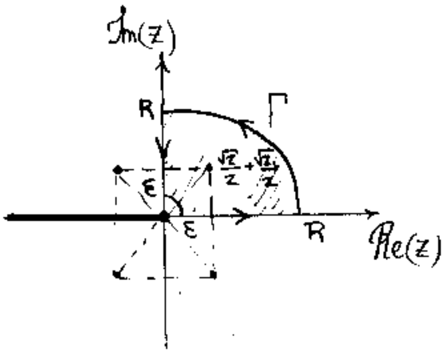
④ $\displaystyle\int_0^{\infty} \frac{\sqrt[2]{x}}{1+x^4}\,dx$ $\qquad \sqrt[2]{z} = e^{\ln \sqrt[2]{z}} = e^{\ln z^{1/2}} = e^{\frac{1}{2}(\ln|z| + i.\arg(z))}$

$-\pi < \arg(z) < \pi$

tomo $\displaystyle f(z) = \frac{\sqrt[2]{z}}{1+z^4} = \frac{e^{1/2(\ln|z| + i.\arg(z))}}{\left(z-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)\left(z+\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)\left(z+\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)\left(z-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)}$



$$\operatorname{Res}\left(f, \frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right) = \lim_{z \to \frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i} \left(\frac{e^{1/2(\ln|1| + i\pi/4)}}{(\sqrt{2})(\sqrt{2}+\sqrt{2}i)(\sqrt{2}i)} \right)$$

$$= \frac{e^{\frac{i\pi}{8}}}{2i.(\sqrt{2}+\sqrt{2}i)}$$

$z^4+1=0$

$z^4=-1$

$z^4 = 1.e^{i\pi}$

$z = 1^{1/4}.e^{i\left(\frac{\pi+2k\pi}{4}\right)} \to \quad e^{-i\pi/4},\ e^{i\pi/4},\ e^{i\frac{3}{4}\pi},\ e^{-i\frac{3}{4}\pi}$

con $0 < \arg z < 2\pi$

$0 < \arg z < 2k\pi \qquad k=0,1,\ldots n-1$

Con $-\pi < \arg z < \pi$

$-\pi k < \arg z < k\pi \qquad k=-1,0,1,\ldots n-1$

$$2\pi i \operatorname{Res}(f, e^{i\frac{\pi}{4}}) = \int = \int_{\varepsilon}^{R} \frac{\sqrt[2]{x}}{1+x^4}\,dx + \int_{\Gamma_R} f(z).dz + \int_R^{\varepsilon} f(z).dz + \int_{\Gamma_\varepsilon} f(z).dz$$

$$-\int_0^{\pi/2} \frac{\varepsilon^{1/2}.e^{i\theta/2}.\varepsilon.e^{i\theta}i}{1+\varepsilon^4.e^{i\theta 4}}.d\theta$$

$$\leqslant \frac{-\varepsilon^{3/2}}{-1+\varepsilon^4}\int_0^{\pi/2} d\theta \underset{\varepsilon \to 0}{\longrightarrow} 0$$

$$\int_0^{\pi/2} \frac{R^{1/2}e^{\frac{i\theta}{2}}.R e^{i\theta}.i.d\theta}{1+R^4e^{i\theta 4}}$$

$$|\quad| \leq \frac{R^{3/2}}{R^4-1}\int_0^{\pi/2} d\theta \underset{R\to\infty}{\longrightarrow} 0$$

$z(r) = r.e^{\frac{i\pi}{2}} \qquad \varepsilon \leqslant r \leqslant R$

$z'(r) = e^{i\frac{\pi}{2}}$

$$-\int_{\varepsilon}^{R} \frac{\sqrt{r.e^{\frac{i\pi}{2}}}}{1+r^4.e^{\frac{i\pi 4}{2}}}.e^{i\pi/2}.dr \to \int |\quad|.dr$$

$|1+r^4.e^{i\pi 2}| \geqslant |1-r^4| = r^4-1$

$$-\int_{\varepsilon}^{R} \frac{r^{1/2}}{r^4-1}.dr \leq \int_{\varepsilon}^{R} \frac{r}{r^4-1}.dr \quad \begin{matrix} r>1>0 \\ r^2>r \\ r>\sqrt{r} \end{matrix} \quad \leq \int \frac{r.dr}{(r^2-1)(r^2+1)} = \int_{\varepsilon^2}^{R^2} \frac{du}{2(u-1)(u+1)}$$

$r^2=u$

$2rdr=du$

Esto revienta cuando $\varepsilon \to 0$

$$\frac{r^{1/2}}{r^4-1} \leqslant \frac{1}{r^3} \qquad \int_\varepsilon^R \frac{1}{r^2}\,dr = \frac{r^{-1}}{-1}\Big]_\varepsilon^R = \frac{1}{R}-\frac{1}{\varepsilon}$$

$r^{7/2} \leqslant r^4-1$

GRAN ROBO

$$\int_R^\varepsilon \frac{\sqrt{z}}{1+z^4}\cdot dz = -\int_\varepsilon^R \frac{(iy)^{1/2}\cdot i\cdot dy}{1+(iy)^4} = -\int_\varepsilon^R \frac{i^{1/2}\cdot i\cdot y^{1/2}}{1+y^4}\cdot dy = -\left(\frac{\sqrt{2}}{2}i-\frac{\sqrt{2}}{2}\right)\int_\varepsilon^R \frac{y^{1/2}}{1+y^4}\cdot dy$$

$z = iy \qquad \varepsilon < y < R$

$z' = i$

$i^4 = i^2\cdot i^2 = 1 \qquad \sqrt{i} = i^{1/2} = z:$

$(e^{i\pi/2})^{1/2} = e^{i\pi/4} =$

$\Rightarrow$

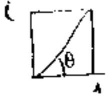
$$\frac{\cancel{2}\pi i\cdot e^{i\pi/8}}{\cancel{2}i\,(\sqrt{2}+\sqrt{2}\,i)} = \underbrace{\left(1+\frac{\sqrt{2}}{2}\right)}_{\frac{2+\sqrt{2}}{2}}\int_\varepsilon^R \frac{x^{1/2}}{1+x^4}\cdot dx + \left(-\frac{\sqrt{2}}{2}\,i\int_\varepsilon^R \frac{x^{1/2}}{1+x^4}\cdot dx\right)$$

$$\frac{2}{(2+\sqrt{2})}\cdot\frac{\pi}{(\sqrt{2}+\sqrt{2}\,i)}\cdot\cos(\pi/8) + i\cdot\operatorname{sen}(\pi/8)\frac{\pi}{\sqrt{2}+\sqrt{2}\,i}$$

$$''\int_\varepsilon^R \frac{x^{1/2}}{1+x^4}\cdot dx$$

$$\frac{2\cdot\pi\cdot\left(e^{i\pi/8}\right)}{(2+\sqrt{2})\cdot\sqrt{2}\left(e^{i\pi/4}\right)\sqrt{2}}$$

$$\frac{\cancel{2}\pi\, e^{-i\pi/8}}{(2+\sqrt{2})\,\cancel{2}} = \boxed{\cos(\pi/8)\cdot\frac{\pi}{(2+\sqrt{2})} = \lim_{\substack{\varepsilon\to 0\\ R\to\infty}}\int_\varepsilon^R \frac{x^{1/2}}{1+x^4}\cdot dx}$$



$\sqrt{2}\,e^{i\pi/4}$