PRACTICA 3

1. i) $f: \mathbb{R}^3 \to \mathbb{R}^3$

$f(x,y,z) = (2x-7z, 0, 3y+2z)$

$f(x+x_1, y+y_1, z+z_1) = (2x+2x_1-7z-7z_1, 0, 3y+3y_1+2z+2z_1)$

$(2x-7z, 0, 3y+2z) + (2x_1-7z_1, 0, 3y_1+2z_1)$

$= f(x,y,z) + f(x_1,y_1,z_1)$

$f[\lambda.(x,y,z)] = (2\lambda x - 7\lambda z, 0, 3\lambda y + 2\lambda z)$

$\lambda(2x-7z, 0, 3y+2z)$

f es TL

ii) $f: \mathbb{R}^2 \to \mathbb{R}^3$ $\quad f(x,y) = (x-y, 2y, 1+x)$

$f(x+x_1, y+y_1) = (x+x_1-y-y_1, 2y+2y_1, 1+x+x_1)$

$(x-y, 2y, 1+x) + (x_1-y_1, 2y_1, x_1)$

f no es TL

iii) $f: \mathbb{C} \to \mathbb{C}$ $\quad f(z) = \bar{z}$ $\quad \mathbb{C}$ con $F = \mathbb{R}$

$f(z+z_1) = (\overline{z+z_1})$

$z = a+bi$
$z_1 = a_1 + b_1 i$

$a+bi+a_1+b_1 i = a+a_1+(b+b_1)i$

$(a+a_1)-(b+b)i = a-bi+a_1-b_1 i$

$= f(z) + f(z_1)$

$\lambda \in \mathbb{R}$ $\quad f(\lambda z) = \overline{\lambda z} = \lambda a - \lambda bi = \lambda(a-bi)$

$\lambda . f(z)$

f es TL $(F = \mathbb{R})$

$\lambda \in \mathbb{C}$ $\quad f(\lambda z) = \underbrace{(c+di)}_{\lambda}.(a+bi) = a.c + a.di + c.bi - db$

$(a.c - db) + (a.d + cb)i$

$(a.c - d.b) - (a.d + cb)i$

$= a.c - db - a.di - cbi$

$c(a-bi) - d(b+ai)$

$c.f(z) - d(b+ai) \neq \lambda . f(z)$

f no es TL $(F = \mathbb{C})$

iv) $f: \mathbb{R}^{2\times 2} \to \mathbb{R}$ $\quad f\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = a_{11}.a_{22} - a_{12}.a_{21}$

$$f\begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{pmatrix} = (a_{11}+b_{11})(a_{22}+b_{22}) - (a_{12}+b_{12})\cdot(a_{21}+b_{21})$$

$$a_{11}.a_{22} + b_{11}.a_{22} + a_{11}\,b_{22} + b_{11}.b_{22}$$

$$- a_{12}.a_{21} - b_{12}.a_{21} - a_{12}.b_{21} - b_{12}.b_{21}$$

$$= (a_{11}.a_{22} - a_{12}.a_{21}) + (b_{11}.b_{22} - b_{12}.b_{21})$$

$$+ (b_{11}.a_{22} - b_{12}.a_{21}) + (a_{11}.b_{22} - a_{12}.b_{21})$$

$\boxed{f \text{ no es TL}}$

v) $f: \mathbb{R}^{2\times 2} \longrightarrow \mathbb{R}^{2\times 3}$ $\quad f\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} a_{22} & 0 & a_{12}+a_{21} \\ 0 & a_{11} & a_{22}-a_{11} \end{pmatrix}$

$$f\begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{pmatrix} = \begin{pmatrix} a_{22}+b_{22} & 0 & a_{12}+b_{12}+a_{21}+b_{21} \\ 0 & a_{11}+b_{11} & a_{22}+b_{22}-a_{11}-b_{11} \end{pmatrix}$$

$$\begin{pmatrix} a_{22} & 0 & a_{12}+a_{21} \\ 0 & a_{11} & a_{22}-a_{11} \end{pmatrix} + \begin{pmatrix} b_{22} & 0 & b_{12}+b_{21} \\ 0 & b_{11} & b_{22}-b_{11} \end{pmatrix}$$

$$= f(A) + f(B)$$

$$f\begin{pmatrix} \lambda a_{11} & \lambda a_{12} \\ \lambda a_{21} & \lambda a_{22} \end{pmatrix} = \begin{pmatrix} \lambda a_{22} & 0 & \lambda a_{12}+\lambda a_{21} \\ 0 & \lambda a_{11} & \lambda a_{22}-\lambda a_{11} \end{pmatrix}$$

$$\lambda . f(A)$$

$\boxed{f \text{ es TL}}$

vi) $f: \mathbb{R}[X] \longrightarrow \mathbb{R}^3$ $\quad f(p) = (p(0), p'(0), p''(0))$

$$f(p+q) = (p+q(0), (p+q)'(0), (p+q)''(0))$$

$$p(0)+q(0),\ p'(0)+q'(0),\ p''(0)+q''(0)$$

$$= f(p) + f(q)$$

$$f(\lambda.p) = (\lambda.p(0),\ \lambda.p'(0),\ \lambda p''(0))$$

$$\lambda.\ f(p)$$

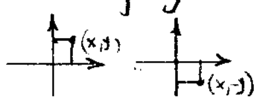
$\boxed{f \text{ es TL}}$

2.

i) $f(x,y) = (x,0)$ eje $\hat{x}$

ii) $f(x,y) = (0,y)$ eje $\hat{y}$

iii) $f(x,y) = (x,-y)$



(reflejo respecto eje x) $x = \frac{x}{2} + \frac{y}{2}$

iv) $f(x,y) = \left(\frac{1}{2}x + \frac{1}{2}y,\ \frac{1}{2}x + \frac{1}{2}y\right)$ recta $y = x$

$\frac{x}{2} = \frac{y}{2}$

$y = \frac{x}{2} + \frac{y}{2}$

v) $f(x,y) = (x.\cos t - y.\operatorname{sen} t,\ x \operatorname{sen} t + y.\cos t)$ $\quad x(1-\cos t) = -y.\operatorname{sen} t$

$x = -y.\operatorname{sen} t / 1-\cos t$

$$y(1-\cos t) = x.\operatorname{sen} t$$
$$y = x.\frac{\operatorname{sen} t}{1-\cos t} \qquad \text{si } t \neq 0, 2\pi$$
$$x = -y.\frac{\operatorname{sen} t}{1-\cos t}$$

$f(x,y) = (-y.k\,,\ x.k)$ dos rectas perpendiculares

3.

i) $tr: K^{n\times n} \longrightarrow K$

$$tr\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ a_{n1} & \cdots & \cdots & a_{nn} \end{pmatrix} = a_{11} + a_{22} + \ldots + a_{nn}$$

$$tr(A+B) = (a_{11}+b_{11}) + (a_{22}+b_{22}) + \ldots + (a_{nn}+b_{nn})$$
$$(a_{11} + a_{22} + \ldots a_{nn}) + (b_{11} + b_{22} + \ldots + b_{nn})$$
$$tr\,A + tr\,B \qquad \blacktriangle$$

$$tr(\lambda A) = \lambda a_{11} + \lambda a_{22} + \ldots + \lambda a_{nn}$$
$$\lambda\ tr(A) \qquad \blacktriangle$$

ii) $t: K^{n\times m} \to K^{m\times n}$, $t(A) = A^t$

$$t\begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{n1} \\ a_{1m} & a_{nm} \end{pmatrix}$$

4×3

4. i) $f:\mathbb{R}^2 \to \mathbb{R}^2$ $\qquad f(1,1)=(-5,3)$ $\qquad B=\{(1,1),(-1,1)\}$

$f(-1,1)=(5,2)$

$\begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$
$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 2 & 0 \end{pmatrix}$

B es una base

$(x,y)=\lambda_1.(1,1)+\lambda_2(-1,1)$ vector genérico en la base B

$x=\lambda_1-\lambda_2$
$y=\lambda_1+\lambda_2$

$\lambda_1 = x+\lambda_2 = x+y-\lambda_1 \Rightarrow \lambda_1=\frac{1}{2}x+y$

$\lambda_2 = y-\lambda_1$ $\qquad \lambda_2=\frac{1}{2}y-x$

$f(x,y)=\lambda_1.f(1,1)+\lambda_2.f(-1,1)$

$f(x,y)=\frac{1}{2}(x+y).(-5,3)+\frac{1}{2}(y-x).(5,2)$

$\left(-\frac{5}{2}x-\frac{5}{2}y+\frac{5}{2}y-\frac{5}{2}x,\ \frac{3}{2}x+\frac{3}{2}y-x+y\right)$

$\boxed{f(x,y)=\left(-5x,\ \frac{5}{2}y+\frac{1}{2}x\right)}$ $\qquad f(5,3)=\left(-25,\frac{17}{2}\right)$
$f(-1,2)=\left(5,\frac{9}{2}\right)$

Si no fuera única $\exists$ $g(x,y)$:

$\begin{pmatrix} -5 & 5 & 0 \\ 3 & 2 & 0 \end{pmatrix}$
$\begin{pmatrix} -1 & 1 & 0 \\ 1 & 2/3 & 0 \end{pmatrix}$
$\begin{pmatrix} -1 & 1 & 0 \\ 0 & 5/3 & 0 \end{pmatrix}$ Li

Como $\{(-5,3),(5,2)\}$ son Li $\Rightarrow$ son base de $\mathbb{R}^2$

$f(x,y)\in\mathbb{R}^2$ se escribe unívocamente como:

$f(x,y)=\lambda_1.(-5,3)+\lambda_2(5,2)$ (Comb. lineal de los vectores de la base) para ciertos λ_1,λ_2

Luego un $f(x,y)$ es único en esa base $\{(-5,3),(5,2)\}$

ii) $f:\mathbb{R}^2\to\mathbb{R}^2$

$f(1,1)=(2,6)$
$f(-1,1)=(2,1)$
$f(2,7)=(5,3)$

$f(x,y)=\frac{1}{2}(x+y).(2,6)+\frac{1}{2}(y-x).(2,1)$

$\left(x+y+y-x;\ 3x+3y+\frac{y}{2}-\frac{x}{2}\right)$

$f(x,y)=\left(2y,\ \frac{5}{2}x+\frac{7}{2}y\right)$ $\quad$ [1]

$(x,y)=\lambda_1.(-1,1)+\lambda_2.(2,7)$

$f(x,y)=\lambda_1.(2,1)+\lambda_2.(5,3)$

$x=-\lambda_1+2\lambda_2$
$y=\lambda_1+7\lambda_2$

$\frac{1}{7}-\frac{2}{63}=\frac{9-2}{63}$ $\quad 1+\frac{2}{7}=\frac{9}{7}$

$\lambda_1=2\lambda_2-x=-x+\frac{2}{7}y-\frac{2}{7}\lambda_1$

$\lambda_2=\frac{y}{7}-\frac{\lambda_1}{7}=$ $\qquad \lambda_1=-x+\frac{2}{7}$

$\lambda_2=\frac{y}{7}-\frac{1}{7}\left(-\frac{7}{9}x+\frac{2}{9}y\right)$ $\qquad \lambda_1=-\frac{7}{9}x+\frac{2}{9}y$

$\lambda_2=\frac{y}{7}+\frac{x}{9}-\frac{2}{63}y=\frac{1}{9}y+\frac{1}{9}x$

$f(x,y)=\left(-\frac{7}{9}x+\frac{2}{9}y\right).(2,1)+\left(\frac{y}{9}+\frac{x}{9}\right).(5,3)$

$\left(-\frac{14}{9}x+\frac{4}{9}y+\frac{5}{9}y+\frac{5}{9}x,\ -\frac{7}{9}x+\frac{2}{9}y+\frac{y}{3}+\frac{x}{3}\right)$

$f(x,y)=\left(y-x,\ \frac{5}{9}y-\frac{4}{9}x\right)$ $\quad$ [2]

Como [1] y [2] son $\neq$ $\Rightarrow$ se sigue que $\nexists!$ $T: \mathbb{R}^2 \to \mathbb{R}^2$ que verifique las condiciones

iii) $f, g : \mathbb{R}^3 \to \mathbb{R}^3$

$$f(-1,0,0) = (1,2,1) \qquad f(2,1,0) = (2,1,0) \qquad f(1,0,1) = (1,2,1)$$

$$\left(\begin{array}{ccc|c} -1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \text{ son LI}$$

$$g(1,1,1) = (1,1,0) \qquad g(2,2,-1) = (3,-1,2) \qquad g(3,2,1) = (0,0,1)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 1 & 2 & 2 & 0 \\ 1 & -1 & 1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & -3 & -1 & 0 \end{array}\right) \text{ son LI}$$

$$(x,y,z) = \lambda_1(-1,0,0) + \lambda_2(2,1,0) + \lambda_3(1,0,1)$$

$$f(x,y,z) = \lambda_1(1,2,1) + \lambda_2(2,1,0) + \lambda_3(1,2,1)$$

$$\begin{aligned} x &= -\lambda_1 + 2\lambda_2 + \lambda_3 \\ y &= \lambda_2 \\ z &= \lambda_3 \end{aligned}$$

$$\lambda_2 = y \qquad \lambda_3 = z \qquad \lambda_1 = -x + 2y + z$$

$$f(x,y,z) = (-x+2y+z)\cdot(1,2,1) + y(2,1,0) + z(1,2,1)$$

$$= (-x+2y+z+2y+z, -2x+4y+2z+y+2z, -x+2y+z+z)$$

$$(-x+4y+2z, -2x+5y+4z, -x+2y+2z)$$

$$(x,y,z) = \lambda_1(1,1,1) + \lambda_2(2,2,-1) + \lambda_3(3,2,1)$$

$$\begin{aligned} x &= \lambda_1 + 2\lambda_2 + 3\lambda_3 \\ y &= \lambda_1 + 2\lambda_2 + 2\lambda_3 \\ z &= \lambda_1 - \lambda_2 + \lambda_3 \end{aligned} \qquad \left(\begin{array}{ccc|c} 1 & 2 & 3 & x \\ 1 & 2 & 2 & y \\ 1 & -1 & 1 & z \end{array}\right) \left(\begin{array}{ccc|c} 1 & 2 & 3 & x \\ 0 & 0 & -1 & y-x \\ 0 & -3 & -2 & z-x \end{array}\right)$$

$$\lambda_3 = x - y$$

$$-\lambda_2 = \frac{z}{3} - \frac{x}{3} + \frac{2x}{3} - \frac{2y}{3}$$

$$\lambda_2 = -\frac{z}{3} - \frac{x}{3} + \frac{2}{3}y$$

$$\lambda_1 = x - 3x + 3y + \frac{2}{3}z + \frac{2}{3}x - \frac{4}{3}y$$

$$\lambda_1 = -\frac{4}{3}x + \frac{5}{3}y + \frac{2}{3}z$$

$$g(x,y,z) = \left(-\frac{4}{3}x + \frac{5}{3}y + \frac{2}{3}z\right)(1,1,0) + \left(-\frac{z}{3} - \frac{x}{3} + \frac{2}{3}y\right)(3,-1,2) + (x-y)(0,0,1)$$

$$= \left(-\frac{4}{3}x + \frac{5}{3}y + \frac{2}{3}z - z - x + 2y, -\frac{4}{3}x + \frac{5}{3}y + \frac{2}{3}z + \frac{z}{3} + \frac{x}{3} - \frac{2}{3}y, -\frac{2}{3}z - \frac{2}{3}x + \frac{4}{3}y + x - y\right)$$

$$g(x,y,z) = \left(-\frac{7}{3}x + \frac{11}{3}y - \frac{1}{3}z, -x + y + z, \frac{1}{3}x + \frac{1}{3}y - \frac{2}{3}z\right)$$

$$f \neq g$$

iv) $f: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$

$$f(1,-1,1) = (2,a,-1)$$
$$f(1,-1,2) = (a^2,-1,1)$$
$$f(1,-1,-2) = (5,-1,-7)$$

hallar $a \in \mathbb{R}$

$$x,y,z = \lambda_1(1,-1,1) + \lambda_2(1,-1,2) + \lambda_3(1,-1,-2)$$

$$\begin{aligned} x &= \lambda_1 + \lambda_2 + \lambda_3 \\ y &= -\lambda_1 - \lambda_2 - \lambda_3 \\ z &= \lambda_1 + 2\lambda_2 - 2\lambda_3 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & x \\ -1 & -1 & -1 & y \\ 1 & 2 & -2 & z \end{array}\right) \to \begin{array}{cccc} 1 & 1 & 1 & x \\ 0 & 0 & 0 & y+x \\ 0 & 1 & -3 & z-x \end{array}$$

no son Li $\Rightarrow$ añado un vector para formar base de $\mathbb{R}^3$ (saco el 3ero)

$$x,y,z = \lambda_1(1,-1,1) + \lambda_2(1,-1,2) + \lambda_3\underbrace{(0,1,1)}_{\text{añadido}}$$

defino $f(0,1,1) = (0,1,1)$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & x \\ -1 & -1 & 1 & y \\ 1 & 2 & 1 & z \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 0 & x \\ 0 & 0 & 1 & y+x \\ 0 & 1 & 1 & z-x \end{array}\right)$$

$$\lambda_3 = y+x$$
$$\lambda_2 = z-x-y-x$$
$$\lambda_2 = z-2x-y$$
$$\lambda_1 = x-z+2x+y$$
$$\lambda_1 = 3x-z+y$$

$$f(x,y,z) = (3x-z+y)(2,a,-1) + (z-2x-y)\cdot(a^2,-1,1) + (y+x)(0,1,1)$$

$$(6x-2z+2y+za^2-2xa^2-ya^2,\ 3xa-za+ya-z+2x+y+y+x$$
$$-3x+z-y+z-2x-y+y+x)$$

$$f(1,-1,-2) = (\underbrace{6+4-2+za^2-2xa^2-ya^2}_{=5}$$

$$(z-2x-y)a^2 = -3$$

5.

1 i) $0 = (2x_1 - 7x_3, 0, 3x_2 + 2x_3)$

$$\left(\begin{array}{ccc|c} 2 & 0 & 7 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 2 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 2 & 0 & 7 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad \begin{array}{l} x_1 = -\frac{7}{2}x_3 \\ x_2 = -\frac{2}{3}x_3 \end{array}$$

$$\boxed{Nu(F) = \langle (-7/2, -2/3, 1) \rangle}$$

$$\begin{array}{l} f(1,0,0) = (2,0,0) \\ f(0,1,0) = (0,0,3) \\ f(0,0,1) = (-7,0,2) \end{array} \Rightarrow \boxed{Im(F) = \langle (2,0,0), (0,0,3), (-7,0,2) \rangle}$$

$$B_{Im(f)} = \{ (2,0,0), (0,0,3) \}$$

$$\begin{pmatrix} 2 & 0 & -7 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 2 & 0 \end{pmatrix} \sim \begin{pmatrix} 2 & 0 & -7 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

ld

f no monomorfismo $(Nu\, F \neq 0)$; f no epimorfismo $(Im\, f \neq \mathbb{R}^3)$
f no isomorfismo

1 iii) $f(z) = \bar{z}$

$$\begin{array}{l} f: \mathbb{C} \to \mathbb{C} \\ K = \mathbb{R} \end{array} \qquad \begin{array}{l} \bar{z} = 0 \\ a - bi = 0 \end{array} \Leftrightarrow a, b = 0 \Rightarrow \boxed{Nu(f) = 0}$$

$$B_{\mathbb{C}_{\mathbb{R}}} = \{ (1,0), (0,i) \} \qquad \begin{array}{l} f(1,0) = 1 \\ f(0,i) = -i \end{array} \qquad \boxed{Im(f) = \langle (1), (-i) \rangle}$$

genera $\mathbb{C}$

$$\left.\begin{array}{l} f \text{ es monomorfismo} \\ f \text{ es epimorfismo} \end{array}\right\} \Rightarrow f \text{ es isomorfismo}$$

$$\begin{array}{l} f(1,0) = (1,0) \\ f(0,i) = (0,-i) \end{array} \Rightarrow \begin{array}{l} f^{-1}(1,0) = (1,0) \\ f^{-1}(0,-i) = (0,i) \end{array}$$

1 v) $f(a_{11}, a_{12}, a_{21}, a_{22}) = (a_{22}, 0, a_{12} + a_{21}, 0, a_{11}, a_{22} - a_{11})$

$$\begin{array}{l} a_{22} = 0 \\ a_{12} + a_{21} = 0 \qquad a_{12} = -a_{21} \\ a_{11} = 0 \\ a_{22} - a_{11} = 0 \Rightarrow a_{22} = a_{11} = 0 \end{array}$$

$$Nu(F) = (0, a_{12}, -a_{12}, 0) \Rightarrow Nu(F) = \langle (0,1,-1,0) \rangle$$

$$\begin{array}{l} f(1,0,0,0) = (0,0,0,0,1,-1) = \omega_1 \\ f(0,1,0,0) = (0,0,1,0,0,0) = \omega_2 \\ f(0,0,1,0) = (0,0,1,0,0,0) = \omega_3 \\ f(0,0,0,1) = (1,0,0,0,0,1) = \omega_4 \end{array} \quad Im(f) = \langle (\vec{\omega_1}), (\vec{\omega_2}), (\vec{\omega_4}) \rangle$$

$$\dim \mathbb{R}^4 = \dim Nu(f) + \dim Im(f)$$

$$\left.\begin{array}{l} f \text{ no es monomorfismo} \\ f \text{ no es epimorfismo} \end{array}\right\} \Rightarrow f \text{ no es isomorfismo}$$

1 vi) $f(p) = (p(0), p'(0), p''(0))$

$$f: \mathbb{R}[X] \longrightarrow \mathbb{R}^3 \qquad \begin{array}{l} p(0) = 0 \\ p'(0) = 0 \\ p''(0) = 0 \end{array} \qquad Nu(f) = 0$$

p es un vector de $\mathbb{R}[X]$

2 i) $f(x,y) = (x,0)$

$x = 0$
y libre

$f(1,0) = (1,0)$
$f(0,1) = (0,0)$

$$\boxed{\begin{array}{l} Nu(f) = \langle(0,1)\rangle \\ Im(f) = \langle(1,0)\rangle \end{array}}$$

$\left.\begin{array}{l} f \text{ no es monomorfismo} \\ f \text{ no es epimorfismo} \end{array}\right\} \Rightarrow f$ no es isomorfismo

2 ii) $f(x,y) = (0,y)$

$y = 0$
libre

$f(1,0) = (0,0)$
$f(0,1) = (0,1)$

$$\boxed{\begin{array}{l} Nu(f) = \langle(1,0)\rangle \\ Im(f) = \langle(0,1)\rangle \end{array}}$$

$\left.\begin{array}{l} f \text{ no es monomorfismo} \\ f \text{ no es epimorfismo} \end{array}\right\} \Rightarrow f$ no es isomorfismo

2 iii) $f(x,y) = (x,-y)$

$f(1,0) = (1,0)$
$f(0,1) = (0,-1)$

$x = 0$
$-y = 0$

$\Rightarrow$ $$\boxed{\begin{array}{l} Nu(f) = 0 \\ Im(f) = \langle(1,0),(0,-1)\rangle \end{array}}$$

$\left.\begin{array}{l} f \text{ es monomorfismo} \\ f \text{ es epimorfismo} \end{array}\right\}$

$\Rightarrow f$ es isomorfismo

$f^{-1}(1,0) = (1,0)$
$f^{-1}(0,-1) = (0,1)$

$(x,y) = \lambda_1 \cdot (1,0) + \lambda_2 (0,-1)$

$x = \lambda_1$
$y = -\lambda_2$

$f^{-1}(x,y) = x \cdot (1,0) + (-y) \cdot (0,1)$

$$\boxed{f^{-1}(x,y) = (x,-y)}$$

2 iv) $f(x,y) = \left(\frac{1}{2}(x+y), \frac{1}{2}(x+y)\right)$

$\frac{1}{2}(x+y) = 0$
$\searrow x = -y$

$f(1,0) = \left(\frac{1}{2}, \frac{1}{2}\right)$

$f(0,1) = \left(\frac{1}{2}, \frac{1}{2}\right)$

$$\boxed{\begin{array}{l} Nu(f) = \langle(1,-1)\rangle \\ Im(f) = \langle\left(\frac{1}{2},\frac{1}{2}\right)\rangle \end{array}}$$

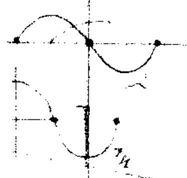
$\left.\begin{array}{l} f \text{ no es monomorfismo} \\ f \text{ no es epimorfismo} \end{array}\right\} \Rightarrow f$ no es isomorfismo

2 v) $f(x,y) = (x.\cos t - y.\text{sen}t,\ x.\text{sen}t + y.\cos t)$

$$\begin{aligned} x.\cos t - y.\text{sen}t &= 0 \\ x.\text{sen}t + y.\cos t &= 0 \end{aligned} \qquad \begin{aligned} x.\cos t &= y.\text{sen}t \\ x.\text{sen}t &= -y.\cos t \end{aligned}$$

$$\begin{aligned} f(1,0) &= (\cos t, \text{sen}t) \\ f(0,1) &= (-\text{sen}t, \cos t) \end{aligned}$$

si $\begin{cases} t \neq 0, n.\pi & n \in N \\ t \neq \pi/2, (2n+1)\frac{\pi}{2} & n \in N \end{cases}$



$$\left(\begin{array}{cc|c} \cos t & -\text{sen}t & 0 \\ \text{sen}t & \cos t & 0 \end{array}\right)$$

si $t \neq 0$
si $t \neq \pi/2$
$$\left(\begin{array}{cc|c} \cos t.\text{sen}t & -\text{sen}^2 t & 0 \\ \text{sen}t.\cos t & \cos^2 t & 0 \\ \cos t.\text{sen}t & -\text{sen}^2 t & 0 \\ 0 & 1 & 0 \end{array}\right)$$

son Li

$$\boxed{\begin{array}{l} Nu(f) = 0 \\ Im(f) = \langle (1, tg\,t), (-tg\,t, 1) \rangle \end{array}}$$

$\left.\begin{array}{l} f \text{ es monomorfismo} \\ f \text{ es epimorfismo} \end{array}\right\} \Rightarrow f$ es isomorfismo

$$\begin{aligned} f^{-1}(1, tg\,t) &= \left(\frac{1}{\cos t}, 0\right) \\ f^{-1}(-tg\,t, 1) &= \left(0, \frac{1}{\text{sen}t}\right) \end{aligned}$$

6. $f: \mathbb{R}^3 \longrightarrow \mathbb{R}^4 \qquad f(x_1,x_2,x_3) = (x_1+x_2, x_1+x_3, 0, 0)$

$g: \mathbb{R}^4 \longrightarrow \mathbb{R}^2 \qquad g(x_1,x_2,x_3,x_4) = (x_1-x_2, 2x_1-x_2)$

<u>f:</u>

$\left.\begin{array}{l} x_1+x_2 = 0 \\ x_1+x_3 = 0 \end{array}\right\}$

$x_1 = -x_2$
$x_1 = -x_3$

$\Rightarrow x_2 = x_3 = -x_1$

$Nu\,f = (x_1, -x_1, -x_1)$

$$\begin{aligned} f(1,0,0) &= (1,1,0,0) \\ f(0,1,0) &= (1,0,0,0) \\ f(0,0,1) &= (0,1,0,0) \end{aligned}$$

$Nu(f) = \langle (1,-1,-1) \rangle$

$Im(f) = \langle (1,0,0,0), (0,1,0,0) \rangle$

$$\boxed{\begin{array}{l} f \text{ no es mono, no es epi} \\ f \text{ no es iso} \end{array}}$$

<u>g:</u>

$$\begin{aligned} x_1 - x_2 &= 0 \\ 2x_1 - x_2 &= 0 \end{aligned}$$

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 & 0 \end{pmatrix}$$
$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{array}{l} x_2 = 0 \\ x_1 = 0 \end{array}$$

$x_3 =$
$x_4 =$ libres

$Nu(g) = (0, 0, x_3, x_4)$

$$\begin{aligned} g(1,0,0,0) &= (1,2) \\ g(0,1,0,0) &= (-1,-1) \\ g(0,0,1,0) &= (0,0) \\ g(0,0,0,1) &= (0,0) \end{aligned}$$

$Nu(g) = \langle (0,0,1,0), (0,0,0,1) \rangle$

$Im(g) = \langle (1,2), (-1,-1) \rangle$

$$\boxed{\begin{array}{l} g \text{ no es mono, no es epi} \\ g \text{ no es iso} \end{array}}$$

<u>$g \circ f$:</u>

$g \circ f: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$

$g \circ f(x_1,x_2,x_3) = (x_1+x_2-x_1-x_3,\ 2x_1+2x_2-x_1-x_3)$

$g \circ f(x_1,x_2,x_3) = (x_2-x_3,\ x_1+2x_2-x_3)$

$$\begin{aligned} x_2 - x_3 &= 0 \\ x_1 + 2x_2 - x_3 &= 0 \end{aligned}$$

$Nu(g \circ f) = (x_1, -x_1, -x_1) \qquad Nu(g \circ f) = \langle (1,-1,-1) \rangle$

$Im(g \circ f) = \langle (0,1), (1,2) \rangle$

$$\begin{aligned} g \circ f(1,0,0) &= (0,1) \\ g \circ f(0,1,0) &= (1,2) \\ g \circ f(0,0,1) &= (-1,-1) \end{aligned}$$

$x_2 = x_3$
$x_1 + 2x_2 - x_2 = 0$
$x_1 = -x_2$

$$\boxed{\begin{array}{l} g \circ f \text{ no es mono, no es epi} \\ g \circ f \text{ no es iso} \end{array}}$$

8.

i) epimorfismo si $F: V \to W \Rightarrow F$ es epi si $F(V) = W$

Pero $\boxed{\text{no es posible}}$ si $F: \mathbb{R}^2 \to \mathbb{R}^3$ pues:

$F(\mathbb{R}^2) \neq \mathbb{R}^3$

con comb. lineales de $v \in \mathbb{R}^2$ no se puede formar cualquier $v \in \mathbb{R}^3$

$$\dim \mathbb{R}^2 = \dim Nu(F) + \dim Im\, F$$
$$2 = 0 + 2$$

en el mejor de los casos W tiene dimensión 2

ii)
$$V_1 = (1,0,1,0)$$
$$V_2 = (1,1,1,0)$$
$$V_3 = (1,1,1,1)$$

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

son Li

$f: \mathbb{R}^2 \to \mathbb{R}^4$ $\{V_1, V_2, V_3\} \subset Im\, f$

$$\dim \mathbb{R}^2 = \dim Nu(f) + \dim Im(f)$$
$$2 = 0 + 2$$

$\boxed{\text{no puede haber}}$ f porque $\{V_1, V_2, V_3\}$ tiene dim 3

$\langle V_1, V_2, V_3 \rangle$ tiene dim 3

iii) $f: \mathbb{R}^3 \to \mathbb{R}^2$

f mono $\Rightarrow Nu(f) = 0 \Rightarrow$
$$\dim \mathbb{R}^3 = Nu(f) + Im(f)$$
$$3 = 0 + 3$$

Pero $Im(f) \not\subset \mathbb{R}^2 \Rightarrow \boxed{\nexists \text{ monomorfismo}}$

iv) $S = \{ X \in \mathbb{R}^4 : x_1 + x_2 + x_3 = 0 \}$ $\qquad x_1 = -x_2 - x_3$

$T = \left\{ X \in \mathbb{R}^4 : \begin{array}{l} 2x_1 + x_4 = 0 \\ x_2 - x_3 = 0 \end{array} \right\}$ $\qquad \begin{array}{l} x_1 = -x_4/2 \\ x_2 = x_3 \end{array}$

$S = (-x_2 - x_3, x_2, x_3, x_4)$ $\qquad S = \langle (-1,1,0,0), (0,0,1,0), (0,0,0,1) \rangle$

$T = (-\frac{x_4}{2}, x_3, x_3, x_4)$ $\qquad T = \langle (-1/2, 0, 0, 1), (0,1,1,0) \rangle$

$f: \mathbb{R}^4 \to \mathbb{R}^4$
$$\dim \mathbb{R}^4 = \dim Nu(f) + \dim Im(f)$$
$$4 = 0 + 4$$

- $f(S) = T \Rightarrow f(S) = \{ t \in W : f(s) = t \text{ para } s \in S \}$

$s \in S$

$$s = \lambda_1(-1,1,0,0) + \lambda_2(0,0,1,0) + \lambda_3(0,0,0,1)$$

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \to \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

son Li

$$f(s) = \lambda_1 . f(-1,1,0,0) + \lambda_2 . f(0,0,1,0) + \lambda_3 . f(0,0,0,1)$$

$$f(S) = \langle [f(-1,1,0,0)], [f(0,0,1,0)], [f(0,0,0,1)] \rangle$$

$\dim 3 \neq \dim 2 \to T$

si f es iso $\Rightarrow$

$f(-1,1,0,0) = (\quad)$
$f(0,0,1,0) = (\quad)$
$f(0,0,0,1) = (\quad)$
$f(\quad) = (\quad)$

son generadores de $Im(f)$ y son Li

$\boxed{\nexists\ f: \mathbb{R}^4 \to \mathbb{R}^4 \text{ iso}}$

v) $f: \mathbb{R}^3 \longrightarrow \mathbb{R}^4$ $\quad \begin{aligned} Im(f) &= \$ \\ Nu(f) &= \Pi \end{aligned}$

a) $$\begin{aligned} \dim \mathbb{R}^3 &= \dim Nu(f) + \dim Im(f) \\ 3 &= \quad 1 \qquad\quad 3 \end{aligned}$$

$\$: \quad x_1 + x_2 - x_3 + 2x_4 = 0$

$x_1 = -x_2 + x_3 - 2x_4$

$(-x_2 + x_3 - 2x_4, x_2, x_3, x_4)$

$\$ = \langle (-1,1,0,0)\ (1,0,1,0)\ (-2,0,0,1) \rangle \qquad \Pi = \langle (1,2,1) \rangle$

$\boxed{\nexists\ f}$ por dimensiones

b)

$3 = 1 + 2$

$\$: \left.\begin{aligned} x_1 + x_2 &= 0 \\ x_3 + x_4 &= 0 \end{aligned}\right\}$

$\$: \quad (x_1, -x_1, x_3, -x_3)$

$\$ = \langle (1,-1,0,0), (0,0,1,-1) \rangle \qquad \Pi = \langle (1,-2,1) \rangle$

$$\begin{aligned} f(1,0,0) &= (1,-1,0,0) \\ f(0,1,0) &= (0,0,1,-1) \\ f(1,-2,1) &= (0,0,0,0) \end{aligned}$$

$\boxed{\exists\ f: \mathbb{R}^3 \longrightarrow \mathbb{R}^4}$

$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ son Li

9. $f: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$

i) $(1,1,0) \in Nu(f) \qquad \dim Im(f) = 1$

$\underline{dim}$:

$$\begin{aligned} \mathbb{R}^3 &= Nu(f) + Im(f) \\ 3 &\qquad 2 \qquad\quad 1 \end{aligned}$$

$$\boxed{\begin{aligned} f(1,0,0) &= (1,0,0) \\ f(1,1,1) &= (0,0,0) \\ f(1,1,0) &= (0,0,0) \end{aligned}}$$

$(x,y,z) = \lambda_1 (1,0,0) + \lambda_2 (1,1,1) + \lambda_3 (1,1,0)$

$$\begin{aligned} x &= \lambda_1 + \lambda_2 + \lambda_3 \\ y &= \quad\ \ \lambda_2 + \lambda_3 \\ z &= \quad\ \ \lambda_2 \end{aligned} \qquad \begin{aligned} \lambda_1 &= x - z - y + z = x - y \\ \lambda_3 &= y - z \end{aligned}$$

$f(x,y,z) = (x-y, 0, 0) \Leftarrow f(x,y,z) = (x-y)(1,0,0) + \underbrace{\ldots}_{=0} + \underbrace{\ldots}_{=0}$

$$\begin{aligned} Nu(f) &= (x,x,z) \\ Nu(f) &= \langle (1,1,0), (0,0,1) \rangle \\ Im(f) &= \langle (1,0,0) \rangle \end{aligned}$$

$$\begin{aligned} f(100) &= (1,0,0) \\ f(010) &= (-1,0,0) \\ f(001) &= (0,0,0) \end{aligned}$$

ii) $Nu(f) \cap Im(f) = \langle (1,1,2) \rangle$

$\underline{dim}$

$$\begin{aligned} \mathbb{R}^3 &= Nu(f) + Im(f) \\ 3 &= \quad 1 \quad + \quad 2 \end{aligned}$$

$$\boxed{\begin{aligned} f(1,0,0) &= (1,0,0) \\ f(1,1,0) &= (1,1,2) \\ f(1,1,2) &= (0,0,0) \end{aligned}}$$

iii) $f \neq 0 \quad Nu(f) \subseteq Im(f)$

dim:
$\mathbb{R}^3 = Nu(f) + Im(f)$
$3 = 1 \quad 2$

$$\boxed{\begin{aligned} f(1,0,0) &= (1,0,0) \\ f(0,1,0) &= (0,1,0) \\ f(0,0,1) &= (0,0,0) \end{aligned}}$$

iv) $f \neq 0 \quad \underset{\mathbb{R}^3 \to \mathbb{R}^3}{f \circ f} = 0$

$\forall v \in \mathbb{R}^3$

$f[f(v)] = 0 \Rightarrow \langle f(v) \rangle = Nu(f)$

$f(v+u) = f(v) + f(u) = 0$
$(1,1,0) + (-1,-1,0) = 0$

$$\boxed{\begin{aligned} f(1,0,0) &= (1,1,0) \\ f(1,1,0) &= (0,0,0) \\ f(1,1,1) &= (-1,-1,0) \end{aligned}}$$

$(x,y,z) = \lambda_1(1,0,0) + \lambda_2(1,1,0) + \lambda_3(1,1,1)$

$x = \lambda_1 + \lambda_2 + \lambda_3 \qquad \lambda_1 = x - y + z - z$
$y = \lambda_2 + \lambda_3 \qquad \lambda_2 = y - z$
$z = \lambda_3 \qquad \lambda_3 = z$

$f(x,y,z) = (x-y-z, x-y-z, 0)$

$f \circ f(x,y,z) = [x-y-z-(x-y-z), x-y-z-x+y+z, 0] = [0,0,0]$

$f(x,y,z) = (x-y).(1,1,0) + (y-z).(0,0,0) - z(1,1,0)$

v) $f \neq I \quad f \circ f = I \equiv f(v) = v$

$\forall v \in \mathbb{R}^3$

$f \circ f(v) = v \qquad f[f(v)] = (x,y,z) = v$

$\vec{u} + \vec{v} + \vec{w} = (x,y,z)$

$f(1,0,0) = (1,1,1)$
$f(0,1,0) = (1,1,0)$
$f(0,0,1) = (1,0,0)$

$f(1,1,1) = (3,2,1)$
$f(1,1,0) = (2,2,1)$
$f(1,0,0)$

$f[f(x,y,z)] = (x,y,z)$
$f[f(u) + f(v) + f(w)] = (x,y,z)$

$f[f(u)] + f[f(v)] + f[f(w)] = (x,y,z)$
$f[f(1,0,0)] + f[f(0,1,0)] + f[f(0,0,1)] = (1,1,1)$
$f[(1,1,1)] + f[(1,1,0)] + f[(1,0,0)] = (1,1,1)$

$u_1 + v_1 + w_1 = x$
$u_2 + v_2 + w_2 = y$
$u_3 + v_3 + w_3 = z$

$1 + 0 + 0 = x$
$0 + 1 + 0 = y$
$0 + 0 + 1 = z$

vi) $Nu(f) \neq \{0\}$
$Im(f) \neq \{0\}$

$Nu(f) \cap Im(f) = \{0\} \Rightarrow Nu(f) \oplus Im(f)$

$\langle (1,1,1) \rangle \quad \langle (1,0,0), (1,1,0) \rangle$

$$\boxed{\begin{aligned} f(1,1,1) &= (0,0,0) \\ f(1,1,0) &= (1,0,0) \\ f(1,0,0) &= (1,1,0) \end{aligned}}$$

10. $S = \langle (1,1,0,1),(2,1,0,1) \rangle \subseteq \mathbb{R}^4$

i) $\dim \mathbb{R}^4 = Nu(f) + Im(f)$

$4 = 2 + 2$

$$\begin{pmatrix}1&2&0\\1&1&0\\0&0&0\\1&1&0\end{pmatrix} \to \begin{pmatrix}1&2&0\\0&-1&0\\0&0&0\\0&0&0\end{pmatrix} \text{ son LI}$$

$$\boxed{\begin{aligned} f(1,1,0,1)&=(0,0)\\ f(2,1,0,1)&=(0,0)\\ f(0,0,1,1)&=(1,0)\\ f(0,0,1,0)&=(0,1)\end{aligned}}$$

$$\left(\begin{array}{cccc|c}1&2&0&0&0\\1&1&0&0&0\\0&0&1&1&0\\1&1&1&0&0\end{array}\right) \to \left(\begin{array}{cccc|c}1&2&0&0&0\\0&-1&0&0&0\\0&0&1&1&0\\0&0&1&0&0\end{array}\right)$$

$$\left(\begin{array}{cccc|c}1&2&0&0&0\\0&-1&0&0&0\\0&0&1&1&0\\0&0&0&-1&0\end{array}\right) \text{ Li}$$

ii) $\begin{pmatrix}1&2&x_1\\1&1&x_2\\0&0&x_3\\1&1&x_4\end{pmatrix}$ $\left(\begin{array}{cc|c}1&2&x_1\\0&-1&x_2-x_1\\0&0&x_3\\0&0&x_4-x_2\end{array}\right)$ $\left(\begin{array}{cc|c}1&0&x_1+2x_2-2x_1\\0&-1&x_2-x_1\\0&0&x_3\\0&0&x_4-x_2\end{array}\right)$ $\left(\begin{array}{cc|c}1&0&2x_2-x_1\\0&1&x_1-x_2\\0&0&x_3\\0&0&x_4-x_2\end{array}\right)$

$\left(\begin{array}{cc|c}0&1&x_1-x_2\\0&0&x_2-x_4\\0&0&x_3\\1&1&x_4\end{array}\right) \to$ $\left(\begin{array}{cc|c}0&1&x_1-x_2\\0&0&x_2-x_4\\0&0&x_3\\1&0&x_4-x_1-x_2\end{array}\right)$

$\begin{cases} x_3 = 0 \\ x_4 - x_2 = 0 \end{cases}$

$(x_1, x_4, 0, x_4)$

$S = \langle (1,0,0,0),(0,1,0,1) \rangle$

$[2(1,0,0,0)+(0,1,0,1)]$,

iii) $\underbrace{\langle (1,1,0,1),(2,1,0,1)\rangle}_{\text{solución homogéneo}} + \underbrace{(0,1,1,2)}_{\text{sol. particular}}$

$A.X = 0$ $\qquad$ $A.X = b$

$A.X_h = 0$
$A.X_p = b$

$A(X_h+X_p) = b$

$(x_1,x_2,x_3,x_4) = \lambda_1(1,1,0,1)+\lambda_2(2,1,0,1)+\lambda_3(0,0,1,1)+\lambda_4(0,0,1,0)$

$x_1 = \lambda_1 + 2\lambda_2$
$x_2 = \lambda_1 + \lambda_2$
$x_3 = \lambda_3 + \lambda_4$
$x_4 = \lambda_1 + \lambda_2 + \lambda_3$

$x_1 - x_2 = \lambda_2$
$\lambda_1 = x_2 - x_1 + x_2 = 2x_2 - x_1$
$x_4 = -x_1 + 2x_2 + x_1 - x_2 + \lambda_3$

$\lambda_4 = x_3 - x_4 + x_2$
$\lambda_3 = x_4 - x_2$

$f(x_1,x_2,x_3,x_4) = (2x_2-x_1).(0,0) + (x_1-x_2).(0,0) + (x_4-x_2).(1,0) + (x_3-x_4+x_2).(0,1)$

$f(x_1,x_2,x_3,x_4) = (x_4 - x_2,\ x_3 - x_4 + x_2)$

$f(0,1,1,2) = (1,\ 1-2+1)$
$(1,0)$

$M_E f \equiv \underset{\mathbb{R}^{2\times4}}{A} = \begin{pmatrix}0&-1&0&1\\0&1&1&-1\end{pmatrix}$

$\begin{pmatrix}0&-1&0&1\\0&1&1&-1\end{pmatrix}.\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$ homogéneo

$\underset{2\times4}{\begin{pmatrix}0&-1&0&1\\0&1&1&-1\end{pmatrix}}.\underset{4\times1}{\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}} = \begin{pmatrix}1\\0\end{pmatrix}$

no homogéneo

$\left(\begin{array}{cccc|c}0&-1&0&1&0\\0&1&1&-1&0\end{array}\right) \to \left(\begin{array}{cccc|c}0&-1&0&1&0\\0&0&1&0&0\end{array}\right)$ $\quad x_2 = x_4$, $x_3 = 0$

$(x_1, x_4, 0, x_4) = \langle (1,0,0,0),(0,1,0,1) \rangle$

$\left(\begin{array}{cccc|c}0&-1&0&1&1\\0&1&1&-1&0\end{array}\right) \to \left(\begin{array}{cccc|c}0&-1&0&1&1\\0&0&1&0&1\end{array}\right) \to$ $x_3 = 1$, $x_2 = -1 + x_4 \to$

$(x_1, x_4-1, 1, x_4) = \langle (1,0,0,0),(0,1,0,1) \rangle + (0,-1,1,0)$

$$\begin{cases} x_3 - 1 = 0 \\ x_2 + 1 - x_1 = 0 \end{cases}$$

11.

$\mathbb{V}$ ev. dim n

$\alpha_B : \mathbb{V} \to \mathbb{R}^n \qquad \alpha_B(v) = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ coordenadas de v en la base B

es monomorfismo: $0 = (\lambda_1, \lambda_2, \ldots, \lambda_n) \Leftrightarrow \lambda_1 = \lambda_2 = \ldots = \lambda_n = 0$

$\Rightarrow Nu(\alpha_B) = 0$

es epimorfismo: $\alpha_B(\mathbb{V}) = \mathbb{R}^n$

$$v \in \mathbb{V} = \lambda_1 . e_1 + \lambda_2 . e_2 + \ldots + \lambda_n . e_n$$
$$\alpha_B(v) = \lambda_1 \alpha_B(e_1) + \lambda_2 . \alpha_B(e_2) + \ldots + \lambda_n . \alpha_B(e_n)$$
$$= \lambda_1 (1, 0, \ldots 0)$$
$$\alpha_B(\mathbb{V}) = (\lambda_1, \lambda_2, \ldots, \lambda_n) = \mathbb{R}^n$$

$\Rightarrow$ es isomorfismo

12. $f : \mathbb{V} \to \mathbb{V} \qquad f \circ f = f$ si f es proyector

i) f proyector : $\Leftarrow)$

$$f[f(v)] = f(v) \quad \forall v \in Im(f)$$

$\Rightarrow) \quad f(\underbrace{f[v]}_{u}) - f(v) = 0 \qquad u, v \in Im(f)$

$$f(u) - f(v) = 0$$
$$f[u - v] = 0$$
$$\to (u - v) \in Nu(f)$$

Si $u - v = 0 \Rightarrow Nu(f) = 0 \Rightarrow u = v \Rightarrow f(v) = u \Rightarrow \boxed{f(v) = v \quad \forall v \in Im f}$

si $u - v \neq 0 \Rightarrow \dim Nu(f) \geq 1 \Rightarrow v \notin Im f$

ii) $f : \mathbb{R}^3 \to \mathbb{R}^3$

a) $Nu(f)$: $x_1 + x_2 + x_3 = 0 \qquad Im f = \langle (-2, 1, 1) \rangle$

$$x_1 = -x_2 - x_3$$
$$(-x_2 - x_3, x_2, x_3)$$
$$Nu(f) = \langle (-1, 1, 0), (-1, 0, 1) \rangle$$

$$f(-1, 1, 0) = (0, 0, 0)$$
$$f(-1, 0, 1) = (0, 0, 0)$$

como es proyector necesito $f(v) = v$

$$f(-2, 1, 1) = (-2, 1, 1)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \text{ld} \to$$ NO SE PUEDE

$$(x, y, z) = \lambda_1(-1, 1, 0) + \lambda_2(-1, 0, 1) + \lambda_3(0, 1, 0)$$

$x = -\lambda_1 - \lambda_2 \qquad \lambda_1 = -z - x$

$y = \lambda_1 + \lambda_3 \qquad \lambda_2 = z$

$z = \lambda_2 \qquad \lambda_3 = y + z + x$

$$f(x, y, z) = (-z - x) . (0, 0, 0) + z(0, 0, 0) + (y + z + x)(-2, 1, 1)$$

Al trabajar con proyectores tengo la restricción de que $f(x,y,z) = (x,y,z) \quad \forall (x,y,z) \in \mathbb{R}^3 \Rightarrow$ No puedo seleccionar vectores al azar

b)

$$3x_1 - x_3 = 0$$
$$x_1 = \frac{1}{3}x_3$$

$$Im(f) = \langle 1,1,1 \rangle$$

$$\left(\frac{1}{3}x_3, x_2, x_3\right)$$

$$Nu(f) = \left\langle \left(\frac{1}{3},0,1\right), (0,1,0) \right\rangle$$

$$\boxed{\begin{aligned} f(1/3,0,1) &= (0,0,0) \\ f(0,1,0) &= (0,0,0) \\ f(1,1,1) &= (1,1,1) \end{aligned}}$$

$$\left(\begin{array}{ccc|c} 1/3 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -2 & 0 \end{array}\right) \text{ Li}$$

iii) $f: \mathbb{V} \to \mathbb{V}$ $\qquad \mathbb{V} = Nu(f) \oplus Im(f)$

$$\left.\begin{array}{l} Nu(f) \\ Im(f) \end{array}\right\} \text{son s.ev. de } \mathbb{V} \Rightarrow$$

$$\dim \mathbb{V} = \dim Nu(f) + \dim Im(f) \;\wedge\; \dim(Nu(f) + Im(f)) = \dim Nu(f) + \dim Im(f) - \dim\left[Nu(f) \cap Im(f\right.$$

$$\Rightarrow \dim(Nu(f) + Im(f)) = \dim \mathbb{V} - \dim(Nu(f) \cap Im(f))$$

si $\exists \; Nu(f) \cap Im(f) \Rightarrow \exists \; v \in (Im(f) \cap Nu(f))$:

$$f(v) = v \;\wedge\; f(v) = 0$$
$$f(v) + f(v) = v$$
$$2f(v) = v$$
$$f(2v) = v \qquad \text{con } v \in Im(f)$$

no cumple propiedad (1)

iv) $\mathbb{S} \subset \mathbb{V}$, $\mathbb{T} \subset \mathbb{V}$ $\qquad \mathbb{V} = \mathbb{S} \oplus \mathbb{T}$ $\qquad P: \mathbb{V} \to \mathbb{V}$ con

$$Nu(P) = \mathbb{S}$$
$$Im(P) = \mathbb{T}$$

13. $f: V \to V \qquad |f|_{BB'}$

i) $M_{BB'}(f) = \left([f(v_1)]^t_{B'} \quad [f(v_2)]^t_{B'} \quad \dots \quad [f(v_n)]^t_{B'} \right)$

a) $\boxed{M_{BB'}(f) = \begin{pmatrix} 3 & -2 & 1 \\ 5 & 1 & -1 \\ 1 & 3 & 4 \end{pmatrix}}$ $\qquad F(x_1,x_2,x_3) = (3x_1-2x_2+x_3,\ 5x_1+x_2-x_3,\ x_1+3x_2+4x_3$

b) $B = \{(1,2,1),(-1,1,3),(2,1,1)\}$ $\qquad F(1,2,1) = (0,6,11$

$B' = \{(1,1,0),(1,2,3),(2,3,4)\}$ $\qquad F(-1,1,3) = ($ $\qquad F(2,1,1) = ($

$$M_{BB'}(f) = \left(\begin{bmatrix} 0 \\ 6 \\ 11 \end{bmatrix}_{B'} \begin{bmatrix} -2 \\ -7 \\ 14 \end{bmatrix}_{B'} \begin{bmatrix} 5 \\ 10 \\ 9 \end{bmatrix}_{B'} \right) = \boxed{\begin{pmatrix} 1 & -26 & 6 \\ 13 & -34 & 11 \\ -7 & 29 & -6 \end{pmatrix}}$$

$$(0,6,11) = \lambda_1(1,1,0) + \lambda_2(1,2,3) + \lambda_3(2,3,4)$$

$$\begin{aligned} 0 &= \lambda_1 + \lambda_2 + 2\lambda_3 \\ 6 &= \lambda_1 + 2\lambda_2 + 3\lambda_3 \\ 11 &= 3\lambda_2 + 4\lambda_3 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 1 & 2 & 3 & 6 \\ 0 & 3 & 4 & 11 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 6 \\ 0 & 3 & 4 & 11 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & 1 & -7 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 1 & -6 \\ 0 & 1 & 0 & 13 \\ 0 & 0 & 1 & -7 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 13 \\ 0 & 0 & 1 & -7 \end{array}\right)$$

$\lambda_3 = -7$
$\lambda_2 = 13$
$\lambda_1 = 1$

$$(-2,-7,14) = \lambda_1(1,1,0) + \lambda_2(1,2,3) + \lambda_3(2,3,4)$$

$$\begin{aligned} -2 &= \lambda_1 + \lambda_2 + 2\lambda_3 \\ -7 &= \lambda_1 + 2\lambda_2 + 3\lambda_3 \\ 14 &= 3\lambda_2 + 4\lambda_3 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 1 & 2 & 3 & -7 \\ 0 & 3 & 4 & 14 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 0 & 1 & 1 & -5 \\ 0 & 3 & 4 & 14 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 1 & 29 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & -34 \\ 0 & 0 & 1 & 29 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 0 & -26 \\ 0 & 1 & 0 & -34 \\ 0 & 0 & 1 & 29 \end{array}\right)$$

$\lambda_3 = 29$
$\lambda_2 = -34$
$\lambda_1 = -26$

$$(5,10,9) = \lambda_1(110) + \lambda_2(123) + \lambda_3(234)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 1 & 2 & 3 & 10 \\ 0 & 3 & 4 & 9 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & 1 & 1 & 5 \\ 0 & 3 & 4 & 9 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & -6 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & -6 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 2 & -6 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & -6 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & -6 \end{array}\right)$$

$\lambda_3 = -6$
$\lambda_2 = 11$
$\lambda_1 = 6$

14. $B = \{V_1, V_2, V_3\}$

$B' = \{\omega_1, \omega_2, \omega_3, \omega_4\}$

$f: \mathbb{R}^3 \longrightarrow \mathbb{R}^4$

$$M_{BB'}|f| = \begin{pmatrix} 1 & -2 & 1 \\ -1 & 1 & -1 \\ 2 & 1 & 4 \\ 3 & -2 & 5 \end{pmatrix}$$

i) $f(\underbrace{3V_1 + 2V_2 - V_3}_{U})$

$$\boxed{M_{BB'}(f) \cdot [v]_B = [f(v)]_{B'}}$$

$[U]_B = (3, 2, -1)$

$$\underset{4\times3}{\begin{pmatrix} 1 & -2 & 1 \\ -1 & 1 & -1 \\ 2 & 1 & 4 \\ 3 & -2 & 5 \end{pmatrix}} \cdot \underset{3\times1}{\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}} = \underset{4\times1}{\begin{pmatrix} -2 \\ 0 \\ 4 \\ 0 \end{pmatrix}}$$

$$\boxed{[f(U)]_{B'} = (-2, 0, 4, 0)}$$

$$\boxed{f(U) = -2\omega_1 + 4\omega_3}$$

ii) base del $N_u(f)$

$f(v) = 0 \Rightarrow v \in N_u(f) \therefore$

$$\begin{pmatrix} 1 & -2 & 1 \\ -1 & 1 & -1 \\ 2 & 1 & 4 \\ 3 & -2 & 5 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ -1 & 1 & -1 & 0 \\ 2 & 1 & 4 & 0 \\ 3 & -2 & 5 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 5 & 2 & 0 \\ 0 & 4 & 2 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$c = 0$
$b = 0$
$a = 0$

$[a, b, c]_B = 0, 0, 0$

$a, b, c = 0 \cdot V_1 + 0 \cdot V_2 + 0 \cdot V_3 \Rightarrow \boxed{N_u(f) = 0}$

base de $Im(f)$

$[f(V_1)]_{B'} = (1, -1, 2, 3) \qquad f(V_1) = \omega_1 - \omega_2 + 2\omega_3 + 3\omega_4$

$[f(V_2)]_{B'} = (-2, 1, 1, -2) \qquad f(V_2) = -2\omega_1 + \omega_2 + \omega_3 - 2\omega_4$

$[f(V_3)]_{B'} = (1, -1, 4, 5) \qquad f(V_3) = \omega_1 - \omega_2 + 4\omega_3 + 5\omega_4$

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ -1 & 1 & -1 & 0 \\ 2 & 1 & 4 & 0 \\ 3 & -2 & 5 & 0 \end{array}\right)$$ son Li $\Rightarrow$ son Li $\Rightarrow$

$$\boxed{Im(f) = \langle (\omega_1 - \omega_2 + 2\omega_3 + 3\omega_4), (-2\omega_1 + \omega_2 + \omega_3 - 2\omega_4), (\omega_1 - \omega_2 + 4\omega_3 + 5\omega_4)}$$

son base

iii) $f^{-1}(\underbrace{\omega_1 - 3.\omega_3 - \omega_4}_{u})$

$[u]_{B'} = (1,0,-3,-1)$

$(M_{BB'}\, f)^{-1}.$

$(M_{B'B}\, f^{-1}).[u]_{B'} = [f^{-1}(u)]_B$

$\boxed{f^{-1} \nexists \text{ aquí porque } f \text{ no es isomorfismo}}$

$M_{BB'}(f).[v]_B = [f(v)]_{B'}$

$[v]_B = (M_{BB'}\, f)^{-1}.[f(v)]_{B'}$

$[v]_B = M_{B'B}\, f^{-1}.[f(v)]_{B'}$

$[f^{-1}(u)]_B = M_{B'B}\, f^{-1}.[u]_{B'}$

$$\begin{tikzcd} v \arrow[r, bend left, "f"] & f(v) \arrow[l, bend left, "f^{-1}"] \end{tikzcd}$$

$f: V \to W$

base B base B'

base B' base B

$f^{-1}: W \to V$