Ejercicio 2

$$U(x,y) = \cos x \cdot (e^{-y} + e^{y}) + x^2 - y^2$$

$$\frac{\partial U}{\partial x} = -\operatorname{sen} x \cdot (e^{-y} + e^{y}) + 2x$$

$$\frac{\partial U}{\partial y} = \cos x \cdot (-e^{-y} + e^{y}) - 2y$$
$$= \cos x \cdot (-e^{-y} + e^{y}) - 2y$$

$$\frac{\partial^2 U}{\partial x^2} = (e^{-y} + e^{y}) \cdot -\cos x + 2$$

$$\frac{\partial^2 U}{\partial y^2} = \cos x \cdot (e^{-y} + e^{y}) - 2$$

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = -(e^{-y} + e^{y}) \cdot \cos x + 2 + \cos x (e^{-y} + e^{y}) - 2$$

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$$

Luego, $U(x,y)$ es armónica porque verifica la ecuación de Laplace

Necesito una $f: \mathbb{C} \to \mathbb{C}$ holomorfa, por ello requeriré que sea diferenciable en el sentido complejo; lo cual implica que cumpla las ecuaciones de Cauchy-Riemann:

$$\frac{\partial U}{\partial x} = \frac{\partial V}{\partial y} \quad \wedge \quad \frac{\partial U}{\partial y} = -\frac{\partial V}{\partial x}$$

donde $U \equiv \operatorname{Re}[f(z)] \wedge V \equiv \operatorname{Im}[f(z)]$

$-y = \phi$
$-dy = d\phi$

$\int e^{-y} \cdot dy$

$-\int e^{\phi} \cdot d\phi$

$-e^{\phi = -y}$

$-e^{-y}$

$$-\operatorname{sen} x \cdot (e^{-y} + e^{y}) + 2x = \frac{\partial V}{\partial y}$$

$$\to -\int \operatorname{sen} x \cdot (e^{-y} + e^{y}) \cdot dy + \int 2x \cdot dy = V + C(x)$$

constante de integración [dependerá si es que depende de el una variable de x]

$$-\int \operatorname{sen} x \cdot e^{-y} dy - \int \operatorname{sen} x \cdot e^{y} dy + 2x \cdot y = V + C$$

$$-\operatorname{sen} x \cdot -e^{-y} \quad - e^{y} \operatorname{sen} x + 2xy = V + C$$

$$\operatorname{sen} x \cdot e^{-y} - e^{y} \cdot \operatorname{sen} x + 2xy = V + C$$

La V obtenida se deriva con respecto a X

$$V = \operatorname{sen} x . e^{-y} - \operatorname{sen} x . e^{y} + 2xy + C(x)$$

$$\frac{\partial V}{\partial x} = e^{-y} . \cos x - \cos x . e^{y} + 2y + C'(x)$$

igualo con $-\frac{\partial U}{\partial y}$ y obtengo:

$$\left(e^{-y} - e^{y}\right) . \cos x + 2y + C'(x) = -\left[\cos x\left(-e^{-y} + e^{y}\right) - 2y\right]$$

$$\left(e^{-y} - e^{y}\right) . \cos x + 2y + C'(x) = \cos x . \left(e^{-y} - e^{y}\right) + 2y$$

$$C'(x) = 0 \quad \rightarrow \quad C(x) = C \quad \left[\text{no depende de las variables}\right]$$

con $C = \operatorname{Re}(C) + i \operatorname{Im}(C)$

Finalmente

$$\boxed{f(x+iy) = \left[\cos x . \left(e^{-y} + e^{y}\right) + x^2 - y^2 + \operatorname{Re}(C)\right] + i\left[\operatorname{sen} x . \left(e^{-y} - e^{y}\right) + 2xy + \operatorname{Im}(C)\right]}$$

es la f holomorfa pedida