① $\displaystyle\sum_{n=1}^{\infty} \frac{n.3^n}{n^2+2}.(z-2)^{2n}$ defino $(z-2)^2 \equiv W$

* Radio de convergencia

$$\lim_{n\to\infty} \left| \frac{(n+1).3^{n+1}}{\left[(n+1)^2+2\right].\left[3^n.n\right]} . \frac{W^{n+1}.(n^2+2)}{W^n} \right| = \lim_{n\to\infty} \frac{|W|.3.(n+1)(n^2+2)}{(n^2+2n+3).n}$$

$$\lim_{n\to\infty} \left| \frac{3.|W|.(n^3+n^2+2n+2)}{n^3+2n^2+3n} \right| = 3.|W| < 1 \quad \rightarrow \quad R = \frac{1}{3}$$

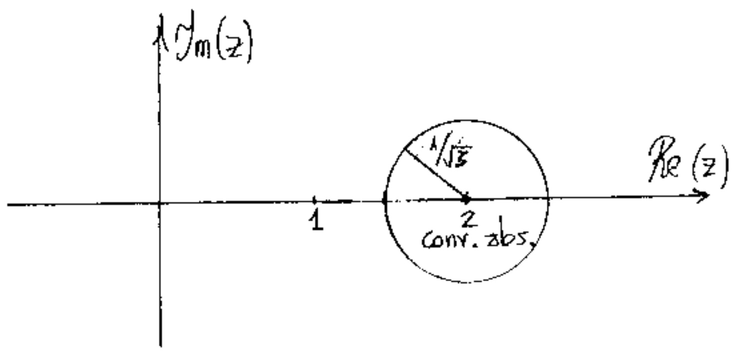
$$\left|(z-2)^2\right| < \frac{1}{3} \longrightarrow \sqrt{(x-2)^2+y^2}.\sqrt{(x-2)^2+y^2} < \frac{1}{3}$$

$$\left|z^2-4z+4\right| < \frac{1}{3} \qquad (x-2)^4+y^2(x-2)^2 2+y^4 < \frac{1}{9}$$

$$\left|x^2-y^2+i2xy-4x-4iy+4\right| < 1/3 \qquad \left((x-2)^2+y^2\right)^2 < 1/9$$

$$\sqrt{(x^2-y^2-4x+4)^2+(2xy-4y)^2} < 1/3 \qquad (x-2)^2+y^2 < 1/3$$

$$< 1/9 \qquad \downarrow \; |z-2| < 1/\sqrt{3}$$



* Conv. absoluta en el borde del disco

$$\sum_{n=1}^{\infty} \frac{n.3^n}{n^2+2}.\left(\frac{1}{\sqrt{3}}\right)^{2n} = \sum_{n=1}^{\infty} \frac{n}{n^2+2}$$ diverge $\Rightarrow$ no converge abs. en $|z-2| = \frac{1}{\sqrt{3}}$

* Convergencia Condicional

$$\left|\sum_{n=0}^{\infty} 3^n.(z-2)^{2n}\right| = \left|\sum_{n=0}^{\infty} \left[3(z-2)^2\right]^n\right| = \left|\frac{1-\left[3(z-2)^2\right]^{N+1}}{1-3(z-2)^2}\right| \leq \frac{1+\left|3(z-2)^2\right|^{N+1}}{\left|1-3(z-2)^2\right|}$$

vale si $3(z-2)^2 \neq 1$

$(z-2)^2 \neq \frac{1}{3}$

$z-2 = 1/\sqrt{3}$

$z = 2+\frac{1}{\sqrt{3}}\ ,\ 2-\frac{1}{\sqrt{3}}$

$$\leq \frac{2}{\left|1-3(z-2)^2\right|} = M$$

no depende de n

$|z-2| = \frac{1}{\sqrt{3}}$

$|z-2|^2 = \left|(z-2)^2\right| = \frac{1}{3}$

$\rightarrow \quad \left|3(z-2)^2\right| = 1$

$\sum_{n=0}^{\infty} \frac{n}{n^2+2}$ donde $\frac{n}{n^2+2} = a_n$ es decreciente

$\frac{n}{n^2+2} \longrightarrow 0$

$\frac{n}{n^2+2} \geq 0 \quad \forall n \in \mathbb{N}$

$\Rightarrow$ vale Dirichlet y $\sum_{N=0}^{\infty}$ converge condicionalmente en $|z-2| = \frac{1}{\sqrt{3}}$ salvo en $\begin{cases} 2+1/\sqrt{3} \\ 2-1/\sqrt{3} \end{cases}$

* $z = 2+1/\sqrt{3}\ ,\ 2-1/\sqrt{3}$

$$\sum_{N=0}^{\infty} \frac{n.3^n}{n^2+2}.\left(2\pm 1/\sqrt{3}-2\right)^{2n} = \sum_{N=0}^{\infty} \frac{n.\cancel{3^n}}{(n^2+2)}\frac{1}{\cancel{3^n}} \longrightarrow \text{diverge en } z = 2\pm\frac{1}{\sqrt{3}}$$

$n^2 < n^2+2 < 2n^2$

$\frac{1}{n^2} > \frac{1}{n^2+2} > \frac{1}{2}\cdot\frac{1}{n^2}$

$\frac{n}{n^2} > \frac{n}{n^2+2} > \frac{1}{2}\cdot\frac{n}{n^2}$

$\sum \frac{1}{n} > \sum \frac{n}{n^2+2} > \frac{1}{2}\sum \cdot \frac{1}{n}$

▲ diverge　　　　diverge ▲