Ejercicio 2

(a) $f:\Omega \to \mathbb{C}$ holomorfa, $\Omega \subset \mathbb{C}$ abierto

$f = U + i.V$ donde $U = Re(F)$

→ si F es holomorfa U,V cumplen condiciones de Cauchy-Riemann ⇒

$$U_x = V_y$$
$$-U_y = V_x \to U_y = -V_x$$

$$\to \quad U_{xx} = V_{yx} \quad \wedge \quad U_{yy} = -V_{xy}$$

justificar mejor que $u,v \in C^2$

Sin embargo si F es holomorfa $U,V \in C^2 \to$ $U_{xy} = U_{yx}$, $V_{xy} = V_{yx}$

(las derivadas cruzadas son iguales)

$\therefore$

$$U_{xx} + U_{yy} = V_{yx} + (-V_{xy}) = V_{yx} - V_{yx} = 0$$

$$\Rightarrow \boxed{U_{xx} + U_{yy} = 0} \to U \text{ es armónica}$$

✓

(b) $U(x,y) = e^{x^2-y^2} \cdot \cos(2xy)$

$$\frac{\partial U}{\partial x} = e^{x^2-y^2}.2x.\cos 2xy + (-e^{x^2-y^2}.\operatorname{sen}(2xy).2y$$
$$2.e^{x^2-y^2}[x.\cos(2xy) - \operatorname{sen}(2xy).y]$$
$$\frac{\partial^2 U}{\partial x^2} = 2.e^{x^2-y^2}.2x[x.\cos(2xy) - \operatorname{sen}(2xy).y]$$
$$+2.e^{x^2-y^2}.[\cos(2xy) - x.\operatorname{sen}(2xy).2y - (\cos(2xy)).2y^2$$

$$\frac{\partial U}{\partial y} = -e^{x^2-y^2}.2y.\cos(2xy) + e^{x^2-y^2}.-\operatorname{sen}(2xy).2x$$
$$= -e^{x^2-y^2}.2[+y.\cos(2xy) + x.\operatorname{sen}(2xy)]$$
$$\frac{\partial^2 U}{\partial y^2} = +2e^{x^2-y^2}.2y.[y.\cos(2xy) + x.\operatorname{sen}(2xy)]$$
$$-e^{x^2-y^2}.2.[\cos(2xy) - y.\operatorname{sen}(2xy).2x + x.\cos(2xy).2x$$

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = \cancel{4e^{x^2-y^2}.x^2\cos(2xy)} - \cancel{4xy\,e^{x^2-y^2}\operatorname{sen}(2xy)} +$$
$$2e^{x^2-y^2}.\cos(2xy) - \cancel{4xy\,e^{x^2-y^2}\operatorname{sen}(2xy)} - \cancel{4y^2e^{x^2-y^2}\cos(2xy)}$$

$$\cancel{4y^2e^{x^2-y^2}.\cos(2xy)} + \cancel{4e^{x^2-y^2}xy\operatorname{sen}(2xy)}$$
$$-2e^{x^2-y^2}.\cos(2xy) + \cancel{4xy\,e^{x^2-y^2}\operatorname{sen}(2xy)}$$
$$- 4x^2.e^{x^2-y^2}.\cos(2xy) = 0$$

$$\Rightarrow \boxed{U(x,y) = e^{x^2-y^2}.\cos(2xy) \text{ es armónica}}$$