PRACTICA 6

1.

$$A = \begin{pmatrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 1&1&0&0&0&0&0&0&0 \end{pmatrix}$$

$|A| = |A^t| \Rightarrow$ pero no sirve demasiado esto

$$|A-\lambda I| = |A-\lambda I| = \begin{vmatrix} -\lambda&0&0&0&0&0&0&0&0 \\ 0&-\lambda&0&0&0&0&0&0&0 \\ 0&0&-\lambda&0&0&0&0&0&1 \\ 0&0&0&-\lambda&0&0&0&0&0 \\ 0&0&1&0&-\lambda&0&0&0&0 \\ 0&0&0&0&0&-\lambda&0&0&0 \\ 0&0&0&1&0&0&-\lambda&0&0 \\ 0&0&0&0&0&1&0&-\lambda&0 \\ 1&1&0&0&0&0&0&0&-\lambda \end{vmatrix}$$

$$|A-\lambda I| = -\lambda \begin{vmatrix} -\lambda&0&0&0&0&0&0&0 \\ 0&-\lambda&0&0&0&0&0&0 \\ 0&0&-\lambda&0&0&0&0&0 \\ 0&0&-0&-\lambda&0&0&0&0 \\ 0&0&0&0&-\lambda&0&0&0 \\ 0&0&1&0&0&-\lambda&0&0 \\ 0&0&0&0&1&0&-\lambda&0 \\ 1&1&0&0&0&0&0&-\lambda \end{vmatrix} + 1 \begin{vmatrix} -\lambda&0&0&0&0&0&0&0 \\ 0&-\lambda&0&0&0&0&0&0 \\ 0&0&0&-\lambda&0&0&0&0 \\ 0&0&1&0&-\lambda&0&0&0 \\ 0&0&0&0&0&-\lambda&0&0 \\ 0&0&0&1&0&0&-\lambda&0 \\ 0&0&0&0&0&1&0&-\lambda \\ 1&1&0&0&0&0&0&0 \end{vmatrix}$$

$$|A-\lambda I| = -\lambda \cdot \lambda^8 + (-1) \cdot \begin{vmatrix} -\lambda&0&0&0&0&0&0&0 \\ 0&-\lambda&0&0&0&0&0&0 \\ 1&1&0&0&0&0&0&0 \\ 0&0&0&-\lambda&0&0&0&0 \\ 0&0&1&0&-\lambda&0&0&0 \\ 0&0&0&0&0&-\lambda&0&0 \\ 0&0&0&1&0&0&-\lambda&0 \\ 0&0&0&0&0&1&0&-\lambda \end{vmatrix} = \boxed{-\lambda^9 = 0}$$

→ Polinomio característico

$\lambda^9 = 0$ $\quad \underline{m_1 = 9}, \; \lambda_1 = 0 \quad \underline{r_1 = 4}$

tamaño superbloques

q. de bloques

un superbloque

$$J = \begin{pmatrix} 0&1&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0 \end{pmatrix}$$

$Nu(A) = Nu(A - \lambda_1 . I)$

$x_9 = 0 = x_3 = x_4 = x_6$

$x_1 + x_2 = 0 \rightarrow x_1 = -x_2$

$S_{\lambda_1} = \langle (-x_2, x_2, 0, 0, x_5, 0, x_7, x_8, 0) \rangle$

$\langle (-1,1,0,0,0,0,0,0,0), (0,0,0,0,1,0,0,0,0), (0,0,0,0,0,0,1,0,0), (0,0,0,0,0,0,0,1,0) \rangle$

$A = C . J . C^{-1}$

$A . C = C . J$ $\qquad$ Sea $J = M_{BB} f$ $\qquad$ $B =$ base de Jordan

$$A.C = C.J \Rightarrow A.[v_1\ v_2\ v_3 \dots v_9] = [v_1\ v_2 \dots v_9].\left([f(v_1)]_B \dots [f(v_9)]_B\right)$$

$$\begin{aligned}
f(v_1) = A.v_1 &= 0 \\
A.v_2 &= 1.v_1 \\
A.v_3 &= 1.v_2 \\
A.v_4 &= 0 \\
A.v_5 &= 1.v_4 \\
A.v_6 &= 0 \\
A.v_7 &= 1.v_6 \\
A.v_8 &= 0 \\
A.v_9 &= 1.v_8
\end{aligned}$$

$$\begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}
\qquad
\begin{matrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
1 & 0 & 0 \\
e_9 & e_3 & e_5
\end{matrix}$$

$x_9 = 0$
$x_3 = 0$
$x_4 = 0$
$x_6 = 0$
$x_1 = -x_2$

$\langle 1-x_2, x_2, 0, 0, x_5, 0, x_7, x_8, 0 \rangle$ $\quad x_2 = 1$

$v_2 = (0, 0, 0, 0, 0, 0, 0, 0, 0)$

$(x_1, -x_1, 0, 0, x_5, 0, x_7, x_8, 0)$

$(1, -1, 0, 0, 0, 0, 0, 0, 0)$

$N_u(A - 0.I) =$ dim. 4 $, \{e_8, e_7, e_5, e_2 - e_1\}$ $\qquad x_9 = 0$

$$N_u(A - 0I^2) = N_u(A^2) = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \rightarrow$$

dim 7

$x_9 = 0$
$x_1 = -x_2$

$(x_1, -x_1, x_3, x_4, x_5, x_6, x_7, x_8, 0)$

$\{e_1, e_3, e_4, e_5, e_6, e_7, e_8\}$

$$N_u(A - 0I)^3 = N_u(A^3) = \begin{pmatrix}
0 & & \cdots & & 0 \\
0 & \ddots & & & 0 \\
0 & & & & 0 \\
0 & & & & 0 \\
1 & 1 & 0 & & 0 \\
0 & & & & 0 \\
0 & & & & 0 \\
0 & & & \ddots & 0 \\
0 & & \cdots & & 0
\end{pmatrix}$$

dim 8

$x_1 = -x_2$

$(x_1, -x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9)$

$\{e_1, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}$

$$N_u(A - 0I)^4 = N_u(A^4) = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \ddots & & & & & & & 0 \\
0 & & & & & & & & 0 \\
0 & & & & & & & & 0 \\
0 & & & & & & & & 0 \\
0 & & & & & & & & 0 \\
0 & & & & & & & & 0 \\
0 & & & & & & & \ddots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} = 0 \in \mathbb{R}^{9\times 9}$$

dim 9

$\{e_1, e_2, \dots, e_9\}$

2.

$$A = (a_{ij}) \in \mathbb{C}^{5\times5} \text{ con } a_{ij} = \begin{cases} 0 & \text{si } i \leq j \\ 1 & \text{si } i > j \end{cases}$$

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$|A-\lambda I| = \begin{vmatrix} -\lambda & 0 & 0 & 0 & 0 \\ 1 & -\lambda & 0 & 0 & 0 \\ 1 & 1 & -\lambda & 0 & 0 \\ 1 & 1 & 1 & -\lambda & 0 \\ 1 & 1 & 1 & 1 & -\lambda \end{vmatrix} = -\lambda^5 = 0 \Rightarrow \lambda^5 = 0$$

$\lambda_1 = 0$ $\quad \underbrace{r_1 = 1}_{\text{bloques}}$

$m_1 = 5$

superbloques

$Nu(A-\lambda I)$

$$\left(\begin{array}{ccccc|c} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{array}\right)$$

$x_1 = 0$
$x_2 = 0$
$x_3 = 0$
$x_4 = 0$

$Nu(A) = \langle (0,0,0,0,1) \rangle$

dim 1

$$\boxed{J = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}}$$

$A v_1 = 0 . v_1$
$A v_2 = v_1$
$A . v_3 = v_2$
$A . v_4 = v_3$
$A . v_5 = v_4$

$$\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{array} \qquad \begin{array}{ccccc} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ v_1 & v_2 & v_3 & v_4 & v_5 \end{array}$$

$x_1 = x_2 = x_3 = 0$, $x_4 = 1$, x_5 libre

$$Nu(A^2) = 0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 3 & 2 & 1 & 0 & 0 \end{pmatrix}$$

$x_1 = 0$, $x_2 = 0$, $x_3 = 0$

$Nu(A^2) = \langle (0,0,0,1,0), (0,0,0,0,1) \rangle$ $\qquad e_4, e_5$

$$Nu(A^3) = 0 \quad \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$x_1 = 0$, $x_2 = 0$

$Nu(A^3) = \langle (0,0,1,0,0), (0,0,0,1,0) (0,0,0,0,1) \rangle$ $\qquad e_3, e_4, e_5$

$$\mathcal{N}u(A^4) = \begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 1&0&0&0&0 \end{pmatrix} \qquad x_1 = 0$$

$\dim 4$

$$\mathcal{N}u(A^4) = \langle e_3, e_4, e_5, e_2 \rangle$$

$$\mathcal{N}u(A^5) = 0 \in \mathbb{C}^{5\times 5} \Rightarrow \mathcal{N}u(A^5) = \langle e_3, e_4, e_5, e_2, e_1 \rangle$$

$v_1 = e_1$; $v_2 = \mathcal{N}u(A).e_1 = (0,1,1,1,1)$; $v_3 = \mathcal{N}u(A).(0,1,1,1,1) = (0,0,1,2,3)$;

$V_4 = \mathcal{N}u(A).(0,0,1,2,3) = (0,0,0,1,3)$

$$A = \begin{pmatrix} 0&0&0&0&0\\ 1&0&0&0&0\\ 1&1&0&0&0\\ 1&1&1&0&0\\ 1&1&1&1&0 \end{pmatrix} \begin{pmatrix} 1\\0\\0\\0\\0 \end{pmatrix}$$

$$\begin{array}{cccc} 0&0&0&0\\ 1&0&0&0\\ 1&1&0&0\\ 1&2&1&0\\ 1&3&3&1\\ V_2&V_3&V_4&V_5 \end{array}$$

$W_1 = \mathcal{N}u$

$$\boxed{C = \begin{pmatrix} 1&0&0&0&0\\ 0&1&0&0&0\\ 0&1&1&0&0\\ 0&1&2&1&0\\ 0&1&3&3&1 \end{pmatrix}}$$

$A.V_1 = 0.V_1$
$A.V_2 = 0.V_2 + 1.V_1$
$A.V_3 = 1.V_2$
$A.V_4 = 1.V_3$
$A.V_5 = 1.V_4$

$$\begin{array}{c|c} C & J \\ \begin{matrix} 1&0&0&0&0\\ 0&1&0&0&0\\ 0&1&1&0&0\\ 0&1&2&1&0\\ 0&1&3&3&1 \end{matrix} & \begin{matrix} 0&1&&&\\ 0&0&1&&\\ 0&&0&1&\\ 0&&&0&1\\ 0&&&&0 \end{matrix} \end{array}$$

Corresponde a

$$J = \begin{pmatrix} 0&0&0&0&0\\ 1&0&0&0&0\\ 0&1&0&0&0\\ 0&0&1&0&0\\ 0&0&0&1&0 \end{pmatrix}$$

$F(v_1) = A.V_1 = A.e_1 = V_2$
$A.V_2 = A(e_2+e_3+e_4+e_5) = V_3$
$A.V_3 = V_4 =$
$A.V_4 = V_5 = e_5$
$A.V_5 = 0$

3.

$$A = \begin{pmatrix} 0&0&0&0&0&0\\ 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0 \end{pmatrix}$$

A es diagonal

i)

$$\begin{vmatrix} -\lambda&0&0&0&0&0\\ 1&-\lambda&0&0&0&0\\ 0&1&-\lambda&0&0&0\\ 0&0&1&-\lambda&0&0\\ 0&0&0&1&-\lambda&0\\ 0&0&0&0&1&-\lambda \end{vmatrix} = \lambda^6 = 0$$

$\lambda_1 = 0$ $\quad r_i = 1$
$m_1 = 6$

$$\mathcal{N}u(A - 0I) = \mathcal{N}u(A) \rightarrow \left(\begin{matrix} 0&0&0&0&0&0\\ 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0 \end{matrix}\,\middle|\,\begin{matrix}0\\0\\0\\0\\0\\0\end{matrix}\right)$$

$\mathcal{N}u(A) = \langle 0,0,0,0,0,1 \rangle = \langle e_6 \rangle$

$S_{\lambda_1 = 0} = \langle e_6 \rangle$

$$J=\begin{pmatrix}0&&&&&\\&0&&&&\\&&0&&&\\&&&0&&\\&&&&0&\\&&&&&0\end{pmatrix}$$

$$\mathcal{N}_u(A^2)=\mathcal{N}_u\begin{pmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\end{pmatrix} \qquad \mathcal{N}_u(A^2)=\langle(0,0,0,0,1,0),(0,0,0,0,0,1)\rangle$$
$$\langle e_5, e_6\rangle$$

$$\mathcal{N}_u(A^3)=\mathcal{N}_u\begin{pmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\end{pmatrix}$$

$\mathcal{N}_u(A^3)=\langle e_4, e_5, e_6\rangle$ y

$\mathcal{N}_u(A^4)=\langle e_3, e_4, e_5, e_6\rangle$

$\mathcal{N}_u(A^5)=\langle e_2, e_3, e_4, e_5, e_6\rangle$

$\mathcal{N}_u(A^6)=\langle e_1, e_2, e_3, e_4, e_5, e_6\rangle$ dim 6

$$\boxed{A^7=0\in K^{6\times 6}\Rightarrow A \text{ es nilpotente}}$$

$$\left.\begin{array}{r}V_1=e_1+e_6\\ V_2=\mathcal{N}_u(A).V_1=e_2\\ \mathcal{N}_u(A).V_2=e_3\\ \dots\\ \mathcal{N}_u(A).V_k=e_k\end{array}\right\}\Rightarrow \quad C=\begin{pmatrix}1&\cdots&&&&0\\0&1&&&&\\0&&1&&&\\0&&&1&&\\0&&&&1&\\1&\cdots&&&&1\end{pmatrix} \wedge \; C^{-1}=\begin{pmatrix}1&\cdots&&&&0\\0&1&&&&0\\0&&1&&&0\\0&&&1&&0\\0&&&&1&0\\-1&\cdots&&&&1\end{pmatrix}$$

$[T(v_1)]_B$

|Elijo en forma adecuada una base|

$A.V_1=0.V_1$

$A.V_2=1.V_1$

$A.V_3=1.V_2$

$B=\{e_1+e_6, e_2, e_3, e_4, e_5, e_6\}\rightarrow$ obtengo $J=A$

$V_1=e_6 \Rightarrow V_2=e_5$

$V_3=e_4$

$V_4=e_3$

$V_5=e_2$

$V_6=e_1$

$$\left(\begin{array}{cccccc|c}0&0&0&0&0&0&0\\1&0&&&&0&0\\0&1&0&&&0&0\\0&&1&0&&0&0\\0&&&1&0&0&0\\0&0&0&0&1&0&1\end{array}\right)\begin{matrix}0\\0\\0\\0\\1\\0\end{matrix}$$

busco que:

$$A=C\cdot \underset{\substack{\parallel\\ J \text{ para } A}}{A'}\cdot C^{-1}$$

$$\begin{pmatrix}0&0&0&0&0&1\\0&&&&1&0\\0&&&1&&0\\0&&1&&&0\\0&1&&&&0\\1&0&0&0&0&0\end{pmatrix}\cdot\begin{pmatrix}0&1&&&&\\&0&1&&&\\&&0&1&&\\&&&0&1&\\&&&&0&1\\&&&&&0\end{pmatrix}\cdot\begin{pmatrix}0&&&&&1\\0&&&&1&\\0&0&&1&&\\0&&1&&&\\0&1&&&&\\1&&&&&\end{pmatrix}=\begin{pmatrix}0&0&0&0&0&0\\1&0&&&&0\\0&1&0&&&0\\0&&1&0&&0\\0&&&1&0&0\\0&0&0&0&1&0\end{pmatrix}$$

$$\Rightarrow \boxed{A \text{ y } A' \text{ son semejantes}}$$

6. i)

$$A = \begin{pmatrix} 1 & 1 & -1 \\ -3 & -3 & 3 \\ -2 & -2 & 2 \end{pmatrix} \qquad J = M^{-1}.A.M$$

$$\begin{vmatrix} 1-\lambda & 1 & -1 \\ -3 & -3-\lambda & 3 \\ -2 & -2 & 2-\lambda \end{vmatrix} = \left[-(1-\lambda)(3+\lambda)(2-\lambda) - 6 - 6\right] - \left[2(-3-\lambda) - 3(2-\lambda) - 6(1-\lambda)\right]$$

$$-[3-3\lambda+\lambda-\lambda^2][2-\lambda]$$

$$-[6-6\lambda+2\lambda-\lambda^2 2-3\lambda+3\lambda^2-\lambda^2+\lambda^3] - 12 \quad +2(3+\lambda)+3(2-\lambda)+6(1-\lambda)$$

$$-(6-7\lambda+\lambda^3) - 12 \qquad + 6 + 2\lambda + 6 - 3\lambda + 6 - 6\lambda = 0$$

$$-\lambda^3 + 7\lambda \cancel{-18} \cancel{+18} \cancel{-7\lambda} = 0$$

$$-\lambda^3 = 0 \quad \Rightarrow \quad \boxed{\lambda = 0}$$

$$\lambda^3 = 0$$

$$\begin{pmatrix} 1 & 1 & -1 \\ -3 & -3 & 3 \\ -2 & -2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} . \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \Rightarrow \quad x_1 + x_2 - x_3 = 0 \qquad V = (x_1, x_2, x_1+x_2)$$

$$x_1 = x_3 - x_2 \qquad V = (x_3 - x_2, x_2, x_3)$$

$$\$_{\lambda_1=0} = \langle (1,0,1), (-1,1,0) \rangle$$

Elijo $V_1 = 2(1,0,1) + 3(-1,1,0)$

$\lambda_1 = 0$ $\quad$ $\underbrace{r_1 = 2}_{\text{bloques}}$

$m_1 = 3$

un superbloque

$$\left\{ \begin{array}{l} A.V_1 = 0.V_1 \\ A.V_2 = 1.V_1 + 0.V_2 \\ A.V_3 = 0.V_3 \end{array} \right.$$

$$\boxed{J = \left(\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right)}$$

$$C = \begin{pmatrix} -1 & -1 & 0 \\ 3 & 0 & 1 \\ 2 & 0 & 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & -1 \\ -3 & -3 & 3 & 3 \\ -2 & -2 & 2 & 2 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{array}\right) \left(\begin{array}{ccc|c} 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$x_1 + x_2 - x_3 = -1$$

$$x_1 = -1 + x_3 - x_2$$

$$V_2 = \langle (-1 + x_3 - x_2, x_2, x_3) \rangle$$

$$V_2 = (-1, 0, 0)$$

Elijo $V_3 = (0,1,0)$ Li con V_1 y V_2

$Nu(A - 0.I)$: dim 2 $\qquad \{e_1 + e_3, -e_1 + e_2\}$

$Nu(A^2) = 0$

ii) $A = \begin{pmatrix} 3 & 0 & 8 \\ 3 & -1 & 6 \\ -2 & 0 & -5 \end{pmatrix}$

$$\begin{vmatrix} 3-\lambda & 0 & 8 \\ 3 & -1-\lambda & 6 \\ -2 & 0 & -5-\lambda \end{vmatrix} = \left[(3-\lambda).(-1-\lambda)(-5-\lambda)\right] - \left[-16.(-1-\lambda)\right] = \left[(-3+\lambda-3\lambda+\lambda^2)(-5-\lambda)\right] - 16(1+\lambda) =$$

$$= +15+10\lambda-5\lambda^2+3\lambda+2\lambda^2-\lambda^3-16-16\lambda = 0$$
$$-\lambda^3-3\lambda^2-3\lambda-1 = 0$$

$\lambda_1 = -1 \qquad (\lambda+1)^3 = 0$

$\lambda_1 = -1 \quad \underbrace{r_1 = 2}_{2 \text{ bloques}}$

$m_1 = 3$

1 superbloque de 3x3

$Nu(A-\lambda_1 I)$

$$\begin{pmatrix} 4 & 0 & 8 \\ 3 & 0 & 6 \\ -2 & 0 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 2 \\ 1 & 0 & 2 \\ 1 & 0 & 2 \end{pmatrix} \rightarrow$$

$x_1 + 2x_3 = 0$

$x_1 = -2x_3$

$(x_1, x_2, -\frac{1}{2}x_1)$

$S_{\lambda_1} = \langle (1, 0, -\frac{1}{2}), (0,1,0) \rangle \leftarrow 2 = \dim Nu(A-\lambda_1 I)$

$$\boxed{J = \left(\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{array}\right)}$$

$J = C^{-1}.A.C$

Cálculo de la base

$$\begin{cases} A.V_1 = -1.V_1 \\ A.V_2 = 1.V_1 - 1V_2 \\ A.V_3 = -1.V_3 \end{cases}$$

elijo $V_1 = (1, 3/4, -1/2)$

$$\begin{pmatrix} 3 & 0 & 8 \\ 3 & -1 & 6 \\ -2 & 0 & -5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3/4 \\ -1/2 \end{pmatrix} = \begin{pmatrix} -1 \\ -3/4 \\ 1/2 \end{pmatrix} = -1 \begin{pmatrix} 1 \\ 3/4 \\ -1/2 \end{pmatrix}$$

$AV_2 + V_2 = V_1$

$(A+I).V_2 = V_1 \longrightarrow$

$$\left(\begin{array}{ccc|c} 4 & 0 & 8 & 1 \\ 3 & 0 & 6 & 3/4 \\ -2 & 0 & -4 & -1/2 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1/4 \\ 1 & 0 & 2 & 1/4 \\ 1 & 0 & 2 & 1/4 \end{array}\right) \rightarrow \quad 1 \quad 0 \quad 2 \mid 1/4$$

$x_1 + 2x_3 = 1/4$

$x_1 = 1/4 - 2x_3$

$V_2 = (1/4 - 2x_3, x_2, x_3)$

$V_2 = (1/4, 0, 0)$

$$\boxed{C = \begin{pmatrix} 1 & 1/4 & 0 \\ 3/4 & 0 & 1 \\ -1/2 & 0 & 0 \end{pmatrix}}$$

$\downarrow \downarrow \downarrow$

$v_1 \; v_2 \; v_3 \qquad B_j = \{v_1, v_2, v_3\}$

iii) $A = \begin{pmatrix} -4 & 2 & 10 \\ -4 & 3 & 7 \\ -3 & 1 & 7 \end{pmatrix}$

$$\begin{vmatrix} -4-\lambda & 2 & 10 \\ -4 & 3-\lambda & 7 \\ -3 & 1 & 7-\lambda \end{vmatrix} = \left[(-4-\lambda).(3-\lambda).(7-\lambda) - 42 - 40\right] - \left[-30(3-\lambda) - 8(7-\lambda) + 7(-4-\lambda)\right]$$

$(-12-3\lambda+4\lambda+\lambda^2)(7-\lambda) \qquad 30(3-\lambda) + 8(7-\lambda) + 7(4+\lambda)$

$-84+7\lambda+7\lambda^2+12\lambda-\lambda^2-\lambda^3-82 \quad +90 \quad -30\lambda+56-8\lambda+28+7\lambda = 0$

$-\lambda^3 - 12\lambda + 6\lambda^2 + 8 = 0$

$-\lambda^3 - 2^2.3.\lambda + 3.\lambda^2.2 + 2^3$

$(\lambda-2)^3 = 0$

$\lambda_1 = 2 \qquad \underbrace{r_1 = 1}_{\text{bloques}}$

$m_1 = 3$

$$Nu(A-2I) \Rightarrow \begin{pmatrix} -6 & 2 & 10 \\ -4 & 1 & 7 \\ -3 & 1 & 5 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} -6 & 2 & 10 & 0 \\ 6 & -6/4 & -42/4 & 0 \\ 6 & -2 & -10 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -6 & 2 & 10 & 0 \\ 0 & 1/2 & -1/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$-6x_1 + 2x_2 + 10x_3 = -6x_1 + 12x_2 = 0$$

$$\frac{x_2}{2} = \frac{x_3}{2} \Rightarrow x_2 = x_3 \qquad x_1 = 2x_2$$

$$(2x_2, x_2, x_2)$$

$$V_1 = \langle (2,1,1) \rangle = S_{\lambda_1} \text{ de valor } 3$$

$$\boxed{J = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}}$$

base

$$\begin{cases} A.V_1 = 2.V_1 \\ A.V_2 = 1.V_1 + 2V_2 \\ A.V_3 = 1V_2 + 2V_3 \end{cases}$$

$$\boxed{C = \begin{pmatrix} 2 & 0 & -1 \\ 1 & 1 & -3 \\ 1 & 0 & 0 \end{pmatrix}}$$

$$(A - 2I).V_2 = V_1$$

$$\left(\begin{array}{ccc|c} -6 & 2 & 10 & 2 \\ -4 & 1 & 7 & 1 \\ -3 & 1 & 5 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} -6 & 2 & 10 & 2 \\ 6 & -6/4 & -42/4 & -6/4 \\ 6 & -2 & -10 & -2 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} -6 & 2 & 10 & 2 \\ 0 & 1/2 & -1/2 & 1/2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$6x_1 = 2x_2 + 10x_3 - 2$$

$$x_1 = \frac{1}{3}x_2 + \frac{5}{3}x_3 - \frac{1}{3} =$$

$$\frac{1}{2}x_2 = \frac{1}{2} + \frac{1}{2}x_3$$

$$\frac{1}{3} + \frac{1}{3}x_3 + \frac{5}{3} \qquad -\frac{1}{3}$$

$$x_2 = 1 + x_3$$

$$x_1 = 2x_3 \qquad (2x_3, 1+x_3, x_3)$$

$$V_2 = (0,1,0)$$

$$(A-2I).V_3 = V_2$$

$$\left(\begin{array}{ccc|c} -6 & 2 & 10 & 0 \\ -4 & 1 & 7 & 1 \\ -3 & 1 & 5 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} -6 & 2 & 10 & 0 \\ 6 & -6/4 & -42/4 & -6/4 \\ 6 & -2 & -10 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -6 & 2 & 10 & 0 \\ 0 & 1/2 & -1/2 & -3/2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$6x_1 = 2x_2 + 10x_3 \qquad \frac{x_2}{2} - \frac{x_3}{2} = -\frac{3}{2}$$

$$6x_1 = 12x_3 - 6 \qquad x_2 - x_3 = -3$$

$$x_1 = 2x_3 - 1 \qquad x_2 = -3 + x_3$$

$$V_3 = (2x_3 - 1, x_3 - 3, x_3)$$

$$V_3 = (-1, -3, 0)$$

$$A.C = C.J$$

$$\boxed{A = C.J.C^{-1}}$$

iv) $$A = \begin{pmatrix} -2 & 8 & 6 \\ -4 & 10 & 6 \\ 4 & -8 & -4 \end{pmatrix}$$

$$(-20 - 8\lambda + \lambda^2)(-4-\lambda)$$

$$-20 - 10\lambda + 2\lambda + \lambda^2$$

$$384 - [240 - 24\lambda + 128 + 32\lambda + 96 + 48\lambda]$$

$$384 - 464 - 56\lambda$$

$$\begin{vmatrix} -2-\lambda & 8 & 6 \\ -4 & 10-\lambda & 6 \\ 4 & -8 & -4-\lambda \end{vmatrix} = [(-2-\lambda).(10-\lambda).(-4-\lambda) + 192 + 192] - [24(10-\lambda) + 32(4+\lambda) + 48(2+\lambda)]$$

$$80 + 32\lambda - 4\lambda^2 + 20\lambda + 8\lambda^2 - \lambda^3 - 80 - 56\lambda$$

$$\frac{-4 \pm \sqrt{16 - 4 \cdot -1 \cdot -4}}{-2} \rightarrow 2 \text{ doble}$$

$$-4\lambda + 4\lambda^2 - \lambda^3 = 0$$

$$\lambda(-4 + 4\lambda - \lambda^2) = 0$$

$$\lambda_1 = 0 \qquad m_1 = 1 \qquad r_1 = 1$$

$$\lambda_2 = 2 \qquad m_2 = 2 \qquad r_2 = 2$$

$\underline{\underline{\lambda_1=0}}$

$$\begin{pmatrix} -2 & 8 & 6 & | & 0 \\ -4_2 & 10_5 & 6_{-3} & | & 0 \\ 4 & -8 & -4 & | & 0 \end{pmatrix} \to \begin{pmatrix} -2 & 8 & 6 & | & 0 \\ 0 & 3 & 3 & | & 0 \\ 0 & 2 & 2 & | & 0 \end{pmatrix} \to \begin{pmatrix} 1 & -4 & -3 & | & 0 \\ 0 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$X_2 = -X_3 \qquad X_1 = 3X_3 + 4X_2 = 3X_3 - 4X_3 = -X_3$

$-X_3, -X_3, X_3$

$S_{\lambda_1=0} = \langle (-1,-1,1) \rangle$

$\underline{\underline{\lambda_2=2}}$

$$\begin{pmatrix} -4 & 8 & 6 & | & 0 \\ -4 & 8 & 6 & | & 0 \\ 4 & -8 & -6 & | & 0 \end{pmatrix} \to \begin{pmatrix} -4 & 8 & 6 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \to \quad 1 \ -2 \ -3/2 \ | \ 0$$

$X_1 = 2X_2 + 3/2\, X_3$

$\left(2X_2 + \frac{3}{2}X_3, X_2, X_3\right)$

$S_{\lambda_2=2} = \langle (2,1,0), (\frac{3}{2},0,1) \rangle$

$$\boxed{J = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}}$$

A era diagonalizable

bastaría ver que se satisface [1]

$$[1] \begin{cases} A.V_1 = 0 V_1 \\ A.V_2 = 2.V_2 \\ A.V_3 = 2.V_3 \end{cases} \to \quad (A-2I).V_2 = 0 \Rightarrow \text{Calculo } Nu(A-2I)$$

$2V_2$

$$\begin{pmatrix} -2 & 8 & 6 & | & 4 \\ -4 & 10 & 6 & | & 2 \\ 4 & -8 & -4 & | & 0 \end{pmatrix} \downarrow \begin{pmatrix} -2 & 8 & 6 & | & 4 \\ -2 & 2 & 0 & | & -2 \\ 0 & 2 & 2 & | & 2 \end{pmatrix} \downarrow \begin{pmatrix} -2 & 8 & 6 & | & 4 \\ 0 & -6 & -6 & | & -6 \\ 0 & 2 & 2 & | & 2 \end{pmatrix} \downarrow \begin{pmatrix} 1 & -4 & -3 & | & -2 \\ 0 & 1 & 1 & | & 1 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$Nu(A-2I) = \langle (2-X_3, 1-X_3, X_3) \rangle$

$X_2 + X_3 = 1 \qquad X_2 = 1 - X_3$

$X_1 = -2 + 3X_3 + 4X_2$

$4(1-X_3)$

$X_1 = 4 - 4X_3 + 3X_3 - 2 = 2 - X_3$

$$\boxed{C = \begin{pmatrix} -1 & 2 & 3/2 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}}$$

8.

$$A=\begin{pmatrix}3&0&8&a\\3&-1&6&0\\-2&0&-5&0\\0&0&0&-1\end{pmatrix}$$

$$\begin{vmatrix}3-\lambda&0&8&a\\3&-1-\lambda&6&0\\-2&0&-5-\lambda&0\\0&0&0&-1-\lambda\end{vmatrix}=-2\begin{vmatrix}0&8&a\\-1-\lambda&6&0\\0&0&-1-\lambda\end{vmatrix}+(-5-\lambda)\begin{vmatrix}3-\lambda&0&a\\3&-1-\lambda&0\\0&0&-1-\lambda\end{vmatrix}$$

$$=-2\cdot\left[(-1-\lambda)^2\cdot 8\right]-(5+\lambda)\cdot\left[(3-\lambda)\cdot(-1-\lambda)^2\right]$$

$$=16(-1-\lambda)^2-5(3-\lambda)(-1-\lambda)^2-\lambda(3-\lambda)(-1-\lambda)^2$$

$$=(-1-\lambda)^2\cdot\left[16-5(3-\lambda)-\lambda(3-\lambda)\right]=(1+\lambda)^2\cdot\left[\lambda^2+2\lambda+1\right]$$

$$16-15+5\lambda-3\lambda+\lambda^2$$

$$\frac{-2\pm\sqrt{4-4\cdot1\cdot1}}{2}$$

$\lambda_1=-1 \quad m_{\lambda_1}=4 \quad r_{\lambda_1}:2 \qquad (1+\lambda)^2(\lambda+1)^2=(1+\lambda)^4$

$\underline{\lambda_1}: -1$

$$\begin{pmatrix}4&0&8&a\\3&0&6&0\\-2&0&-4&0\\0&0&0&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}=\begin{pmatrix}0\\0\\0\\0\end{pmatrix} \qquad J=\left(\begin{array}{cc|cc}-1&1&0&0\\0&-1&0&0\\\hline 0&0&-1&1\\0&0&0&-1\end{array}\right)$$

$$\left(\begin{array}{cccc|c}4&0&8&a&0\\0&0&0&-a&0\\0&0&0&a&0\\0&0&0&0&0\end{array}\right) \qquad \left(\begin{array}{cccc|c}4&0&8&a&0\\0&0&0&a&0\\0&0&0&0&0\\0&0&0&0&0\end{array}\right)$$

$x_4=0$

$x_1=-\frac{8}{4}x_3=-2x_3$

$(-2x_3, x_2, x_3, 0)$

$(-2\alpha, \beta, \alpha, 0)$

$$\left(\begin{array}{cccc|c}4&0&8&a&-2\alpha\\3&0&6&0&0\\-2&0&-4&0&\alpha\\0&0&0&0&0\end{array}\right)$$

$$\left(\begin{array}{cccc|c}4&0&8&a&-2\alpha\\0&0&0&-a&2\alpha\\0&0&0&a&0\\0&0&0&0&0\end{array}\right) \quad \begin{matrix}2\alpha\\ \shortparallel\\ 0\end{matrix}$$

$$\left(\begin{array}{cccc|c}4&0&8&a&-2\alpha\\0&0&0&-a&\frac{4}{3}\beta+2\alpha\\0&0&0&0&\frac{4}{3}\beta+2\alpha\\0&0&0&0&0\end{array}\right) \begin{matrix}\\=0\\=0\\ \\ \end{matrix}$$

$x_4=0$

$0=-\frac{4}{3}\frac{\beta}{a}-\frac{2\alpha}{a} \longrightarrow \frac{2\alpha}{a}=-\frac{4}{3}\frac{\beta}{a}$

$\alpha=-\frac{2}{3}\beta$

$\beta=-\frac{3}{2}\alpha$

$x_1=-\frac{2\alpha}{4}-\frac{a\,x_4}{4}-\frac{8x_3}{4}$

$x_1=-2x_3-\frac{\alpha}{2}-\frac{a}{4}\left(-\frac{4}{3}\frac{\beta}{a}-\frac{2\alpha}{a}\right)$

$x_1=-2\underbrace{x_3}_{=\gamma}-\cancel{\frac{\alpha}{2}}+\frac{\beta}{3}+\cancel{\frac{\alpha}{2}}$

$(-2\gamma+\frac{1}{3}\beta, \varepsilon, \gamma, 0)$

$(-2\alpha, -\frac{3}{2}\alpha, \alpha, 0)$

$\alpha=1 \longrightarrow (-2, -3/2, 1, 0)$

$(-2\gamma+\frac{1}{3}\beta, \varepsilon, \gamma, 0)$

$\gamma=1 \longrightarrow (-2, 0, 1, 0)$

$\beta=1 \longrightarrow (1/3, 0, 0, 0)$

$\varepsilon=1 \longrightarrow (0, 1, 0, 0)$

$$B=\begin{pmatrix}-2&1/3&0&-2\\0&0&1&-3/2\\1&0&0&1\\0&0&0&0\end{pmatrix}$$