(5) $y'' - y' - 6y = 3e^{4t}$ $\qquad y(0)=1$

$y'(0)=0$

$$\mathcal{L}(y'') - \mathcal{L}(y') - \mathcal{L}(6y) = \mathcal{L}(3e^{4t})$$

$$s^2\mathcal{L}(y) - s\,y(0) - y'(0) - [s\mathcal{L}(y) - y(0)] - 6\mathcal{L}(y) = 3\cdot\mathcal{L}(e^{4t})$$

$$s^2\mathcal{L}(y) - s\,y(0) - y'(0) - s\mathcal{L}(y) + y(0) - 6\mathcal{L}(y) = 3\cdot\frac{1}{s-4}$$

$$s^2\mathcal{L}(y) - s - 0 - s\mathcal{L}(y) + 1 - 6\mathcal{L}(y) = \frac{3}{s-4}$$

$$\mathcal{L}(y)[s^2 - s - 6] - s + 1 = \frac{3}{s-4}$$

$$\mathcal{L}(y) = \left(\frac{3}{s-4} - 1 + s\right)\left(\frac{1}{s^2-s-6}\right)$$

$$\mathcal{L}(y) = \left(\frac{3 - s + 4 + s^2 - 4s}{s-4}\right)\cdot\left(\frac{1}{s^2-s-6}\right)$$

NOTAS

$$\mathcal{L}(f^n)(s) = s^n\mathcal{L}(f)(s) - s^{n-1}f(0) - s^{n-2}f'(0) \ldots - f^{n-1}(0)$$

$$\mathcal{L}(e^{zt}) = \frac{1}{s-z} \qquad z\in\mathbb{C} \qquad s - \mathrm{Re}(z) > 0$$

$$\frac{1\pm\sqrt{1-4.1.-6}}{2}$$

$$\frac{1\pm 5}{2} \nearrow 3 \quad \searrow -2$$

$$5 \pm 25 - 4.7.1$$

$$\mathcal{L}(y) = \frac{s^2 - 5s + 7}{(s-4)(s^2-s-6)}$$

$$\mathcal{L}(y) = \frac{s^2 - 5s + 7}{(s-4)(s-3)(s+2)}$$

Fracciones simples

$$\frac{A}{s-4} + \frac{B}{s-3} + \frac{C}{s+2}$$

$$= A(s-3)(s+2) + B(s-4)(s+2) + C(s-4)(s-3)$$

$$A(s^2 - 3s + 2s - 6) + B(s^2 - 4s + 2s - 8) + C(s^2 - 4s - 3s + 12)$$

$$A(s^2 - s - 6) + B(s^2 - 2s - 8) + C(s^2 - 7s + 12)$$

$$As^2 - As - 6A + Bs^2 - 2Bs - B8 + Cs^2 - C7s + 12C$$

$$s^2(A+B+C) - s(A + 2B + 7C) + 12C - 6A - B8$$

$1 = A + B + C$ $\qquad A = 1 - B - C$

$5 = A + 2B + 7C$ $\qquad 5 = 1 - B - C + 2B + 7C = 1 + B + 6C \rightarrow B = 4 - 6C$

$7 = 12C - 6A - B8$ $\qquad 7 = 12C - 6(1-B-C) - B8$

$7 = 12C - 6 + 6B + 6C - B8$

$7 = 18C - 2B - 6$

$13 = 18C - 2B$

$A = 1 + \frac{1}{5} - \frac{7}{10} = \frac{1}{2}$ $\qquad B = 4 - \frac{6.7}{10} = 4 - \frac{21}{5} = -\frac{1}{5}$ $\qquad 13 = 18C - 8 + 12C$

$21 = 30C \rightarrow C = 7/10$

$$\mathcal{L}[y] = \frac{1/2}{s-4} - \frac{1/5}{s-3} + \frac{7/10}{s+2}$$

Vale si $s>4$

$$\mathcal{L}[y] = (1/2)\cdot\left(\frac{1}{s-4}\right) - (1/5)\left(\frac{1}{s-3}\right) + (7/10)\cdot\left(\frac{1}{s+2}\right)$$

$s-4>0 \quad s>4$
$s-3>0 \quad s>3$
$s+2>0 \quad s>-2$

$$\mathcal{L}[y] = (1/2)\cdot\mathcal{L}^{-1}\left(\frac{1}{s-4}\right) - (1/5)\cdot\mathcal{L}^{-1}\left(\frac{1}{s-3}\right) + (7/10)\mathcal{L}^{-1}\left(\frac{1}{s+2}\right)$$

$$\mathcal{L}[y] = 1/2\,\mathcal{L}(e^{4t}) - 1/5\,\mathcal{L}(e^{3t}) + 7/10\,\mathcal{L}(e^{-2t})$$

$$\mathcal{L}[y] = \mathcal{L}\left(\frac{e^{4t}}{2}\right) - \mathcal{L}\left(\frac{e^{3t}}{5}\right) + \mathcal{L}\left(\frac{e^{-2t}\cdot 7}{10}\right)$$

$$\mathcal{L}[y] = \mathcal{L}\left[\frac{e^{4t}}{2} - \frac{e^{3t}}{5} + \frac{7\cdot e^{-2t}}{10}\right]$$

$$\boxed{y(t) = \frac{e^{4t}}{2} - \frac{e^{3t}}{5} + \frac{7\cdot e^{-2t}}{10}}$$