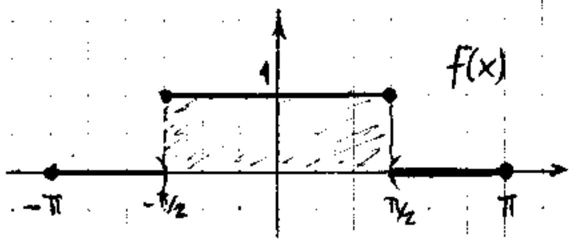
③ a) $[-\pi,\pi]$

$$f(x) = \begin{cases} 1 & , x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ 0 & , x \in \left[-\pi, -\pi/2\right) \cup \left(\pi/2, \pi\right] \end{cases}$$



$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x).\cos(nx).dx = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \cos(nx).dx = \frac{1}{\pi}\left(+\frac{\operatorname{sen}(nx)}{n}\right]_{-\pi/2}^{\pi/2}\Bigg)$$

$$a_n = \frac{1}{n\pi}\left[\operatorname{sen}\left(\tfrac{\pi}{2}n\right) - \operatorname{sen}\left(-\tfrac{\pi}{2}n\right)\right]$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x).dx = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} dx = \frac{1}{\pi}\left(\pi/2+\pi/2\right) = 1$$

$$a_n = \frac{2\operatorname{sen}\left(n\frac{\pi}{2}\right)}{n\pi}$$

$$b_n = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} 1.\operatorname{sen}(nx).dx = 0$$ por ser el integrando impar

$$\boxed{f \sim \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{n\pi}.\operatorname{sen}(n\pi/2).\cos(nx)}$$

f es 2π periódica, f continua a trozos en $[-\pi,\pi]$ la serie de Fourier converge puntualmente a

$$\frac{f(x^+)+f(x^-)}{2}$$

donde f tenga derivadas laterales finitas. Luego, como los puntos de salto son $x=-\pi/2$ y $x=\pi/2$ analizaremos qué sucede allí.

en $-\pi/2$

$$f'(x^+) = \lim_{t\to 0^+} \frac{f(x+t)-f(x^+)}{t} = 0$$

$$f'(x^-) = \lim_{t\to 0^+} \frac{f(x-t)-f(x^-)}{t} = 0$$

Para el punto $x = \pi/2$ la situación es la misma.

f' es continua a trozos. Luego

$$S_N(x) \underset{\text{Converge}}{\rightarrow} \begin{cases} f(x) & \text{en } [-\pi, -\pi/2) \cup (-\pi/2, \pi/2) \cup (\pi/2, \pi] \\ 1/2 & \text{en } x = \{\frac{\pi}{2}, -\frac{\pi}{2}\} \end{cases}$$

La convergencia es puntual.

Sin embargo como $S_N(x)$ converge a algo discontinuo la convergencia no puede ser uniforme.

b) $$\sum_{N=0}^{\infty} \frac{(-1)^N}{2N+1} = \frac{\pi}{4}$$

$N = n-1$
$N+1 = n$

$$\sum_{n=1} \frac{(-1)^{n-1}}{2n-1} = \frac{\pi}{4}$$

$$-\sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1} = \frac{\pi}{4}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1} = \frac{-\pi}{4}$$

$$\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \pi/4$$

$$-\frac{1}{1} + \frac{1}{3} - \frac{1}{5} + \frac{1}{7} - \cdots = -\pi/4$$

$$f \sim \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \operatorname{sen}\left(\frac{n\pi}{2}\right) \cdot \cos(nx)$$

Esta serie ~~cabalga~~ galopa por n impar con n par $\operatorname{sen} \frac{n\pi}{2} = 0 \Rightarrow$

$$f \sim \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{(2n-1)\pi} \operatorname{sen}\left[(2n-1)\frac{\pi}{2}\right] \cdot \cos(2n-1)x$$

n	$\left(\operatorname{sen} \frac{n\pi}{2}\right)$	
1	$\operatorname{sen} \pi/2$	1
2	$\operatorname{sen} \pi$	0
3	$\operatorname{sen} \frac{3}{2}\pi$	-1
4	$\operatorname{sen} 2\pi$	0
5	$\operatorname{sen} \frac{5}{2}\pi$	1
6	$\operatorname{sen} 3\pi$	0

$$f \sim \frac{1}{2} + \sum_{N=1} \frac{2 \cdot (-1)^{N+1}}{\pi(2N-1)} \cdot \cos[(2N-1)x]$$

$\Rightarrow$ Evalúo en $x=0 \rightarrow f(x=0) = 1 \Rightarrow$

$$1 \sim \frac{1}{2} + \sum_{N=1}^{\infty} -\frac{2(-1)^N}{\pi(2N-1)}$$

$$\frac{1}{2} = -\frac{2}{\pi} \sum_{N=1}^{\infty} \frac{(-1)^N}{(2N-1)}$$

$$\frac{\pi}{4} = -\sum_{N=1}^{\infty} \frac{(-1)^N}{(2N-1)}$$

Cambio de índice
$N-1 = n \rightarrow$

$$= -\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2(n+1)-1}$$

$$\boxed{\frac{\pi}{4} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}}$$