$\boxed{1}$. $\displaystyle\sum_{N=0}^{\infty} \frac{1}{(N+1)(\sqrt{2})^N} \cdot \left(\frac{z}{z+1}\right)^N$

* Radio de Convergencia:

$\frac{z}{z+1} = W \Rightarrow$

$$\lim_{n\to\infty} \left| \frac{1 \cdot (N+1)(\sqrt{2})^N}{(N+2)(\sqrt{2})^{N+1}} \cdot \frac{W^{N+1}}{W^N} \right| = \lim_{n\to\infty} \frac{(N+1)}{(\sqrt{2})(N+2)} |W| = \frac{|W|}{\sqrt{2}} < 1$$

$$|W| < \sqrt{2}$$

Radio de convergencia $|W| = \sqrt{2}$

$$\frac{|z|}{|z+1|} < \sqrt{2}$$

$$\sqrt{x^2+y^2} < \sqrt{2}\sqrt{(x+1)^2+y^2}$$

$$x^2+y^2 < 2(x^2+2x+1+y^2)$$

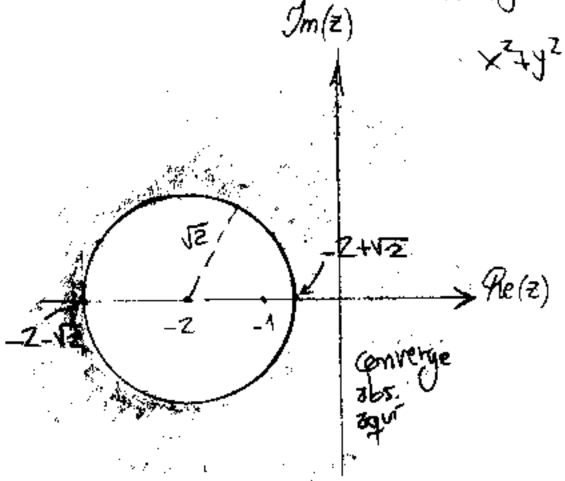
$$x^2+y^2 < 2x^2+4x+2+2y^2$$

$$0 < x^2+y^2+4x+2$$

$$0 < x^2+2(x).2+4+y^2+2-4$$

$$2 < (x+2)^2+y^2$$

Converge abs. $\forall \{z \in \mathbb{C} : |z+2| > \sqrt{2}\}$



* Veamos que conv. abs. en $z=0$ y

$$\sum_{N=0}^{\infty} \left| \frac{1}{(N+1)(\sqrt{2})^N} (1)^N \right| = \sum_{N=0}^{\infty} \frac{}{(N+1)} \cdot \left(\frac{1}{\sqrt{2}}\right)^N \Rightarrow \text{conv. abs. pues}$$

$$(n+1) > 1$$

$$\frac{1}{n+1} < 1$$

$\left(\frac{1}{n+1}\right) R^N < R^N$ con $R<1$ Converge

* <u>Comportamiento en el círculo de convergencia</u>

$$\sum_{N=0}^{\infty} \frac{1}{(n+1)(\sqrt{2})^N} \cdot \left(\frac{z}{z+1}\right)^N = \sum_{N=0}^{\infty} \frac{1}{N+1} \cdot \left(\frac{z}{\sqrt{2}z+\sqrt{2}}\right)^N$$

es 1 este módulo

$$\left|\sum_{N=0}^{\infty} \left(\frac{z}{\sqrt{2}(z+1)}\right)^N\right| = \left|\frac{1 - \left(\frac{z}{\sqrt{2}z+\sqrt{2}}\right)^{N+1}}{1 - \frac{z}{\sqrt{2}(z+1)}}\right| \leq \frac{1 + \left|\frac{z}{\sqrt{2}(z+1)}\right|^{N+1}}{\left|1 - \frac{z}{\sqrt{2}(z+1)}\right|} \leq \frac{2}{\left|1 - \frac{z}{\sqrt{2}(z+1)}\right|} \equiv M[z]$$

$z=-1$ no $\notin$ al círculo

no depende de n

$$\frac{\sqrt{2}z + \sqrt{2} - z}{\sqrt{2}(z+1)} = \frac{z(\sqrt{2}-1)+\sqrt{2}}{\sqrt{2}(z+1)} \neq 0 \iff z \neq \frac{-\sqrt{2}}{(\sqrt{2}-1)}$$

$$z \neq \frac{-\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

$$z \neq \frac{-2-\sqrt{2}}{2-1} = -2-\sqrt{2}$$

$a_n = \frac{1}{n+1} \searrow 0$ [tiende a cero decreciente]

$a_n \geq 0 \ \forall n \in \mathbb{N}$

$\frac{1}{n+1} \geq 0$

$\left|\sum_{N=0}^{\infty} \left(\frac{z}{\sqrt{2}(z+1)}\right)^N\right| \leq M[z]$ convergente

$\Rightarrow \sum_{N=0}^{\infty}$ converge condicionalmente en $|z+2| = \sqrt{2}$ salvo en $z = -2-\sqrt{2}$

$a_{n+1} < a_n$

$\frac{1}{n+2} < \frac{1}{n+1}$

$n+1 < n+2$

$1 < 2$

$\rightarrow$ vale $\forall n \in \mathbb{N}$

* $z = -2-\sqrt{2}$

$$\sum_{N=0}^{\infty} \frac{1}{(N+1)\cdot(\sqrt{2})^N} \left(\frac{-2-\sqrt{2}}{-2-\sqrt{2}+1}\right)^N = \sum_{N=0}^{\infty} \frac{1}{N+1}$$

diverge

$\Rightarrow$ en $z = -2-\sqrt{2}$ $\sum_{N=0}^{\infty}$ diverge

$$\frac{1}{\sqrt{2}}\left(\frac{-2-\sqrt{2}}{-1-\sqrt{2}}\right) = \frac{-2-\sqrt{2}}{-\sqrt{2}-2} = 1$$