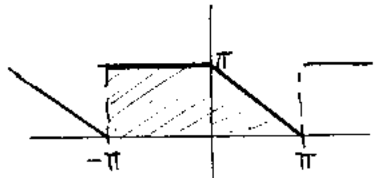
① $f = \begin{cases} \pi & -\pi \leq x < 0 \\ \pi - x & 0 \leq x < \pi \end{cases}$



$$a_n = \frac{1}{\pi}\left(\int_{-\pi}^{0} \pi . \cos(nx)\, dx + \int_{0}^{\pi} (\pi - x) . \cos(nx) . dx\right)$$

$$a_n = \frac{1}{\pi}\left(\pi . \frac{\operatorname{sen}(nx)}{n}\Big]_{-\pi}^{0} + \int_0^{\pi} \pi \cos(nx)\, dx - \int_0^{\pi} x . \cos(nx)\, dx \right) \qquad \begin{matrix} u = x & dv = \cos(nx) \\ du = dx & v = \frac{\operatorname{sen}(nx)}{n} \end{matrix}$$

$$\frac{1}{\pi}\left[0 + 0 - \left[\frac{\operatorname{sen}(nx)}{n} . x \right]_0^{\pi} - \int_0^{\pi} \frac{\operatorname{sen}(nx) . dx}{n} \right]$$

$$a_n = \frac{1}{\pi}\left[- \frac{\cos(nx)}{n^2}\Big|_0^{\pi} \right] = \frac{-1}{n^2 \pi} . [\cos(n\pi) - 1] = \frac{1 - \cos(n\pi)}{n^2 \pi}$$

$$b_n = \frac{1}{\pi}\left(\int_{-\pi}^{0} \pi . \operatorname{sen}(nx) . dx + \int_0^{\pi} (\pi - x) . \operatorname{sen}(nx) . dx \right)$$

$$b_n = \frac{1}{\pi}\left(\int_{-\pi}^{0} \pi . \operatorname{sen}(nx) . dx + \int_0^{\pi} \pi . \operatorname{sen}(nx) . dx - \int_0^{\pi} x . \operatorname{sen}(nx) . dx \right) \qquad \begin{matrix} u = x & dv = \operatorname{sen}(nx) \\ du = dx & v = \frac{-\cos(nx)}{n} \end{matrix}$$

$$\begin{matrix} x = -x \\ dx = -dx \end{matrix} \left(- \int_{\pi}^{0} \pi . \operatorname{sen}(nx)\, dx + \int_0^{\pi} \pi . \operatorname{sen}(nx) . dx - \left[- \frac{x . \cos(nx)}{n}\Big|_0^{\pi} + \int_0^{\pi} \frac{\cos nx}{n} . dx \right] \right)$$

$= 0$

$$\frac{1}{\pi}\left(- \left[- \frac{\pi . \cos(n\pi)}{n} + \frac{1}{n^2} . \operatorname{sen}(nx) \right]_0^{\pi} \right) \quad 0 . \frac{\cos 0}{n}$$

$$b_n = \frac{\cos(n\pi)}{n}$$

$$a_0 = \frac{1}{\pi}\left(\int_{-\pi}^{0} \pi . dx + \int_0^{\pi} (\pi - x) . dx \right) = \frac{1}{\pi}\left[(\pi . \pi) + \pi . \pi - \frac{x^2}{\pi 2}\Big|_0^{\pi} \right] = 2\pi - \frac{\pi}{2} = \frac{3}{2}\pi$$

$$\boxed{f \sim \frac{3}{4}\pi + \sum_{n=1}^{\infty} \frac{1 - (-1)^n}{n^2 \pi} . \cos(nx) + \frac{\cos(n\pi)}{n} . \operatorname{sen}(nx)}$$

$$f \sim \frac{3\pi}{4} + \frac{1}{\pi}\sum_{n=1}^{\infty} \frac{1-(-1)^n}{n^2}.\cos(nx) + \frac{(-1)^n}{n}.\operatorname{sen}(nx)$$

si $x=0$ (donde f es continua vale) ↘

$$\pi \sim \frac{3\pi}{4} + \frac{1}{\pi}\sum_{n=1}^{\infty} \frac{1-(-1)^n}{n^2}$$

↘ Camina por n impar: $n=1,3,5$

$$\frac{\pi}{4} \sim + \frac{1}{\pi}\sum_{n=0}^{\infty} \frac{2}{(2n+1)^2} \Rightarrow \frac{\pi^2}{8} = \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$$

$$\sum_{n=1}^{\infty} \frac{1}{(2n+1)^2} = \sum_{n=0} \frac{1}{(2n+1)^2} - 1$$

$$\boxed{\sum_{n=1}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8} - 1}$$

f continua en $(-\pi,\pi)$; 2π periódica, derivadas laterales continuas a trozos ⇒ se integra ...

$\forall\, x \in [0,\pi)$

$$\int_{-\pi}^{x}(\pi - x).dx \sim \frac{3\pi}{4}\int_{\pi}^{x} dx + \frac{1}{\pi}\sum \frac{1-(-1)^n}{n^2}\int_{-\pi}^{x}\cos(nx).dx + \frac{(-1)^n}{n}\int_{-\pi}^{x}\operatorname{sen}(nx).dx$$

$$\int_{-\pi}^{0}\pi\, dx + \int_{0}^{x}(\pi - x).dx \sim \frac{3\pi}{4}(x+\pi) + \frac{1}{\pi}\sum \frac{1-(-1)^n}{n^3}.\operatorname{sen}(nx) + \frac{(-1)^n}{n^2}.-\left[\cos(nx)\Big|_{-\pi}^{x}\right)$$

$$\pi(\pi) + \pi\int_0^x dx - \int_0^x x\,dx \qquad \frac{3\pi}{4}x + \frac{3}{4}\pi^2 \qquad + \frac{(-1)^n}{n^2}\left(-\cos(nx) + \cos(n\pi)\right)$$

$$\pi^2 + \pi x - \frac{x^2}{2}\Big]_0^x$$

$$\pi^2 + \pi x - \frac{1}{2}x^2 \sim \frac{3\pi x}{4} + \frac{3\pi^2}{4} + \frac{1}{\pi}\sum \frac{1-(-1)^n}{n^3}\operatorname{sen}(nx) - \frac{(-1)^n}{n^2}\cos(nx) + \frac{1}{n^2}$$

$$\pi^2 \sim \frac{3}{4}\pi^2 - \sum \frac{(-1)^n}{n^2} + \sum \frac{1}{n^2}$$

$$\frac{\pi^2}{4} = -\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} + \sum_{n=1}^{\infty} \frac{1}{n^2} = \sum_{n=1}^{\infty} \frac{1-(-1)^n}{n^2} =$$

$$-\left(\sum_{n=1}^{\infty}\frac{1}{n^2} - \sum_{n=1}^{\infty}\frac{2}{(2n-1)^2}\right)$$

→ pares ⊕
impares ⊖

$$\frac{\pi}{4} = \frac{1}{\pi}\left(\sum_{n=1}^{\infty}\frac{1}{n^2} - \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\right)$$

pares e impares + pares ⊖ impares ⊕

$$\frac{\pi^2}{4} = \sum_{n=1}^{\infty}\frac{1}{n^2} - \left(\sum_{n=1}\frac{1}{(2n)^2} - \sum_{n=1}\frac{1}{(2n-1)^2}\right)$$

$$\frac{\pi^2}{4} = \sum_{n=1}^{\infty}\frac{1}{n^2} - \frac{1}{4}\sum_{n=1}^{\infty}\frac{1}{n^2} + \sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$$

$$\frac{\pi^2}{4} - \frac{\pi^2}{8} = \left(1 - \frac{1}{4}\right)\sum_{n=1}^{\infty}\frac{1}{n^2} \qquad \begin{matrix}\parallel \\ \frac{\pi^2}{8}\end{matrix}$$

$$\frac{4}{3}\left(\frac{\pi^2}{8}\right) = \boxed{\frac{\pi^2}{6} = \sum_{n=1}^{\infty}\frac{1}{n^2}}$$

$$\frac{\pi^2}{4} - \frac{\pi^2}{6} = -\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$$

$$\frac{\pi^2}{12} = -\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$$

$$-\frac{\pi^2}{12} = \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$$

pares ⊕ impares ⊖

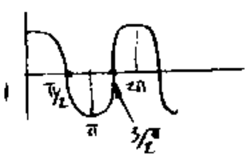
$$-\frac{\pi^2}{12} = \sum_{n=1}^{\infty}\frac{1}{4n^2} - \frac{1}{(2n+1)^2}$$

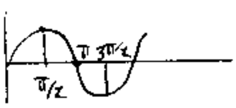
$x = \pi/2 \rightarrow$

$$\frac{\pi}{2} = \frac{3}{4}\pi + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n^2}\cdot\cos\left(\frac{n\pi}{2}\right) \quad \sum_{n=1}^{\infty}\frac{(-1)^n}{n}\cdot\operatorname{sen}\left(\frac{n\pi}{2}\right)$$

n par = 0, n impar ≠ 0 | n par = ±1, n impar = 0 | n par = 1/n, n impar −1/n | n par = 0

$$\sum_{n=1}^{\infty}$$





$\operatorname{sen}\left(\frac{n\pi}{2}\right)$ → n par = 0; n impar = 1, −1

$\left(\frac{\pi}{2}\right)$ $\left(\frac{3\pi}{2}\right)$ $\frac{5\pi}{2}$