① $$\sum_{N=1}^{\infty} \frac{(-1)^N}{(3^N+e^N)\sqrt{N+1}} \cdot \left(\frac{2+z}{2i-z}\right)^{N-1}$$

* Buscamos el radio de convergencia

Sea $\dfrac{2+z}{2i-z} = W \Rightarrow$

$$\lim_{N\to\infty} \left| \frac{(-1)^{N+1} \cdot W^N \cdot (3^N+e^N)\sqrt{N+1}}{(3^{N+1}+e^{N+1})\sqrt{N+2} \cdot W^{N-1} \cdot (-1)^N} \right| = \lim_{N\to\infty} \left| \frac{\sqrt{N+1}}{\sqrt{N+2}} \cdot \frac{(3^N+e^N)}{(3^{N+1}+e^{N+1})} \cdot W \right| = \lim_{n\to\infty} \frac{\sqrt{n+1}}{\sqrt{n+2}} \cdot \frac{3^N(1+(e/3)^N)}{3^N(3+\frac{e^{N+1}}{3^N})} \cdot |W|$$

$$= \frac{1}{3} \cdot |W| < 1 \iff |W| < 3 \Rightarrow$$

$\underline{\underline{R=3}}$ para W

$$\left|\frac{2+z}{2i-z}\right| < 3 \quad \rightarrow \quad |2+z| < 3 \cdot |2i-z|$$

$$\sqrt{(x+2)^2+y^2} < \sqrt{9} \cdot \sqrt{(y-2)^2+x^2}$$

$$x^2+4x+4+y^2 < 9 \cdot (y^2-4y+4+x^2)$$

$$x^2+4x+4+y^2 < 9y^2-36y+36+9x^2$$

$$0 < 8y^2+32-4x+8x^2-36y$$

$$-32 < 8x^2-4x+8y^2-36y$$

$$-4 < x^2 - \frac{x}{2} + y^2 - \frac{9}{2}y$$

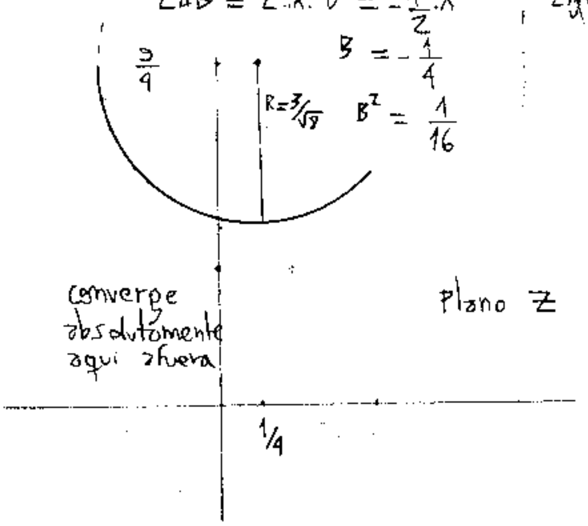
$$-4 < \left[x-\frac{1}{4}\right]^2 - \frac{1}{16} + \left[y-\frac{9}{4}\right]^2 - \frac{81}{16}$$

Completando cuadrados

$2AB = 2 \cdot x \cdot B = -\frac{1}{2} \cdot x$ | $2AB = \frac{-9}{2}y$

$B = -\frac{1}{4}$ | $B = -\frac{9}{4}$

$B^2 = \frac{1}{16}$ | $B^2 = \frac{81}{16}$



$$\frac{9}{8} < \left(x-\frac{1}{4}\right)^2 + \left(y-\frac{9}{4}\right)^2$$

converge absolutamente $\forall z \in \mathbb{C}$:

$$\frac{9}{8} < \left[\mathrm{Re}(z)-\frac{1}{4}\right]^2 + \left[\mathrm{Im}(z)-\frac{9}{4}\right]^2$$

Sea $z = 0 \rightarrow$

$$\underbrace{\sum_{N=1}^{\infty} \frac{(-1)^N}{(3^N+e^N)\sqrt{N+1}} \cdot \left(\frac{2}{2i}\right)^{N-1}}_{\equiv \phi} \rightarrow \left| \frac{(-1)^N}{(3^N+e^N)\cdot\sqrt{N+1}} \cdot \left(\frac{2}{2i}\right)^{N-1} \right| = \frac{1}{(3^N+e^N)(N+1)^{1/2}}$$

$$3^N \leq (3^N+e^N)(N+1)^{1/2} \rightarrow \sum_{N=1}^{\infty} |\phi| \leq \sum_{N=1}^{\infty} \frac{1}{3^N}$$

$$\frac{1}{3^N} \geq \frac{1}{(3^N+e^N)\cdot(N+1)^{1/2}}$$

converge abs.

Para $z = 0$ converge absolutamente

* Que pasa en el borde del circulo :

$$\left| z - \left(\frac{1}{4} + \frac{9}{4}i\right) \right| = \frac{3}{\sqrt{8}} \quad ; \text{ es decir } W = 3$$

$$\sum_{N=1}^{\infty} \frac{(-1)^N}{(3^N+e^N)(N+1)^{1/2}} \left(\frac{2+z}{2i-z}\right)^{N-1} \rightarrow \sum_{N=1}^{\infty} \frac{3^{N-1}}{(3^N+e^N)\cdot(N+1)^{1/2}} = \sum_{N=1}^{\infty} \frac{(1/3)\, 3^N}{3^N\left[1+\left(\frac{e}{3}\right)^N\right]\cdot(N+1)^{1/2}} = \frac{1}{3}\sum_{N=1}^{\infty} \frac{1}{(N+1)^{1/2}\left[1+\left(\frac{e}{3}\right)^N\right]}$$

$$1 \leq 1+\left(\frac{e}{3}\right)^N \leq 2$$

$$1 \geq \frac{1}{\left[1+\left(\frac{e}{3}\right)^N\right]} \geq \frac{1}{2}$$

$$\frac{1}{(n+1)^{1/2}} \geq \frac{1}{\left[1+\left(\frac{e}{3}\right)^N\right](n+1)^{1/2}} \geq \frac{1}{2(n+1)^{1/2}}$$

$$\sum_{N=1}^{\infty} \frac{1}{(n+1)^{1/2}} \geq \sum_{N=1}^{\infty} \frac{1}{\left(1+\left(\frac{e}{3}\right)^N\right)(n+1)^{1/2}} \geq \sum_{N=1}^{\infty} \frac{1}{2(n+1)^{1/2}}$$

▲ diverge

$$n \leq n+1 \leq n^2$$

$$\sqrt{n} \leq \sqrt{n+1} \leq n$$

$$\frac{1}{n^{1/2}} \geq \frac{1}{(n+1)^{1/2}} \geq \frac{1}{n}$$

$$\sum_{N=1}^{\infty} \frac{1}{n^{1/2}} \geq \sum_{N=1}^{\infty} \frac{1}{(n+1)^{1/2}} \geq \sum_{N=1}^{\infty} \frac{1}{n}$$

▲ diverge ▲ diverge

Con $|W| = 3$; es decir $\left| z - \frac{(1+9i)}{4} \right| = \frac{3}{\sqrt{8}}$ la serie no converge absolutamente

* Veo convergencia condicional

$$\sum_{N=1}^{\infty} \frac{(-1)^N}{(3^N+e^N)\cdot\sqrt{N+1}} \cdot \left(\frac{2+z}{2i-z}\right)^{N-1} = \sum_{N=1}^{\infty} \frac{3^{N-1}\cdot(-1)^{N-1}\cdot(-1)}{3^{N-1}\cdot(3^N+e^N)\cdot(N+1)^{\frac{1}{2}}} \cdot \left(\frac{2+z}{2i-z}\right)^{N-1}$$

$$= -\sum_{N=1}^{\infty} \frac{(1/3)\, 3^N}{(3^N+e^N)(N+1)^{1/2}} \left(\frac{-2-z}{(2i-z)3}\right)^{N-1}$$

$$= -\frac{1}{3}\left(\sum_{N=1}^{\infty} \frac{1}{\left[1+(e/3)^N\right](n+1)^{1/2}} \cdot \left(\frac{-2-z}{6i-3z}\right)^{N-1}\right)$$

La convergencia de lo de adentro es la de lo de afuera pues la convergencia de una serie no se ve afectada al multiplicar o dividir por alguna constante.

serie geométrica

$$\left|\sum_{N=1}^{\infty}\left(\frac{-2-z}{6i-3z}\right)^{N-1}\right| = \left|\sum_{n=0}^{\infty}\left(\frac{-2-z}{6i-3z}\right)^{n}\right| = \left|\frac{1-\left(\frac{-2-z}{6i-3z}\right)^{n+1}}{1-\frac{-2-z}{6i-3z}}\right| \leq \frac{1+\left|\frac{-2-z}{6i-3z}\right|^{N+1}}{\left|1+\frac{2+z}{6i-3z}\right|}$$

$N-1=n$

$$\left|\frac{-2-z}{6i-3z}\right| = \frac{|-1|}{|3|}\underbrace{\left|\frac{2+z}{2i-z}\right|}_{=3 \text{ porque estoy en el borde}} = \frac{1}{3}\cdot 3 = 1$$

$$\leq \frac{2}{\left|1+\frac{2+z}{6i-3z}\right|} \equiv M[z] \leftarrow \text{no depende de } n$$

$\forall z \in \left\{z \in \mathbb{C} : \left|\frac{2+z}{2i-z}\right| = 3\right\}$ salvo $1+\frac{2+z}{6i-3z}=0$

$$2+z = -6i+3z$$
$$-2z = -2-6i$$
$$\boxed{z = 1+3i}$$

Luego $a_n = \frac{1}{\left[1+\left(\frac{e}{3}\right)^N\right]\cdot(n+1)^{1/2}} \longrightarrow 0$ y es decreciente y es $a_n > 0 \ \forall n$

$\Rightarrow$ se cumple el criterio de Dirichlet

$\sum$ converge condicionalmente en el borde de $\left|z-\left[\frac{1+9i}{4}\right]\right| = \frac{3}{\sqrt{8}}$ salvo en $z = 1+3i$ donde diverge pues ↓

$*$ $z = 1+3i$

$$\frac{2+1+3i}{2i-1-3i} = \frac{3+3i}{-1-i} = \frac{3(1+i)}{-1(1+i)} = -3 \rightarrow$$

$$\sum = \sum_{N=1}^{\infty}\frac{(-1)^N\cdot(-1)^{N-1}\cdot 3^{N-1}}{(3^N+e^N)\cdot\sqrt{N+1}} = \sum_{N=1}^{\infty}\frac{(-1)^{N+N-1}\cdot 3^N\cdot 1/3}{3^N\left(1+\left[\frac{e}{3}\right]^N\right)(n+1)^{1/2}} = -\frac{1}{3}\cdot\underbrace{\sum_{N=1}^{\infty}\frac{1}{\left(1+\left[\frac{e}{3}\right]^N\right)(n+1)^{1/2}}}_{\text{diverge}}$$

* <u>a_n es decreciente</u>

$$a_n = \frac{1}{\left[1+\left(\frac{e}{3}\right)^N\right]\cdot(n+1)^{1/2}}$$

$$2 > 1+\left(\frac{e}{3}\right)^N > 1+\left(\frac{e}{3}\right)^{N+1} > 1$$

$$1 > \left(\frac{e}{3}\right)^N > \left(\frac{e}{3}\right)^N\cdot\frac{e}{3} > 0$$

$$1 > \frac{e}{3}$$

$$3 > e$$

$\Rightarrow \forall n \in \mathbb{N}$ vale

$$\frac{1}{2} < \frac{1}{\left(1+\left[\frac{e}{3}\right]^N\right)} < \frac{1}{\left(1+\left[\frac{e}{3}\right]^{N+1}\right)} < 1$$

$$(n+1)^{1/2} < (n+2)^{1/2}$$

$$n+1 < n+2$$

$$1 < 2$$

$\Rightarrow \forall n \in \mathbb{N}$ vale

$$\frac{1}{(n+1)^{1/2}} > \frac{1}{(n+2)^{1/2}} = \frac{1}{[(n+1)+1]^{1/2}}$$

$$\frac{1}{(n+1)^{1/2}} > \frac{1}{(n+1)^{1/2}\left(1+\left(\frac{e}{3}\right)^N\right)} > \frac{1}{(n+2)}$$

$$\frac{1}{(n+2)^{1/2}\cdot\left(1+\left(\frac{e}{3}\right)^{N+1}\right)} > \frac{1}{(n+2)^{1/2}\left(1+\left(\frac{e}{3}\right)^N\right)}$$

$\Rightarrow$ a_n es decreciente