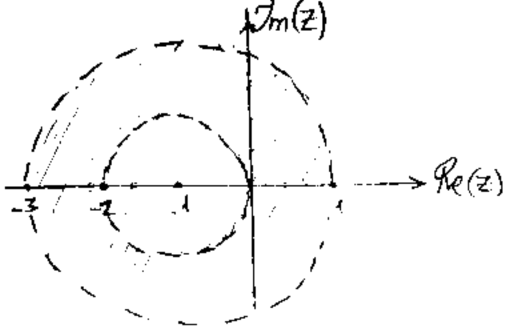
③ $\dfrac{1}{z^3+6z^2+9z}$

$1<|z+1|<2 \qquad f(z)=\dfrac{1}{z^3+6z^2+9z}=\dfrac{1}{z(z^2+6z+9)}=-\dfrac{1}{z(z+3)^2}$



$\dfrac{-6\pm\sqrt{36-4.1.9}}{2}=-3$

$(z+3)^2=z^2+6z+9$

sea $\quad z+1=U \;\rightarrow\; 1<|U|<2 \;\rightarrow\; 1<|U| \;,\; \dfrac{|U|}{2}<1 \;\rightarrow$

$f(z)=\dfrac{1}{z(z+3)^2} \;\rightarrow\; f(U)=\dfrac{1}{(U-1).(U+1)^2}=\dfrac{-1}{(U+1)^2(1-U)}=\dfrac{1}{(U+1)^2}\cdot\left(\dfrac{1}{1-U}\right)=\left(\dfrac{1}{1+U}\right)^2\cdot\sum_{N=0}^{\infty}U^n$

con $1<|U|$

$1<|z+1| \;\rightarrow\; \dfrac{1}{|z+1|}<1$

$\dfrac{|z+1|}{2}<1$

$f(z)=\dfrac{1}{z(z+3)^2}=\dfrac{A}{z}+\dfrac{Bx+C}{(z+3)^2}+\dfrac{D}{z+3}=\dfrac{1}{9}\cdot\dfrac{1}{z}-\dfrac{1}{3}\cdot\dfrac{1}{(z+3)^2}-\dfrac{1}{9}\cdot\dfrac{1}{z+3}$

$=\dfrac{1}{9}\cdot\dfrac{1}{(z+1-1)}-\dfrac{1}{3}\cdot\dfrac{1}{(z+1+2)^2}-\dfrac{1}{9}\cdot\dfrac{1}{(z+1+2)}$

$f(z)=-\dfrac{1}{9(z+1)}\cdot\dfrac{1}{\left(1-\frac{1}{z+1}\right)}-\dfrac{1}{12}\cdot\dfrac{1}{\left[1+\frac{(z+1)}{2}\right]^2}-\dfrac{1}{18}\cdot\dfrac{1}{\left[1+\left(\frac{z+1}{2}\right)\right]}$

$$\boxed{f(z)=-\dfrac{1}{9(z+1)}\cdot\sum_{N=0}^{\infty}\dfrac{1}{(z+1)^n}-\dfrac{1}{18}\sum_{n=0}^{\infty}(-1)^n\cdot\left(\dfrac{z+1}{2}\right)^n-\dfrac{1}{12}\sum_{N=1}^{\infty}\left(\dfrac{z+1}{2}\right)^{n-1}n.(-1)^n}$$