PRACTICA 4

1.

i) $|A| = (-3.5) - (4.2) = \boxed{-23}$ ii) $\det(A) = \boxed{0}$ iii) $(0+(-2)+60) - (0+4+12) = 58-16 = \boxed{42}$

iv) $|A| = (10+8+3) - (12+4+5) = \boxed{0}$

$$\begin{pmatrix} 2 & -1 & 3 \\ -1 & 1 & -2 \\ 1 & -1 & 5 \end{pmatrix} \to \begin{pmatrix} 0 & -1 & 1 \\ 0 & 1 & -1 \\ 4 & -1 & 5 \end{pmatrix} \to \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 1 & -1/4 & 5/4 \end{pmatrix}$$

v)
$$\begin{vmatrix} 2 & 3 & -2 & 5 \\ 4 & -5 & 0 & 6 \\ 2 & 0 & -1 & 7 \\ 6 & 3 & -4 & 8 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 3 & -2 & 5 \\ 2 & -8 & 2 & 1 \\ 0 & -3 & 1 & 2 \\ 4 & 0 & -2 & 3 \end{vmatrix} = 2\begin{vmatrix} 4 & 6 & -4 & 10 \\ 0 & -11 & 4 & -4 \\ 0 & -3 & 1 & 2 \\ 4 & 0 & -2 & 3 \end{vmatrix}$$

$$= 2\begin{vmatrix} 4 & 6 & -4 & 10 \\ 0 & -11 & 4 & -4 \\ 0 & -3 & 1 & 2 \\ 0 & -6 & 2 & -7 \end{vmatrix} = \begin{vmatrix} 2 & 3 & -2 & 5 \\ 0 & -11 & 4 & -4 \\ 0 & -3 & 1 & 2 \\ 0 & -6 & 2 & -7 \end{vmatrix}$$

$$= 2\begin{vmatrix} -11 & 4 & -4 \\ -3 & 1 & 2 \\ -6 & 2 & -7 \end{vmatrix} - 3\underbrace{\begin{vmatrix} 0 & 4 & -4 \\ 0 & 1 & 2 \\ 0 & 2 & -7 \end{vmatrix}}_{=0} - 2\underbrace{\begin{vmatrix} 0 & -11 & -4 \\ 0 & -3 & 2 \\ 0 & -6 & 7 \end{vmatrix}}_{=0} - 5\underbrace{\begin{vmatrix} 0 & -11 & 4 \\ 0 & -3 & 1 \\ 0 & -6 & 2 \end{vmatrix}}_{=0}$$

$$= 2\begin{vmatrix} -11 & 4 & -4 \\ -3 & 1 & 2 \\ -3 & 1 & -9 \end{vmatrix} = 2\begin{vmatrix} -11 & 4 & -4 \\ -3 & 1 & 2 \\ 0 & 0 & -11 \end{vmatrix}$$

$$= 2\left[(121+0+0) - (12.11)\right] = \boxed{-22}$$

vi)
$$\begin{vmatrix} 5 & 4 & -2 & 5 \\ 2 & -3 & 0 & 6 \\ 0 & 0 & 2 & 0 \\ -4 & 3 & 3 & 8 \end{vmatrix} = \begin{vmatrix} 5 & 2 & 0 & -4 \\ 4 & -3 & 0 & 3 \\ 2 & 0 & 2 & 3 \\ 5 & 6 & 0 & 8 \end{vmatrix} = 2\begin{vmatrix} 5 & 2 & -4 \\ 4 & -3 & 3 \\ 5 & 6 & 8 \end{vmatrix} = 2\begin{vmatrix} 5 & 2 & -4 \\ 4 & -3 & 3 \\ 0 & 4 & 12 \end{vmatrix}$$

$$= 2\left([-180+0-64] - [0+96+60]\right)$$

$$= \boxed{-800}$$

2. i) $A \in K^{n\times n}$

$$\det(A) = \begin{vmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & a_{22} & a_{23} & \cdots & a_{2n} \\ 0 & 0 & a_{33} & \cdots & a_{3n} \\ \cdots & \cdots & \cdots & \ddots & \vdots \\ 0 & 0 & 0 & & a_{nn} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} & \cdots & a_{2n} \\ 0 & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & a_{nn} \end{vmatrix} = a_{11}.a_{22}.\begin{vmatrix} a_{33} & a_{34} & \cdots & a_{3n} \\ 0 & a_{44} & \cdots & a_{4n} \\ \vdots & \vdots & & \\ 0 & 0 & \cdots & a_{nn} \end{vmatrix}$$

y siguiendo este proceso en forma iterativa se tiene:

$$\boxed{\det(A) = \prod_{i=1}^{n} a_{ii}}$$

ii) $\det A = \det B$ si B se obtiene de A mediante operaciones elementales

$$\det(A) = \begin{vmatrix} 0 & 0 & \cdots & 0 & a_1 \\ 0 & 0 & \cdots & a_2 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & a_{n-1} & \cdots & 0 & 0 \\ a_n & 0 & \cdots & 0 & 0 \end{vmatrix} = \begin{vmatrix} a_n & 0 & \cdots & 0 & a_1 \\ 0 & a_{n-1} & \cdots & a_2 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & a_{n-1} & \cdots & 0 & 0 \\ a_n & 0 & \cdots & 0 & 0 \end{vmatrix} \begin{array}{l} F_n + F_1 \\ F_{n-1} + F_2 \\ \\ \\ \\ \end{array}$$

$$= \begin{vmatrix} a_n & 0 & \cdots & 0 & a_1 \\ 0 & a_{n-1} & \cdots & a_2 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 2a_{n-1} & \cdots & a_2 & 0 \\ 2a_n & 0 & \cdots & 0 & a_1 \end{vmatrix} \begin{array}{l} \\ \\ \\ F_{n-1}+F_2 \\ F_n+F_1 \end{array} = \begin{vmatrix} a_n & 0 & \cdots & 0 & a_1 \\ 0 & a_{n-1} & \cdots & a_2 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_2 & 0 \\ 0 & 0 & \cdots & 0 & a_1 \end{vmatrix} \begin{array}{l} \\ \\ \\ 2F_2 - F_{n-1} \\ 2F_1 - F_n \end{array}$$

$$\Rightarrow \boxed{\det(A) = (a_n)\cdot(a_{n-1})\cdots(a_2)(a_1)}$$

3. i) $A \in K^{n\times n}$
$B \in K^{m\times m}$
$C \in K^{n\times m}$
$M \in K^{(n+m)\times(n+m)}$

$$M = \left(\begin{array}{c|c} A & C \\ \hline 0 & B \end{array}\right)$$

4.

$$\det(A)=\begin{vmatrix} 1 & 2 & 3 & 4 & \dots & n \\ -1 & 0 & 3 & 4 & \dots & n \\ -1 & -2 & 0 & 4 & \dots & n \\ \dots & \dots & \dots & \dots & \dots & \dots \\ -1 & -2 & -3 & -4 & \dots & 0 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & 4 & \dots & n \\ 0 & 2 & 6 & 8 & \dots & 2n \\ 0 & 0 & 3 & 8 & \dots & 2n \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & n \end{vmatrix} \begin{matrix} \\ F_2+F_1 \\ F_3+F_1 \\ \\ F_n+F_1 \end{matrix}$$

$$\det(A) = 1.2.3.\dots.n = \boxed{n!}$$

5.

$$A=\begin{pmatrix} t & 0 & 0 & \cdots & 0 & 0 & a_0 \\ -1 & t & 0 & \cdots & 0 & 0 & a_1 \\ 0 & -1 & t & \cdots & 0 & 0 & a_2 \\ 0 & 0 & -1 & \cdots & 0 & 0 & a_3 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & -1 & t & a_{n-2} \\ 0 & 0 & 0 & \cdots & 0 & -1 & t+a_{n-1} \end{pmatrix} \quad n\times n$$

$$A^t=\begin{pmatrix} t & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & t & -1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & t & -1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & -1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & t & -1 \\ a_0 & a_1 & a_2 & a_3 & & a_{n-2} & t-a_{n-1} \end{pmatrix}$$

$$\det(A)= t.\underbrace{\begin{vmatrix} t & -1 & 0 & \dots & 0 \\ 0 & t & -1 & \dots & 0 \\ \vdots & \vdots & & & \\ a_1 & a_2 & a_3 & \dots & t-a_{n-1} \end{vmatrix}}_{(n-1)\times(n-1)} + 1.\underbrace{\begin{vmatrix} 0 & -1 & 0 & \dots & 0 & 0 \\ 0 & t & -1 & \dots & 0 & 0 \\ \vdots & & & \ddots & & \\ 0 & 0 & 0 & \dots & t & -1 \\ a_0 & a_2 & a_3 & \dots & a_{n-2} & t-a_{n-1} \end{vmatrix}}_{(n-1)\times(n-1)} + 0.A_{13} - 0.A_{14} + \dots \pm 0.A_{1n}$$

$$t.t\begin{vmatrix} t & -1 & \dots & 0 & 0 \\ 0 & t & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & t & -1 \\ a_2 & a_3 & \dots & a_{n-2} & t-a_{n-1} \end{vmatrix} + 1.\,a_0\underbrace{\begin{vmatrix} -1 & 0 & \dots & 0 & 0 \\ t & -1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & & \vdots \\ 0 & 0 & \dots & t & -1 \end{vmatrix}}_{(n-2)\times(n-2)}$$

$$+t1\underbrace{\begin{vmatrix} 0 & -1 & 0 & & 0 \\ \vdots & \vdots & \vdots & & \\ 0 & 0 & 0 & & \\ a_1 & a_3 & a_4 & & t-a_{n-1} \end{vmatrix}}_{(n-2)\times(n-2)}$$

$$= t^2\underbrace{\begin{vmatrix} t & -1 & \dots & 0 & 0 \\ 0 & t & \dots & 0 & 0 \\ \dots & & \ddots & & \dots \\ 0 & 0 & \dots & t & -1 \\ a_2 & a_3 & \dots & a_{n-2} & t-a_{n-1} \end{vmatrix}}_{(n-2)\times(n-2)} + ta_1.\underbrace{\begin{vmatrix} -1 & 0 & & 0 & 0 \\ t & -1 & & 0 & 0 \\ \vdots & \vdots & \ddots & & \vdots \\ 0 & 0 & & t & -1 \end{vmatrix}}_{(n-3)\times(n-3)} + a_0\underbrace{\begin{vmatrix} -1 & 0 & \dots & 0 & 0 \\ t & -1 & \dots & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & t & -1 \end{vmatrix}}_{(n-2)\times(n-2)}$$

$$= t^3\underbrace{\begin{vmatrix} t & -1 & \dots & 0 & 0 \\ 0 & t & \dots & 0 & 0 \\ \vdots & & \ddots & & \\ 0 & 0 & & t & -1 \\ a_3 & a_4 & \dots & a_{n-2} & t-a_{n-1} \end{vmatrix}}_{(n-3)\times(n-3)} + t^2 1\begin{vmatrix} 0 & -1 & \dots & 0 & 0 \\ 0 & t & & 0 & 0 \\ \vdots & \vdots & \ddots & & \\ 0 & 0 & \dots & t & -1 \\ a_2 & a_4 & \dots & a_{n-2} & t-a_{n-1} \end{vmatrix} + t.a_1\left|\quad\right| + a_0\left|\quad\right|$$

$$+t^2a_2\underbrace{\begin{vmatrix} -1 & & 0 & 0 \\ t & & 0 & 0 \\ & & & \\ 0 & & t & -1 \end{vmatrix}}_{(n-3)\times(n-3)} \quad (n-4)\times(n-4)$$

$t\,a_{n-1}t^2 + a_{n-3}\,t\,a_{n-2}$

$(t^3 - t^2a_{n-1}) + a_{n-3} - [-t\,a_{n-2}]$

$t^n - t^{n-1}$

$$\det(A) = t^{n-3}\begin{vmatrix} t & -1 & 0 \\ 0 & t & -1 \\ a_{n-3} & a_{n-2} & t-a_{n-1} \end{vmatrix} + t^{n-3}a_{n-3}\begin{vmatrix} -1 & 0 \\ t & -1 \end{vmatrix} + \dots + t.a_1\begin{vmatrix} -1 & 0 \\ t & -1 \end{vmatrix} + a_0\begin{vmatrix} -1 & 0 \\ t & -1 \end{vmatrix}$$

$$\det(A) = t^{n-2}\cdot\Big(\overbrace{[t^2 - t.a_{n-1}] - [-a_{n-2}]}^{t^2 + a_{n-2} - t.a_{n-1}}\Big) + t^{n-3}.a_{n-3} + \dots + t.a_1 + a_0$$

$$\det(A) = t^n + t^{n-2}.a_{n-2} - t^{n-1}.a_{n-1} + t^{n-3}.a_{n-3} + \dots + t.a_1 + a_0$$

$$\boxed{|A| = t^n - t^{n-1}.a_{n-1} + t^{n-2}.a_{n-2} + t^{n-3}.a_{n-3} + \dots + t a_1 + a_0}$$

6. $A.\begin{pmatrix}1\\2\\1\end{pmatrix} = \begin{pmatrix}1\\2\\7\end{pmatrix}$ $\quad \det(A) = 3$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 3$$

$$\left.\begin{aligned} a_{11} + 2a_{12} + a_{13} &= 1 \\ a_{21} + 2a_{22} + a_{23} &= 2 \\ a_{31} + 2a_{32} + a_{33} &= 7 \end{aligned}\right\} [1]$$

$$M = \begin{pmatrix} a_{12} & a_{22} & a_{32} \\ 1 & 2 & 7 \\ a_{11}+2a_{13} & a_{21}+2a_{23} & a_{31}+2a_{33} \end{pmatrix}$$

$$\det(M) = \begin{vmatrix} a_{12} & a_{22} & a_{32} \\ 1 & 2 & 7 \\ a_{11} & a_{21} & a_{31} \end{vmatrix} + \begin{vmatrix} a_{12} & a_{22} & a_{32} \\ 1 & 2 & 7 \\ 2a_{13} & 2a_{23} & 2a_{33} \end{vmatrix}$$

$$\det(M) = \begin{vmatrix} a_{12} & a_{22} & a_{32} \\ 1 & 2 & 7 \\ a_{11} & a_{21} & a_{31} \end{vmatrix} + \frac{1}{2}\begin{vmatrix} a_{12} & a_{22} & a_{32} \\ 1 & 2 & 7 \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

mejor probamos reemplazar [1] $\Rightarrow$

$$\det(M) = \begin{vmatrix} a_{12} & a_{22} & a_{32} \\ a_{11}+2a_{12}+a_{13} & a_{21}+2a_{22}+a_{23} & a_{31}+2a_{32}+a_{33} \\ a_{11}+2a_{13} & a_{21}+2a_{23} & a_{31}+2a_{33} \end{vmatrix}$$

$$= \begin{vmatrix} a_{12} & a_{22} & a_{32} \\ a_{11}+a_{12}+a_{13} & a_{21}+a_{22}+a_{23} & a_{31}+a_{32}+a_{33} \\ a_{12}+a_{13} & a_{23}+a_{22} & a_{32}+a_{33} \end{vmatrix} \begin{matrix} F_2-F_1 \\ F_3-F_2 \end{matrix}$$

$$= \begin{vmatrix} a_{12} & a_{22} & a_{32} \\ a_{11} & a_{21} & a_{31} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} \begin{matrix} F_2-F_3 \\ F_3-F_1 \end{matrix}$$

$$= \begin{vmatrix} a_{12} & a_{22} & a_{32} \\ a_{11}-a_{12} & a_{21}-a_{22} & a_{31}-a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} \begin{matrix} F_1-F_2 \\ F_2-F_1 \end{matrix} = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{11}-a_{12} & a_{21}-a_{22} & a_{31}-a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} F_2+F_1$$

$$= -\begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} F_1-F_2 = -\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \boxed{-3}$$

7. $A, B \in \mathbb{R}^{2\times2}$ $\quad A = \begin{pmatrix}1 & 3\\2 & 1\end{pmatrix}$ $\quad B = \begin{pmatrix}2 & -1\\1 & 3\end{pmatrix}$

$A.C = C.B$, $C \in \mathbb{R}^{2\times2}$ $\Rightarrow$

$$\begin{pmatrix}1 & 3\\2 & 1\end{pmatrix}\cdot\begin{pmatrix}c_{11} & c_{12}\\c_{21} & c_{22}\end{pmatrix} = \begin{pmatrix}c_{11} & c_{12}\\c_{21} & c_{22}\end{pmatrix}\cdot\begin{pmatrix}2 & -1\\1 & 3\end{pmatrix}$$

$$\begin{pmatrix}c_{11}+3c_{21} & c_{12}+3c_{22}\\2c_{11}+c_{21} & 2c_{12}+c_{22}\end{pmatrix} = \begin{pmatrix}2c_{11}+c_{12} & -c_{11}+3c_{12}\\2c_{21}+c_{22} & -c_{21}+3c_{22}\end{pmatrix}$$

Voy a plantear la igualdad de sus determinantes $\Rightarrow$

$$\Rightarrow \quad [1] = [2]$$

$$\begin{vmatrix} c_{11}+3c_{21} & c_{12}+3c_{22} \\ 2c_{11}+c_{21} & 2c_{12}+c_{22} \end{vmatrix} = \begin{vmatrix} 2c_{11}+c_{12} & -c_{11}+3c_{12} \\ 2c_{21}+c_{22} & -c_{21}+3c_{22} \end{vmatrix}$$

[1]: $(c_{11}+3c_{21}).(2c_{12}+c_{22}) - (c_{12}+3c_{22}).(2c_{11}+c_{21}) =$

$$\cancel{2c_{11}c_{12}} + 6c_{21}c_{12} + \cancel{3c_{21}c_{22}} + c_{11}c_{22} - \cancel{2c_{11}c_{12}} - 6c_{11}c_{22} - c_{12}c_{21} - \cancel{3c_{21}c_{22}} =$$

$$5c_{21}c_{12} - 5c_{11}c_{22} = 5\begin{vmatrix} c_{12} & c_{11} \\ c_{22} & c_{21} \end{vmatrix}$$

[2]: $(2c_{11}+c_{12})(-c_{21}+3c_{22}) - (2c_{21}+c_{22}).(-c_{11}+3c_{12}) =$

$$-\cancel{2c_{11}c_{21}} - c_{12}c_{21} + 6c_{22}c_{11} + \cancel{3c_{12}c_{22}} + \cancel{2c_{11}c_{21}} + c_{11}c_{22} - 6c_{12}c_{21} - \cancel{3c_{12}c_{22}} =$$

$$+7c_{11}c_{22} - 7c_{12}c_{21} = 7\begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix}$$

Necesito: $\begin{vmatrix} c_{12} & c_{11} \\ c_{22} & c_{21} \end{vmatrix} = \begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix}$ pero

$$\downarrow$$

$$= \begin{vmatrix} c_{12} & c_{22} \\ c_{11} & c_{21} \end{vmatrix} = -\begin{vmatrix} c_{11} & c_{21} \\ c_{12} & c_{22} \end{vmatrix} = -\begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} \quad \text{pero}$$

$$-\begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} \neq \begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} \quad \text{pues } -\det(C) \neq \det(C) \text{ salvo si } C=(0)$$

Luego si C es no inversible $\Rightarrow \det(C)=0 \ \wedge \ \Rightarrow \exists\, C=(0) : A.C = C.B$

8. i) $V_1 = (a,b,c)$, $V_2 = (d,e,f)$ $\qquad \varphi: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$

$$\varphi(x,y,z) = a.e.z + x.b.f + dyc - x.ec - z.b.d - a.f.y$$

1) $\varphi(\bar{x}+\bar{y}) = \varphi(x_0+x_1, y_0+y_1, z_0+z_1) = a.e(z_0+z_1) + (x_0+x_1)b.f + (y_0+y_1).dc - (x_0+x_1).e.c$

$$- (z_0+z_1).b.d - (y_0+y_1).a.f$$

$$= (a.e.z_0 + x_0.bf + y_0.dc - x_0.ec - z_0.b.d - y_0.a.f) + (a.e.z_1 + x_1.b.f$$

$$+ y_1.d.c - x_1.e.c. - z_1.b.d - y_1.a.f = \varphi(x_0,y_0,z_0) + \varphi(x_1,y_1,z_1)$$

2) $\varphi(\lambda\bar{x}) = \varphi(\lambda x, \lambda y, \lambda z) = \lambda.a.e.z + \lambda x bf + \lambda dyc - \lambda xec - \lambda zbd - \lambda afy$

$$= \lambda.\varphi(x,y,z) \qquad \Rightarrow \boxed{\varphi \text{ es TL}}$$

ii) $\{V_1,V_2\}$ son Li $\Rightarrow$ $\left(\begin{array}{cc|c} a & d & 0 \\ b & e & 0 \\ c & f & 0 \end{array}\right)$

$$\lambda_1.(a,b,c) + \lambda_2(d,e,f) = 0 \iff \lambda_1,\lambda_2 = 0$$

$$aez + bfx + dcy - ecx - bdz - afy = 0$$

$$[\ (bf-ec)x + (dc-af)y + (ae-bd)z = 0 \qquad \Rightarrow$$

$$x = [-(dc-af)y - (ae-bd)z].\frac{1}{(bf-ec)}$$

$$= \left[(-dc+af).y+(-ae+bd)z,\ y,\ z\right]$$

$$= y\left[(af-dc),1,0\right] + z\left[(bd-ae),0,1\right]$$

$$= \langle (af-dc,1,0),(bd-ae,0,1) \rangle$$

$$(af, 1+dc, dc),$$

9.

$$\begin{array}{c}[1]\\ \begin{vmatrix} b_{11} & 1+b_{12} & 2+b_{13} \\ b_{21} & 1+b_{22} & 2+b_{23} \\ b_{31} & 2+b_{32} & 3+b_{33} \end{vmatrix}\end{array} = \begin{array}{c}[2]\\ \begin{vmatrix} -b_{11} & 1-b_{12} & 2-b_{13} \\ -b_{21} & 1-b_{22} & 2-b_{23} \\ -b_{31} & 2-b_{32} & 3-b_{33} \end{vmatrix}\end{array}$$

$$b_{11}.(1+b_{22}).(3+b_{33}) + b_{31}(1+b_{12})(2+b_{23}) + b_{21}(2+b_{32})(2+b_{13}) - b_{31}(1+b_{22}).(2+b_{13}) - b_{21}(1+b_{12}).(3+b_{33}) - b_{11}(2+b_{32}).(2+b_{23})$$

$$= b_{11}.(-1+b_{22}).(3-b_{33}) + b_{31}(-1+b_{12})(2-b_{23}) + b_{21}(-2+b_{32})(2-b_{13}) - b_{31}(-1+b_{22})(2-b_{13}) - b_{11}(-2+b_{32})(2-b_{23}) - (3-b_{33})(1-b_{12})b_{21}$$

$$3.b_{11} + b_{11}b_{22}.3 + b_{11}b_{33} + b_{11}b_{22}b_{33} \qquad -b_{11}3 + b_{11}b_{22}.3 + b_{11}b_{33}$$

Δ esto es muy complicado

en [1] $\begin{vmatrix} b_{11} & b_{21} & b_{31} \\ 1+b_{12} & 1+b_{22} & 2+b_{32} \\ 2+b_{13} & 2+b_{23} & 3+b_{33} \end{vmatrix} = \begin{vmatrix} b_{11} & b_{21} & b_{31} \\ b_{12} & b_{22} & b_{32} \\ 2+b_{13} & 2+b_{23} & 3+b_{33} \end{vmatrix} + \begin{vmatrix} b_{11} & b_{21} & b_{31} \\ 1 & 1 & 2 \\ 2+b_{13} & 2+b_{23} & 3+b_{33} \end{vmatrix}$

en [2] $\begin{vmatrix} -b_{11} & -b_{21} & -b_{31} \\ 1-b_{12} & 1-b_{22} & 2-b_{32} \\ 2-b_{13} & 2-b_{23} & 3-b_{33} \end{vmatrix} = \begin{vmatrix} -b_{11} & -b_{21} & -b_{31} \\ -b_{12} & -b_{22} & -b_{32} \\ 2-b_{13} & 2-b_{23} & 3-b_{33} \end{vmatrix} + \begin{vmatrix} -b_{11} & -b_{21} & -b_{31} \\ 1 & 1 & 2 \\ 2-b_{13} & 2-b_{23} & 3-b_{33} \end{vmatrix}$

en [1] $= \begin{vmatrix} b_{11} & b_{21} & b_{31} \\ b_{12} & b_{22} & b_{32} \\ b_{13} & b_{23} & b_{33} \end{vmatrix} + \begin{vmatrix} b_{11} & b_{21} & b_{31} \\ b_{12} & b_{22} & b_{32} \\ 2 & 2 & 3 \end{vmatrix} + \begin{vmatrix} b_{11} & b_{21} & b_{31} \\ 1 & 1 & 2 \\ b_{13} & b_{23} & b_{33} \end{vmatrix} + \begin{vmatrix} b_{11} & b_{21} & b_{31} \\ 1 & 1 & 2 \\ 2 & 2 & 3 \end{vmatrix}$

en [2] $= \begin{vmatrix} -b_{11} & -b_{21} & -b_{31} \\ -b_{12} & -b_{22} & -b_{32} \\ -b_{13} & -b_{23} & -b_{33} \end{vmatrix} + \begin{vmatrix} -b_{11} & -b_{21} & -b_{31} \\ -b_{12} & -b_{22} & -b_{32} \\ 2 & 2 & 3 \end{vmatrix} + \begin{vmatrix} -b_{11} & -b_{21} & -b_{31} \\ 1 & 1 & 2 \\ -b_{13} & -b_{23} & -b_{33} \end{vmatrix} + \begin{vmatrix} -b_{11} & -b_{21} & -b_{31} \\ 1 & 1 & 2 \\ 2 & 2 & 3 \end{vmatrix}$

$$= -\begin{vmatrix} b_{11} & b_{21} & b_{31} \\ b_{12} & b_{22} & b_{32} \\ b_{13} & b_{23} & b_{33} \end{vmatrix} + \begin{vmatrix} b_{11} & b_{21} & b_{31} \\ b_{12} & b_{22} & b_{32} \\ 2 & 2 & 3 \end{vmatrix} + \begin{vmatrix} b_{11} & b_{21} & b_{31} \\ 1 & 1 & 2 \\ b_{13} & b_{23} & b_{33} \end{vmatrix} - \begin{vmatrix} b_{11} & b_{21} & b_{31} \\ 1 & 1 & 2 \\ 2 & 2 & 3 \end{vmatrix}$$

Juntando [1] y [2]

$$\cancel{k}\begin{vmatrix} b_{11} & b_{21} & b_{31} \\ b_{12} & b_{22} & b_{32} \\ b_{13} & b_{23} & b_{33} \end{vmatrix} = -\cancel{k}\begin{vmatrix} b_{11} & b_{21} & b_{31} \\ 1 & 1 & 2 \\ 2 & 2 & 3 \end{vmatrix}$$

$$\begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = (3b_{11} + 4b_{21} + 2b_{31}) - (2b_{31} + 3b_{21} + 4b_{11}) =$$

$$\boxed{\det(B) = -b_{11} + b_{21}} \Rightarrow \text{si } b_{11} \neq b_{21} \quad |B| \neq 0 \text{ y}$$

$$\Rightarrow B \text{ es inversible}$$

10. i) $A \in \mathbb{R}^{4\times4}$

$$\begin{vmatrix} a & b & c & d \\ b & -a & d & -c \\ c & -d & -a & b \\ d & c & -b & -a \end{vmatrix} = a \cdot \begin{vmatrix} -a & d & -c \\ -d & -a & b \\ c & -b & -a \end{vmatrix} - b \begin{vmatrix} b & d & -c \\ c & -a & b \\ d & -b & -a \end{vmatrix} + c \begin{vmatrix} b & -a & -c \\ c & -d & b \\ d & c & -a \end{vmatrix}$$

$$-d \begin{vmatrix} b & -a & d \\ c & -d & -a \\ d & c & -b \end{vmatrix}$$

$$|A| = a \begin{vmatrix} -a & d & -c \\ -d & -a & b \\ c & -b & -a \end{vmatrix} - b \begin{vmatrix} b & d & c \\ c & -a & b \\ d & -b & -a \end{vmatrix} + c \begin{vmatrix} b & c & d \\ -a & -d & c \\ -c & b & -a \end{vmatrix} - d \begin{vmatrix} b & -a & d \\ c & -d & -a \\ d & c & -b \end{vmatrix}$$

$$|A| = a[-a^3 - \cancel{dbc} + \cancel{cdb} - c^2a - ad^2 - ab^2] - b[a^2b + bd^2 + c^2b - \cancel{adc} + \cancel{adc} + b^3]$$
$$+ c[\cancel{bda} - c^3 - \cancel{abd} - cd^2 - a^2c - b^2c] - d[b^2d + c^2d + a^2d + d^3 + \cancel{abc} - \cancel{abc}]$$
$$= a^2[-a^2 - c^2 - d^2 - b^2] - b^2[a^2 + d^2 + c^2 + b^2] + c^2[-c^2 - d^2 - a^2 - b^2]$$
$$-d^2[b^2 + c^2 + a^2 + d^2] = -(a^2 + b^2 + c^2 + d^2) \cdot (a^2 + b^2 + c^2 + d^2) =$$

$$|A| = -(a^2+b^2+c^2+d^2)^2 \Rightarrow$$

si $A.x = 0 \Rightarrow$ si $|A| \neq 0 \Rightarrow \exists$ unica solución
si $|A| = 0 \Rightarrow \nexists$ solución

$$-(a^2+b^2+c^2+d^2)^2 = 0$$
$$a^2+b^2+c^2+d^2 = 0$$
$$a^2 = -\underbrace{(b^2+c^2+d^2)}_{>0} \iff b, c, d = 0 \wedge a = 0$$

$\boxed{A.X = 0 \text{ tiene solución unica} \iff a \neq 0 \vee b \neq 0 \vee c \neq 0 \vee d \neq 0}$

ii) $A \in \mathbb{C}^{4\times4}$

$$a = \sqrt{-(b^2+c^2+d^2)} = i \cdot \sqrt{(b^2+c^2+d^2)}$$

En $\mathbb{C}^{4\times4}$ no es válido pues $\exists$ un plano de no-soluciones dado por

$$\left(i\sqrt{b^2+c^2+d^2}, b, c, d\right)$$

$$\left\langle \left(i\sqrt{b^2+c^2+d^2}, b, 0, 0\right), \left(i\sqrt{b^2+c^2+d^2}, 0, c, 0\right), \left(i\sqrt{b^2+c^2+d^2}, 0, 0, d\right) \right\rangle$$

11.

$\alpha_1, \alpha_2, \dots, \alpha_n \in \mathbb{R} \neq 0$ y $\alpha_i = \alpha_{i+1} \; \forall i$

$e^{\alpha_1 x}, \dots, e^{\alpha_n x}$ son Li en $\mathbb{R}$

Sea $H(x) = c_1 . e^{\alpha_1 x} + c_2 . e^{\alpha_2 x} + \dots + c_n . e^{\alpha_n x}$ con $\alpha_1 \neq \alpha_2 \neq \dots \neq \alpha_n \neq 0$

$$H' = \alpha_1 . c_1 . e^{\alpha_1 x} + \alpha_2 . c_2 . e^{\alpha_2 x} + \dots + \alpha_n . c_n . e^{\alpha_n x}$$

$$H'' = \alpha_1^2 . c_1 e^{\alpha_1 x} + \alpha_2^2 . c_2 . e^{\alpha_2 x} + \dots + \alpha_n^2 . c_n . e^{\alpha_n x}$$

...

$$H^{(n-1)}(x) = \alpha_1^{n-1} . c_1 . e^{\alpha_1 x} + \alpha_2^{n-1} . c_2 . e^{\alpha_2 x} + \dots + \alpha_n^{n-1} . c_n . e^{\alpha_n x}$$

$$H^{n-1}(x) = \Big(\underbrace{\alpha_1^{n-1} . c_1 . e^{\alpha_1}}_{\lambda_1} + \underbrace{\alpha_2^{n-1} . c_2 . e^{\alpha_2}}_{\lambda_2} + \dots + \underbrace{\alpha_n^{n-1} c_n . e^{\alpha_n}}_{\lambda_n}\Big) . e^x = 0$$

$$\lambda_1 + \lambda_2 + \dots + \lambda_n = 0$$

$$\Leftrightarrow \quad \lambda_1 = \lambda_2 = \dots = \lambda_n = 0 \quad \Rightarrow$$

$$\boxed{e^{\alpha_1 x}, \dots, e^{\alpha_n x} \text{ son Li}}$$

Luego no se puede tener una base para el espacio vectorial de funciones $e^{\alpha_i x}$ pues siempre se puede hallar otro vector ($e^{\alpha_i x}$) Li con los ya tomados

12.

i) $\begin{pmatrix} 2 & 3 \\ 5 & 1 \end{pmatrix}$ $\qquad \det \underbrace{\begin{pmatrix} 2 & 3 \\ 5 & 1 \end{pmatrix}}_{\equiv A} = -13$ $\qquad \text{Adj } A = \begin{pmatrix} 1 & -3 \\ -5 & 2 \end{pmatrix}$

cofactores

$A_{11} = +1$

$A_{12} = -5$

$A_{21} = -3$

$A_{22} = +2$

$$A^{-1} = \frac{1}{-13} . \begin{pmatrix} 1 & -3 \\ -5 & 2 \end{pmatrix}$$

ii) $\begin{pmatrix} 2 & -3 & 3 \\ -5 & 4 & 0 \\ 0 & -2 & 2 \end{pmatrix}$ $\qquad \det(A) = (16+0+30) - (0+30+0) = 16$

$$\text{Adj } A = \begin{pmatrix} 8 & 10 & 10 \\ 0 & 4 & -10 \\ -12 & -15 & -7 \end{pmatrix}^t = \begin{pmatrix} 8 & 0 & -12 \\ 10 & 4 & -15 \\ 10 & -10 & -7 \end{pmatrix}$$

$A_{11} = +8$ $\qquad A_{31} = +(-12)$

$A_{12} = -(-10)$ $\qquad A_{32} = -(+15)$

$A_{13} = +10$ $\qquad A_{33} = +(-7)$

$A_{21} = -0$

$A_{22} = +4$

$A_{23} = -10$

$$A^{-1} = \frac{1}{16} \begin{pmatrix} 8 & 0 & -12 \\ 10 & 4 & -15 \\ 10 & -10 & -7 \end{pmatrix}$$

iii) $\begin{pmatrix} -1 & 1 & 6 & 5 \\ 1 & 1 & 2 & 3 \\ -1 & 2 & 5 & 4 \\ 2 & 1 & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} -1 & 1 & 6 & 5 \\ 0 & 2 & 8 & 8 \\ 0 & 3 & 7 & 7 \\ 2 & 1 & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} -1 & 1 & 6 & 5 \\ 0 & 2 & 8 & 8 \\ 0 & 3 & 7 & 7 \\ 0 & 3 & 12 & 11 \end{pmatrix}$ $\rightarrow \begin{pmatrix} -1 & 1 & 6 & 5 \\ 0 & 2 & 8 & 8 \\ 0 & 0 & -5 & -4 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{matrix} \\ \\ -F_4 + F \\ -\frac{3}{2} F_2 + F_4 \end{matrix}$

$$\det(A) = -1 . \begin{vmatrix} 2 & 8 & 8 \\ 3 & 7 & 7 \\ 3 & 12 & 11 \end{vmatrix} = \begin{vmatrix} 2 & 8 & 8 \\ 3 & 7 & 7 \\ -3 & -12 & -11 \end{vmatrix} = \begin{vmatrix} 2 & 8 & 8 \\ 3 & 7 & 7 \\ 0 & -5 & -4 \end{vmatrix}$$

$$\det(A) = (-56 + 0 - 120) - (0 - 96 - 70) = -10$$

$A_{11} = + \begin{vmatrix} 2 & 8 & 8 \\ 0 & -5 & -4 \\ 0 & 0 & -1 \end{vmatrix}$ $\quad A_{12} = - \begin{vmatrix} 0 & 8 & 8 \\ 0 & -5 & -4 \\ 0 & 0 & -1 \end{vmatrix}$ $\quad A_{13} = + \begin{vmatrix} 0 & & \\ 0 & & \\ 0 & & \end{vmatrix}$ $\quad A_{14} = - \begin{vmatrix} 0 & & \\ 0 & & \\ 0 & & \end{vmatrix}$

$A_{21} = - \begin{vmatrix} 1 & 6 & 5 \\ 0 & -5 & -4 \\ 0 & 0 & -1 \end{vmatrix}$ $\quad A_{22} = + \begin{vmatrix} -1 & 6 & 5 \\ 0 & -5 & -4 \\ 0 & 0 & -1 \end{vmatrix}$ $\quad A_{23} = - \begin{vmatrix} -1 & 1 & 5 \\ 0 & 0 & -4 \\ 0 & 0 & -1 \end{vmatrix}$ $\quad A_{24} = + \begin{vmatrix} -1 & 1 & 6 \\ 0 & 0 & -5 \\ 0 & 0 & 0 \end{vmatrix}$

$A_{31} = + \begin{vmatrix} 1 & 6 & 5 \\ 2 & 8 & 8 \\ 0 & 0 & -1 \end{vmatrix}$ $\quad A_{32} = - \begin{vmatrix} -1 & 6 & 5 \\ 0 & 8 & 8 \\ 0 & 0 & -1 \end{vmatrix}$ $\quad A_{33} = + \begin{vmatrix} -1 & 6 & 5 \\ 0 & 0 & -4 \\ 0 & 0 & -1 \end{vmatrix}$ $\quad A_{34} = - \begin{vmatrix} -1 & 1 & 6 \\ 0 & 2 & 8 \\ 0 & 0 & 0 \end{vmatrix}$

$(-8)-(-12) = 4$

$A_{41} = - \begin{vmatrix} 1 & 6 & 5 \\ 2 & 8 & 8 \\ 0 & -5 & -4 \end{vmatrix}$ $\quad A_{42} = + \begin{vmatrix} -1 & 6 & 5 \\ 0 & 8 & 8 \\ 0 & -5 & -4 \end{vmatrix}$ $\quad A_{43} = - \begin{vmatrix} -1 & 1 & 5 \\ 0 & 2 & 8 \\ 0 & 0 & -4 \end{vmatrix}$ $\quad A_{44} = + \begin{vmatrix} -1 & 1 & 6 \\ 0 & 2 & 8 \\ 0 & 0 & -5 \end{vmatrix}$

$-\left[(32+0-50)-(0-40-48)\right] \qquad (32)-(40)$

$$\operatorname{Adj} A = \begin{pmatrix} 10 & 0 & 0 & 0 \\ -5 & -5 & 0 & 0 \\ 4 & -8 & 0 & 0 \\ -6 & -8 & -8 & 5 \end{pmatrix}^t = \begin{pmatrix} 10 & -5 & 4 & -6 \\ 0 & -5 & -8 & -8 \\ 0 & 0 & 0 & -8 \\ 0 & 0 & 0 & 5 \end{pmatrix}$$

$$A^{-1} = -\frac{1}{10} \begin{pmatrix} 10 & -5 & 4 & -6 \\ 0 & -5 & -8 & -8 \\ 0 & 0 & 0 & -8 \\ 0 & 0 & 0 & 5 \end{pmatrix}$$

13. A es inversible $\Rightarrow$ $|A| \neq 0$

$$\det(\mathrm{Adj}\,A) \qquad \mathrm{adj}.(A) = |A|.A^{-1}$$

$$\det(\underbrace{|A|}_{k \in \mathbb{R}}.A^{-1}) = |A|^n.\det(A^{-1})$$

Si A no es inversible $\Rightarrow$ $\det(\mathrm{Adj}A) = 0$

14.

i) $\left(\begin{array}{cc|c} 3 & -1 & 3 \\ 1 & 7 & 4 \end{array}\right)$ $\underbrace{}_{\equiv A}$

$21 - (-1)$
$(-2\cdot1)-(4)$

$$X_1 = \frac{25}{22}$$

$$X_2 = \frac{9}{22}$$

ii) $\left(\begin{array}{ccc|c} 3 & -2 & 1 & 0 \\ -1 & 1 & 2 & 1 \\ 2 & 1 & 4 & 2 \end{array}\right)$

$$X_1 = \frac{-1}{-13} = 1/13 \qquad \begin{matrix}(12-8-1) \\ -(2+6+8)\end{matrix}$$

$$X_2 = \frac{-4}{-13} = 4/13 \qquad \begin{pmatrix} 0 & -2 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 4 \end{pmatrix}$$

$$X_3 = \frac{-5}{-13} = 5/13 \qquad \begin{matrix}(0+1\,-8)- \\ (2+0+-8)\end{matrix}$$

$\begin{pmatrix} 3 & -2 & 0 \\ -1 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix}$ $\begin{matrix}(6+0-4) \\ -(1+3+4)\end{matrix}$ $\qquad$ $\begin{pmatrix} 3 & 0 & 1 \\ -1 & 1 & 2 \\ 2 & 2 & 4 \end{pmatrix}$ $\begin{matrix}(12-2+0) \\ -(2+12+0) \\ 10-14\end{matrix}$

iii) $\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ -1 & -1 & -4 & 1 & -1 \\ 1 & -1 & -1 & -1 & 4 \\ 5 & 1 & -3 & 2 & 0 \end{array}\right)$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & -3 & 2 & -1 \\ 0 & -2 & -2 & -2 & 4 \\ 5 & 1 & -3 & 2 & 0 \end{array}\right) \begin{matrix} -F_1+F_3 \\ -5F_1+F_4 \rightarrow \end{matrix} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & -3 & 2 & -1 \\ 0 & -2 & -2 & -2 & 4 \\ 0 & -4 & -8 & -3 & 0 \end{array}\right)$$

$$-2F_3+F_4 \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & -3 & 2 & -1 \\ 0 & -2 & -2 & -2 & 4 \\ 0 & 0 & -4 & 1 & -8 \end{array}\right) \quad -\tfrac{4}{3}F_2+F_4 \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & -3 & 2 & -1 \\ 0 & -2 & -2 & -2 & 4 \\ 0 & 0 & 0 & -5/3 & -20/3 \end{array}\right)$$

$$\det(A) = 10$$

$$X_1 = \frac{\begin{vmatrix} 0 & 1 & 1 & 1 \\ -1 & 0 & -3 & 2 \\ 4 & -2 & -2 & -2 \\ -20/3 & 0 & 0 & -5/3 \end{vmatrix}}{10}$$

$$X_2 = \frac{\begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & -1 & -3 & 2 \\ 0 & 4 & -2 & -2 \\ 0 & -20/3 & 0 & -5/3 \end{vmatrix}}{10}$$

$$X_3 = \frac{\begin{vmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & -1 & 2 \\ 0 & -2 & 4 & -2 \\ 0 & 0 & -20/3 & -5/3 \end{vmatrix}}{10}$$

$$X_4 = \frac{\begin{vmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & -3 & -1 \\ 0 & -2 & -2 & 4 \\ 0 & 0 & 0 & -20/3 \end{vmatrix}}{10} = \frac{40}{10} = \boxed{4}$$

15.

$$0 = \begin{vmatrix} 1 & b & c \\ 2 & e & f \\ 5 & h & i \end{vmatrix} = \begin{vmatrix} 1 & 2 & 5 \\ b & e & h \\ c & f & i \end{vmatrix} =$$

$$0 = \begin{vmatrix} a & 2 & c \\ d & 4 & f \\ g & 10 & i \end{vmatrix} = \begin{vmatrix} a & d & g \\ 2 & 4 & 10 \\ c & f & i \end{vmatrix} = 2 \begin{vmatrix} a & d & g \\ 1 & 2 & 5 \\ c & f & i \end{vmatrix}$$

$$0 = \begin{vmatrix} a & b & -1 \\ d & e & -2 \\ g & h & -5 \end{vmatrix} = \begin{vmatrix} a & d & g \\ b & e & h \\ -1 & -2 & -5 \end{vmatrix}$$

$$X_1 = \frac{\begin{vmatrix} 1 & b & c \\ 2 & e & f \\ 5 & h & i \end{vmatrix}}{|A|} \qquad X_2 = \frac{2\begin{vmatrix} a & 1 & c \\ d & 2 & f \\ g & 5 & i \end{vmatrix}}{|A|} \qquad X_3 = \frac{-1\begin{vmatrix} a & b & 1 \\ d & e & 2 \\ g & h & 5 \end{vmatrix}}{|A|}$$

$\Rightarrow$

$$a X_1 + b X_2 + c X_3 = 1$$
$$d X_1 + e X_2 + f X_3 = 2$$
$$g X_1 + h X_2 + i X_3 = 5$$

$= ei + 2hc + 5bf - 5ec - fh - 2b$

$= 2ai + 5dc + gf - 2gc - 5fa - ri$

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \cdot \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & b & c \\ 2 & e & f \\ 5 & h & i \end{pmatrix} + \begin{pmatrix} a & 2 & c \\ d & 4 & f \\ g & 10 & i \end{pmatrix} + \Bigg($$

$$\begin{vmatrix} 1 & b & c \\ 2 & e & f \\ 5 & h & i \end{vmatrix} = 1 \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} 2 & f \\ 5 & i \end{vmatrix} + c \begin{vmatrix} 2 & e \\ 5 & h \end{vmatrix} = 0$$

$$\begin{vmatrix} a & b & -1 \\ d & e & -2 \\ g & h & -5 \end{vmatrix} = a \begin{vmatrix} e & -2 \\ h & -5 \end{vmatrix} - b \begin{vmatrix} d & -2 \\ g & -5 \end{vmatrix} + -1 \begin{vmatrix} d & e \\ g & h \end{vmatrix} = 0$$

$$\begin{vmatrix} a & 2 & c \\ d & 4 & f \\ g & 10 & i \end{vmatrix} = a \begin{vmatrix} 4 & f \\ 10 & i \end{vmatrix} - 2 \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & 4 \\ g & 10 \end{vmatrix} = 0$$

$$a \begin{vmatrix} 2 & e \\ 5 & h \end{vmatrix} - b \begin{vmatrix} 2 & d \\ 5 & g \end{vmatrix} - 1 \begin{vmatrix} d & e \\ g & h \end{vmatrix} = 0$$

$$a \begin{vmatrix} 2 & e \\ 5 & h \end{vmatrix} - b \begin{vmatrix} 2 & d \\ 5 & g \end{vmatrix} = \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$\begin{vmatrix} e & f \\ h & i \end{vmatrix} = b \begin{vmatrix} 2 & f \\ 5 & i \end{vmatrix} - c \begin{vmatrix} 2 & e \\ 5 & h \end{vmatrix}$$

$$2a \begin{vmatrix} 2 & f \\ 5 & i \end{vmatrix} - 2 \begin{vmatrix} d & f \\ g & i \end{vmatrix} - 2c \begin{vmatrix} 2 & d \\ 5 & g \end{vmatrix} = 0$$

$$a \begin{vmatrix} 2 & f \\ 5 & i \end{vmatrix} - c \begin{vmatrix} 2 & d \\ 5 & g \end{vmatrix} = \begin{vmatrix} d & f \\ g & i \end{vmatrix}$$

$$\det(A) = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$|A| = \cancel{a.b \begin{vmatrix} 2 & f \\ 5 & i \end{vmatrix}} - ac \begin{vmatrix} 2 & e \\ 5 & h \end{vmatrix} - ab \begin{vmatrix} 2 & f \\ 5 & i \end{vmatrix} - bc \begin{vmatrix} 2 & d \\ 5 & g \end{vmatrix} + ca \begin{vmatrix} 2 & e \\ 5 & h \end{vmatrix} - cb \begin{vmatrix} 2 & d \\ 5 & g \end{vmatrix}$$

$$\boxed{|A| = -2.b.c. \begin{vmatrix} 2 & d \\ 5 & g \end{vmatrix} = -2.b.c.(2g - 5d)}$$