and, in units where $c \neq 1$ and the ρ_r is an energy density,

$$\rho_r = g\frac{\pi^2}{30}\frac{(k_B T)^4}{(\hbar c)^3},$$

where k_B is Boltzmann's constant converting between kelvins and en
(e.g., $k_B = 1.38 \times 10^{-16}$ erg/K) and g is the number of degrees of fi
the zero rest mass particles making up the radiation. For photons $g =$
two possible polarizations. More realistically, including three species of
turns out to lead to a formula like (18.24) but with an effective g of abo
is valid for $k_B T <\sim 1$ MeV and will be adequate for our purposes (see,
and Turner, 1990).

Inserting (18.23) into (18.20), we find that it can be easily integrate

$$\rho_r(t) = \rho_r(t_0)\left[\frac{a(t_0)}{a(t)}\right]^4,$$

or, equivalently in terms of the temperature $T(t)$ from (18.24),

$$T(t) = T(t_0)\left[\frac{a(t_0)}{a(t)}\right].$$

Thus the time dependence of the radiation energy density and its temp
fixed by the scale factor. Temperature varies inversely with the scale fa
universe began in a big bang where $a = 0$, the temperature began at i
cooled as the universe expanded. Many important large-scale proper
matter in the universe, such as the primordial abundances of the eleme
understood as arising from the cooling of an initial thermal equilibri
high temperature. Some of them are briefly described in Box 18.1.

Matter dominates radiation now, but the converse was true in the earl
For any densities now there was some early time when $a(t)$ was smalle
and ρ_r was bigger than ρ_m. The universe then was *radiation dominated*
numbers in (17.1) and (17.2) and the relations (18.22) and (18.25) this
when $a(t_0)/a(t) \sim 10^3$, when the universe was about 10^{-3} of its pr
Thus, over most of its history to date, the universe's matter dominates

Vacuum

The energy density of the vacuum, ρ_v, is the final case. At the time
there is no fundamental theory that fixes the value of the vacuum e
even its sign is predicted. To avoid having to consider a multiplicity o
will restrict attention in this text to a vacuum energy that is (i) consta
and time and (ii) positive as indicated by present observations (Chapt
first law of thermodynamics (18.20) then implies a pressure

$$p_v = -\rho_v,$$

BOX 18.1 The Thermal History of the Universe

The temperature is infinite at the big bang that begins the FRW models. At a sufficiently early moment the universe was hot enough that matter was dissociated into its most basic constituents. The abundances of the various kinds of elementary particles were then plausibly set by the conditions of thermal equilibrium. Consider by way of example the abundances of neutrino species. Neutrinos (ν's) and antineutrinos ($\bar{\nu}$'s) interact with electrons (e^-'s) and positrons (e^+'s) through reactions such as

$$\nu + \bar{\nu} \longleftrightarrow e^+ + e^-, \qquad \nu + e^- \longleftrightarrow \nu + e^-, \qquad \text{etc.} \tag{a}$$

If there were too many ν's and $\bar{\nu}$'s, they would annihilate by the first reaction. If there were too few, they would be produced by the annihilation of electrons and positrons. In thermal equilibrium the number density of these particles is such that these reactions balance. It is the same with all other kinds of particles at the high temperatures of sufficiently early moments.[a]

As the universe expands, the temperature drops according to (18.26). The number of reactions per unit time, Γ, drops with temperature for processes such as (a). The densities of particles are dropping because of expansion, energies per particle are declining, and cross sections typically decrease with energy. If the rate Γ drops below the expansion rate of the universe,

$$\Gamma(t) < \dot{a}(t)/a(t) \equiv H(t), \tag{b}$$

then the reaction is no longer fast enough to maintain thermal equilibrium in the face of the expanding universe. The abundances "freeze out" at their value when (b) happens. For neutrinos this happens at a temperature of about 1 MeV at a time of about 1 s after the big bang (see Example 18.2). A cosmic neutrino background with blackbody spectrum is then one of the predictions of the big bang, although currently it is not possible to detect it.

Understanding in detail the relic abundances of particles and nuclei left over from the big bang is a rich and fascinating subject that is important both for cosmology and for the theory of the elementary particles. The particle theory required to understand it in detail puts it beyond the scope of this book. However, we can mention three significant transitions in the thermal history of the universe.

Baryosynthesis ($T \sim 10^{14}$ GeV, $t \sim 10^{-34}$ s). Total baryon number is a conserved quantity for elementary particle interactions below the energy scale of 10^{14} GeV characteristic of the unification of the strong and electroweak forces. Protons and neutrons are familiar examples of baryons, each with a baryon number of $+1$. Antiprotons and antineutrons have baryon numbers of -1. Baryons and antibaryons are not equally represented in today's universe. If they were, there would be a lot more action from matter-antimatter annihilation than observations detect. Were baryon number conserved, this asymmetry could only be attributed to the initial condition of the universe. But above 10^{14} GeV, it may not be conserved, and the abundances of baryons and antibaryons could be set by the conditions of thermal equilibrium, just as the abundances of neutrino species were at much lower energy, as discussed before. Small differences between the interactions of matter and antimatter (for which there is evidence at accelerator energies) could have led to asymmetries in the abundances of baryons and antibaryons in thermal equilibrium. When the temperature dropped below 10^{14} GeV, these would have "frozen out," leading to the asymmetry observed today.

Nucleosynthesis ($T \sim .1$ MeV, $t \sim 3$ min). When the temperature drops through the range of a few tenths of a MeV, the free protons and neutrons combine to make light nuclei. The primordial abundances of the elements is thereby fixed—approximately 75% H, 24% ^{4}He by mass and much smaller but important fractions of other light elements. These abundances are sensitive to the number density of baryons and provide an important test of big bang cosmology. (See Box 19.1 on p. 400 for more details.)

Recombination ($T \sim .3$ eV, $t \sim 4 \times 10^5$ yr). When the temperature drops below a few tenths of an electron volt, electrons and nuclei combine to make atoms, in particular atomic hydrogen.[b] The universe then becomes transparent to radiation and the remaining photons constitute the cosmic background radiation.

Baryon number asymmetry, the primoidial abundances of the elements, and the cosmic background radiation are relics of the thermal history of the universe. These relics yield a great deal of information about the early epochs in which they were made.

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[a] Gravitons may not be in thermal equilibrium because the same coupling constant G that governs their interactions also governs the rate of expansion of the universe.

[b] To say that electrons and nuclei *re*combine is misleading because they were never combined before this moment, but it is the standard terminology, and this event is called recombination.