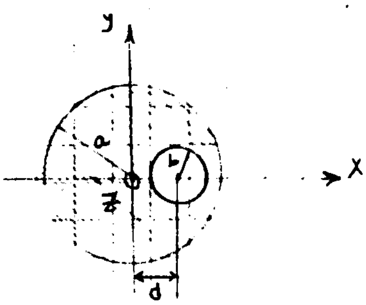
En este problema se puede usar superposición porque al considerar el efecto de cada hilo por separado puedo olvidarme del otro.
Es decir la presencia de ② no hace que ① varíe el φ_1. φ_1 es el mismo con ② o sin ②.
La presencia de ② no cambia la distribución de carga en ①

Podríamos aplicar superposición de la siguiente forma:

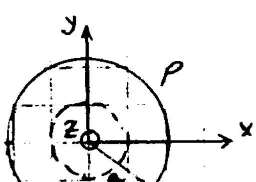


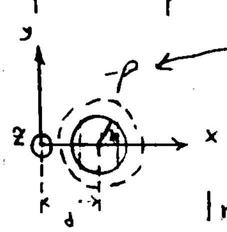
donde el hilo en A tiene el mismo φ que el cilindro ②, mientras que en B tiene el mismo φ que el cilindro ①.

11.



a. Podemos utilizar superposición para este caso.





← tomamos una densidad negativa

$|r\hat{r} - d\hat{x}| > b$ $\qquad$ $|r\hat{r} - d\hat{x}| < b$

$r<a$

$$E\cdot \cancel{2\pi r l} = \overset{2}{\cancel{4\pi}}\,\rho\cdot\pi\cdot r^2\cancel{l}$$

$$E = 2\pi\rho\, r\hat{r}$$

$$E = 2\pi\rho\cdot\sqrt{x^2+y^2}\cdot(\cos\varphi\,\hat{x} + \operatorname{sen}\varphi\,\hat{y})$$

$$E = 2\pi\cdot\rho\cdot(x\hat{x} + y\hat{y})$$

$$E\cdot \cancel{2\pi r l} = \overset{2}{\cancel{4\pi}}\,\rho\cdot\pi\cdot b^2\cdot l$$

$$E\cdot\sqrt{(x-d)^2+y^2} = -2\rho\pi b^2$$

$$E = \frac{-2\rho\pi b^2}{\sqrt{(x-d)^2+y^2}}$$

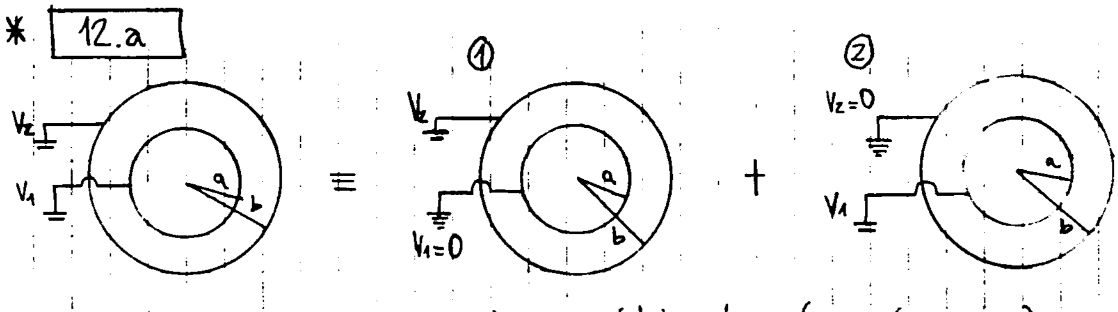
$$E\cdot \cancel{2\pi r l} = \overset{2}{\cancel{4\pi}}\,\rho\cdot\pi\cdot r^2\cdot l$$

$$E = -2\rho\pi\cdot\sqrt{(x-d)^2+y^2}$$

$$E = -2\pi\rho\left([x-d]\hat{x} + y\hat{y}\right)$$

dentro de la cavidad $\longrightarrow$ $\boxed{\vec{E} = 2\pi\rho\cdot(d\hat{x})}$

$*$ $\boxed{12.a}$



$\varnothing(\text{cilindro}) = -4\pi(\text{radio}).\sigma.\ln r$ (expresión genérica)

①

$/\!/\sigma_1$ interior

interior		extenor	
$r<a$	$V = 0$	$r<b$	$V = \overbrace{-4\pi b\,\sigma_2 \ln b}^{=V_2} \rightarrow -\dfrac{V_2}{\ln b} = 4\pi b\sigma_2$
$r>a$	$V = -4\pi a\sigma_1 \ln r + C_2$	$r>b$	$V = -4\pi b\,\sigma_2 \ln r + \underset{0}{\underset{\shortparallel}{C_4}}$
	$C_2 = 4\pi a\sigma_1 \ln a$		

$$r<a \qquad V = 0 + V_2 + C_1 \qquad \{\, V(a) = +V_2 + C_1$$

$$a<r<b \qquad V = -4\pi a\sigma_1 \ln r + V_2 + C_2 \qquad \begin{cases} V(a) = -4\pi a\sigma_1 \ln a + V_2 + C_2 \\ V(b) = -4\pi a\sigma_1 \ln b + V_2 + C_2 \end{cases}$$

$$r>b \qquad V = -4\pi a\sigma_1 \ln r + V_2 \cdot \frac{\ln r}{\ln b} + C_3 \qquad \{\, V(b) = -4\pi a\sigma_1 \ln b + V_2 + C_3$$

$$\rightarrow C_2 = C_3$$

$$V(a) = 0 = V_2 + C_1 \rightarrow \boxed{C_1 = -V_2} \qquad 0 = C_2 - C_1 - 4\pi a\sigma_1 \ln a$$

$$0 = C_2 + V_2 - 4\pi a\sigma_1 \ln a$$

$$V_2 = -4\pi a\sigma_1 \ln b + V_2 + C_2 \qquad \rightarrow \quad V_2 = 4\pi a\sigma_1(\ln a - \ln b)$$

$$C_2 = 4\pi a\sigma_1 \ln b \qquad 4\pi a\sigma_1 = \frac{V_2}{(\ln a - \ln b)}$$

①

$$r<a \qquad V = 0$$

$$a<r<b \qquad V = -\frac{V_2 . \ln r}{[\ln a - \ln b]} + V_2 + \frac{V_2 \ln b}{[\ln a - \ln b]} = V_2\left(\frac{-\ln r + \ln a \cancel{- \ln b} \cancel{+ \ln b}}{\ln a - \ln b}\right)$$

$$r>b \qquad V = -\frac{V_2 . \ln r}{[\ln a - \ln b]} + V_2 . \frac{\ln r}{\ln b} + \frac{V_2 . \ln b}{[\ln a - \ln b]} = \ldots$$

$$V_2\left(\frac{-\ln r . \ln b + \ln a . \ln r - \ln b . \ln r + (\ln b)^2}{(\ln a - \ln b)(\ln b)}\right)$$

$$V = V_2\left[\frac{\ln a . \ln r - 2 . \ln r . \ln b + (\ln b)^2}{(\ln a - \ln b)(\ln b)}\right]$$

(2)

$$\frac{-V_1}{2\ln a} = 2\pi\sigma_1 a$$

interior

$r<a \quad V = V_1 = -4\pi\sigma_1 a \ln a$

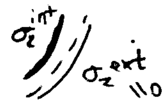
$r>a \quad V = -4\pi\sigma_1 a . \ln r = \frac{V_1}{\ln a} . \ln r$

exterior

$r<b \quad V = -4\pi b \sigma_2 \ln r + C_1$

$r>b \quad V = 0$

$C_1 = 4\pi b \sigma_2 . \ln b$



la por conexión a tierra

$$\sigma_2 = -\sigma_1 . \frac{a}{b} = \frac{V_1}{4\pi . \ln a . b} \qquad Q^N = 0 = \sigma_2 . b + \sigma_1 . a = 0$$

$\sigma_2 = -\sigma_1 \frac{a}{b}$

$r<a \quad V = V_1 - \frac{V_1}{\ln a}(\ln r - \ln b) + C_1 \qquad V(a) = V_1 - \frac{V_1}{\ln a}(\ln a - \ln b) + C_1$

$a<r<b \quad V = \frac{V_1}{\ln a} . \ln r - \frac{V_1}{\ln a}(\ln r - \ln b) + C_2 \qquad V(a) = V_1 \ln b / \ln a + C_2$

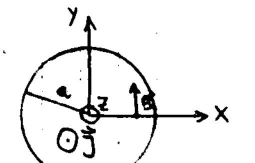
$V(b) = V_1 . \frac{\ln b}{\ln a} + C_2 = 0 \rightarrow C_2 = C_3 \equiv C$

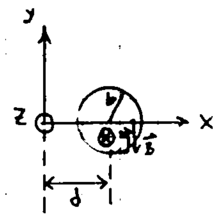
$r>b \quad V = V_1 . \frac{\ln r}{\ln a} + C_3$

$V(b) = V_1 . \frac{\ln b}{\ln a} + C_3 = 0 \qquad C = -V_1 . \frac{\ln b}{\ln a}$

El cilindro no puede estar hecho de material conductor, porque de estarlo el campo eléctrico $\vec{E}$ sería nulo en su interior.

b. Aquí también podemos superponer





$r<a$

$$I = \int \vec{J}.d\vec{S}$$

$$I = J.\pi.r^2$$

$$B.\cancel{2\pi r} = \frac{\cancel{4\pi}^{2}}{c}.J.\pi.r^{\cancel{2}}$$

$$B = \frac{2}{c}J\pi r\,\hat{\varphi}$$

$$B = \frac{2}{c}J\pi\sqrt{x^2+y^2}\,\hat{\varphi}$$

$$B = \frac{2}{c}J\pi\sqrt{x^2+y^2}\,(-\operatorname{sen}\varphi\,\hat{x} + \cos\varphi\,\hat{y})$$

$$\vec{B} = \frac{2}{c}J\pi(-y\,\hat{x} + x\,\hat{y})$$

$$(x-d)^2 + y^2 < b^2$$

$$B\,\cancel{2\pi r} = -\frac{\cancel{4\pi}^{2}}{c}J\pi r^{\cancel{2}}$$

$$B = -\frac{2}{c}J\pi r.(\operatorname{sen}\varphi\,\hat{x} + \cos\varphi\,\hat{y})$$

$$\vec{B} = +\frac{2}{c}J\pi\sqrt{(x-d)^2+y^2}\,(\operatorname{sen}\varphi\,\hat{x} - \cos\varphi\,\hat{y})$$

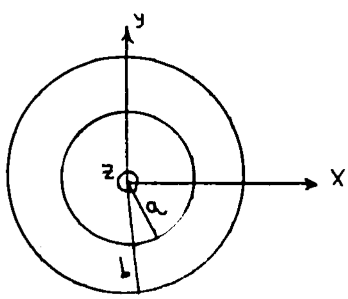
$$\vec{B} = \frac{2}{c}J\pi r\left(\frac{y}{r}\hat{x} - \frac{(x-d)}{r}\hat{y}\right)$$

$$\vec{B} = \frac{2}{c}J\pi\left(y\,\hat{x} - [x-d]\,\hat{y}\right)$$

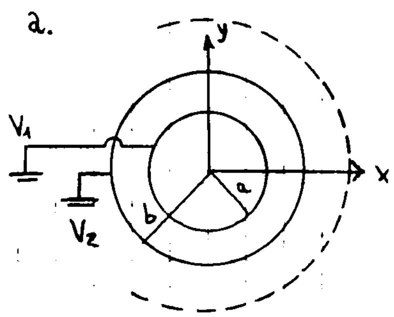
$$\boxed{\vec{B} = \frac{2}{c}J\pi.d\,\hat{y}}$$

Se ha podido observar que la cavidad ha efectivamente congelado el campo; los valores son iguales a los existentes sin la cavidad y $\vec{E} \perp \vec{B}$

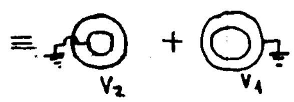
12.



a.



Los cilindros son conductores



cilindro # ② ①

	$r>b$	$r<b$
②	$-4\pi\sigma_2 b.\ln r + C_1$	C_2
	$r>a$	$r<a$
①	$-4\pi\sigma_1 a.\ln r + C_3$	C_4

$r>b \quad V = -4\pi\sigma_2 b\ln r - 4\pi\sigma_1 a\ln r + C_1 + C_3$

$b>r>a \quad V = C_2 - 4\pi\sigma_1 a\ln r + C_3$

$r<a \quad V = C_4$

$r<a \quad V = C_4$

$r=a \quad V_1 = C_4 \rightarrow \quad V = V_1 \; (r<a)$

$a<r<b \quad V = C_2 + C_3 - 4\pi\sigma_1 a \ln r \rightarrow \begin{cases} V(r=a) = V_1 = C_2+C_3-4\pi\sigma_1 a \ln a \\ V(r=b) = V_2 = C_2+C_3-4\pi\sigma_1 a \ln b \end{cases}$

$r>b \quad V = -4\pi\sigma_2 \ln r . b - 4\pi\sigma_1 \ln r . a + C_1 + C_3$ $\qquad V_1 - V_2 = 4\pi\sigma_1 a \left(\ln \frac{b}{a}\right)$

$V(r=b) = V_2 = -4\pi\sigma_2 . \ln b . b - 4\pi\sigma_1 . \ln b . a + C_1 + C_3 .$

$r>b \quad V = -4\pi . \ln r (\sigma_2 . b + \sigma_1 . a) + V_2 + 4\pi . \ln b . (\sigma_2 b + \sigma_1 a)$

$a<r<b \quad V = -4\pi . \ln r . \sigma_1 . a + V_2 + 4\pi . \sigma_1 . a . \ln b$

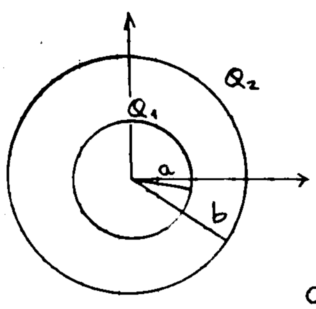
$\quad " \quad + V_1 + 4\pi\sigma_1 . a \ln a$

$r<a \quad V = V_1$

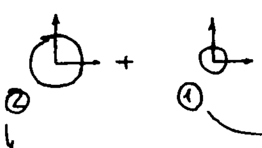
$$\boxed{\begin{array}{ll} r>b & V = 4\pi . (\sigma_2 . b + \sigma_1 . a) . \ln\left(\frac{b}{r}\right) + V_2 \\ a<r<b & V = 4\pi . \sigma_1 . a . \ln\left(\frac{b}{r}\right) + V_2 \\ & \quad 4\pi . \sigma_1 . a . \ln\left(\frac{a}{r}\right) + V_1 \\ r<a & V = V_1 \end{array}}$$

b. Ahora tenemos cilindros cargados (no conductores).

Podemos superponer porque la carga es fija



	cilindro radio a	cilindro radio b
$r>b$	$-4\pi a \sigma_1 \ln r + C_1$	$-4\pi b \sigma_2 \ln r + C_3$
$a<r<b$	$-4\pi a \sigma_1 \ln r + C_1$	C_4
$r<a$	C_2	C_4



$\sigma_1 = \frac{Q_{T1}}{2\pi a L} \qquad \sigma_2 = \frac{Q_{T2}}{2\pi b L}$

$\frac{2\pi a \sigma_1}{L} = Q_1 \qquad \frac{2\pi b . \sigma_2}{L} = Q_2$

$E . \cancel{2}\cancel{\pi} r \cancel{l} = 4\pi \cancel{2}\cancel{\pi} b \cancel{l} \sigma_2$ $\rightarrow$ análogamente al ②

$E = \frac{4\pi b \sigma_2}{r}$

$E = 0$

cilindro 1

$Q_1 = \frac{Q_T}{L}$

$\sigma_1 = \frac{Q_T}{2\pi a L}$

$Q_1 = \frac{\sigma_1 2\pi a \cancel{L}}{\cancel{L}}$

$r<a \quad V = \overbrace{C_2 + C_4}^{= D_1} \quad \rightarrow V(r=a) = D_1$

$a<r<b \quad V = -4\pi a \sigma_1 \ln r + \overbrace{C_1 + C_4}^{= D_2} \rightarrow \begin{array}{l} V(r=a) = -4\pi a . \sigma_1 . \ln a + D_2 \\ V(r=b) = -4\pi a \sigma_1 \ln b + D_2 \end{array}$

$r>b \quad V = -4\pi a \sigma_1 \ln r - 4\pi b \sigma_2 \ln r + \overbrace{C_1 + C_3}^{= D_3} \rightarrow V(r=b) = D_3 - 4\pi a \sigma_1 \ln b - 4\pi b \sigma_2 \ln b$

Pero $\quad D_1 = -4\pi a \sigma_1 \ln a \rightarrow$ puedo tomar $D_2 = 0$.

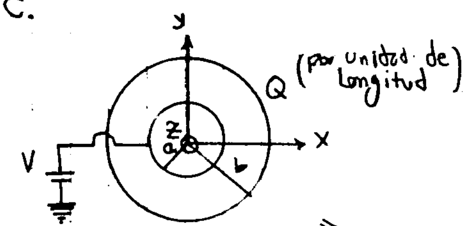
$D_3 - D_2 = 4\pi b \sigma_2 \ln b$

$D_2 - D_1 = 4\pi a \sigma_1 \ln a$

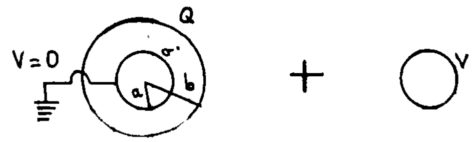
$$\boxed{\begin{array}{ll} r>b & V = -4\pi a\sigma_1 \ln r + 4\pi b\sigma_2 \ln\left(\frac{b}{r}\right) = -2Q_1 \ln r + 2Q_2 \ln\left(\frac{b}{r}\right) \\ a<r<b & V = -4\pi a\sigma_1 \ln r = -2Q_1 \ln r \\ r<a & V = -4\pi a \sigma_1 \ln a = -2Q_1 \ln a \end{array}}$$

Si los cilindros fuesen conductores se inducen σ' sobre cada uno de ellos debidas a la influencia del otro. AL superponer las configuraciones se debe tener en cuenta.

C.



$$\sigma = \frac{Q^T}{L.2\pi.b} \rightarrow Q = \frac{Q^T}{L} = 2\pi b.\sigma$$



	corteza cond.	corteza ext	corteza cond.
$r<a$	$-4\pi a\sigma'\ln a$		$V = -4\pi a\sigma \ln a$
$r<b$		$-2.Q.\ln b$	
$r>a$	$-4\pi a\sigma'\ln r$		$-4\pi.a.\sigma.\ln r = \frac{V}{\ln a}.\ln r$
$r>b$		$-2.Q.\ln r$	

$$\sigma = -\frac{V}{4\pi a.\ln a}$$

$r<a$ $\quad -4\pi a\sigma'\ln a - 2Q\ln b + C_1$

$a<r<b$ $\quad -4\pi a\sigma'.\ln r - 2Q\ln b + C_2$

$r>b$ $\quad -4\pi a.\sigma'.\ln r - 2Q\ln r + C_3$

$$V(a) = \left.\begin{array}{l} -4\pi a\sigma'\ln a - 2Q\ln b + C_1 \\ -4\pi a\sigma'\ln a - 2Q\ln b + C_2 \end{array}\right\} \rightarrow C_1 = C_2 = 0$$

$$V(b) = \left.\begin{array}{l} -4\pi a\sigma'\ln b - 2Q\ln b + C_2 \\ -4\pi a\sigma'\ln b - 2Q\ln b + C_3 \end{array}\right\} \rightarrow C_2 = C_3 = 0$$

pero $V(a) = 0 = -4\pi a\sigma'\ln a - 2Q\ln b$

$$\sigma' = -\frac{\cancel{2}Q}{\cancel{4}\pi a}\cdot\frac{\ln b}{\ln a} = \frac{Q}{2\pi a}\cdot\frac{\ln b}{\ln a}$$

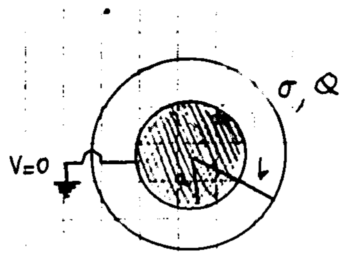
$$\boxed{\begin{array}{ll} r<a & V \\ a<r<b & -2Q\ln b + 2Q.\frac{\ln b}{\ln a}.\ln r + \frac{V}{\ln a}.\ln r \\ r>b & -2Q\ln r + 2Q.\frac{\ln b}{\ln a}.\ln r + \frac{V}{\ln a}.\ln r \end{array}}$$

La corteza conductora interior contribuye con $V(r) = V.\frac{\ln r}{\ln a}$ pese a que se induce una carga por unidad de longitud de:

$$Q^{ind}_{\substack{\text{sobre}\\ a}} = Q.\frac{\ln b}{\ln a}$$

que es compensada por la batería a fin de mantener V constante.
Luego la corteza exterior aporta $-2Q.\ln r$ afuera y $2Q\ln b$ adentro.

13.



a. El casquete esférico induce una carga $-Q^{ind}$ sobre la esfera conductora que se equilibra exactamente con la $+Q$ del casquete.
El $+Q^{ind}$ se va por tierra

$C_3 = -C_2$

$V_\infty = 0$

$r>b$	$V = +\frac{4\pi a^2\sigma'}{r} + C_3 + C_2 + \frac{4\pi b^2\sigma}{r}$	$V(r=b) = C_3 + C_2 + 4\pi \frac{a^2}{b}\sigma' + 4\pi b\sigma$
$a<r<b$	$V = C_1 + \frac{4\pi a^2\sigma'}{r} + C_3$	$\begin{cases} V(r=a) = 0 = C_1 + C_3 + 4\pi a\sigma' \\ V(r=b) = C_1 + C_3 + 4\pi \frac{a^2}{b}\sigma' \end{cases}$
$r<a$	$V = C_4 + C_1$	$V(r=a) = 0 = C_1 + C_4$; $C_1 = -C_4$

$\sigma = \frac{Q}{4\pi b^2}$ $\equiv$

$r>a$	$V = -\frac{Q}{r} + C_7 = \frac{-4\pi a^2\sigma'}{r} + C_7$	$r>b$	$V = \frac{Q}{r} = \frac{4\pi b^2\sigma}{r} + C_7$
$r<a$	$V = C_1$	$r<b$	$V = C_1$

$$\boxed{\begin{array}{ll} r>b & V = \frac{4\pi}{r}(a^2\sigma' + b^2\sigma) \\ a<r<b & V = \frac{4\pi}{r}a^2\sigma' - 4\pi a\sigma' \\ r<a & V = 0 \end{array}}$$

$C_2 + \cancel{C_3} + \cancel{4\pi \frac{a^2}{b}\sigma'} + 4\pi b\sigma = C_1 + \cancel{C_3} + \cancel{4\pi \frac{a^2}{b}\sigma'}$

$C_1 = C_2 + 4\pi b\sigma$

$C_1 = -C_3 - 4\pi a\sigma'$

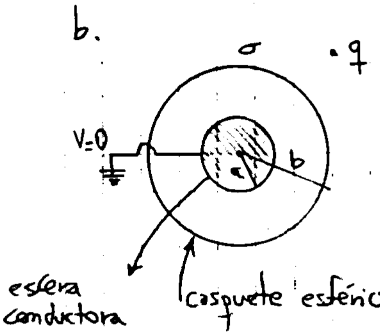
$0 = 4\pi b\sigma + 4\pi a\sigma'$

$\sigma' = -\frac{b}{a}\cdot\sigma$

$r>b$ $\begin{cases} V = \frac{4\pi}{r}\sigma(-ab + b^2) \\ V = \frac{4\pi}{r}\left(a^2\left(-\frac{b}{a}\right)\sigma + b^2\sigma\right) \end{cases}$

$a<r<b$ $\begin{cases} V = \frac{4\pi}{r}a^2\left(-\frac{b}{a}\sigma\right) - 4\pi a(\sigma)\left(-\frac{b}{a}\right) \\ V = -\frac{4\pi a}{r}b\sigma + 4\pi\sigma b \end{cases}$

b.



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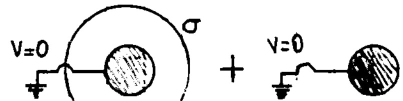
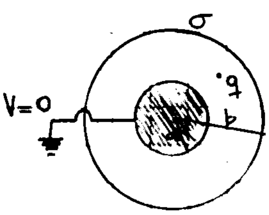


▲ se puede separar así porque la carga q y el casquete no se perturban entre sí



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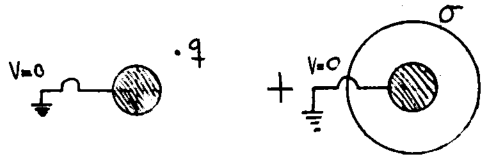

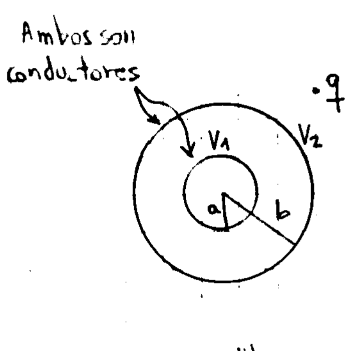

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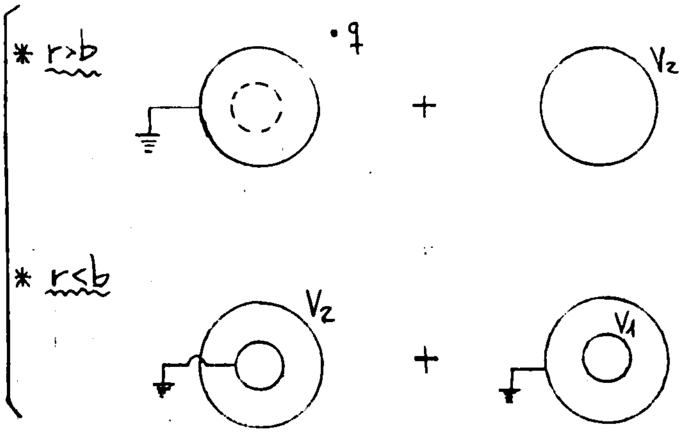

$\equiv$

Un conductor aísla del exterior (a tierra o no a tierra)