PRACTICA 5

1.

1) $A = \begin{pmatrix} 1 & 3 \\ -3 & -1 \end{pmatrix}$

$AV = \lambda V$
$(A - \lambda I).V = 0$

$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 3 \\ -3 & -1-\lambda \end{vmatrix} = (1-\lambda)(-1-\lambda) + 3 = -1+\lambda-\lambda+\lambda^2+3$

Polinomio Característico $\rightarrow \boxed{\lambda^2+2 = 0}$

$\lambda^2 = -2$

$\lambda = \sqrt{-2} \nearrow i\sqrt{2} \searrow -i\sqrt{2}$

$\underline{\lambda_1 = i\sqrt{2}}$

$\begin{pmatrix} 1-i\sqrt{2} & 3 \\ -3 & -1-i\sqrt{2} \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

son Ld $\begin{cases} (1-i\sqrt{2})x + 3y = 0 \\ -3x + (-1-i\sqrt{2})y = 0 \end{cases}$

$y = \frac{(-1+i\sqrt{2})}{3}x$

$-3x + (-1-i\sqrt{2}).\frac{(-1+i\sqrt{2})}{3}x = 0$

$-3x + \left(\frac{1+i\sqrt{2}-i\sqrt{2}-i^2\sqrt{2}^2}{3}\right)x = 0$

$-3x + \frac{1+2}{3}x = 0$

$-3x + \frac{3}{3}x = 0$

$0.x = 0$

$\underline{\lambda_2 = -i\sqrt{2}}$

$\begin{pmatrix} 1+i\sqrt{2} & 3 \\ -3 & -1+i\sqrt{2} \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{bmatrix} (1+i\sqrt{2})x + 3y = 0 \\ -3x + (-1+i\sqrt{2})y = 0 \end{bmatrix}$

$\boxed{\text{en } \mathbb{C}}$ en $\mathbb{R}$ $\nexists$ λ_1, λ_2

Autovalores: $\lambda_1: i\sqrt{2}$ $\lambda_2: -i\sqrt{2}$

Autovectores: $V_1 = \left(1, \frac{-1+i\sqrt{2}}{3}\right)$ $V_2 = \left(1, \frac{-1-i\sqrt{2}}{3}\right)$ $\left(x, -\frac{1+i\sqrt{2}}{3}x\right)$

$(3, -1+i\sqrt{2}), (3, -1-i\sqrt{2})$

ii) $A = \begin{pmatrix} 1 & 3 \\ 3 & -1 \end{pmatrix}$

$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 3 \\ 3 & -1-\lambda \end{vmatrix} = -(1-\lambda)(1+\lambda) - 9 = -1+\lambda^2-9 = 0$

$\boxed{\lambda^2 - 10 = 0}$

$\lambda^2 = 10$

$\lambda = \begin{cases} \sqrt{10} \\ -\sqrt{10} \end{cases}$

$\underline{\lambda_1: \sqrt{10}}$

$\begin{pmatrix} 1-\sqrt{10} & 3 \\ 3 & -1-\sqrt{10} \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{bmatrix} (1-\sqrt{10})x + 3y = 0 \\ 3x + (-1-\sqrt{10})y = 0 \end{bmatrix}$

$V_1 = \left(1, \frac{-1+\sqrt{10}}{3}\right)$

$y = -\frac{(1-\sqrt{10})}{3}x$

$3x + (1+\sqrt{10}).\frac{(1-\sqrt{10})}{3}x$

$3x + \frac{1-10}{3}x = 0$

$3x - 3x = 0$

$0.x = 0$

$\underline{\lambda_2: -\sqrt{10}}$

$\begin{pmatrix} 1+\sqrt{10} & 3 \\ 3 & -1+\sqrt{10} \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$(1+\sqrt{10})x + 3y = 0$

$y = \frac{-(1+\sqrt{10})}{3}x$

$V_1 = \left(1, -\frac{(1+\sqrt{10})}{3}\right)$

en $\mathbb{R}$ o en $\mathbb{C}$ es lo mismo

iii) $A = \begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix}, \; a \in \mathbb{R}$

$$\begin{vmatrix} -\lambda & a \\ -a & -\lambda \end{vmatrix} = \lambda^2 + a^2 = 0$$

$$\lambda = \pm\sqrt{-a^2} = i.|a|$$
$$\lambda_1 = -i|a|$$
$$\lambda_2 = i|a|$$

$\underline{\underline{\lambda_1}} = -i.|a|$

$$\begin{pmatrix} +i.|a| & a \\ -a & +i|a| \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$i|a|\,x + ay = 0$$

$$y = -\frac{i.|a|.x}{a}$$

$$\boxed{\lambda_1 = -i|a|\;,\; V_1 = \left(1, -\frac{|a|}{a} i\right)}$$

$\underline{\underline{\lambda_2}} = i|a|$

$$\begin{pmatrix} -i|a| & a \\ -a & -i|a| \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-i|a|x + ay = 0$$

$$\boxed{\lambda_2 = i|a|\;,\; V_2 = \left(1, \frac{i|a|}{a}\right)}$$

$$y = \frac{i|a|\,x}{a}$$

iv) $A = \begin{pmatrix} 0 & 2 & 1 \\ -2 & 0 & 3 \\ -1 & -3 & 0 \end{pmatrix}$

$$\begin{vmatrix} -\lambda & 2 & 1 \\ -2 & -\lambda & 3 \\ -1 & -3 & -\lambda \end{vmatrix} = (-\lambda^3 - 6 + 6) - (\lambda + 4\lambda + 9\lambda) = \boxed{-\lambda^3 - 14\lambda}$$

Polinomio característico

$$-\lambda(\lambda^2 + 14) = 0$$

$$\lambda_1 = 0$$
$$\lambda_2 = i\sqrt{14}$$
$$\lambda_3 = -i\sqrt{14}$$

$\underline{\lambda_1 = 0}$

$$\begin{pmatrix} 0 & 2 & 1 \\ -2 & 0 & 3 \\ -1 & -3 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 & 1 \\ -2 & 0 & 3 \\ 0 & -6 & -3 \end{pmatrix} \rightarrow \left(\begin{array}{ccc|c} -2 & 0 & 3 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \rightarrow$$

$$-2x + 3z = 0$$
$$2y + z = 0 \;,\; z = -2y$$

$$\frac{3}{2}z = x = -\frac{3}{\cancel{2}} \cdot \cancel{2}y = -3y$$

$$V_1 = (-3y, y, -2y) = (-3, 1, -2)$$

$\underline{\lambda_2 = i\sqrt{14}}$

$$\begin{pmatrix} -i\sqrt{14} & 2 & 1 \\ -2 & -i\sqrt{14} & 3 \\ -1 & -3 & -i\sqrt{14} \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-2\left(\frac{-i\sqrt{14}}{2}\right)$

$$\begin{pmatrix} -i\sqrt{14} & 2 & 1 \\ 0 & -5 & -\frac{3}{2}i\sqrt{14} + 1 \\ 0 & 6 - i\sqrt{14} & 2i\sqrt{14} + 3 \end{pmatrix}$$

$$\begin{pmatrix} -i\sqrt{14}/2 & 1 & 1/2 \\ 0 & -1 & -3/10\, i\sqrt{14} + 1/5 \\ 0 & 1 - \frac{1}{6}i\sqrt{14} & \frac{1}{3}i\sqrt{14} + \frac{1}{2} \end{pmatrix} \rightarrow \begin{pmatrix} -\frac{1}{2}i\sqrt{14} & 1/6\, i\sqrt{14} & -1/3\, i\sqrt{14} \\ 0 & -1/6\, i\sqrt{14} & -\frac{1}{6.6} i^2 14 + 1/30 \cdot i\sqrt{14} \\ 0 & 1 - 1/6\, i\sqrt{14} & 1/3\, i\sqrt{14} + 1/2 \end{pmatrix}$$

$$+\frac{\cancel{3}.\cancel{14}}{\cancel{10}.6}$$

$$\begin{pmatrix} 0 & -\frac{1}{6} i\sqrt{14} & \frac{7}{10} + \frac{1}{30} i\sqrt{14} \\ 0 & 1 & -\frac{1}{5} + \frac{3}{10} i\sqrt{14} \end{pmatrix} \to \begin{pmatrix} -\frac{i\sqrt{14}}{2} & \frac{1}{6} i\sqrt{14} & -\frac{1}{3} i\sqrt{14} \\ 0 & -\frac{1}{6} i\sqrt{14} & \frac{1}{30} i\sqrt{14} + \frac{7}{10} \\ 0 & \frac{1}{6} i\sqrt{14} & -\frac{1}{30} i\sqrt{14} - \frac{7}{10} \end{pmatrix} \Rightarrow$$

$$-\frac{1}{30} i\sqrt{14} + \frac{\overset{1}{\cancel{3}}}{\underset{5}{\cancel{10}}} \cdot \frac{i^2 \cdot \overset{7}{\cancel{14}}}{\underset{2}{\cancel{6}}}$$

$$\left(-\frac{1}{6} i\sqrt{14}\right) y = \left(-\frac{7}{10} - \frac{1}{30} i\sqrt{14}\right) z$$

$$y = \left(\frac{-\frac{7}{10} - \frac{1}{30} i\sqrt{14}}{-\frac{1}{6} i\sqrt{14}}\right) z$$

$$y = \left(\frac{-\frac{7}{10} i\sqrt{14} - \frac{1}{30} i^2 14}{-\frac{1}{6} i^2 14}\right) z = \frac{-\frac{7}{10} i\sqrt{14} + \frac{7}{15}}{\frac{7}{3}}$$

$$y = \left(-\frac{3}{10} i\sqrt{14} + \frac{1}{5}\right) z$$

$$\left(-\frac{i\sqrt{14}}{2}\right) x + \left(\frac{1}{6} i\sqrt{14}\right)\left(-\frac{3}{10} i\sqrt{14} + \frac{1}{5}\right) z - \left(\frac{1}{3} i\sqrt{14}\right) z = 0$$

$$-\frac{i\sqrt{14}}{2} x = -\left(\frac{1}{6} i\sqrt{14}\right)\left(-\frac{3}{10} i\sqrt{14} + \frac{1}{5}\right) z + \frac{1}{3} i\sqrt{14}\, z$$

$$x = \left(\frac{1}{\underset{2}{\cancel{6}}} \cdot \frac{\overset{1}{\cancel{3}}}{10} i^2 14 - \frac{1}{5} \cdot \frac{1}{6} i\sqrt{14}\right) z + \frac{1}{3} i\sqrt{14}\, z$$

$$-\frac{14}{20} z + \left(-\frac{1}{30} i\sqrt{14} + \frac{1}{3} i\sqrt{14}\right) z$$

$$V_2 = \left(\frac{3}{5} + \frac{i\sqrt{14}}{10}, \frac{1}{5} - \frac{i \cdot 3\sqrt{14}}{10}, 1\right)$$

$$X = \frac{\left(-\frac{7}{10} + \frac{3}{10} i\sqrt{14}\right) z \cdot 2}{-i\sqrt{14}}$$

$$X = \left(\frac{-\frac{7}{10}\cdot 3\, i\sqrt{14} + \frac{\overset{1}{\cancel{3}}}{\underset{5}{\cancel{10}}} i^2 14}{-14}\right) z$$

$$X = \left(+\frac{1}{10} i\sqrt{14} + \frac{3}{5}\right) z$$

2.

$U = \{u_1, u_2, \dots, u_n\}$ base de K^n

$f: K^n \to K^n$ $\quad M_U f = A$

$B = \{v_1, v_2, \dots, v_n\}$ base de K^n : $\quad M_B f = D \rightarrow$ diagonal

C_{UB}

ii)

$$A = \begin{pmatrix} 1 & 3 \\ 3 & -1 \end{pmatrix} = M_U f \Rightarrow A = \begin{pmatrix} f(1,0)^t & f(0,1)^t \end{pmatrix}$$

$$A = C . D . C^{-1}$$

$$M_{UU}f = \begin{pmatrix} 3 & 3 \\ -1+\sqrt{10} & -1-\sqrt{10} \end{pmatrix} \cdot \begin{pmatrix} \sqrt{10} & 0 \\ 0 & -\sqrt{10} \end{pmatrix} \cdot C^{-1}$$

$$M_{UU}f = C_{BU} \quad . \quad M_{BB}f \quad . \quad C_{UB}$$

$f(1,0) = (1,3)$
$f(0,1) = (3,-1)$

$$\begin{pmatrix} 1 & 3 \\ 3 & -1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+3y \\ 3x-y \end{pmatrix}$$

$f(v_1) = (\sqrt{10}, 0)$
$f(v_2) = (0, \sqrt{10})$

$$f(v_1) = \begin{pmatrix} x_1 + 3x_2 \\ 3x_1 - x_2 \end{pmatrix} = \begin{pmatrix} \sqrt{10} \\ 0 \end{pmatrix} \quad \text{con } v_1 = x_1, x_2$$

$$f(v_2) = \begin{pmatrix} y_1 + 3y_2 \\ 3y_1 - y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -\sqrt{10} \end{pmatrix} \quad \text{con } v_2 = y_1, y_2$$

$$\begin{cases} y_1 + 3y_2 = 0 \\ 3y_1 - y_2 = -\sqrt{10} \end{cases} \qquad \begin{cases} x_1 + 3x_2 = \sqrt{10} \\ 3x_1 - x_2 = 0 \end{cases}$$

$y_1 = -3y_2$

$-9y_2 - y_2 = -\sqrt{10}$

$y_2 = \frac{-\sqrt{10}}{-10}$

$y_2 = \frac{1}{\sqrt{10}}$

$x_1 : \frac{x_2}{3}$

$\frac{x_2}{3} + 3x_2 = \sqrt{10}$

$\frac{10}{3}x_2 = \sqrt{10}$

$x_2 = \frac{\sqrt{10}}{10} . 3$

$x_2 = \frac{3}{\sqrt{10}}$

$$v_2 = \left(\frac{-3}{\sqrt{10}}, \frac{1}{\sqrt{10}} \right) \qquad v_1 = \left(\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} \right)$$

son Li (v_1, v_2)

$$\left(\begin{array}{cc|c} 1/\sqrt{10} & -3/\sqrt{10} & 0 \\ 3/\sqrt{10} & 1/\sqrt{10} & 0 \end{array}\right)$$

$$\left(\begin{array}{cc|c} 1/\sqrt{10} & -3/\sqrt{10} & 0 \\ 0 & 10/\sqrt{10} & 0 \end{array}\right) \Rightarrow \text{son li}$$

$B = \{v_1, v_2\}$

pero

$v_2 = (-3, 1)$
$v_1 = (1, 3)$

$$A^{-1} = \frac{1}{|A|} \cdot Adj. \Rightarrow (C_{BU})^{-1} = \frac{1}{-6\sqrt{10}} \cdot \begin{pmatrix} -1-\sqrt{10} & 1-\sqrt{10} \\ -3 & 3 \end{pmatrix}^t$$

$$|C_{BU}| = -3 - 3\sqrt{10} - (-3 + 3\sqrt{10}) = -6\sqrt{10}$$

$$\boxed{C_{UB} = \frac{1}{-6\sqrt{10}} \begin{pmatrix} -1-\sqrt{10} & -3 \\ 1-\sqrt{10} & 3 \end{pmatrix}}$$

Sí, es posible hallar una base de K^n

1) $A = \begin{pmatrix} 1 & 3 \\ -3 & -1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 3 \\ -3 & -1 \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ -1+i\sqrt{2} & -1-i\sqrt{2} \end{pmatrix}$$

son Li → no hay base de autovectores

pero no son base de $\mathbb{R}^2$ ⇒ no se puede hallar $M_{BB}f$

iii) $A = \begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix}$, $a \in \mathbb{R}$

$$\begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix} = \begin{pmatrix} a & a \\ -|a|i & |a|i \end{pmatrix}$$

3. A, C y $D \in K^{n\times n}$: $A = C.D.C^{-1}$ $\quad \forall n \in \mathbb{N}$.

$$A^n = C.D^n.C^{-1}$$

$$A^n = C.D.\overbrace{C^{-1}.\underbrace{C}.D.\overbrace{C^{-1}}^{I}.}^{I}.\underbrace{\quad}_{2}.......\overbrace{\quad}^{I}\underbrace{C.D.C^{-1}}_{n} = C.D^n.C^{-1}$$

4. $f: \mathbb{R}^3 \to \mathbb{R}^3$

$$f(x,y,z) = (-x-2y+6z,\ 4y,\ -x-3y+4z)$$

$$f(1,0,0) = (-1,0,-1) \qquad f(0,1,0) = (-2,4,-3) \qquad f(0,0,1) = (6,0,4)$$

$$M_{EE}f = \begin{pmatrix} -1 & -2 & 6 \\ 0 & 4 & 0 \\ -1 & -3 & 4 \end{pmatrix}$$

i) B de $\mathbb{R}^3$: $M_{BB}f$ diagonal

$$M_{EE}f = C_{BE} \,.\, M_{BB}f \,.\, C_{EB}^{-1}$$

$$\begin{pmatrix} -1 & -2 & 6 \\ 0 & 4 & 0 \\ -1 & -3 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 2 & -18 \\ 0 & 0 & 6 \\ 1 & 1 & -13 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix} \cdot C_{EB}^{-1}$$

<u>autovalores</u> y <u>autovectores de A</u>

$$\begin{vmatrix} -1-\lambda & -2 & 6 \\ 0 & 4-\lambda & 0 \\ -1 & -3 & 4-\lambda \end{vmatrix} = (-1-\lambda).(4-\lambda)^2 + 2.0 + 6.(4-\lambda) = (4-\lambda).[6+(-1-\lambda)(4- \text{[illegible]}$$

$$6+(-4+\lambda-4\lambda \text{[illegible]}$$

$$(4-\lambda).(2+3\lambda+\lambda^2 \text{[illegible]}$$

$$-(1+\lambda).(16-8\lambda+\lambda^2)$$

$$-[16-8\lambda+\lambda^2+16\lambda-8\lambda^2+\lambda^3]+24-6\lambda$$

$$-[16+8\lambda-7\lambda^2+\lambda^3]+24-6\lambda = 8-14\lambda+7\lambda^2-\lambda^3$$

$$\begin{array}{l|l} -\lambda^3+7\lambda^2-14\lambda+8 & \lambda-4 \\ \hline -\lambda^3 \quad 4\lambda^2 & -\lambda^2+3\lambda-2 \\ \quad 3\lambda^2-14\lambda+8 & \\ \quad 3\lambda^2-12\lambda & \\ \qquad -2\lambda \quad 8 & \\ \qquad -2\lambda \quad 8 & \\ \qquad\quad 0 & \end{array}$$

$$= (-\lambda^2+3\lambda-2).(\lambda-4)$$
$$= -(\lambda-1).(\lambda-2).(\lambda-4)$$

$$\frac{-3 \pm \sqrt{9-4(-1)(-2)}}{-2}$$

$$\lambda_1 = \frac{-3+1}{-2} \qquad \lambda_2 = \frac{-3-1}{-2}$$
$$1 \qquad\qquad 2$$

$$\lambda_1 = 1 \qquad \lambda_2 = 2 \qquad \lambda_3 = 4$$

<u>λ_1</u>

$$\begin{pmatrix} -2 & -2 & 6 \\ 0 & 3 & 0 \\ -1 & -3 & 3 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & -3 \\ 0 & 3 & 0 \\ -1 & -3 & 3 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & -3 \\ 0 & 3 & 0 \\ 0 & -2 & 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -3 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$x+y-3z=0 \Rightarrow x = z.3$$
$$y=0$$
$$(3z,0,z) \Rightarrow V_1 = (3,0,1)$$

<u>λ_2</u>

$$\begin{pmatrix} -3 & -2 & 6 \\ 0 & 2 & 0 \\ -1 & -3 & 2 \end{pmatrix}$$

$$y=0$$
$$-3x = -6z$$
$$x = 2z$$

$$V_2 = (2,0,1)$$

$\underline{\lambda_3}$

$$\begin{pmatrix} -5 & -2 & 6 \\ 0 & 0 & 0 \\ -1 & -3 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x = -3y$

$-5x - 2y + 6z = 0$
$15y - 2y + 6z = 0$
$13y = -6z$
$y = -\frac{6}{13} z$

$-\frac{13}{6} y = z$

$V_3 = \left(-3, 1, -\frac{13}{6}\right)$

$V_3 = (-18, 6, -13)$

Si $B = \{\omega_1, \omega_2, \omega_3\} \Rightarrow$

$f(\omega_1) = (1, 0, 0)$
$f(\omega_2) = (0, 2, 0)$
$f(\omega_3) = (0, 0, 4)$

$\underline{\underline{\omega_1:}}$

$-x - 2y + 6z = 1$
$4y = 0 \rightarrow y = 0$
$-x - 3y + 4z = 0$

$-4z + 6z = 1$
$2z = 1$
$z = 1/2$

$4z = x \rightarrow x = 2$

$\omega_1 = (2, 0, 1/2)$

$\underline{\underline{\omega_2:}}$

$-x - 1 + 6z = 0$
$4y = 2 \rightarrow y = 1/2$ $\quad x = -1 + 6z$
$-x - \frac{3}{2} + 4z = 0$

$x = -3/2 + 4z$

$0 = -2z - 5/2$
$z = -5/4$

$x = -13/2$

$\omega_2 = \left(-\frac{13}{2}, \frac{1}{2}, -\frac{5}{4}\right)$

$\underline{\underline{\omega_3:}}$

$-x + 6z = 0$
$4y = 0 \rightarrow y = 0$
$-x + 4z = 4$

$6z = x$

$-6z + 4z = 4$
$z = \frac{4}{-2}$

$\omega_3 = (-12, 0, -2)$

$$\boxed{B = \left\{ (2, 0, 1/2), (-13/2, 1/2, -5/4), (-12, 0, -2) \right\}}$$

ii)

$$\begin{pmatrix} -1 & -2 & 6 \\ 0 & 4 & 0 \\ -1 & -3 & 4 \end{pmatrix}^n = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}^n \cdot \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

$$\boxed{\begin{pmatrix} -1 & -2 & 6 \\ 0 & 4 & 0 \\ -1 & 3 & 4 \end{pmatrix}^n = \begin{pmatrix} 3 & 2 & -18 \\ 0 & 0 & 6 \\ 1 & 1 & -13 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 4^n \end{pmatrix} \cdot \begin{pmatrix} 1 & -4/3 & -2 \\ -1 & 7/2 & 3 \\ 0 & 1/6 & 0 \end{pmatrix}}$$

$$C^{-1} = \frac{1}{(12) - (18)} \cdot \begin{pmatrix} -6 & +6 & 0 \\ +8 & -21 & -1 \\ 12 & -18 & 0 \end{pmatrix}^t$$

iii) $\quad P \in \mathbb{R}^{3\times 3}$: $\quad P^2 = \begin{pmatrix} -1 & -2 & 6 \\ 0 & 4 & 0 \\ -1 & -3 & 4 \end{pmatrix} = A^1$

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}^2 = \underset{C}{\begin{pmatrix} 3 & 2 & -18 \\ 0 & 0 & 6 \\ 1 & 1 & -13 \end{pmatrix}} \cdot \underset{D}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}} \cdot \underset{C^{-1}}{\begin{pmatrix} 1 & -4/3 & -2 \\ -1 & 7/2 & 3 \\ 0 & 1/6 & 0 \end{pmatrix}}$$

Sea: $\quad J = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 2 \end{pmatrix} \Rightarrow$

$A = C \cdot J^2 \cdot C^{-1} \Rightarrow$
$P^2 = C \cdot J^2 \cdot C^{-1} \Rightarrow$
$P = C \cdot J \cdot C^{-1} \quad \therefore$

$$\boxed{P = \begin{pmatrix} 3 & 2 & -18 \\ 0 & 0 & 6 \\ 2 & 1 & -13 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & -4/3 & -2 \\ -1 & 7/2 & 3 \\ 0 & 1/6 & 0 \end{pmatrix}}$$

5. $f: K^n \to K^n$ proyector $\quad \dim Im(f) = s$

$$f \circ f = f$$

$$K^n = Nu(f) \oplus Im(f)$$

$$f(v) = v \quad \forall\, v \in Im(f)$$

$\dim(K^n)$

$n = \quad s$

f es diagonalizable si

$M_{EE}\, f = A$ tiene n autovectores Li

$$f[f] = f$$

$$f[f] - f = 0$$
$$f[f(v)] - f[v] = 0$$
$$f[f(v) - v] = 0 \Rightarrow (f(v) - v) \in Nu\, f$$

$$f(v) = A.v$$
$$[1] \quad f(v) = f \circ f\,(v) \quad \forall\, v$$
$$f(v) = f[f(v)] = f[A.v] = A.A.v$$

de [1] se tiene:

$$A.v = A.A.v$$
$$A.v = A^2.v$$
$$(A - A^2).v = 0 \Rightarrow \lambda I \cong A^2$$

puede verse como

$$A.v = A^2 v = \lambda v$$
$$(A^2 - \lambda I).v = 0$$

f proyector $\Rightarrow \forall\, v \in Im(f)$ [no al $Nu(f)$] se da $\quad f(v) = v$

$$A.v = v$$
$$(A - I).v = 0 \quad \forall\, v \in Im(f) \Rightarrow$$

$\lambda = 1$ es único autovalor

$$\underset{\dim n}{K^n} = \underbrace{Nu(f)}_{\dim (n-s)} \oplus \underbrace{Im(f)}_{\dim s}$$

$$\dim(Nu(A - I)) = s$$

$$rg(Nu(A - I)) = s$$

6. $A=\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$

$$\begin{vmatrix} a-\lambda & b \\ 0 & c-\lambda \end{vmatrix} = (a-\lambda)(c-\lambda) = 0 \qquad \begin{matrix}\lambda_1 = a \\ \lambda_2 = c\end{matrix}$$

$\underline{\underline{\lambda_1}}=a$

$$\begin{pmatrix} 0 & b \\ 0 & c-a \end{pmatrix}\cdot\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \left\{\begin{matrix} by=0 \\ (c-a)\cdot y=0 \end{matrix}\right. \qquad V_1=(1,0)$$

$\underline{\underline{\lambda_2}}=c$

$$\begin{pmatrix} a-c & b \\ 0 & 0 \end{pmatrix}\cdot\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad (a-c)x+by=0 \qquad V_2=\left(1, \frac{c-a}{b}\right)$$

$$y=\left(\frac{a-c}{b}\right)\cdot -x$$

V_1 y V_2 ; $\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & \frac{c-a}{b} & 0 \end{array}\right)$

son Li $\iff c\neq a \implies$ Puedo formar base de autovectores $\implies$ A es diagonalizable

$b=0 \implies$ $\exists$ base de autovectores $\implies$ A es diagonalizable

$V_1=(1,0)$
$V_2=(0,1)$

$\boxed{\text{si } c\neq a \implies A \text{ diagonalizable}}$

7.

$$A=\begin{pmatrix} k & 1 & k+k^2 & -k^2 \\ 0 & k+1 & 0 & k \\ 0 & 1 & k & 1 \\ 0 & 0 & 0 & k+1 \end{pmatrix}$$

$$|A-\lambda I| = \begin{vmatrix} k-\lambda & 1 & k+k^2 & -k^2 \\ 0 & k+1-\lambda & 0 & k \\ 0 & 1 & k-\lambda & 1 \\ 0 & 0 & 0 & k+1-\lambda \end{vmatrix} = (k-\lambda)\cdot\begin{vmatrix} k+1-\lambda & 0 & k \\ 1 & k-\lambda & 1 \\ 0 & 0 & k+1-\lambda \end{vmatrix}$$

$$-1\cdot 0 + (k+k^2)\cdot 0 + k^2\cdot 0 = (k-\lambda)^2\cdot(k+1-\lambda)^2 = 0$$

$\lambda=k$ (doble)
$\lambda=k+1$ (doble)

$\underline{\underline{\lambda=k}}$

$$\begin{pmatrix} 0 & 1 & k+k^2 & -k^2 \\ 0 & 1 & 0 & k \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}\cdot\begin{pmatrix} x \\ y \\ z \\ \omega \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & k+k^2 & -k^2 \\ 0 & 1 & 0 & k \\ 0 & 0 & 0 & 1-k \\ 0 & 0 & 0 & 1 \end{pmatrix}\cdot\begin{pmatrix} x \\ y \\ z \\ \omega \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & k+k^2 & -k^2 \\ 0 & 0 & -k-k^2 & k+k^2 \\ 0 & 0 & 0 & 1-k \\ 0 & 0 & 0 & 1 \end{pmatrix}\cdot\begin{pmatrix} x \\ y \\ z \\ \omega \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left\{\begin{matrix} y+(k+k^2)z-k^2\omega=0 \\ (-k-k^2)z+(k+k^2)\cdot\omega=0 \\ (1-k)\omega=0 \\ \omega=0, \end{matrix}\right. \rightarrow \begin{matrix} y=-(k+k^2)z \\ y+(k+k^2)z=0 \\ -(k+k^2)z=0 \end{matrix}$$

$$V_1=(x, -(k+k^2)z, z, 0)$$

$$V_1 = \alpha_1(1,0,0,0) + \alpha_2(0,-(k+k^2),1,0) \qquad [1]$$

$\underline{\underline{\lambda = k+1}}$

$$\begin{pmatrix} 1 & 1 & k+k^2 & -k^2 \\ 0 & 0 & 0 & k \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ \omega \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{aligned} y + z + \omega &= 0 \\ x + y + (k+k^2)z - k^2\omega &= 0 \\ k.\omega &= 0 \end{aligned} \qquad \begin{aligned} y &= -z - \omega \\ x &= +z + \omega - (k+k^2)z \end{aligned}$$

$$V_2 = (z+\omega - [k+k^2]z, -z-\omega, z, \omega)$$

$$V_2 = \alpha_1(1,-1,0,1) + \alpha_2(1-[k+k^2], -1, 1, 0)$$

Necesito poder formar base de $\mathbb{R}^4$ con los autovectores $\Rightarrow$ de [1] y su obtención

$$z \neq 0 \Rightarrow \quad \begin{aligned} -(k+k^2).z &= 0 \\ -(k+k^2) &= 0 \\ k^2 + k &= 0 \\ k(k+1) &= 0 \end{aligned} \quad \begin{cases} k=0 \\ k=-1 \end{cases} \Rightarrow$$

$\underline{\underline{k=0}}$

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -1 \end{pmatrix} \Rightarrow \text{son Li}$$

$\underline{\underline{k=-1}}$

conduce a la misma matriz

$\Rightarrow$ $\boxed{A \text{ es diagonalizable si } k=0 \vee k=-1}$

8. $\{a_n\}_{n\in \mathbb{N}_0}$:

$$\begin{aligned} a_0 &= 4 \\ a_1 &= 9 \\ &\vdots \\ a_{n+2} &= 5a_{n+1} - 6a_n \quad \forall n \in \mathbb{N}_0 \end{aligned}$$

i) $A = \begin{pmatrix} 0 & 1 \\ -6 & 5 \end{pmatrix}$

$$A.\begin{pmatrix} a_n \\ a_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -6 & 5 \end{pmatrix} \cdot \begin{pmatrix} a_n \\ a_{n+1} \end{pmatrix} = \begin{pmatrix} a_{n+1} \\ -6a_n + 5a_{n+1} \end{pmatrix} = \begin{pmatrix} a_{n+1} \\ a_{n+2} \end{pmatrix}$$

ii) $A^n . \begin{pmatrix} 4 \\ 9 \end{pmatrix} =$

$$\begin{vmatrix} -\lambda & 1 \\ -6 & 5-\lambda \end{vmatrix} = (-\lambda)(5-\lambda) + 6 = -5\lambda + \lambda^2 + 6 = 0$$

$$\frac{5 \pm \sqrt{25 - 4.1.6}}{2} = \begin{cases} 3 = \lambda_2 \\ 2 = \lambda_1 \end{cases} \text{ autovalores}$$

$\lambda_1 = 2$

$$\begin{pmatrix} -2 & 1 \\ -6 & 3 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad -2x + y = 0 \qquad V_1 = (1,2)$$

$\lambda_2 = 3$

$$\begin{pmatrix} -3 & 1 \\ -6 & 2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad V_2 = (1,3)$$

$$\begin{pmatrix}0&1\\-6&5\end{pmatrix}=\begin{pmatrix}1&1\\2&3\end{pmatrix}\cdot\begin{pmatrix}2&0\\0&3\end{pmatrix}\cdot\begin{pmatrix}3&-1\\-2&1\end{pmatrix} \Rightarrow A^n=\begin{pmatrix}1&1\\2&3\end{pmatrix}\cdot\begin{pmatrix}2^n&0\\0&3^n\end{pmatrix}\begin{pmatrix}3&-1\\-2&1\end{pmatrix}$$

$$\frac{1}{1}\cdot\begin{pmatrix}3&-2\\-1&1\end{pmatrix}^t=\begin{pmatrix}3&-1\\-2&1\end{pmatrix}$$

$$A^n=\begin{pmatrix}2^n&3^n\\2^{n+1}&3^{n+1}\end{pmatrix}\cdot\begin{pmatrix}3&-1\\-2&1\end{pmatrix}=\begin{pmatrix}3.2^n-2.3^n&-2^n+3^n\\3.2^{n+1}-2.3^{n+1}&-2^{n+1}+3^{n+1}\end{pmatrix}$$

$$A^n\cdot\begin{pmatrix}4\\9\end{pmatrix}=\begin{pmatrix}3.2^n-2.3^n&-2^n+3^n\\3.2^{n+1}-2.3^{n+1}&-2^{n+1}+3^{n+1}\end{pmatrix}\cdot\begin{pmatrix}4\\9\end{pmatrix}=\begin{pmatrix}12.2^n-8.3^n-9.2^n+9.3^n\\12.2^{n+1}-8.3^{n+1}-9.2^{n+1}+9.3^{n+1}\end{pmatrix}$$

$$A^n\cdot\begin{pmatrix}4\\9\end{pmatrix}=\begin{pmatrix}3.2^n+3^n\\3.2^{n+1}+3^{n+1}\end{pmatrix} \underset{n=0}{\Rightarrow} A^0\cdot\begin{pmatrix}4\\9\end{pmatrix}=\begin{pmatrix}3+1\\6+3\end{pmatrix}=\begin{pmatrix}4\\9\end{pmatrix}=\begin{pmatrix}a_0\\a_1\end{pmatrix}$$

$$a_n=5a_{n-1}-6a_{n-2}$$
$$a_{n+1}=5a_n-6a_{n-1}$$
$$a_{n+2}=5a_{n+1}-6a_n$$

$$n=1 \quad A\cdot\begin{pmatrix}4\\9\end{pmatrix}=\begin{pmatrix}9\\21\end{pmatrix}=\begin{pmatrix}a_1\\a_2\end{pmatrix}$$

$$n=0 \Rightarrow a_2=5a_1-6a_0=45-6.4=21$$

$$\therefore \quad \boxed{A^n\cdot\begin{pmatrix}4\\9\end{pmatrix}=\begin{pmatrix}a_n\\a_{n+1}\end{pmatrix}}$$

iii)

$$\begin{pmatrix}0&1\\-6&5\end{pmatrix}=\begin{pmatrix}1&1\\2&3\end{pmatrix}\cdot\begin{pmatrix}2&0\\0&3\end{pmatrix}\cdot\begin{pmatrix}3&-1\\-2&1\end{pmatrix}$$

$$\begin{pmatrix}3&-1\\-2&1\end{pmatrix}\cdot\begin{pmatrix}0&1\\-6&5\end{pmatrix}=\begin{pmatrix}2&0\\0&3\end{pmatrix}\cdot\begin{pmatrix}3&-1\\-2&1\end{pmatrix}$$

$$\begin{pmatrix}3&-1\\-2&1\end{pmatrix}\cdot\begin{pmatrix}0&1\\-6&5\end{pmatrix}\cdot\begin{pmatrix}1&1\\2&3\end{pmatrix}=\begin{pmatrix}2&0\\0&3\end{pmatrix} \Rightarrow \boxed{P=\begin{pmatrix}3&-1\\-2&1\end{pmatrix}}$$

iv) $\boxed{a_n=3.2^n+3^n}$

9. $\{a_n\}_{n\in\mathbb{N}_0}$ con $a_{n+2}=\dfrac{a_{n+1}+a_n}{2}$

i) $a_0=1$, $a_1=1/2$

$$\underset{n=0}{} \quad a_2=\frac{a_1+a_0}{2}=\frac{1/2+1}{2}=\frac{3/2}{2}=\frac{3}{4}$$

$$a_3=\frac{a_2+a_1}{2}=\frac{3/4+1/2}{2}=\frac{5}{8}$$

Armamos el sistema $\rightarrow$ $\begin{pmatrix}0&1\\1/2&1/2\end{pmatrix}\cdot\begin{pmatrix}a_n\\a_{n+1}\end{pmatrix}=\begin{pmatrix}a_{n+1}\\a_{n+2}\end{pmatrix} \Rightarrow$

a_{n+1} a_n
a_n a_{n+2}

$$\begin{vmatrix}-\lambda&1\\1/2&1/2-\lambda\end{vmatrix}=(-\lambda).(1/2-\lambda)-\frac{1}{2}=\lambda^2-\frac{1}{2}\lambda-\frac{1}{2}=$$

$$\frac{1/2\pm\sqrt{1/4-4.1.-1/2}}{2}=\frac{1/2\pm 3/2}{2} \begin{matrix}\nearrow 1=\lambda_2\\ \searrow -1/2=\lambda_1\end{matrix}$$

$\lambda_1=-1/2$

$$\begin{pmatrix}1/2&1\\1/2&1\end{pmatrix}\cdot\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

$$1/2\,x+y=0 \qquad V_1=(1,-1/2)$$

$\underline{\lambda_2 = 1}$ $\begin{pmatrix}-1 & 1\\ 1/2 & -1/2\end{pmatrix}\cdot\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$ $\quad -x+y=0 \quad V_2=(1,1)$

$n=0 \quad A\cdot\begin{pmatrix}a_0\\ a_1\end{pmatrix}=\begin{pmatrix}a_1\\ a_2\end{pmatrix}$

$n=1 \quad A\cdot\begin{pmatrix}a_1\\ a_2\end{pmatrix}=\begin{pmatrix}a_2\\ a_3\end{pmatrix} \quad \Rightarrow \quad n \quad A^{n+1}\cdot\begin{pmatrix}a_0\\ a_1\end{pmatrix}=\begin{pmatrix}a_{n+1}\\ a_{n+2}\end{pmatrix}$

$A^2\cdot\begin{pmatrix}a_0\\ a_1\end{pmatrix}=\begin{pmatrix}a_2\\ a_3\end{pmatrix}$

$$A^{n+1}=\begin{pmatrix}1 & 1\\ -1/2 & 1\end{pmatrix}\cdot\begin{pmatrix}-1/2 & 0\\ 0 & 1\end{pmatrix}^{n+1}\cdot\begin{pmatrix}2/3 & -2/3\\ 1/3 & 2/3\end{pmatrix}$$

$$A^{n+1}=\begin{pmatrix}1 & 1\\ -1/2 & 1\end{pmatrix}\cdot\begin{pmatrix}(-1/2)^{n+1} & 0\\ 0 & 1\end{pmatrix}\cdot\begin{pmatrix}2/3 & -2/3\\ 1/3 & 2/3\end{pmatrix}=\begin{pmatrix}(-1/2)^{n+1} & 1\\ (-1/2)^{n+2} & 1\end{pmatrix}\cdot\begin{pmatrix}2/3 & -2/3\\ 1/3 & 2/3\end{pmatrix}$$

$$A^{n+1}=\begin{pmatrix}(-1/2)^{n+1}\frac{2}{3}+1/3 & -(-1/2)^{n+1}2/3+2/3\\ (-1/2)^{n+2}\frac{2}{3}+1/3 & -(-1/2)^{n+2}\frac{2}{3}+2/3\end{pmatrix} \Rightarrow$$

$$A^n\begin{pmatrix}1\\ 1/2\end{pmatrix}=\begin{pmatrix}\left(-\frac{1}{2}\right)^{n+1}\frac{2}{3}+\frac{1}{3} & -\left(-\frac{1}{2}\right)^{n+1}\frac{2}{3}+\frac{2}{3}\\ \left(-\frac{1}{2}\right)^{n+2}\frac{2}{3}+\frac{1}{3} & -\left(-\frac{1}{2}\right)^{n+2}\frac{2}{3}+\frac{2}{3}\end{pmatrix}\cdot\begin{pmatrix}1\\ 1/2\end{pmatrix}=\begin{pmatrix}\left(-\frac{1}{2}\right)^{n+1}\frac{2}{3}+\frac{1}{3} & -\left(-\frac{1}{2}\right)^{n+1}\frac{1}{3}+\frac{1}{3}\\ \left(-\frac{1}{2}\right)^{n+2}\frac{2}{3}+\frac{1}{3} & -\left(-\frac{1}{2}\right)^{n+2}\frac{1}{3}+\frac{1}{3}\end{pmatrix}=$$

$$\begin{pmatrix}\frac{1}{3}\left(-\frac{1}{2}\right)^{n+1}+\frac{2}{3}\\ \frac{1}{3}\left(-\frac{1}{2}\right)^{n+2}+\frac{2}{3}\end{pmatrix}=\begin{pmatrix}a_{n+1}\\ a_{n+2}\end{pmatrix}$$

$-\frac{1}{3}+\frac{2}{3}=\frac{1}{3}$

$\frac{3}{2}-\frac{3}{1}=\frac{6-6}{4}$

$$\frac{2}{3}\left(-\frac{1}{2}\right)^{n+2}+\frac{4}{3}-\frac{2}{3}-\frac{1}{3}\left(-\frac{1}{2}\right)^{n+1}=a_n$$

$-\frac{1}{6}+\frac{2}{3}=\frac{1+4}{6}$

$$\frac{\cancel{2}^{1}}{3}\cdot\frac{1}{\cancel{4}_{2}}\left(-\frac{1}{2}\right)^n+\frac{2}{3}+\frac{1}{6}\left(-\frac{1}{2}\right)^n=\boxed{a_n=\frac{1}{3}\left(-\frac{1}{2}\right)^n+\frac{2}{3}}$$

$a_0=1$

$a_1=1/2$

ii)

$$A^n\cdot\begin{pmatrix}0\\ 3\end{pmatrix}=\begin{pmatrix}\frac{2}{3}\left(-\frac{1}{2}\right)^{n+1}+\frac{1}{3} & -\left(-\frac{1}{2}\right)^{n+1}\frac{2}{3}+\frac{2}{3}\\ \frac{2}{3}\left(-\frac{1}{2}\right)^{n+2}+\frac{1}{3} & -\left(-\frac{1}{2}\right)^{n+2}\frac{2}{3}+\frac{2}{3}\end{pmatrix}\cdot\begin{pmatrix}0\\ 3\end{pmatrix}=\begin{pmatrix}-2\left(-\frac{1}{2}\right)^{n+1}+2\\ -2\left(-\frac{1}{2}\right)^{n+2}+2\end{pmatrix}=\begin{pmatrix}a_{n+1}\\ a_{n+2}\end{pmatrix}$$

$$-\cancel{4}^{1}\left(-\frac{1}{2}\right)^n\cdot\frac{1}{\cancel{4}_{1}}+4+2\cdot\left(-\frac{1}{2}\right)\cdot\left(-\frac{1}{2}\right)^n-2=a_n$$

$$\boxed{-2\left(-\frac{1}{2}\right)^n+2=a_n}$$

10.

$$x_{n+1} = 6x_n + 2y_n$$
$$y_{n+1} = 2x_n + 3y_n$$

$$\begin{pmatrix} 6 & 2 \\ 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix}$$

$n=0$

si $n=0$ $\quad \begin{pmatrix} 6 & 2 \\ 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$

$n=0 \quad A \cdot \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$

$n=1 \quad A \cdot \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = A \cdot A \cdot \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = A^2 \cdot \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$

$$A = \begin{pmatrix} 6 & 2 \\ 2 & 3 \end{pmatrix} \qquad \begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = A^{n+1} \cdot \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

$$\begin{vmatrix} 6-\lambda & 2 \\ 2 & 3-\lambda \end{vmatrix} = (6-\lambda)\cdot(3-\lambda) - 4 = 18 - 3\lambda - 6\lambda + \lambda^2 - 4 = 14 - 9\lambda + \lambda^2 = 0$$

$$\frac{9 \pm \sqrt{81 - 4 \cdot 1 \cdot 14}}{2} \quad \lambda_2 = 7, \quad \lambda_1 = 2$$

$\lambda_1 = 2$

$$\begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad 4x + 2y = 0 \quad y = -\frac{4}{2}x = -2x \quad V_1 = (1, -2)$$

$\lambda_2 = 7$

$$\begin{pmatrix} -1 & 2 \\ 2 & -4 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad -x + 2y = 0 \quad y = \frac{1}{2}x \quad V_2 = (1, 1/2)$$

$$A = \begin{pmatrix} 1 & 1 \\ -2 & 1/2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 \\ 0 & 7 \end{pmatrix} \cdot \begin{pmatrix} 1/5 & -2/5 \\ 4/5 & 2/5 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ -2 & 1/2 & 0 & 1 \end{array}\right) \quad \left(\begin{array}{cc|cc} 4 & 4 & 4 & 0 \\ -4 & 1 & 0 & 2 \end{array}\right) \quad \left(\begin{array}{cc|cc} 4 & 4 & 4 & 0 \\ 0 & 5 & 4 & 2 \end{array}\right) \quad \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & 4/5 & 2/5 \end{array}\right) \quad \left(\begin{array}{cc|cc} 1 & 0 & 1-4/5 & -2/5 \\ 0 & 1 & 4/5 & 2/5 \end{array}\right)$$

$1 - \frac{4}{5}$

$$A^{n+1} = \begin{pmatrix} 1 & 1 \\ -2 & 1/2 \end{pmatrix} \cdot \begin{pmatrix} 2^{n+1} & 0 \\ 0 & 7^{n+1} \end{pmatrix} \cdot \begin{pmatrix} 1/5 & -2/5 \\ 4/5 & 2/5 \end{pmatrix}$$

$$A^{n+1} = \begin{pmatrix} 2^{n+1} & 7^{n+1} \\ -2\,2^{n+1} & \frac{1}{2}7^{n+1} \end{pmatrix} \cdot \begin{pmatrix} 1/5 & -2/5 \\ 4/5 & 2/5 \end{pmatrix} = \begin{pmatrix} \frac{1}{5}2^{n+1} + \frac{4}{5}7^{n+1} & -\frac{2}{5}2^{n+1} + \frac{2}{5}7^{n+1} \\ -\frac{2}{5}2^{n+1} + \frac{4}{10}7^{n+1} & \frac{4}{5}2^{n+1} + \frac{1}{5}7^{n+1} \end{pmatrix}$$

$$A^{n+1} \cdot \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix}$$

$$A^n \cdot \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} x_n \\ y_n \end{pmatrix} \Rightarrow A^n \cdot \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} (\frac{1}{5}2^n + \frac{4}{5}7^n)x_0 + (-\frac{2}{5}2^n + \frac{2}{5}7^n)y_0 \\ x_0 + y_0 \end{pmatrix}$$

$$\boxed{\begin{aligned} x_n &= \left(\frac{1}{5}2^n + \frac{4}{5}7^n\right)x_0 + \left(-\frac{2}{5}2^n + \frac{2}{5}7^n\right)y_0 \\ y_n &= \left(-\frac{2}{5}2^n + \frac{2}{5}7^n\right)x_0 + \left(\frac{4}{5}2^n + \frac{1}{5}7^n\right)y_0 \end{aligned}}$$

x_2 :

y_2 :

$x_3 : 296\,x_0 + \frac{1097}{5}\,y_0$

$6x_0$

11.

i)

$$x' = 6x + 2y \qquad x(0) = 3$$
$$y' = 2x + 3y \qquad y(0) = -1$$

$$\begin{pmatrix} 6 & 2 \\ 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}' \qquad A.\bar{x} = \bar{x}' \qquad A.e^{\lambda x} = \lambda . e^{\lambda x}$$

$$X(t) = (x(t), y(t)) = c_1 . e^{2t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + c_2 . e^{7t} \begin{pmatrix} 1 \\ 1/2 \end{pmatrix}$$

$$\boxed{\begin{aligned} x &= c_1 . e^{2t} + c_2 . e^{7t} \\ y &= -c_1 . e^{2t} . 2 + 1/2 . c_2 . e^{7t} \end{aligned}}$$

ii)

$$\{ f \in C^\infty : f'' = f \} = \langle e^x, e^{-x} \rangle$$

$$f' = g \qquad \left\{ \begin{aligned} f' &= g \\ g' &= f \end{aligned} \right.$$
$$f'' = g'$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} f \\ g \end{pmatrix} = \begin{pmatrix} f' \\ g' \end{pmatrix}$$

$$\begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - 1 = 0 \Rightarrow \lambda^2 = \left\{ \begin{aligned} 1 \\ -1 \end{aligned} \right.$$

$\underline{\lambda_1 = 1}$

$$\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} f \\ g \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{aligned} -f + g &= 0 \\ f - g &= 0 \end{aligned}$$

$$f = g$$

$\underline{\lambda_1 = -1}$

$$V_1 = (f, f) \qquad E_{\lambda_1} = \langle (1,1) \rangle$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow$$

$$f + g = 0$$
$$f = -g$$

$\boxed{\langle e^x, e^{-x} \rangle}$ verifica

$$\boxed{(f, g) = c_1 . e^{-x} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 . e^x \begin{pmatrix} 1 \\ 1 \end{pmatrix}} \qquad V_2 = (f, -f) \qquad E_{\lambda_2} = \langle (1,-1) \rangle$$

$$\rightarrow f = c_1 e^{-x} + c_2 . e^x \Rightarrow f'' = c_1 . e^{-x} + c_2 e^x = f$$
$$g = -c_1 . e^{-x} + c_2 . e^x = f'$$

12.

$\delta : C^\infty(\mathbb{R}) \longrightarrow C^\infty(\mathbb{R})$ derivación

$$T(v) = (v')$$

$$T(\underset{\substack{\parallel \\ v}}{e^{\lambda x}}) = e^{\lambda x} . \lambda = \lambda . e^{\lambda v} \Rightarrow T(v) = \lambda . v \quad \forall \lambda \in \mathbb{R}$$

$\Rightarrow \exists \infty$ autovalores

13.

$A \in K^{n \times n}$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & & & \\ a_{31} & & \ddots & & \\ \vdots & & & & \\ a_{n1} & & & & a_{nn} \end{pmatrix} \qquad A.v = \lambda . v$$

i) $\det(A-\lambda I)=0$

$$\begin{vmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & & \\ \vdots & & \ddots & \\ a_{n1} & & & a_{nn}-\lambda \end{vmatrix} = \begin{vmatrix} a_{11}-\lambda & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22}-\lambda & & \\ \vdots & & \ddots & \\ a_{1n} & & & a_{nn}-\lambda \end{vmatrix}$$

$|A|=|A^t| \qquad |A-\lambda I| = |(A-\lambda I)^t|$

si $(A-\lambda I).V=0 \Rightarrow \underset{\text{para } \lambda=\lambda_1}{|A-\lambda I|=0}$ pero $\underset{\text{para } \lambda=\lambda_1}{|A-\lambda I| = |A-\lambda I|^t}$

$\Rightarrow (A-\lambda_1 I)^t . V = 0 = (A-\lambda_1).V$

Luego A y A^t tienen los mismos autovalores.

$A.V = \lambda V$

$A^t.V = \lambda.V$

V_1 es autovector $\Rightarrow \lambda.V_1$ es autovector

$\begin{pmatrix} a & b \\ c & d \end{pmatrix} . \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \lambda_1 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad \begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0 = (a-\lambda).(d-\lambda) - c.b$

$\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix} . \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \lambda \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad \begin{vmatrix} 1-\lambda & 0 \\ 2 & 3-\lambda \end{vmatrix} = (1-\lambda).(3-\lambda) = 0 \qquad \lambda_1 = 1,\ \lambda_2 = 3$

$\equiv A$

$\lambda_1 = 1$: $\begin{pmatrix} 0 & 0 \\ 2 & 2 \end{pmatrix} . \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \qquad 2x_1 + 2x_2 = 0,\quad x_1 = -x_2 \qquad V_1 = (1,-1)$

$\lambda_2 = 3$: $\begin{pmatrix} -2 & 0 \\ 2 & 0 \end{pmatrix} . \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \qquad -2x_1 = 0 \qquad V_2 = (0,1)$

con A^t

$\lambda_1 = 1$: $\begin{pmatrix} 0 & 2 \\ 0 & 2 \end{pmatrix} . \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \qquad V_1 = (1,0)$

$\lambda_2 = 3$: $\begin{pmatrix} -2 & 2 \\ 0 & 0 \end{pmatrix} . \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \qquad -2x_1 + 2x_2 = 0,\quad x_1 = x_2 \qquad V_2 = (1,1)$

A autovectores $(1,-1), (0,1)$

A^t autovectores $(1,0), (1,1)$

$\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix} . \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} . \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix} . \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} . \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

$\boxed{\text{$A$ y A^t tienen mismos autovalores pero diferentes autovectores para } A = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}}$

ii) A es inversible $\Rightarrow |A^{-1}| \neq 0$

$\Rightarrow \exists A^{-1}$

$A.V = \lambda.V$ si $\lambda = 0$ es autovalor $\Rightarrow A.V = 0$

$\Rightarrow A^{-1}.A.V = A^{-1}.0 \Rightarrow V = 0$ pero $V \neq 0$ por ser autovector

$$A.x = \lambda.x \quad \Rightarrow \quad \begin{aligned} A^{-1}.A x &= A^{-1}.\lambda.x \\ x &= \lambda.A^{-1}.x \\ \frac{1}{\lambda}.x &= A^{-1}.x \end{aligned}$$

$\frac{1}{\lambda}$ autovalor de A
con autovector x

14. $A \in \mathbb{C}^{2\times 2}$ $\quad P_{\mathbb{C}}[X]$ $\quad$ *hallaremos* $P(A)$

1) $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

a) $P = X - 1 \qquad P(A) = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

b) $P = X^2 - 1 \qquad P(A) = \underbrace{\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}}_{\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}$

c) $P = (X-1)^2 \qquad P(A) = \left[\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right]^2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

ii) $A = \begin{pmatrix} i & 0 \\ 1 & -i \end{pmatrix}$

a) $P = X^3 - iX^2 + 1 + i$

$$P = \begin{pmatrix} i & 0 \\ 1 & -i \end{pmatrix} \cdot \begin{pmatrix} i & 0 \\ 1 & -i \end{pmatrix} \cdot \begin{pmatrix} i & 0 \\ 1 & -i \end{pmatrix} - i \begin{pmatrix} i & 0 \\ 1 & -i \end{pmatrix} \cdot \begin{pmatrix} i & 0 \\ 1 & -i \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$$

$$\begin{pmatrix} i & 0 \\ 1 & -i \end{pmatrix} \cdot \begin{pmatrix} i^2 & 0 \\ 0 & i^2 \end{pmatrix} \quad - \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix} + \begin{pmatrix} 1+i & 0 \\ 0 & 1+i \end{pmatrix}$$

$$P(A) = \begin{pmatrix} -i & 0 \\ -1 & i \end{pmatrix} + \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} + \begin{pmatrix} 1+i & 0 \\ 0 & 1+i \end{pmatrix} = \begin{pmatrix} 1+i & 0 \\ -1 & 1+3i \end{pmatrix}$$

15.

$A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \qquad P_A[\lambda] = (2-\lambda)(3-\lambda) + 1 = 6 - 3\lambda + \lambda^2 - 2\lambda + 1$

$\lambda^2 - 5\lambda + 7 = 0$

$5 \pm \sqrt{25-4.}$

$-\lambda^2 + 5\lambda - 7 = 0$

$A^2 = 5A - 7$ (*)

$A^4 - 4A^3 - A^2 + 2A - 5I_2$

lo puedo pensar como $\quad P(X) = X^4 - 4X^3 - X^2 + B$

$X^2(X^2 - 4X - 1) + B$

$P[A] =$

$A^4 - 4A^3 - 5A + 7 + 2A - 5I$

$A^4 - 4A^3 - 3A + 2$

$A^4 - 4A.(5A - 7) - 3A + 2$

$A^4 - 20A^2 + 28A - 3A + 2$

$-20(5A - 7) + 25A + 2$

$(5A-7)(5A-7) - 100A + 140 + 2 + 25A$

$25A^2 - 70A + 49 - 100A + 142 + 25A$

$= 25(5A - 7) - 145A + 191$

$125A - 175 - 145A + 191$

$P[A] = -20A + 16 = \begin{pmatrix} -40 & 20 \\ -20 & -60 \end{pmatrix} + \begin{pmatrix} 16 & 0 \\ 0 & 16 \end{pmatrix} = \begin{pmatrix} -24 & 20 \\ -20 & -44 \end{pmatrix}$

$\begin{pmatrix} 4 & -2 \\ 2 & 6 \end{pmatrix} - \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} -1 & -2 \\ 2 & 1 \end{pmatrix} \equiv B$

$$P[A] = \begin{pmatrix} 3 & -5 \\ 5 & 8 \end{pmatrix} \cdot \left[\begin{pmatrix} 3 & -5 \\ 5 & 8 \end{pmatrix} - \begin{pmatrix} 8 & -4 \\ 4 & 12 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right] + \begin{pmatrix} -1 & -2 \\ 2 & 1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ 5 & 8 \end{pmatrix}$$

$$P(A) = \begin{pmatrix} 3 & -5 \\ 5 & 8 \end{pmatrix} \cdot \begin{pmatrix} -6 & -1 \\ 1 & -5 \end{pmatrix} + \begin{pmatrix} -1 & -2 \\ 2 & 1 \end{pmatrix}$$

$$P(A) = \begin{pmatrix} -23 & 22 \\ -22 & -45 \end{pmatrix} + \begin{pmatrix} -1 & -2 \\ 2 & 1 \end{pmatrix} = \boxed{\begin{pmatrix} -24 & 20 \\ -20 & -44 \end{pmatrix}}$$

ii) A^{1000} para $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 1 & -\lambda & 0 \\ 0 & 1 & -\lambda \end{vmatrix} = \lambda^2(1-\lambda) = 0 \qquad A^2(1-A) = 0$$

$$\lambda^2 - \lambda^3 = 0$$

$$\Rightarrow A^2 - A^3 = 0 \qquad \lambda_1 = 0$$

$$A^2 = A^3 \qquad \lambda_1 = 1$$

$\underline{\lambda_1 = 0}$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right) \qquad \begin{matrix} x = 0 \\ y = 0 \end{matrix} \qquad V_1 = (0,0,1)$$

$\underline{\lambda_2 = 1}$

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix} \to \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad \begin{matrix} x = y \\ y = z \end{matrix} \qquad V_2 = (y,y,y)$$

$$V_2 = (1,1,1)$$

$$A^2 \cdot (1-A) = 0$$

$$A^2 \cdot \begin{pmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} = 0$$

$$\begin{vmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 0$$

$$A^3 = A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

$$A^4 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$A^n = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \cdot \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}}_{A^{n-1}} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\boxed{A^{1000} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}}$$

iii) $A^n = \begin{pmatrix} 4 & -1 \\ 1 & 2 \end{pmatrix}^n \quad \forall n \in N$ $\qquad \dfrac{6 \pm \sqrt{36 - 4.1.9}}{2} = \to 3$

$$\begin{vmatrix} 4-\lambda & -1 \\ 1 & 2-\lambda \end{vmatrix} = (4-\lambda).(2-\lambda) + 1 = 8 - 2\lambda - 4\lambda + \lambda^2 + 1 = 9 - 6\lambda + \lambda^2 \Rightarrow$$

$\underline{\lambda = 3}$

$$\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad V_1 = (1,1)$$

$$A^2 - 6A + 9 = 0$$

$$A^2 = 6A - 9$$

$$A^2 . A^2 = (6A - 9).(6A - 9)$$

$$A^4 = 36A^2 - 54A - 54A + 81$$

$$A^4 = 36A^2 - 108A + 81$$

$$36(6A - 9) - 108A + 81$$

$$216A - 324 + 81 - 108A$$

$$A^4 = 108A - 243$$

$$27.4.A - 54.4 + 27$$

$$4(27A - 54) + 27$$

$$A^5 = 108(6A - 9) - 243A$$

$$648A - 972 - 243A$$

$$A^2 = 6A - 9$$

$$A^4 = (6A - 9)^2 = \left[\begin{pmatrix} 24 & -6 \\ 6 & 12 \end{pmatrix} - \begin{pmatrix} 9 & 0 \\ 0 & 9 \end{pmatrix}\right]^2$$

$$\begin{pmatrix} 15 & -6 \\ 6 & 3 \end{pmatrix}^2$$

$$A^6 = (6A - 9)^3$$

$$A^8 = (6A - 9)^4$$

$$A^4 = B^2$$

$$A^8 = B^4$$

$$\begin{vmatrix} 15-\lambda & -6 \\ 6 & 3-\lambda \end{vmatrix} = (15-\lambda)(3-\lambda) + 36$$

$$45 - 3\lambda + 36$$

$$-15\lambda + \lambda^2$$

$$\lambda^2 - 18\lambda + 81 = 0$$

$$B^2 - 18B + 81 = 0 \qquad 18 \pm \ 324 - 4.1.81$$

$$A^3 = 6(6A - 9) - 9A$$

$$36A - 54 - 9A$$

$$A^3 = 27A - 54$$

$$A^4 = 27(6A - 9) - 54A$$

$$162A - 243 - 54A$$

$$A^4 = 108A - 243$$

$$A^5 = 108(6A - 9) - 243A$$

$$648A - 972 - 243A$$

$$A^5 = 405A - 972$$

$$\begin{pmatrix}1 & -1\\ 1 & 2\end{pmatrix} = \begin{pmatrix}1 & 0\\ 1 & -1\end{pmatrix}\begin{pmatrix}3 & 1\\ 0 & 3\end{pmatrix}\begin{pmatrix}1 & 0\\ 1 & -1\end{pmatrix} \qquad \begin{pmatrix}1 & 0\\ 1 & -1\end{pmatrix}^{-1} = \begin{pmatrix}1 & 0\\ 1 & -1\end{pmatrix}$$

$$B^n = \begin{pmatrix}3^n & n.3^{n-1}\\ 0 & 3^n\end{pmatrix}$$

$$A^n = \begin{pmatrix}1 & 0\\ 1 & -1\end{pmatrix}\cdot\begin{pmatrix}3 & 1\\ 0 & 3\end{pmatrix}^n\cdot\begin{pmatrix}1 & 0\\ 1 & -1\end{pmatrix} \Rightarrow \begin{pmatrix}3 & 1\\ 0 & 3\end{pmatrix}^2 = \begin{pmatrix}3 & 1\\ 0 & 3\end{pmatrix}\cdot\begin{pmatrix}3 & 1\\ 0 & 3\end{pmatrix} = \begin{pmatrix}9 & 6\\ 0 & 9\end{pmatrix}$$

$$B = \begin{pmatrix}3 & 1\\ 0 & 3\end{pmatrix}^3 = \begin{pmatrix}9 & 6\\ 0 & 9\end{pmatrix}\begin{pmatrix}3 & 1\\ 0 & 3\end{pmatrix} = \begin{pmatrix}27 & 27\\ 0 & 27\end{pmatrix}$$

$$\boxed{A^n = \begin{pmatrix}1 & 0\\ 1 & -1\end{pmatrix}\cdot\begin{pmatrix}3^n & n.3^n\\ 0 & 3^n\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\ 1 & -1\end{pmatrix}}$$

iV) $A = \begin{pmatrix}1 & 3\\ -1 & 4\end{pmatrix}$

$$\begin{vmatrix}1-\lambda & 3\\ -1 & 4-\lambda\end{vmatrix} = (1-\lambda).(4-\lambda)+3 = 4-4\lambda-\lambda+\lambda^2+3$$

$$7-5\lambda+\lambda^2 = 0$$

$$\left(\begin{array}{cc|cc}1 & 3 & 1 & 0\\ -1 & 4 & 0 & 1\end{array}\right) \quad \left(\begin{array}{cc|cc}1 & 3 & 1 & 0\\ 0 & 7 & 1 & 1\end{array}\right) \quad \left(\begin{array}{cc|cc}1/3 & 0 & 1/3\ 1/7 & -1/7\\ 0 & 1 & 1/7 & 1/7\end{array}\right) \quad \left(\begin{array}{cc|cc}1 & 0 & 3.4/21 & -3/7\\ 0 & 1 & 1/7 & 1/7\end{array}\right)$$

$$\begin{pmatrix}1 & 3\\ -1 & 4\end{pmatrix}\cdot\begin{pmatrix}1 & 3\\ -1 & 4\end{pmatrix} = \begin{pmatrix}-2 & 15\\ -5 & 13\end{pmatrix}$$

$$A^2 = 5A - 7$$

$$\frac{A}{7} = \frac{5A - 7}{7}$$

5

$$A^{-1} = \begin{pmatrix}4/7 & -3/7\\ 1/7 & 1/7\end{pmatrix}$$

$$(4/7-\lambda).(1/7-\lambda) + 3/49 = 0$$

$$4/49 - \lambda/7 - \lambda 4/7 + \lambda^2 + 3/49 = 0$$

$$\lambda^2 - \lambda 5/7 + 1/7 = 0$$

$$(A^{-1})^2 - \frac{5}{7}A^{-1} + \frac{1}{7} = 0$$

$$A.A^{-1}A^{-1} - \frac{5}{7}A\,A^{-1} + \frac{1}{7}A = 0$$

$$A^{-1} - \frac{5}{7}I + \frac{1}{7}A = 0$$

$$\boxed{A^{-1} = 5/7\,I - 1/7\,A}$$

16.

$\mathbb{V}$ $\quad f: \mathbb{V} \to \mathbb{V}$

$\dim \mathbb{V} = n$ $\qquad$ f es isomorfismo $\Leftrightarrow$ f es epimorfismo $Im(f) = \mathbb{V}$; f es mono $Nu(f) = 0$

$$\chi_f = P_f[\lambda] = P_A[\lambda] = a_0 + a_1\lambda + a_2\lambda^2 + \ldots + a_n\lambda^n$$

$$f(X) = A.x \qquad A \in K^{n\times n}$$

$\bar{X} \in K^n$

x vector columna de K^n

Por Cayley-Hamilton;

$$I a_0 + a_1.A + a_2 A^2 + \ldots + a_n A^n = 0$$

17. $\mathbb{V}$ ev

$f: \mathbb{V} \to \mathbb{V}$ $\qquad$ $\mathbb{S}, \mathbb{T} \subset \mathbb{V}$

$\dim(\mathbb{S}) = s$

$\dim(\mathbb{T}) = t$ $\swarrow$ itálica

$\mathbb{S}$ y $\mathbb{T}$ son f-invariantes $\qquad$ $\mathbb{S} \oplus \mathbb{T} = \mathbb{V} \Rightarrow s + t = n$

$\dim \mathbb{V} = n$

$$\Rightarrow \begin{cases} f(s) \in \mathbb{S} & \forall s \in \mathbb{S} \\ f(t) \in \mathbb{T} & \forall t \in \mathbb{T} \end{cases}$$

Sea $\overbrace{\{V_1, V_2, \dots, V_s\}}^{s \text{ elementos}} = B_{\mathbb{S}} \wedge \overbrace{\{V_{s+1}, V_{s+2}, \dots, V_n\}}^{t \text{ elementos}} = B_{\mathbb{T}}$

$$\begin{aligned}
f(V_1) &= \alpha_1 V_1 + \alpha_2 V_2 + \dots + \alpha_s V_s \\
f(V_2) &= \beta_1 V_1 + \beta_2 V_2 + \dots + \beta_s V_s \\
&\dots \\
f(V_s) &= \omega_1 V_1 + \omega_2 V_2 + \dots + \omega_s V_s \\
f(V_{s+1}) &= \alpha_{s+1} V_{s+1} + \dots + \alpha_n . V_n \\
&\dots \\
f(V_n) &= \omega_{s+1} V_{s+1} + \dots + \omega_n . V_n
\end{aligned}$$

$\Rightarrow$ tomo $B = \{V_1, V_2, \dots, V_s, V_{s+1}, \dots, V_n\}$ base de $\mathbb{V}$ $\Rightarrow$

$$[f]_B = \left([f(V_1)]_B^t \, [f(V_2)]_B^t \dots [f(V_s)]_B^t \, [f(V_{s+1})]_B^t \dots [f(V_n)]_B^t \right)$$

$$[f]_B = \left(\begin{array}{cccc|ccc} \alpha_1 & \beta_1 & & \omega_1 & & & \\ \alpha_2 & \beta_2 & \ddots & \omega_2 & & 0 & \\ \vdots & \vdots & & \vdots & & & \\ \alpha_s & \beta_s & \dots & \omega_s & & & \\ \hline & & & & \alpha_{s+1} & \dots & \omega_n \\ & 0 & & & \vdots & \ddots & \vdots \\ & & & & \alpha_{s+1} & \dots & \omega_n \end{array} \right) = \left(\begin{array}{c|c} A_1 & 0 \\ \hline 0 & A_2 \end{array} \right)$$

donde $\begin{cases} A_1 \in K^{s \times s} \\ A_2 \in K^{t \times t} \end{cases}$

Si $B = \{V_1, V_2, \dots, V_s, V_{s+1}, \dots, V_n\}$ base de autovectores $\Rightarrow$

$$[f]_B = \left(\begin{array}{cccc|ccc} \lambda_1 & 0 & \dots & 0 & & & \\ 0 & \lambda_2 & \dots & \vdots & & 0 & \\ \vdots & \vdots & \ddots & \vdots & & & \\ 0 & 0 & \dots & \lambda_s & & & \\ \hline & & & & \lambda_{s+1} & \dots & 0 \\ & 0 & & & \vdots & \ddots & \vdots \\ & & & & 0 & \dots & \lambda_n \end{array} \right)$$

$$\Rightarrow P_f[\lambda] = (\lambda - \lambda_1).(\lambda - \lambda_2) \dots (\lambda - \lambda_s).(\lambda - \lambda_{s+1}) \dots (\lambda - \lambda_n)$$

$$P_f[\lambda] = P_{A_1}[\lambda] . P_{A_2}[\lambda]$$

18. i) $f: \mathbb{R}^2 \to \mathbb{R}^2 \qquad f(x,y) = (x+3y, 3x-2y)$

$$[f]_E = \begin{pmatrix} 1 & 3 \\ 3 & -2 \end{pmatrix}$$

$$\begin{vmatrix} 1-\lambda & 3 \\ 3 & -2-\lambda \end{vmatrix} = (1-\lambda)(-2-\lambda) - 9 = -2+2\lambda+\lambda^2-\lambda-9 = \lambda^2+\lambda-11$$

$$\lambda^2+\lambda-11 = 0$$

$$\frac{-1 \pm \sqrt{1-4.1.-11}}{2} \quad \lambda_1 = \frac{-1+\sqrt{45}}{2} \quad \lambda_2 = \frac{-1-\sqrt{45}}{2}$$

$\lambda_1 = \dfrac{-1-\sqrt{45}}{2}$

$$\begin{pmatrix} 1+\frac{1}{2}+\frac{\sqrt{45}}{2} & 3 \\ 3 & -2+\frac{1}{2}+\frac{\sqrt{45}}{2} \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\boxed{V_1 = \left(\frac{-2}{1+\sqrt{5}}, 1 \right)}$$

$$\left(\frac{3}{2}+\frac{\sqrt{45}}{2}\right)x + 3y = 0$$

$$x = \frac{-3.2}{3+\sqrt{45}}\,y = \frac{-\cancel{3}.2}{\cancel{3}(1+\sqrt{5})}\,y$$

$\lambda_2 = \dfrac{-1+\sqrt{45}}{2}$

$$\begin{pmatrix} 1+\frac{1}{2}-\frac{\sqrt{45}}{2} & 3 \\ 3 & -2+\frac{1}{2}-\frac{\sqrt{45}}{2} \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \frac{3}{2}-\frac{\sqrt{45}}{2} & 3 \\ 3 & -\frac{3}{2}-\frac{\sqrt{45}}{2} \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = 0 \qquad \boxed{V_2 = \left(\frac{-2}{1-\sqrt{5}}, 1 \right)}$$

$$\frac{3-\sqrt{45}}{6} \begin{pmatrix} 3-\sqrt{45} & 6 \\ 6 & -3-\sqrt{45} \end{pmatrix} \to \begin{pmatrix} 3-\sqrt{45} & 6 \\ 0 & (-3-\sqrt{45}).\frac{(3-\sqrt{45})}{6} - 6 \end{pmatrix} \to \begin{pmatrix} 3-\sqrt{45} & 6 \\ 0 & 0 \end{pmatrix}$$

$$-9-\sqrt{45}.3+3\sqrt{45}+45 \qquad (3-\sqrt{45})x = -6y$$

$$x = \frac{-6}{3(1-\sqrt{5})}\,y$$

$$\begin{array}{cc} V_1 & V_2 \end{array}$$
$$\begin{pmatrix} \frac{-2}{1+\sqrt{5}} & \frac{-2}{1-\sqrt{5}} \\ 1 & 1 \end{pmatrix} \qquad \text{Sea } B = \{V_1, V_2\} \text{ base de } \mathbb{R}^2$$

$$[f]_B = \left(\begin{pmatrix} 1 & 3 \\ 3 & -2 \end{pmatrix} \cdot \begin{pmatrix} \frac{-2}{1+\sqrt{5}} \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 3 \\ 3 & -2 \end{pmatrix} \cdot \begin{pmatrix} \frac{-2}{1-\sqrt{5}} \\ 1 \end{pmatrix} \right) = \begin{pmatrix} \frac{-2}{1+\sqrt{5}}+3 & \frac{-2}{1-\sqrt{5}}+3 \\ \frac{-6}{1+\sqrt{5}}-2 & \frac{-6}{1-\sqrt{5}}-2 \end{pmatrix}$$

$$f(V_1) = A.V_1 = \begin{pmatrix} 1 & 3 \\ 3 & -2 \end{pmatrix} \cdot \begin{pmatrix} -2/1+\sqrt{5} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1+3\sqrt{5}}{1+\sqrt{5}} \\ \frac{-8-2\sqrt{5}}{1+\sqrt{5}} \end{pmatrix}$$

$$f(V_1) = \frac{-1-3\sqrt{5}}{2} \begin{pmatrix} \frac{2}{-(1+3\sqrt{5})} \cdot \frac{1+3\sqrt{5}}{1+\sqrt{5}} \\ \frac{2.(-8-2\sqrt{5})}{(-1-3\sqrt{5}).(1+\sqrt{5})} \end{pmatrix} ; \begin{pmatrix} \frac{2}{(-1+3\sqrt{5})} \cdot \frac{1-3\sqrt{5}}{1-\sqrt{5}} \\ \frac{2}{(-1+3\sqrt{5})} \cdot \frac{-8+2\sqrt{5}}{(1-\sqrt{5})} \end{pmatrix} \frac{-1+3\sqrt{5}}{2} = f(V_2)$$

$$(-1+3\sqrt{5}+\sqrt{5}-3.5)$$
$$\cancel{-16+4\sqrt{5}} \quad 2$$

$$f(V_1) = \left(\frac{-1-3\sqrt{5}}{2}\right).V_1$$

$$[f]_B = \left(\begin{array}{c|c} \frac{-1-3\sqrt{5}}{2} & 0 \\ \hline 0 & \frac{-1+3\sqrt{5}}{2} \end{array}\right)$$

$$\mathbb{R}^2 = \langle V_1, V_2 \rangle = \underbrace{\$\left(\tfrac{-1-3\sqrt{5}}{2}\right)}_{\lambda_1}^{\langle V_1\rangle} \oplus \underbrace{\$\left(\tfrac{-1+3\sqrt{5}}{2}\right)}_{\lambda_2}^{\langle V_2\rangle}$$

Los subespacios invariantes son $\$_{\lambda_1}$ y $\$_{\lambda_2}$

$$\left(\begin{array}{cc|c} 1 & 3 & 0 \\ 3 & -2 & 0 \end{array}\right) \quad \left(\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & -11 & 0 \end{array}\right) \Rightarrow Nu(f) = (0,0) \Rightarrow \begin{array}{l} f(e_1) = (1,3) \\ f(e_2) = (3,-2) \end{array} \Rightarrow Im(f) = \langle (1,3), (3,-2) \rangle = \mathbb{R}^2$$

$f(0,0) = (0,0) \Rightarrow$ $Nu(f)$ es f-invariante pues $f(\text{núcleo}) \in$ núcleo

$Im(f)$ es f-invariante pues $f(\underset{\mathbb{R}^2}{\text{imagen}}) \in \mathbb{R}^2$

todos los $W_i = \langle \underbrace{V_1, V_2}_{\text{autovectores}} \rangle$ son f-invariantes

ii)

$$[f_\theta]_E = \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{pmatrix}$$

f no diagonalizable $\Rightarrow \nexists$ base de autovectores

$$\begin{vmatrix} \cos\theta-\lambda & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta-\lambda \end{vmatrix} = (\cos\theta-\lambda)^2 + \operatorname{sen}^2\theta = \cos^2\theta - 2\cos\theta\lambda + \lambda^2 + \operatorname{sen}^2\theta$$

$$= \lambda^2 - \underbrace{2\cos\theta}\lambda + 1 = 0$$

$$\frac{2.\cos\theta \pm \sqrt{4.\cos^2\theta - 4.1.1}}{2}$$

$$\frac{2.\cos\theta \pm \sqrt{4(\cos^2\theta-1)}}{2} = \frac{2\cos\theta \pm 2.\operatorname{sen}\theta}{2}$$

$$= \cos\theta \pm \operatorname{sen}\theta -$$

$\lambda_1 = \cos\theta + \operatorname{sen}\theta$

$\lambda_2 = \cos\theta - \operatorname{sen}\theta$

$\underline{\lambda_1}$:

$$\begin{pmatrix} \operatorname{sen}\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \operatorname{sen}\theta \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} \operatorname{sen}\theta & -\operatorname{sen}\theta & 0 \\ 0 & 2\operatorname{sen}\theta & 0 \end{array}\right) \Rightarrow \begin{array}{l} \operatorname{sen}\theta.x - \operatorname{sen}\theta.y = 0 \\ 2.\operatorname{sen}\theta.y = 0 \end{array}$$

$\Rightarrow y = 0$ pues $\operatorname{sen}\theta \neq 0 \;\forall\, \theta \neq k\pi$

$\Rightarrow x = 0$

$\Rightarrow$ no hay autovector

$\underline{\lambda_2}$:

$$\begin{pmatrix} +\operatorname{sen}\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \operatorname{sen}\theta \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} \operatorname{sen}\theta & -\operatorname{sen}\theta & 0 \\ 2\operatorname{sen}\theta & 0 & 0 \end{array}\right) \Rightarrow \text{no hay autovector} \Rightarrow$$

$\boxed{\nexists\ B \text{ de autovectores} \quad f \text{ no es diagonalizable}}$

$f(1,0) = (\cos\theta, \operatorname{sen}\theta)$

$f(0,1) = (-\operatorname{sen}\theta, \cos\theta)$

$Im(f)$ tiene dim 2 $\Rightarrow$

$Nu(f)$ tiene dim 0 $\Rightarrow$

Todos los subespacios invariantes son: $\boxed{Nu(f) \text{ y } \mathbb{R}^2}$

iii)

$$\begin{vmatrix} \cos\theta-\lambda & -\text{sen}\,\theta \\ \text{sen}\,\theta & \cos\theta-\lambda \end{vmatrix} = (\cos\theta-\lambda)^2 + \text{sen}^2\theta =$$

$$\cos^2\theta - 2.\lambda.\cos\theta + \lambda^2 + \text{sen}^2\theta =$$
$$1 - 2\lambda\cos\theta + \lambda^2 =$$

$\lambda_1 = \cos\theta + \text{sen}\,\theta$
$\lambda_2 = \cos\theta - \text{sen}\,\theta$

$$\begin{pmatrix} -\text{sen}\,\theta & -\text{sen}\,\theta \\ \text{sen}\,\theta & -\text{sen}\,\theta \end{pmatrix} . \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -\text{sen}\,\theta & -\text{sen}\,\theta \\ 0 & -2\,\text{sen}\,\theta \end{pmatrix} . \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$(-2.\text{sen}\,\theta).y = 0$

$\text{sen}\,\theta.y = 0$

si $\theta = k.\pi$ $(k \in \mathbb{Z})$

$\downarrow$

$V_1 = (x, y)$
$V = \langle (1,0), (0,1) \rangle$

19.

$f: \mathbb{R}^5 \longrightarrow \mathbb{R}^5 \qquad f(x_1, x_2, x_3, x_4, x_5) = (x_2, x_3, x_4, x_5, 0)$

$$[f]_E = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{vmatrix} -\lambda & 1 & 0 & 0 & 0 \\ 0 & -\lambda & 1 & 0 & 0 \\ 0 & 0 & -\lambda & 1 & 0 \\ 0 & 0 & 0 & -\lambda & 1 \\ 0 & 0 & 0 & 0 & -\lambda \end{vmatrix} = -\lambda^5 = 0$$

$\lambda = 0$ (auto valor única)

i)

$$\left(\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow V_1 = (1,0,0,0,0) \qquad S_1 = \langle 1,0,0,0,0 \rangle$$

$\dim S_1 = 1$

$Nu\,(A - \lambda_i I)^2 = 0$

$$\left(\begin{array}{ccccc|c} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \qquad \begin{matrix} x_3 = 0 \\ x_4 = 0 \\ x_5 = 0 \end{matrix} \Rightarrow V_2 = (x_1, x_2, 0, 0, 0)$$

$S_2 = \langle (1,0,0,0,0), (0,1,0,0,0) \rangle$ dim 2

$Nu(A)^3 = 0 \qquad x_5 = x_4 = 0 \Rightarrow S_3 = \langle e_1, e_2, e_3 \rangle$ dim 3

$Nu(A^4) = 0 \qquad x_5 = 0 \Rightarrow S_4 = \langle e_1, e_2, e_3, e_4 \rangle$ dim 4