PRACTICA 2

1.

i) $V = \mathbb{K}^{\mathbb{N}} = \{ \{a_i\}_{i\in\mathbb{N}} = (a_1, a_2, \dots, a_n, \dots) : a_i \subset \mathbb{K}\ \forall i \in \mathbb{N} \}$ las sucesiones de elementos de $\mathbb{K}$

① $\emptyset \in V$ pues $a_n = 0 \in V$ pues $\lim\limits_{n\to\infty} a_n = 0$

② $\left.\begin{array}{l} a_n \to 0 \\ B_n \to 0 \end{array}\right\} \Rightarrow (a_n + B_n) \to 0$

③ $a_n \to 0 \} \Rightarrow k.a_n \to 0$ con $k \in \mathbb{R}$

ii) $V = \mathbb{K}^X = \{ f: X \to \mathbb{K} : f$ es una función $\}$

① $\emptyset \in V$ si $f(x) = 0\ \forall x$

② $f(x) = g(x) = 0 \Rightarrow (f+g)(x) = 0 \wedge 0 \in V$

③ $k.f(x) = k.0 \Rightarrow k.f(x) = 0 \wedge 0 \in V$

2.

puntos origen $(0,0)$

eje x $\mathbb{L} = \lambda(1,0)$

eje y $\mathbb{L} = \lambda.(0,1)$

recta $\mathbb{L} = \lambda.V$ (que pase por el $\vec{0} \in \mathbb{R}^2$)

3.

V es espacio vectorial sobre K

i) $S_1 = \{ v \in \mathbb{R}^3 : v = a(1,0,0) + b(1,1,1)\ ;\ a,b \in \mathbb{R} \}$ $V = \mathbb{R}^3$ $K = \mathbb{R}$

① $0 \in S_1$ $0 = a(1,0,0) + b(1,1,1)$ si $a = b = 0$

② $v_1 + v_2 \in S_1$ $[a_1(1,0,0) + b_1(1,1,1)] + [a_2(1,0,0) + b_2(1,1,1)]$

$(a_1 + a_2)(1,0,0) + (b_1 + b_2).(1,1,1) \to$ con $\left.\begin{array}{l} a_1 + a_2 \in \mathbb{R} \\ b_1 + b_2 \in \mathbb{R} \end{array}\right\}$ garantizado por ser $\mathbb{R}$ un cuerpo

③ $k.v_1 \in S_1$ $k.[a_1(1,0,0) + b_1(1,1,1)]$

$a_1.k(1,0,0) + b_1.k(1,1,1)$ con $a_1.k, b_1.k \in \mathbb{R}$

ii)

4. $A \in \mathbb{K}^{n \times m}$, $S = \{x \in \mathbb{K}^m : A.x = 0\}$

① $\vec{0} \in S$ pues $A.\vec{0} = 0$ donde $\vec{0} \in \mathbb{K}^m$

② Sean $x_1, x_2 \in S \Rightarrow$

$$\begin{array}{l} A.x_1 = 0 \\ A.x_2 = 0 \end{array} \Rightarrow A(x_1 + x_2) = 0 + 0 = 0$$

$$\Rightarrow x_1 + x_2 \in S$$

③ Sea $\lambda \in \mathbb{R}$

$$\lambda . A.x = \lambda 0 \Rightarrow$$
$$A(\lambda x) = 0 \Rightarrow \lambda x \in S$$

Por 1,2,3 S es subespacio vectorial de $\mathbb{K}^m$

5. S, T subespacios de $\mathbb{V}$ (1)

i) $S \cap T$

① $0 \in (S \cap T)$ pues $\left.\begin{array}{l} 0 \in S \\ 0 \in T \end{array}\right\} \Rightarrow 0 \in (S \cap T)$

por (1)

② si $v, u \in (S \cap T)$
$(v+u) \in (S \cap T)$ $\quad \left.\begin{array}{l} u, v \in S \\ u, v \in T \end{array}\right\} \Rightarrow u, v \in (S \cap T)$

por (1) $\left.\begin{array}{l} u + v \in S \\ u + v \in T \end{array}\right\} \Rightarrow (u+v) \in (S \cap T)$

③ si $\lambda \in \mathbb{R}$, $v \in (S \cap T)$
$(\lambda . v) \in (S \cap T)$ $\quad \left.\begin{array}{l} v \in S \\ v \in T \end{array}\right\} \Rightarrow \left.\begin{array}{l} \lambda . v \in S \\ \lambda . v \in T \end{array}\right\} \Rightarrow (\lambda v) \in (S \cap T)$

por (1)

ii) Sean $S = \lambda(1,0)$ $\quad$ S es subespacio de $\mathbb{R}^2$
$T = \lambda(0,1)$ $\quad$ T " " " "

$$(S \cup T) = \{ \bar{x} \in \mathbb{R}^2 : \bar{x} \in S \text{ ó } \bar{x} \in T \}$$

① $0 \in (S \cup T)$ pues $(0,0) \in S$

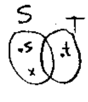
② $\begin{array}{l} (a,0) \in S \\ (0,b) \in T \end{array}$ $\quad a, b \neq 0 \Rightarrow (a,0) + (0,b) \notin S, T \Rightarrow \notin S \cup T$

$\Rightarrow S \cup T$ no es subespacio

iii) Sea S : $\quad \forall s \in S$ se cumple $\underline{s \in T} \vee s \notin T \Rightarrow$
$\hookrightarrow s \in S \cap T$

Sea T : $\quad \forall t \in T$ se cumple $\underline{t \in S} \vee t \notin S$
$\hookrightarrow t \in T \cap S$

Luego supongamos $\exists$ algún $s \in S : s \notin T \Rightarrow S \not\subseteq T$



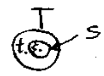
② Sea $(s+t)$ donde $\begin{array}{l} s \in S \wedge s \notin T \\ t \in T \wedge t \notin S \end{array}$ $\Rightarrow$ supongamos $(s+t) \in S \cup T$

$\Rightarrow \underbrace{(s+t) \in S}_{\mathrm{I}}$, o bien $\underbrace{(s+t) \in T}_{\mathrm{II}}$, o bien $\underbrace{(s+t) \in S \cap T}_{\mathrm{III}}$

I) como S es ev $\Rightarrow [(s+t) + (-s)] \in S$ pero $t \in S \Rightarrow$ absurdo
II) como T es ev $\quad [(s+t) + (-t)] \in T$ pero $s \in T \Rightarrow$ absurdo
III) no es válido por hipótesis

$\Rightarrow$ $S \cup T$ no es subespacio vectorial si $S \not\subseteq T$

$\Leftarrow$ Sea $S \subseteq T \Rightarrow s \in T \ \forall s \in S \Rightarrow$



② $(s+t) \in (S \cup T)$ pues $\left.\begin{matrix} s \in T \\ t \in T \end{matrix}\right\}$ y T es s.e.v.

③ $\begin{matrix} \lambda . s \in (S \cup T) \\ \lambda . t \in (S \cup T) \end{matrix}$ pues $\left.\begin{matrix} \lambda . s \in S \\ \lambda . t \in T \end{matrix}\right\}$ por ser S y T s.e.v

① $0 \in S \cup T$ pues $\left.\begin{matrix} 0 \in S \\ 0 \in T \end{matrix}\right\}$ por ser s.e.v.

$\Rightarrow$ $S \cup T$ es s.e.v.

① $0 \in S \cup T \Rightarrow 0 \in S, 0 \in T, 0 \in S \cap T$

② $(s+t) \in S \cup T$ donde $\begin{matrix} s \in S \wedge s \notin T \\ s \in T \wedge s \notin S \end{matrix}$ $\Rightarrow \underset{I}{(s+t) \in S}, \underset{II}{(s+t) \in T}, (s+t) \in S \cap T$

III: no vale por hipótesis

I) lleva a $(s+t)+(-s) \in S$ $t \in S$ absurdo

II) lleva a $s+t+-t \in T$ $s \in T$ absurdo

Luego para que valgan I ó II necesito que $s \in T \ \forall s \in S \Rightarrow$ II es válido $\Rightarrow$ ② se cumple $\Rightarrow$

③ $\begin{matrix} \lambda . s \in S \cup T \\ \lambda . t \in S \cup T \end{matrix}$ $\Rightarrow$ si $s \in S \wedge s \notin T$ no vale con lo cual necesito $\rightarrow S \subseteq T$

$\boxed{\text{NOTA}}$. $T \subseteq S$ es demostrable en forma similar

6.

i) $\langle K^n \rangle = (e_1, e_2, \ldots, e_n)$ donde $\begin{matrix} e_1 = (1,0,0,\ldots,0) \\ e_2 = (0,1,0,\ldots,0) \\ \cdots \\ e_n = (0,0,0,\ldots,1) \end{matrix}$

ii) $K_n[X] = \{ f \in K[x] : f = 0 \text{ ó grado } f < n \}$

$$\begin{matrix} (1,0,0,\ldots,0) \\ (0,t,0,\ldots,0) \\ (0,0,t^2,\ldots,0) \\ (0,0,0,t^3,\ldots,0) \\ (0,0,0,0,\ldots,t^n) \end{matrix}$$

iii) $\langle K[X] \rangle = \{ (1,0), (0,x) \}$

vi) $S_1 = \{ (x,y,z) \in \mathbb{R}^3 : x+y-z=0 ; x-y=0 \}$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 1 & -1 & 0 & 0 \end{array}\right) \quad \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -2 & 1 & 0 \end{array}\right)$$

$x=y$
$x-y=0$
$x+y-2y=0$
$z=2y$

$S_1 = \langle (1,1,2) \rangle$

solución $= \lambda(1,1,2)$

iv) $K^{n \times n} = \left\langle \begin{pmatrix} 1 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ \vdots & & & \vdots \\ 0 & \ldots & \ldots & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ & & \ddots & \\ 0 & \ldots & \ldots & 0 \end{pmatrix}, \ldots \begin{pmatrix} 0 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ & & \ddots & \\ 0 & \ldots & \ldots & 1 \end{pmatrix} \right\rangle$

7. $S = \langle (1,-1,2,1),(3,1,0,-1),(1,1,-1,-1) \rangle \subseteq \mathbb{R}^4$

i) $v = (2,1,3,5) = x_1(1,-1,2,1) + x_2(3,1,0,-1) + x_3(1,1,-1,-1)$

$$\Downarrow$$

$\boxed{v \notin S}$ porque no puedo hallar combinación lineal que satisfaga el sistema

$$\left(\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ -1 & 1 & 1 & 1 \\ 2 & 0 & -1 & 3 \\ 1 & -1 & -1 & 5 \end{array}\right) \left(\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 0 & 4 & 2 & 3 \\ 0 & 2 & 1 & -7 \\ 1 & -1 & -1 & 5 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 0 & 4 & 2 & 3 \\ 0 & 2 & 1 & -7 \\ 0 & -4 & -2 & 3 \end{array}\right) \left(\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 0 & 4 & 2 & 3 \\ 0 & 2 & 1 & -7 \\ 0 & 0 & 0 & 6 \end{array}\right)$$

ii) $S \subseteq \{\bar{x} \in \mathbb{R}^4 : x_1 - x_2 - x_3 = 0\}$?

$\left(\begin{array}{cccc|c} 1 & -1 & -1 & 0 & 0 \end{array}\right)$ $\quad x_1 = x_2 + x_3$

$= (x_2+x_3, x_2, x_3, x_4)$

$= \alpha(1,1,0,0) + \beta(1,0,1,0) + \gamma(0,0,0\ldots$

hay valores α, β, γ que permiten obtener a los generadores de S

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right) \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right) \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$\alpha = -1$
$\gamma = 1$
$\beta = 2$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right) \left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right) \left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$\alpha + \beta = 1$
$\alpha = -1$

incompatibilidad

No puedo generar uno de los generadores de S $\Rightarrow$ $\boxed{S \not\subseteq \{\bar{x} \in \mathbb{R}^4 : x_1 - x_2 - x_3 = 0\}}$

iii) $\underbrace{\{x \in \mathbb{R}^4 :\ x_1 - x_2 - x_3 = 0\}}_{\equiv T} \subseteq S$?

$T = \langle (1,1,0,0); (1,0,1,0); (0,0,0,1) \rangle \qquad S = \langle (1,-1,2,1); (3,1,0,-1); (1,1,-1,1) \rangle$

$$\left(\begin{array}{ccc|c} 1 & 3 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 2 & 0 & -1 & 0 \\ 1 & -1 & -1 & 0 \end{array}\right) \left(\begin{array}{ccc|c} 1 & 3 & 1 & 1 \\ 0 & 4 & 2 & 2 \\ 0 & -6 & -3 & -2 \\ 0 & -4 & -2 & -1 \end{array}\right) \left(\begin{array}{ccc|c} 1 & 3 & 1 & 1 \\ 0 & 4 & 2 & 2 \\ 0 & -6 & -3 & -2 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

$\underbrace{0\ 0\ 0\ |\ 1}_{\text{incompatibilidad}}$

hay un generador de T que no se puede generar con los generadores de S

$\Rightarrow \exists\ t \in T :\ t \not\subset S \Rightarrow \boxed{T \not\subseteq S}$

8. $v_1, v_2, v_3 \in \mathbb{V}$ e.v.

$$v_1 + 3v_2 - v_3 = 0$$
$$2v_1 - v_2 - v_3 = 0$$

Supongamos v_1, v_2, v_3 generan $S \Rightarrow \quad S = \langle v_1, v_2, v_3 \rangle$

pero $\quad v_1 + 3v_2 - v_3 = 0 \qquad\qquad 2v_1 - v_2 - v_3 = 0$

$\qquad v_1 = v_3 - 3v_2 \qquad\qquad v_1 = \frac{v_3}{2} + \frac{v_2}{2}$

$$v_3 - 3v_2 = \frac{v_3}{2} + \frac{v_2}{2}$$
$$\frac{1}{2}v_3 - \frac{5}{2}v_2 = 0$$
$$\frac{1}{2}v_3 = \frac{5}{2}v_2 \Rightarrow v_2 = \frac{1}{5}v_3$$

$$v_1 = v_3 - 3.\frac{1}{5}v_3 = \frac{2}{5}v_3$$
$$v_1 = \frac{2}{5}v_3$$

$$\{v_1, v_2, v_3\} = \left\{\frac{2}{5}v_3, \frac{1}{5}v_3, v_3\right\}$$
$$\{v_1, v_2, v_3\} = v_3\left\{\frac{2}{5}, \frac{1}{5}, 1\right\}$$

todo $s \in S$ se escribe $\qquad s = a v_1 + b v_2 + c v_3$

Pero por lo visto ambos se puede escribir $\qquad s = a.\frac{2}{5}v_3 + b.\frac{1}{5}v_3 + c.1.v_3$

$$s = \underbrace{\left(a.\frac{2}{5} + \frac{b}{5} + c\right)}_{\in \mathbb{R}}.v_3 \Rightarrow$$

$$S = \langle v_3 \rangle$$

9.

i) $\mathbb{V}$ e.v, $v, w \in \mathbb{V}$

$$\langle v, w \rangle = \langle v, w + 5v \rangle$$
$$a v + b w = c v + d w + 5e.v$$
$$a.v + b.w = (5e + c).v + d.w$$

$\boxed{\text{Verdadero}}$ pues $\langle v, w \rangle = \langle v, w \rangle$

ii) $v_1, v_2, v_3, v_4, w \in \mathbb{R}^7 :\ \langle v_1, v_2, w \rangle = \langle v_3, v_4, w \rangle = S \Rightarrow$

todo $s \in S$ se escribe $\qquad s = a v_1 + b v_2 + c w$

$\qquad\qquad\qquad\qquad\qquad s = d v_3 + e v_4 + m w$

$$a v_1 + b v_2 + c w = d v_3 + e v_4 + m w$$
$$a v_1 + b v_2 + (c - m).w = d v_3 + e v_4$$
$$\langle v_1, v_2, w \rangle = \langle v_3, v_4 \rangle \Rightarrow \boxed{\text{Falso}}$$

iii) $v_1, v_2, v_3, w \in \mathbb{V}$ e.v

$\langle v_1, v_2, v_3, w\rangle = \langle v_1, v_2, v_3\rangle \iff w \in \langle v_1, v_2, v_3\rangle$

significa

$\Rightarrow$ todo $s \in S = \langle v_1, v_2, v_3, w\rangle = \langle v_1, v_2, v_3\rangle$

$$s = c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 w$$
$$s = c_5 v_1 + c_6 v_2 + c_7 v_3$$

$$0 = (c_1 - c_5) v_1 + (c_2 - c_6) v_2 + (c_3 - c_7) v_3 + c_4 w$$
$$c_4 . w = (c_5 - c_1) v_1 + (c_6 - c_2) v_2 + (c_7 - c_3) v_3 \Rightarrow$$

$$w = \frac{1}{c_4} . \left[(c_5 - c_1) v_1 + (c_6 - c_2) v_2 + (c_7 - c_3) v_3\right] \quad \text{si} \quad c_4 \neq 0 \Rightarrow$$

$w = \langle v_1, v_2, v_3\rangle$ pues es c.l. de ellos $\Rightarrow$ $\boxed{\text{es verdadero}}$

$\Leftarrow$ $w \in \langle v_1, v_2, v_3\rangle \Rightarrow w = a v_1 + b v_2 + c v_3$

Sea $S = \langle v_1, v_2, v_3\rangle \Rightarrow$

$s \in S$ es tal que $s = c_1 v_1 + c_2 v_2 + c_3 v_3$

pero sea $c_1 = a + d$, $c_2 = b + e$, $c_3 = c + m \Rightarrow$

$$s = (a+d) v_1 + (b+e) v_2 + (c+m) . v_3 =$$
$$s = a v_1 + b v_2 + c v_3 + d v_1 + e v_2 + m v_3$$
$$s = w + d v_1 + e v_2 + m v_3 \Rightarrow$$
$$S = \langle w, v_1, v_2, v_3\rangle \Rightarrow$$

$\langle v_1, v_2, v_3\rangle = \langle w, v_1, v_2, v_3\rangle \Rightarrow$ $\boxed{\text{es verdadero}}$

11.

i)

$$\begin{pmatrix} 1 & 2 & 1 & 5 & 0 \\ 2 & 3 & 1 & 1 & 0 \\ 3 & 1 & 4 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 1 & 5 & 0 \\ 0 & -1 & -1 & -9 & 0 \\ 0 & -5 & 1 & -14 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 1 & 5 & 0 \\ 0 & 5 & 5 & 45 & 0 \\ 0 & -5 & 1 & -14 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 1 & 5 & 0 \\ 0 & 5 & 5 & 45 & 0 \\ 0 & 0 & 6 & 31 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 1 & 5 & 0 \\ 0 & 1 & 1 & 9 & 0 \\ 0 & 0 & 6 & 31 & 0 \end{pmatrix} \Rightarrow$$

$$c = -\frac{31}{6} d$$

$$b + -\frac{31}{6} d + 9d = 0$$
$$b + \frac{23}{6} d = 0$$
$$b = -\frac{23}{6} d$$

Eligiendo $d \neq 0$ tengo que los vectores son L.d

$$a - \frac{46}{6} d - \frac{31}{6} d + 5d = 0$$
$$a = \frac{47}{6} d$$

son linealmente dependientes $\left(\frac{47}{6} d, -\frac{23}{6} d, -\frac{31}{6} d, d\right)$

ii)

$$\begin{pmatrix} 1-i & 2 & 0 \\ i & -1+i & 0 \end{pmatrix} = \begin{pmatrix} 1-i & 2 & 0 \\ 1 & 1+i & 0 \end{pmatrix} = \begin{pmatrix} 1-i & 2 & 0 \\ 1-i & 2 & 0 \end{pmatrix} = \begin{pmatrix} 1-i & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$(1-i).(1+i)$
$1^2 - i^2$
$1 + 1 = 2$

$$(1-i) a + 2b = 0$$
$$(1-i) . a = -2b$$
$$-\frac{1}{2}(1-i) . a = b$$

si $K = \mathbb{R}$

la ecuación solo se verifica si $a = b = 0$

iii) $(1-x)^3, (1-x)^2, 1-x, 1$ en $K[X]$

$(1-3x^2+3x-x^3), (1-2x+x^2), (1-x), (1)$

$$\left(\begin{array}{cccc|c} -1 & 0 & 0 & 0 & 0 \\ -3 & 1 & 0 & 0 & 0 \\ 3 & -2 & -1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right) \rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 3 & -2 & -1 & 0 & 0 \\ -3 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{array}\right)$$

$\boxed{\text{son linealmente independientes}}$

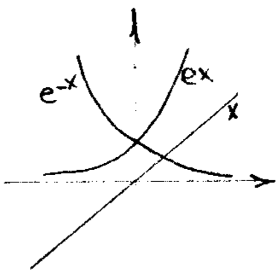
iv) $(\operatorname{sen} x), (\cos x)$ $\mathbb{R}^{\mathbb{R}}$, $K = \mathbb{R}$

$\lambda_1 \cdot \operatorname{sen} x + \lambda_2 \cdot \cos x = 0$

$\lambda_1 = -\lambda_2 \cdot \frac{\cos x}{\operatorname{sen} x}$ si $\operatorname{sen} x \neq 0$

$\boxed{\text{son linealmente dependientes}}$

v) e^x, x, e^{-x} $\mathbb{R}^{\mathbb{R}}$, $K = \mathbb{R}$



$\lambda_1 e^x + \lambda_2 x + \lambda_3 e^{-x} = 0$

$\lambda_2 x + \lambda_1 e^x + \lambda_3 \frac{1}{e^x} = 0$

$\lambda_2 x = \lambda_1 e^x + \lambda_3 \frac{1}{e^x}$

$\left(\begin{array}{ccc|c} x & e^x & e^{-x} & 0 \end{array}\right)$

$x = -e^x - e^{-x} < 0$

13. busco independencia lineal

i) $\alpha(1,2,k)+\beta(1,1,1)+\gamma(0,1,1-k)=\vec{0}\in\mathbb{R}^3 \iff \alpha=\beta=\gamma=0$

$$\alpha\underbrace{\begin{bmatrix}1\\2\\k\end{bmatrix}}_{V_1}+\beta\underbrace{\begin{bmatrix}1\\1\\1\end{bmatrix}}_{V_2}+\gamma\underbrace{\begin{bmatrix}0\\1\\1-k\end{bmatrix}}_{V_3}=\underbrace{(V_1|V_2|V_3)}_{\mathbb{R}^{3\times3}}\cdot\underbrace{\begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix}}_{\mathbb{R}^{3\times1}}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$A=\left(\begin{array}{ccc|c}1&1&0&0\\2&1&1&0\\k&1&1-k&0\end{array}\right)\left(\begin{array}{ccc|c}1&1&0&0\\0&-1&1&0\\k&1&1-k&0\end{array}\right)$$

tiene solución única trivial si $\det A\neq0 \Rightarrow$

$$(\cdot[-1(1-k)]+k+0)-(0+0+1)=-1+k+k-1=2k-2$$

$$2(k-1)\neq0$$
$$k\neq1$$

$$k=0\quad\left(\begin{array}{ccc|c}1&1&0&0\\2&1&1&0\\0&1&1&0\end{array}\right)=\left(\begin{array}{ccc|c}1&1&0&0\\0&-1&1&0\\0&1&1&0\end{array}\right)=\left(\begin{array}{ccc|c}1&1&0&0\\0&-1&1&0\\0&0&2&0\end{array}\right)$$

$\boxed{\text{Si } k\neq1 \Rightarrow \text{el conjunto es L.i}}$

ii)
$$A=\left(\begin{array}{ccc|c}k&3&k&0\\1&-1&2&0\\0&2&-2&0\end{array}\right)$$

$$\det. A=(2k+0+2k)-(0+(-6)+4k)=6\neq0\Rightarrow$$

siempre $\det A\neq0 \Rightarrow$ siempre hay solución única trivial $Ax=0$

$\boxed{\forall k\in\mathbb{R}\Rightarrow \text{el conjunto es L.i.}}$

14. i)
$$\left(\begin{array}{ccc|c}1&1&3&0\\4&-3&-8&0\\-2&-1&-2&0\\1&2&7&0\end{array}\right)=\left(\begin{array}{ccc|c}1&1&3&0\\0&1&4&0\\0&-5&-12&0\\0&1&4&0\end{array}\right)=\left(\begin{array}{ccc|c}1&1&3&0\\0&1&4&0\\0&0&8&0\\0&0&0&0\end{array}\right)$$

↑ la última columna se quita

De los generadores (tres) tres son Li $\Rightarrow$

$$B=\{(1,4,-2,1),(1,-3,-1,2),(3,-8,-2,7)\}$$

$\boxed{\text{dimensión}=3}$

ii)
$$x_1+x_2+x_3+x_4=0$$
$$x_1=-x_2-x_3-x_4$$

$$X=(-x_2-x_3-x_4,\,x_2,\,x_3,\,x_4)$$
$$X=\alpha(-1,1,0,0)+\beta(-1,0,1,0)+\gamma(-1,0,0,1)$$
$$X=\langle(-1,1,0,0),(-1,0,1,0),(-1,0,0,1)\rangle$$

$$\left(\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

los 3 son Li $\Rightarrow$ son una base $\boxed{\text{dim.} = 3}$

iii) ① sea $K = \mathbb{R}$

$\mathbb{C} = \langle (1,0), (0,i) \rangle$ base $\boxed{\text{dim.} = 2}$

$\left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & i & 0 \end{array}\right)$ son Li

② sea $K = \mathbb{C}$

$\underset{}{\overset{\in \mathbb{C}}{(\lambda + \alpha . i)}} . \overset{\in \mathbb{C}}{(a + bi)}$

$\lambda . a + \alpha . a i + \lambda . bi - \alpha . b = (\lambda . a - \alpha . b) + (\alpha . a + \lambda . b) . i \in \mathbb{C}$

17.

i)

$$\begin{pmatrix} 1 & 1 & 1 & 5 & | & 0 \\ 1 & 3 & 1 & 1 & | & 0 \\ 2 & 5 & 4 & 1 & | & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 5 & | & 0 \\ 0 & 2 & 0 & -4 & | & 0 \\ 0 & 3 & 2 & -9 & | & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 5 & | & \\ 0 & 2 & 0 & -4 & | & 0 \\ 0 & 0 & 2 & -3 & | & 0 \end{pmatrix}$$

se tira

$$\boxed{B = \{(1,1,2),\ (1,3,5),\ (1,1,4)\}}$$

ii)

$$S = \langle (X^2+2X+1),\ (X^2+3X+1),\ (X+2) \rangle \qquad \mathbb{R}[X]$$

$$\alpha_1 (X^2+2X+1) + \alpha_2 (X^2+3X+1) + \alpha_3 (X+2) = 0$$

$$\alpha_1 X^2 + \alpha_1 2X + \alpha_1 + \alpha_2 X^2 + \alpha_2 3X + \alpha_2 + \alpha_3 X + \alpha_3 2 = 0$$

$$(\alpha_1 + \alpha_2) X^2 + (2\alpha_1 + 3\alpha_2 + \alpha_3) X + (\alpha_1 + \alpha_2 + \alpha_3 2) = 0$$

$$\alpha_1 + \alpha_2 = 0 \quad \Rightarrow \quad \alpha_1 = -\alpha_2$$

$$2\alpha_1 + 3\alpha_2 + \alpha_3 = 0$$
$$-2\alpha_2 + 3\alpha_2 + \alpha_3 = 0 \quad \Rightarrow \quad \alpha_2 + \alpha_3 = 0 \quad \Rightarrow \quad \alpha_3 = -\alpha_2$$
$$\Rightarrow \alpha_3 = \alpha_1$$

$$\alpha_1 + \alpha_2 + 2\alpha_3 = 0$$
$$\alpha_3 - \alpha_3 + 2\alpha_3 = 0 \quad \Rightarrow \quad 2\alpha_3 = 0 \quad \Rightarrow \quad \alpha_3 = 0$$
$$\Rightarrow \alpha_1 = \alpha_2 = \alpha_3 = 0$$
$$\Rightarrow \boxed{\text{ya conforman una base}}$$

iii)

$$S = \left\langle \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & i \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & i \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \right\rangle \subseteq \mathbb{C}^{2\times 2}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & i & i & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & i & i & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 & | & 0 \\ 0 & i & i & 0 & | & 0 \\ 0 & 0 & -1 & -1 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$$

tiro el último

$$\boxed{B = \left\langle \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & i \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & i \\ 0 & 0 \end{pmatrix} \right\rangle}$$

<u>con $K = \mathbb{C}$</u>

<u>con $K = \mathbb{R}$</u>

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \\ 1 & 1 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 & | & 0 \\ 0 & 1 & 0 & -1 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$$

tiro estos dos

$$\boxed{B = \left\langle \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \right\rangle}$$

16. i)

$$\begin{pmatrix} 1 & 0 & a & e & 0 \\ 1 & 2 & b & f & 0 \\ 1 & 1 & c & g & 0 \\ 1 & 1 & d & h & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & a & e & 0 \\ 0 & 2 & b-a & f-e & 0 \\ 0 & 1 & c-a & g-e & 0 \\ 0 & 0 & d-c & h-g & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & a & e & 0 \\ 0 & 2 & b-a & f-e & 0 \\ 0 & 0 & 2(c-a)-(b-a) & 2(g-e)-(f-e) & 0 \\ 0 & 0 & d-c & h-g & 0 \end{pmatrix}$$

$d-c=0 \Rightarrow d=c$

$h-g \neq 0 \Rightarrow h \neq g$

$f-e=0 \Rightarrow f=e$

$b-a=0 \Rightarrow b=a$

$2(g-e)-(f-e) = 2g-2e-f+e$

$2g-e-f=0$

$2g=e+f$

$2g=2e$

$\boxed{c=d}$ $\boxed{b=a}$ $\boxed{g=e=f} \neq h$

$g=e=f=1 \quad h=2 \quad a=3 \quad d=-1$

$$\begin{pmatrix} 1 & 0 & 3 & 1 & 0 \\ 1 & 2 & 3 & 1 & 0 \\ 1 & 1 & -1 & 1 & 0 \\ 1 & 1 & -1 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 3 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & -1 & -4 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 3 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & -8 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \Rightarrow$$ son L.i $\Rightarrow$ son base de V

$$\boxed{B = \{(1,1,1,1),(0,2,1,1),(3,3,-1,-1),(1,1,1,2)\}}$$

ii)

$\alpha_1(X^3-2X+1)+\alpha_2(X^3+3X)$ $\qquad V=\mathbb{R}_3[X]$

$$\alpha_1(X^3-2X+1)+\alpha_2(X^3+3X)+\alpha_3(aX^3+bX^2+cX+d)+\alpha_4(eX^3+fX^2+gX+h)=0$$

$$(\alpha_1+\alpha_2+\alpha_3.a+\alpha_4 e).X^3+(\alpha_3 b+\alpha_4 f)X^2+(-2\alpha_1+3\alpha_2+c\alpha_3+g\alpha_4)X$$
$$+(\alpha_1+\alpha_3 d+\alpha_4 h)=0$$

buscamos hacer que $\alpha_1=\alpha_2=\alpha_3=\alpha_4=0$ para que sean L.i

1) $\alpha_3.b=-\alpha_4 f$ $\qquad \boxed{b\neq 0,\ f\neq 0}$

$\alpha_3 = -\frac{f}{b}.\alpha_4$

2) $\alpha_1+d.\alpha_3+h.\alpha_4=0$

$\alpha_1 = -h.\alpha_4 + d.\frac{f}{b}\alpha_4$

$\alpha_1 = \left(-h+\frac{df}{b}\right).\alpha_4$ $\qquad \boxed{d.\frac{f}{b}\neq h}$

$\uparrow \boxed{\frac{cf}{b}\neq g}$

$-g+\frac{cf}{b} = 2\left(h-\frac{df}{b}\right)$

$-g+\frac{cf}{b}-2h+\frac{2df}{b}\neq 0$

3) $-2\alpha_1+3\alpha_2+c\alpha_3+g\alpha_4=0$

$$3\alpha_2 = -g\alpha_4+\frac{c.f}{b}\alpha_4-\left(2h-\frac{2d.f}{b}\right)\alpha_4 = \left(-g+\frac{cf}{b}-2h+\frac{2df}{b}\right)\alpha_4$$

4) $\alpha_1+\alpha_2+a.\alpha_3+e\alpha_4=0$

$$\left(-h+\frac{df}{b}\right)\alpha_4+\frac{1}{3}\left(-g+\frac{cf}{b}-2h+\frac{2df}{b}\right)\alpha_4-\frac{a.f}{b}\alpha_4+e\alpha_4=0$$

$$\left[-h+\frac{df}{b}+\frac{1}{3}\left(-g+\frac{cf}{b}-2h+\frac{2df}{b}\right)-\frac{af}{b}+e\right]\alpha_4=0$$

Sean $b=1 \quad f=2 \quad d=1 \quad h=0 \quad c=1 \quad g=0 \quad e\neq\frac{a.f}{b}$

$e=1 \quad a=1$

$$\lambda_1(X^3-2X+1)+\lambda_2(X^3+3X)+\lambda_3(X^3+X^2+X+1)+\lambda_4(X^3+2X^2)=0$$

$$(\lambda_1+\lambda_2+\lambda_3+\lambda_4).X^3+(\lambda_3+2\lambda_4)X^2+(-2\lambda_1+3\lambda_2+\lambda_3)+(\lambda_1+\lambda_3)=0$$

$$\lambda_1 + \lambda_3 = 0 \qquad\qquad -2\lambda_1 + 3\lambda_2 + \lambda_3 = 0 \qquad\qquad \lambda_3 + 2\lambda_4 = 0$$
$$\lambda_1 = -\lambda_3 \qquad\qquad +2\lambda_3 + 3\lambda_2 + \lambda_3 = 0 \qquad\qquad \lambda_4 = \frac{1}{2}\lambda_3$$
$$\lambda_2 = -\lambda_3$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 0$$
$$-\lambda_3 - \cancel{\lambda_3} + \cancel{\lambda_3} + \frac{1}{2}\lambda_3 = 0$$
$$-\frac{1}{2}\lambda_3 = 0 \iff \lambda_3 = 0 \Rightarrow \lambda_1 = \lambda_2 = \lambda_4 = 0 \Rightarrow$$

Los vectores son Li $\therefore$

$$\boxed{B = \{ (x^3 - 2x + 1), (x^3 + 3x), (x^3 + x^2 + x + 1), (x^3 + 2x^2) \}}$$

iii) $\left\{ \begin{pmatrix} 1 & 1 \\ i & 1 \end{pmatrix}, \begin{pmatrix} 0 & i \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 1 & 1 \end{pmatrix} \right\}$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & a & 0 \\ 1 & i & 2 & b & 0 \\ i & 1 & 1 & c & 0 \\ 1 & 1 & 1 & d & 0 \end{array}\right) = \left(\begin{array}{cccc|c} 1 & 0 & 0 & a & 0 \\ 0 & i & 2 & b-a & 0 \\ 0 & 1 & 1 & c-ai & 0 \\ 0 & 1 & 1 & d-a & 0 \end{array}\right) = \left(\begin{array}{cccc|c} 1 & 0 & 0 & a & 0 \\ 0 & i & 2 & b-a & 0 \\ 0 & 0 & i-2 & ci-ai^2-b+a & 0 \\ 0 & 0 & 0 & d-a-c+ai & 0 \end{array}\right)$$

$$d - a - c + ai \neq 0$$

Sean $d = c = 0 \Rightarrow$
$b = 0$
$a = 1$

$i = \sqrt{-1}$
$i^2 = -1$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 1 & i & 2 & 0 & 0 \\ i & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{array}\right) = \left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & i & 2 & -1 & 0 \\ 0 & 1 & 1 & -i & 0 \\ 0 & 1 & 1 & -1 & 0 \end{array}\right) = \left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & i & 2 & -1 & 0 \\ 0 & 0 & i-2 & 1-i^2 & 0 \\ 0 & 0 & 0 & -1+i & 0 \end{array}\right)$$

$$\boxed{B = \left\{ \begin{pmatrix} 1 & 1 \\ i & 1 \end{pmatrix}, \begin{pmatrix} 0 & i \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right\}} \qquad K = \mathbb{C}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & a & 0 \\ 1 & 0 & 2 & b & 0 \\ 0 & 1 & 1 & c & 0 \\ 1 & 1 & 1 & d & 0 \end{array}\right) = \left(\begin{array}{cccc|c} 1 & 0 & 0 & a & 0 \\ 0 & 0 & 2 & b-a & 0 \\ 0 & 1 & 1 & c & 0 \\ 0 & 1 & 1 & d-a & 0 \end{array}\right) = \left(\begin{array}{cccc|c} 1 & 0 & 0 & a & 0 \\ 0 & 1 & 1 & c & 0 \\ 0 & 0 & 2 & b-a & 0 \\ 0 & 0 & 0 & d-a-c & 0 \end{array}\right)$$

$$a = c = b = 0 \ \wedge \ d = 1 \Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

$$\boxed{B = \left\{ \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}} \qquad K = \mathbb{R}$$

15.

1)

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & i & 1 & 0 \\ 0 & 0 & i & 0 \end{array}\right)$$ son L.i

$K = \mathbb{C}$ CASO 1

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & z_1 \\ 0 & i & 1 & z_2 \\ 0 & 0 & i & z_3 \end{array}\right)$$

$$\lambda_1 + \lambda_3 = a_1 + b_1 i$$
$$\lambda_2 i + \lambda_3 = a_2 + b_2 i$$
$$\lambda_3 i = a_3 + b_3 i$$

generan un $z = (z_1, z_2, z_3) \in \mathbb{C}^3$ genérico

$\Rightarrow$ es base de $\mathbb{C}^3$

Se prueba un z cualquiera

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 3+i \\ 0 & i & 1 & -5-2i \\ 0 & 0 & i & i \end{array}\right)$$

$\boxed{\lambda_3 = 1}$

$$\lambda_2 i + 1 = -5-2i$$
$$\lambda_2 i = -6-2i$$
$$(a+bi)i = -6-2i$$
$$ai - b = -6-2i$$
$$\boxed{\lambda_2 = -2+6i}$$
$$\lambda_2 \cdot i = -2i-6$$

$$\lambda_1 + 1 = 3+i$$
$$\boxed{\lambda_1 = 2+i}$$

$$(2+i)(1,0,0) + (-2+6i)(0,i,0) + 1(1,1,i)$$
$$(2+i+0+1,\ 0-2i-6+1,\ 0+0+i)$$
$$(3+i,\ -5-2i,\ i)$$

$K = \mathbb{R}$ CASO 2

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & z_1 \\ 0 & i & 1 & z_2 \\ 0 & 0 & i & z_3 \end{array}\right)$$ son L.i pero: $\lambda_3 i = z_3 \Leftrightarrow Re(z_3) = 0$

$\Rightarrow$ no se puede generar cualquier z con $K = \mathbb{R}$ y $\lambda_i \in \mathbb{R}$.

CONTINÚA $\Downarrow$

ii)

$$\left(\begin{array}{cccccc|c} 1 & 0 & 0 & \cdots & 0 & & 0 \\ 0 & 1 & 0 & \cdots & 0 & & 0 \\ 0 & 0 & 1 & \cdots & 0 & & 0 \\ 0 & 0 & 0 & \cdots & 0 & & 0 \\ \cdots & & & & & & \cdots \\ 0 & 0 & 0 & \cdots & 1 & & 0 \end{array}\right)$$ son Li

$$\left(\begin{array}{ccccc|c} 1 & 0 & 0 & \cdots & 0 & z_1 \\ 0 & 1 & 0 & \cdots & 0 & z_2 \\ 0 & 0 & 1 & \cdots & 0 & z_3 \\ \cdots & & & & & \cdots \\ 0 & 0 & 0 & \cdots & 1 & z_n \end{array}\right)$$

generan $\mathbb{C}^n$ porque para un $z = (z_1, z_2, \dots, z_n)$ genérico la c.l. es

$$\lambda_1 = z_1 \ ; \ \lambda_2 = z_2, \dots, \lambda_n = z_n$$

$\Rightarrow \{e_1, \dots, e_n\}$ es base en $\mathbb{C}^n(\mathbb{C})$

$$\left(\begin{array}{ccccc|c} 1 & 0 & 0 & \cdot & 0 & z_1 \\ 0 & 1 & 0 & \cdot & 0 & z_2 \\ 0 & 0 & 1 & \cdot & 0 & z_3 \\ \cdot & \cdot & \cdot & \cdot & & \\ 0 & 0 & 0 & \cdot & 1 & z_n \end{array}\right)$$

$\Rightarrow$ generan $\mathbb{C}^n$ si y solo si $z_1, z_2, \dots z_n$ tienen $Im(z_i) = 0$

$\Rightarrow$ no generan $\mathbb{C}^n$ si $\mathbb{C}^n(\mathbb{R})$

iii)

$$\begin{array}{ccccccccccc} & & & & & n+1 & & & & 2n-1 & \\ \end{array}$$
$$\left(\begin{array}{cccccccccc|c} 1 & 0 & 0 & \cdot & 0 & i & 0 & 0 & \cdot & 0 & 0 \\ 0 & 1 & 0 & \cdot & 0 & 0 & i & 0 & \cdot & 0 & 0 \\ 0 & 0 & 1 & \cdot & 0 & 0 & 0 & i & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 0 & \cdot & 1 & 0 & 0 & 0 & \cdot & i & 0 \end{array}\right) \quad \text{son Li}$$

Ψ viene del 15. i).

$$\left(\begin{array}{cccccc|c} 1 & 0 & 0 & i & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & i & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & i & 0 \end{array}\right)$$

son Li en $K = \mathbb{R}$ pues, por ejemplo,

$$(1,0,0) \neq \lambda(i,0,0) \quad \forall \lambda \in \mathbb{R}$$

Luego $\dim. \mathbb{C}^3 = 6$.

Porque 6 es el menor número de vectores necesarios para generar $\mathbb{C}^3$.

$$\left(\begin{array}{cccccc|c} 1 & 0 & 0 & i & 0 & 0 & a_1+b_1 i \\ 0 & 1 & 0 & 0 & i & 0 & a_2+b_2 i \\ 0 & 0 & 1 & 0 & 0 & i & a_3+b_3 i \end{array}\right)$$

$$\mathbb{C}^{3\times 6} \times \mathbb{C}^{6\times 1} = \mathbb{C}^{3\times 1}$$

18. i) $S=\langle(1,k,1),(-1,k,1),(0,1,k)\rangle\subset\mathbb{R}^3$

$$\left(\begin{array}{ccc|c}1&-1&0&0\\k&k&1&0\\1&1&k&0\end{array}\right)=\left(\begin{array}{ccc|c}1&-1&0&0\\k&k&1&0\\0&2&k&0\end{array}\right)$$

$\equiv A$

$$\det A=(k^2)-(-k^2+2)=k^2+k^2-2$$
$$2k^2-2=0$$
$$k^2-1=0$$
$$k=\begin{cases}1\\-1\end{cases}$$

$\boxed{\text{si } k\neq 1,-1 \Rightarrow \dim S=3}$ $\Rightarrow$ son Li los vectores y son base

si. $k=-1,1$ $\Rightarrow$ son Ld los vectores

$$\left(\begin{array}{ccc|c}1&-1&0&0\\1&1&1&0\\1&1&1&0\end{array}\right)=\left(\begin{array}{ccc|c}1&-1&0&0\\0&2&1&0\\0&0&0&0\end{array}\right)$$

son Li 2 columnas $\Rightarrow$ $\dim S=2$

$$\left(\begin{array}{ccc|c}1&-1&0&0\\-1&-1&1&0\\1&1&-1&0\end{array}\right)=\left(\begin{array}{ccc|c}1&-1&0&0\\0&-2&1&0\\0&0&0&0\end{array}\right)$$

son Li 2 columnas $\Rightarrow$ $\dim S=2$

$\boxed{\text{si } k=-1,1 \Rightarrow \dim S=2}$

ii) $S=\left\langle\begin{pmatrix}1&k\\1&2\end{pmatrix},\begin{pmatrix}k&1\\0&2k\end{pmatrix},\begin{pmatrix}0&0\\1&0\end{pmatrix}\right\rangle\subset\mathbb{R}^{2\times2}$

$k\neq0$

$$\left(\begin{array}{ccc|c}1&k&0&0\\k&1&0&0\\1&0&1&0\\2&2k&0&0\end{array}\right)=\left(\begin{array}{ccc|c}\multicolumn{3}{c|}{\overbrace{\begin{matrix}1&k&0\\0&1-k^2&0\\0&-k&1\end{matrix}}^{B}}&\begin{matrix}0\\0\\0\end{matrix}\\0&0&0&0\end{array}\right)^{\times k}=$$

$\Rightarrow$ serán Li si $\det B\neq0$ $\Rightarrow$

$$|B|=(1-k^2)\neq0 \qquad k^2\neq1 \qquad k\neq\begin{smallmatrix}1\\-1\end{smallmatrix}$$

si son Li $\Rightarrow$ son base y $\boxed{\dim.S=3 \quad \text{con} \quad k\neq-1,1}$

si son Ld $\Rightarrow$

(*) caso $\underline{k=1}$

$$\left(\begin{array}{cccc}1&1&0&0\\1&1&0&0\\1&0&1&0\\2&2&0&0\end{array}\right)=\left(\begin{array}{ccc|c}1&1&0&0\\0&0&0&0\\0&-1&1&0\\0&0&0&0\end{array}\right)=\left(\begin{array}{ccc|c}1&1&0&0\\0&-1&1&0\\0&0&0&0\\0&0&0&0\end{array}\right)$$

$\Rightarrow$ tengo 2 columnas Li $\Rightarrow$ forman una base

$\Rightarrow \boxed{\dim.S=2}$

↓ se arrojaría el 3er vector si necesitase una base

⊛ $\underline{k=-1}$

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & -2 & 0 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \text{son 2 columnas L.i}$$

$$\boxed{\dim S = 2}$$

⊛ caso $\underline{k=0}$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \text{son 3 columnas L.i}$$

$$\dim. S = 3$$

$$\boxed{\begin{array}{lll} \text{si} & k = -1,1 \Rightarrow & \dim S = 2 \\ & k \neq -1,1 \Rightarrow & \dim S = 3 \end{array}}$$

iii) $S = \{X \in \mathbb{R}^3 : A.X = 0\}$ con $A \in \mathbb{R}^{3\times 3}$

si A es inversible $\Rightarrow$ $X = 0$ es la única solución y $\dim. S = 0$

si A no es inversible tenemos ∞ soluciones

$$A = \begin{pmatrix} 1 & -k & -1 \\ -1 & 1 & k^2 \\ 1 & k & k-2 \end{pmatrix} = \begin{pmatrix} 1 & -k & -1 \\ 0 & 1-k & k^2-1 \\ 0 & k+1 & k-2+k^2 \end{pmatrix}$$

$-k^3+k^2-k-1$

$$\det A = \left[(1-k).(k-2+k^2) - (k+1).(k^2-1)\right]$$

$$\det A = (1-k).(k+2).(k-1) - (k+1)^2(k-1) \Rightarrow \text{planteo } \det A = 0 \Rightarrow$$

$k^2+k-2=0$

$k = -2$
$\;\;\;\;\;\;1$

$$k-2+k^2-k^2+2k-k^3-k^3+k-k^2+1 = 0$$
$$2k-1+2k-2k^3-k^2 = 0$$
$$-2k^3-k^2+4k-1 = 0$$
$$(-2k^2-3k+1).(k-1) = 0$$

$$\begin{array}{l|l} -2k^3-k^2+4k-1 & k-1 \\ \hline -\;\; -2k^3\;\; 2k^2 & -2k^2-3k+1 \\ \;\;\;0\;\; -3k^2+4k & \\ \;\;\;\;\;-3k^2+3k & \\ \;\;\;\;\;-\;\; k-1 & \\ \;\;\;\;\;\;\;\; k-1 & \\ \;\;\;\;\;\;\;\;\;\; 0 & \end{array}$$

$$\frac{+3 \pm \sqrt{9-4(-2)1}}{-4}$$

$$k_2 = \frac{3+\sqrt{17}}{-4}$$

$$k_1 = \frac{3-\sqrt{17}}{-4}$$

Si $k \neq 1, \frac{3+\sqrt{17}}{-4}, \frac{3-\sqrt{17}}{-4}$

$$\Rightarrow \boxed{\dim. S = 0}$$

⊛ Caso $\underline{k=1}$

$$A = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 2 & 0 \end{pmatrix} \Rightarrow \begin{array}{l} 2X_2 = 0 \\ X_1 - X_2 - X_3 = 0 \\ X_1 = X_3 \end{array}$$

$$X = (X_1, 0, X_1)$$
$$X = \alpha(1,0,1)$$

$$S = \langle(1,0,1)\rangle \Rightarrow \boxed{\dim. S = 1}$$

⊛ caso $k \cong 0{,}280$

$$A \cong \begin{pmatrix} 1 & -0{,}28077 & -1 \\ 0 & 0{,}7192 & -0{,}92116 \\ 0 & 1{,}2877 & -1{,}64038 \end{pmatrix} \cong \begin{pmatrix} 1 & -0{,}28077 & -1 \\ 0 & 0{,}7192 & -0{,}92116 \\ 0 & 0 & 0 \end{pmatrix}$$

$$-\frac{0{,}92}{0{,}72}\,0{,}2.X_3 - X_3$$
$$X_1 - 0{,}2X_2 - X_3 = 0$$
$$0{,}72X_2 - 0{,}92X_3 = 0$$
$$X_2 = \frac{0{,}92}{0{,}72}X_3$$

⊛ caso $k \cong 1{,}780$ es similar

$$X = (\xi_1 X_1, \xi_2 X_1, \xi_3 X_1)$$

$$S = \langle(\xi_1, \xi_2, \xi_3)\rangle$$

$$\Rightarrow \boxed{\dim S = 1}$$

S tendrá dimensión 0 ó 1

19.

$S = \langle (-2,1,6),(3,0,-8) \rangle$

$S = \langle \underbrace{(1,k,2k)}_{\equiv v_1}, \underbrace{(-1,-1,k^2-2)}_{\equiv v_2}, \underbrace{(1,1,k)}_{\equiv v_3} \rangle$

$\forall s \in S$ se da: $\quad s = \alpha(-2,1,6) + \beta(3,0,-8)$

$$\begin{pmatrix} -2 & 3 & 0 \\ 1 & 0 & 0 \\ 6 & -8 & 0 \end{pmatrix} = \begin{pmatrix} -2 & 3 & 0 \\ 0 & 3 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} -2 & 3 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

son generadores y son Li $\Rightarrow$ son base de S $\Rightarrow$ $\dim S = 2$

$\Rightarrow$ los v_1, v_2, v_3 son Ld $\Rightarrow$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ k & -1 & 1 & 0 \\ 2k & k^2-2 & k & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & -1+k & 1-k & 0 \\ 0 & k^2 & k-2 & 0 \end{array}\right)$$

sea $k \neq 0$

$$\det = \left(-k + [-2k] + k(k^2-2)\right) - \left(-2k - k^2 + k^2 - 2\right)$$

$$\det = -k - 2k + k^3 - \cancel{2k} + \cancel{2k} + \cancel{k^2} - \cancel{k^2} + 2 = k^3 - 3k + 2$$

$$k^3 - 3k + 2 = 0$$

$$\begin{array}{l|l} k^3 - 3k + 2 & k-1 \\ -\ k^3 - k^2 & \overline{k^2 + k - 2} \\ \hline 0 \quad k^2 - 3k & \\ -\ k^2 - k & \\ \hline 0 \quad -2k + 2 & \\ -\ 2k + 2 & \\ \hline \quad 0 & \end{array} \qquad \begin{array}{l|l} k^2 + k - 2 & k-1 \\ -\ k^2 - k & \overline{k+2} \\ \hline -\ 2k - 2 & \\ \ \ 2k - 2 & \\ \hline \quad 0 & \end{array}$$

$\Rightarrow$ $(k^2+k-2).(k-1) = 0$

$(k+2).(k-1)^2 = 0 \iff \boxed{\begin{matrix} k = 1 \\ k = -2 \end{matrix}}$

caso $k = 0$

sea $k=0 \Rightarrow \quad S = \langle (1,0,0)\ (-1,-1,-2)\ (1,1,0) \rangle$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -2 & 0 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -2 & 0 \end{array}\right)$$

son base pues son Li $\Rightarrow$

$\dim S = 3$

20. i) $\quad \mathbb{V} = \mathbb{R}^3$

$S = \{(x,y,z) : \ 3x - 2y + z = 0\}$

$T = \{(x,y,z) : \ x + z = 0\}$

$\underline{S \cap T}$;

$$\left(\begin{array}{ccc|c} 3 & -2 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 3 & -2 & 1 & 0 \\ -3 & 0 & -3 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 3 & -2 & 1 & 0 \\ 0 & -2 & -2 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 3 & -2 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right)$$

$x_2 = -x_3 = x_1$

$3x_1 + 2x_3 + x_3 = 0$

$x_1 = -x_3$

$x \in (S \cap T)$ es tal que $x = (x_1, x_1, -x_1) = \lambda(1,1,-1)$

$\underline{S+T}$: $\quad S =$

$\lambda = z = -3\underset{\mu}{\underset{\parallel}{x}} + 2y$

$\frac{\lambda}{2} + \frac{3}{2}\mu = y$

$(\mu, \lambda/2 + 3/2\mu, \lambda) = \mu(1, 3/2, 0) + \lambda(0, 1/2, 1)$

$S = \langle (1, 3/2, 0), (0, 1/2, 1) \rangle$

T: $\quad x+z=0 \quad \begin{array}{l} y=\alpha \\ x=\lambda \\ z=-\lambda \end{array} \quad T = \langle(\lambda,\alpha,-\lambda)\rangle = \dots$

$\langle(1,0,-1),(0,1,0)\rangle$

$S+T = \langle(2,3,0),(0,1,2),(1,0,-1)\rangle$

$$\left(\begin{array}{ccc|c} 2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} -6 & 0 & -3 & 0 \\ 6 & 2 & 0 & 0 \\ 0 & 2 & -1 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} -6 & 0 & -3 & 0 \\ 0 & 2 & -3 & 0 \\ 0 & 2 & -1 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} -6 & 0 & -3 & 0 \\ 0 & 2 & -3 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right)$$

son Li

$$\dim(S+T) = \dim(S) + \dim(T) - \dim(S\cap T)$$
$$3 = 2 + 2 - 1$$

$\boxed{\text{suma no es directo}}$

ii) $V=\mathbb{R}^3$

$S=\{(x,y,z): \begin{array}{l} 3x-2y+z=0 \\ x-y=0 \end{array}\}$

$\left(\begin{array}{ccc|c} 3 & -2 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right)$

$z=\alpha$
$y=-z=-\alpha$
$3x-2y+z=0$
$3x+2z+z=0$
$x=-z=-\alpha$

$S=\langle -\alpha,-\alpha,\alpha\rangle$
$S=\langle(-1,-1,1)\rangle$

$T=\langle(1,1,0),(5,7,3)\rangle$

$$\left(\begin{array}{cc|c} 1 & 5 & x_1 \\ 1 & 7 & x_2 \\ 0 & 3 & x_3 \end{array}\right) = \left(\begin{array}{cc|c} 1 & 5 & x_1 \\ 0 & 2 & x_2-x_1 \\ 0 & 3 & x_3 \end{array}\right) = \left(\begin{array}{cc|c} 1 & 5 & x_1 \\ 0 & 6 & 3x_2-3x_1 \\ 0 & 6 & 2x_3 \end{array}\right)$$

$$\left(\begin{array}{cc|c} 1 & 5 & x_1 \\ 0 & 6 & 3x_2-3x_1 \\ 0 & 0 & 2x_3-3x_2+3x_1 \end{array}\right) \Rightarrow 2x_3-3x_2+3x_1=0$$

$\underline{S\cap T}$:

$$\left(\begin{array}{ccc|c} 3 & -2 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 3 & -3 & 2 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 3 & -2 & 1 & 0 \\ 3 & -3 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 3 & -2 & 1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 3 & -2 & 1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right) \Leftrightarrow \begin{array}{l} x_3=0 \\ x_2=0 \\ x_1=0 \end{array} \qquad S\cap T=0$$

$\underline{S+T}$

$S+T=\langle(1,1,0),(5,7,3),(-1,-1,1)\rangle$

$$\left(\begin{array}{ccc|c} 1 & 5 & -1 & 0 \\ 1 & 7 & -1 & 0 \\ 0 & 3 & 1 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 5 & -1 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 3 & 1 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 5 & -1 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$$\underset{3}{\dim(S+T)} = \underset{1}{\dim S} + \underset{2}{\dim T} - \underset{0}{\dim S\cap T}$$

$\boxed{\text{suma es directo}}$

iii) $S=\langle(1,1,3),(1,3,5),(6,12,24)\rangle$

$T=\langle(1,1,0),(5,7,3)\rangle$

$\underline{S\cap T}$ $\quad \alpha_1(1,1,3)+\alpha_2(1,3,5)+\alpha_3(6,12,24) = \lambda_1(1,1,0)+\lambda_2(5,7,3)$

$$\begin{array}{l} \alpha_1+\alpha_2+6\alpha_3 = \lambda_1+5\lambda_2 \\ \alpha_1+3\alpha_2+12\alpha_3 = \lambda_1+7\lambda_2 \\ 3\alpha_1+5\alpha_2+24\alpha_3 = +3\lambda_2 \end{array}$$

$$\begin{aligned} \alpha_1 + \alpha_2 + 6\alpha_3 - \lambda_1 - 5\lambda_2 &= 0 \\ \alpha_1 + 3\alpha_2 + 12\alpha_3 - \lambda_1 - 7\lambda_2 &= 0 \\ 3\alpha_1 + 5\alpha_2 + 24\alpha_3 \qquad - 3\lambda_2 &= 0 \end{aligned}$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 6 & -1 & -5 & 0 \\ 1 & 3 & 12 & -1 & -7 & 0 \\ 3 & 5 & 24 & 0 & -3 & 0 \end{array}\right) = \left(\begin{array}{ccccc|c} 1 & 1 & 6 & -1 & -5 & 0 \\ 0 & 2 & 6 & 0 & -2 & 0 \\ 0 & 2 & 6 & 3 & 12 & 0 \end{array}\right)$$

$$= \left(\begin{array}{ccccc|c} 1 & 1 & 6 & -1 & -5 & 0 \\ 0 & 2 & 6 & 0 & -2 & 0 \\ 0 & 0 & 0 & 3 & 14 & 0 \end{array}\right)$$

$$\lambda_1 = \boxed{-\frac{14}{3}\lambda_2}$$

$$\alpha_2 = \frac{2}{2}\lambda_2 - \frac{6}{2}\alpha_3 = \boxed{\lambda_2 - 3\alpha_3 = \alpha_2}$$

$$\alpha_1 = 5\lambda_2 - \frac{14}{3}\lambda_2 - 6\alpha_3 - \lambda_2 + 3\alpha_3$$

$$\boxed{\alpha_1 = -2/3\lambda_2 - 3\alpha_3}$$

$$(-2/3\lambda_2 - 3\alpha_3)(1,1,3) + (\lambda_2 - 3\alpha_3)\cdot(1,3,5) + \alpha_3(6,12,24) = -14/3\lambda_2(1,1,0) + \lambda_2(5,7,3)$$

$$\Big(-\frac{2}{3}\lambda_2 \cancel{-3\alpha_3} + \lambda_2 \cancel{-3\alpha_3} + 6\alpha_3,\ -2/3\lambda_2 \cancel{-3\alpha_3} + 3\lambda_2 \cancel{-9\alpha_3} + 12\alpha_3,\ -2\lambda_2 - 9\alpha_3 + 5\lambda_2 \cancel{-15\alpha_3} + 24\alpha_3\Big) = \Big(-14/3\lambda_2 + \lambda_2\cdot 5,\ -14/3\lambda_2 + 7\lambda_2,\ 3\lambda_2\Big)$$

$$\left(\frac{1}{3}\lambda_2,\ 7/3\lambda_2,\ 3\lambda_2\right) \qquad = (1/3\lambda_2,\ 7/3\lambda_2,\ 3\lambda_2)$$

$$(1/3,\ 7/3,\ 3)\lambda_2 \qquad = 1/3\lambda_2(1,7,9)$$

$$S \cap T = \langle (1/3, 7/3, 3) \rangle$$

$$\dim(S \cap T) = 1$$

↓ otro modo

$$\left(\begin{array}{ccc|c} 1 & 1 & 6 & 0 \\ 1 & 3 & 12 & 0 \\ 3 & 5 & 24 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 1 & 6 & 0 \\ 0 & 2 & 6 & 0 \\ 0 & 2 & 6 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 1 & 6 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \dim S = 2$$

$$z = \lambda \qquad y = -3\lambda \qquad x = -6\lambda + 3\lambda = -3\lambda$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 6 & x_1 \\ 1 & 3 & 12 & x_2 \\ 3 & 5 & 24 & x_3 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 1 & 6 & x_1 \\ 0 & 2 & 6 & x_2 - x_1 \\ 0 & 2 & 6 & x_3 - 3x_1 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 1 & 6 & x_1 \\ 0 & 2 & 6 & x_2 - x_1 \\ 0 & 0 & 0 & x_3 - 3x_1 - x_2 + x_1 \end{array}\right)$$

$$-2x_1 - x_2 + x_3 = 0$$

$$\left(\begin{array}{cc|c} 1 & 5 & x_1 \\ 1 & 7 & x_2 \\ 0 & 3 & x_3 \end{array}\right) = \left(\begin{array}{cc|c} 1 & 5 & x_1 \\ 0 & 2 & x_2 - x_1 \\ 0 & 3 & x_3 \end{array}\right) = \left(\begin{array}{cc|c} 1 & 5 & x_1 \\ 0 & 6 & 3x_2 - 3x_1 \\ 0 & 6 & 2x_3 \end{array}\right) = \left(\begin{array}{cc|c} 1 & 5 & x_1 \\ 0 & 6 & 3x_2 - 3x_1 \\ 0 & 0 & 2x_3 - 3x_2 + 3x_1 \end{array}\right)$$

$$\dim T = 2$$

$\underline{S \cap T}$:
$$\begin{cases} 3x_1 - 3x_2 + 2x_3 = 0 \\ -2x_1 - x_2 + x_3 = 0 \end{cases}$$

$$\left(\begin{array}{ccc|c} 3 & -3 & 2 & 0 \\ -2 & -1 & 1 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 6 & -6 & 4 & 0 \\ -6 & -3 & 3 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 6 & -6 & 4 & 0 \\ 0 & -9 & 7 & 0 \end{array}\right)$$

$$S \cap T = (-1/9\lambda, +7/9\lambda, \lambda)$$

$$S \cap T = \langle 1/9, \pm 7/9, 1 \rangle \Leftarrow \left(\begin{array}{ccc|c} 3 & -3 & 2 & 0 \\ 0 & -9 & 7 & 0 \end{array}\right) \Rightarrow \quad z = \lambda \quad y = 7/9\lambda \quad x = -\frac{2}{3}\lambda + \frac{3}{3}\left(\frac{7}{9}\lambda\right) \quad x = -1/9\lambda$$

$\underline{\underline{S+T}}$:

$$S+T = \langle (1,1,3), (1,3,5), \underset{\Delta}{(6,12,24)}, (1,1,0), \underset{\Delta}{(5,7,3)} \rangle$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 6 & 1 & 5 & 0 \\ 1 & 3 & 12 & 1 & 7 & 0 \\ 3 & 5 & 24 & 0 & 3 & 0 \end{array}\right) = \left(\begin{array}{ccccc|c} 1 & 1 & 6 & 1 & 5 & 0 \\ 0 & 2 & 6 & 0 & 2 & 0 \\ 0 & 2 & 6 & -3 & -12 & 0 \end{array}\right) = \left(\begin{array}{ccccc|c} 1 & 1 & 6 & 1 & 5 & 0 \\ 0 & 2 & 6 & 0 & 2 & 0 \\ 0 & 0 & 0 & -3 & -14 & 0 \end{array}\right)$$

son Ld

base de $S+T = \langle (1,1,3), (1,3,5), (1,1,0) \rangle$

$\Rightarrow$

$\boxed{\text{la suma no es directa}}$

$$\dim_{S+T} = \dim_S + \dim_T + \dim S\cap T$$

$$3 = 2 + 2 - 1$$

iv) $\mathbb{V} = \mathbb{R}[X]$ $\qquad S = \{ f \in \mathbb{R}[X] : f(1) = 0 \}$

$T = \langle 1, X, X^2, X^3 + 2X^2 - X, X^5 \rangle$

$s \in S$

$s = \langle 0, X, X^2, X^3, X^4, X^5, \dots, X^n \rangle$

$t \in T$

$$t = k_1 1 + k_2 X + k_3 X^2 + k_4 (X^3 + 2X^2 - X) + k_5 X^5$$

23.

i) $S = \langle (1,2,-1,3), (2,3,-2,1), (0,1,0,7) \rangle \qquad V = \mathbb{R}^4$

$S \oplus T = V$

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 2 & 3 & 1 & 0 \\ -1 & -2 & 0 & 0 \\ 3 & 1 & 7 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -5 & 7 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & -5 & 5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -5 & 7 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 0 & \\ 0 & -5 & 5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right)$$

son Li $\Rightarrow \dim S = 3$

$\therefore \dim T = 1$

como $S \oplus T = V \Rightarrow S \cap T = 0 \Rightarrow$ basta hallar un vector Li a $S \oplus T$

debería funcionar $(0,0,1,1)$

$$\left(\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 0 \\ 0 & -5 & 5 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 0 \\ 2 & 3 & 1 & 0 & 0 \\ -1 & -2 & 0 & 1 & 0 \\ 3 & 1 & 7 & 1 & 0 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -5 & 7 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 & 0 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \to \text{son Li}$$

$$\boxed{T = \langle (0,0,1,1) \rangle}$$

ii)

$$A = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix} \qquad a + f + k + p = 0$$

$V = \{ A \in \mathbb{R}^{4\times4} \}$

$S = \{ A \in \mathbb{R}^{4\times4} : a = -f-k-p \}$

$T = \{ A \in \mathbb{R}^{4\times4} : a \neq -f-k-p \}$

$S = \langle (-f-k-p, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p) \rangle$

$S = \langle (-1,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0), (-1,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0), (-1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1), (0,1,0,\dots), (0,0,1,0\dots), \dots \rangle$

$\dim(S) = 15 \Rightarrow \dim T = 1$

$$\begin{matrix} -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$$\boxed{T = \langle (1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0) \rangle}$$

iii) $S = \langle 3, 1+X^2 \rangle$ $\qquad \mathbb{V} = \mathbb{R}_4[X]$

$$\left(\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 3 & 0 \end{array}\right)$$

son Li $\Rightarrow \dim S = 2$

$\dim \mathbb{V} = 5 \Rightarrow \dim \mathbb{T} = 3$

$$\left(\begin{array}{ccccc|c} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 3 & 0 & 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccccc|c} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 3 & 0 & 0 & -1 & 0 \end{array}\right) \to \left(\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 3 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

$\mathbb{T} = \langle (1,0,0,0,0), (0,1,0,0,0), (0,0,1,1,0) \rangle$

$$\boxed{\mathbb{T} = \langle X^4, X^3, X^2+X \rangle}$$

24.

21. $S \cap T = \langle (0,1,1) \rangle$ $\qquad \dim S\cap T = 1$

$S = \{\bar{x} : x_1 + x_2 - x_3 = 0\} \rightarrow \quad x_3 = \alpha \quad x_2 = \beta \quad x_1 = \alpha - \beta$

$T = \langle (1,k,2), (-1,2,k) \rangle$

$(\alpha-\beta, \beta, \alpha)$

$S = \alpha(1,0,1) + \beta(-1,1,0)$

$S = \langle (1,0,1), (-1,1,0) \rangle$

$\dim S = 2$

$\dim S+T = \dim S + \dim T - \dim S\cap T$
$\qquad\qquad\quad 2 \qquad\qquad\quad 1$

$\alpha_1(1,k,2) + \alpha_2(-1,2,k) = \beta_1(1,0,1) + \beta_2(-1,1,0)$

$$\begin{aligned} \alpha_1 - \alpha_2 &= \beta_1 - \beta_2 \\ k\alpha_1 + 2\alpha_2 &= \beta_2 \\ 2\alpha_1 + k\alpha_2 &= \beta_1 \end{aligned}$$

$$\left(\begin{array}{cccc|c} 1 & -1 & -1 & 1 & 0 \\ k & 2 & 0 & -1 & 0 \\ 2 & k & -1 & 0 & 0 \end{array}\right)$$

con $k \neq 0$ $\left(\begin{array}{cccc|c} 1 & -1 & -1 & 1 & 0 \\ 0 & 2+k & k & k-1 & 0 \\ 0 & k+2 & 1 & -2 & 0 \end{array}\right) = \left(\begin{array}{cccc|c} 1 & -1 & -1 & 1 & 0 \\ 0 & k+2 & k & k-1 & 0 \\ 0 & 0 & 1-k & -1-k & 0 \end{array}\right)$

con $k=0$ $\left(\begin{array}{cccc|c} 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & 0 & -1 & 0 \\ 2 & 0 & -1 & 0 & 0 \end{array}\right)$

$\left(\begin{array}{cccc|c} 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & 0 & -1 & 0 \\ 0 & 2 & 1 & -2 & 0 \end{array}\right)$

$\left(\begin{array}{cccc|c} 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right)$

$\beta_1 = \beta_2$

$\alpha_2 = \beta_1/2$

$\alpha_1 = \cancel{-\beta_2} + \cancel{\beta_2} + \alpha_2$

$\alpha_1(1,0,2) + \alpha_1(-1,2,0) = \beta_1(1,0,1) + \beta_1(-1,1,0)$

$\alpha_1(0,2,2) = (\beta_1 - \beta_1, \beta_1, \beta_1)$

$2\alpha_1(0,1,1) = (0,1,1)\beta_1$

$k=0$ sirve

Con $k=1$ $\left(\begin{array}{cccc|c} 1 & -1 & -1 & 1 & 0 \\ 1 & 2 & 0 & -1 & 0 \\ 2 & 1 & -1 & 0 & 0 \end{array}\right) = \left(\begin{array}{cccc|c} 1 & -1 & -1 & 1 & 0 \\ 0 & 3 & 1 & -2 & 0 \\ 0 & 3 & 1 & -2 & 0 \end{array}\right)$

$\left(\begin{array}{cccc|c} 1 & -1 & -1 & 1 & 0 \\ 0 & 3 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow$ intersección es un plano

no sirve

$\beta_1 = \beta_2 \left(\frac{1+k}{1-k}\right)$ con $k \neq 1$

$(0,1,1) = \beta_2\left(\frac{1+k}{1-k}\right)(1,0,1) + \beta_2(-1,1,0)$

$(0,1,1) = \left(\left(\frac{1+k}{1-k}\right)\beta_2 - \beta_2,\ \beta_2,\ \beta_2\left(\frac{1+k}{1-k}\right)\right)$

$\frac{1+k-1+k}{1-k}\beta_2$

$(0,1,1) = \left(\frac{2k}{1-k},\ 1,\ \frac{1+k}{1-k}\right)$

$0 = \frac{2k}{1-k}$ $\qquad 1 = \frac{1+k}{1-k}$

$k=0$ $\qquad 1-k = 1+k$

$\qquad\qquad 2k = 0$

$\qquad\qquad k=0$

$\Downarrow$

necesito $k=0$

$\boxed{k=0}$

24.

i) $\underbrace{\dim S}_{2}$ $\underbrace{\dim T}_{2}$ planos en $\mathbb{R}^3$ $\Longrightarrow$ S, T sev de $\mathbb{R}^3$ luego $S \cap T$ sev de $\mathbb{R}^3$

$S \cap T$ tendrá dimensión 0 o 1

$\dim \underset{3}{(S+T)} = 2 + 2 - 1 \Rightarrow$ necesito $\dim(S \cap T) = 1$

pues $S+T$ está en $\mathbb{R}^3$ por ser la suma $(S+T)$ subespacio de $\mathbb{R}^3$

$\Rightarrow \exists\, v \neq 0 : v \in S \cap T$

$\boxed{\text{verdadero}}$

ii) S, T, W subespacios de $\mathbb{R}^5$

$\dim S, T, W$ $\quad \dim(S \cap T)$

$\dim S = 2, \ \dim T = 2, \ \dim W = 2$

$(S+T) = \underset{2}{(S)} + \underset{2}{(T)} - (S \cap T)$

$S + W = S + W - S \cap W$

$\underbrace{(S+T)+W}_{\leq 5} = \underset{4}{(S+T)} + \underset{2}{W} - \underset{¿}{(S \cap T) \cap W} \quad \Rightarrow$

$\dim(S \cap T \cap W)$ debe ser ≥ 1

$\boxed{\text{verdadero}}$

26.

i) $(x_1, x_2, \dots, x_n) = \alpha_1 e_1 + \alpha_2 e_2 + \dots + \alpha_n e_n$

$x_1 = \alpha_1 + \alpha_2 .0 + \dots + \alpha_n .0$
$x_2 = \alpha_1 .0 + \alpha_2 .1 + \dots + \alpha_n .0$

$x_n = \alpha_1 .0 + \alpha_2 .0 + \dots + \alpha_n .1$

$$\boxed{[v]_E = (x_1, x_2, \dots, x_n)}$$

ii) $(1,2,-1) = \alpha_1 (1,2,-1) + \alpha_2 (0,1,2) + \alpha_3 (0,0,2)$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 2 & 1 & 0 & 2 \\ -1 & 1 & 2 & -1 \end{array}\right) - \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right)$$

$$\boxed{[(1,2,-1)]_B = (1,0,0)}$$

$\alpha_3 = 0$
$\alpha_2 = 0$
$\alpha_1 = 1$

iii) $(1,-1,2) = \alpha_1 (1,2,-1) + \alpha_2 (2,1,3) + \alpha_3 (1,3,-3)$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 2 & 1 & 3 & -1 \\ -1 & 3 & -3 & 2 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -3 & 1 & -3 \\ 0 & 5 & -2 & 3 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -15 & 5 & -15 \\ 0 & 15 & -6 & 9 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -3 & 1 & -3 \\ 0 & 0 & -1 & -6 \end{array}\right)$$

$$\boxed{[v]_B = (-11, 3, 6)}$$

$\alpha_3 = 6$
$\alpha_2 = -3 - 6 = \frac{-9}{-3}$
$\alpha_2 = 3$
$\alpha_1 = 1 - 6 - 6 = -11$
$\alpha_1 = -11$

iv)
$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & x_1 \\ 2 & 1 & 3 & x_2 \\ -1 & 3 & -3 & x_3 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 2 & 1 & x_1 \\ 0 & -3 & 1 & x_2 - 2x_1 \\ 0 & 5 & -2 & x_3 + x_1 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 2 & 1 & x_1 \\ 0 & -3 & 1 & x_2 - 2x_1 \\ 0 & 15 & -6 & 3x_3 + 3x_1 \end{array}\right)$$

$$\alpha_1 = (x_1 + 3x_3 - 7x_1 + 5x_2 - 6x_1 + 4x_2 + 2x_3) = \left(\begin{array}{ccc|c} 1 & 2 & 1 & x_1 \\ 0 & -3 & 1 & x_2 - 2x_1 \\ 0 & 0 & -1 & 3x_3 + 3x_1 + 5x_2 - 10x_1 \end{array}\right)$$
$-12x_1 + 5x_3 + 9x_2$

$-3\alpha_2 = x_2 - 2x_1 + 3x_3 - 7x_1 + 5x_2$
$\alpha_2 = -\frac{1}{3}(-9x_1 + 6x_2 + 3x_3)$ $\qquad \alpha_3 = -(3x_3 - 7x_1 + 5x_2)$

$$\boxed{[v]_B = (-12x_1 + 9x_2 + 5x_3,\ 3x_1 - 2x_2 - x_3,\ 7x_1 - 5x_2 - 3x_3)}$$

$-11 \quad -9 \quad +15 \qquad 3 \quad +2 \quad -6 \qquad 7 \quad +5 \quad -6$

$-11 \qquad 3 \qquad 6$

v) $(2x^2 - x^3) = \alpha_1 (3) + \alpha_2 (1+x) + \alpha_3 (5+x^2) + \alpha_4 (x^3 + x^2)$

$(3 \quad 1+x \quad 5+x^2 \quad x^2+x^3 \;=\; 2x^2 - x^3)$

$\alpha_1 = \frac{(2x^2 - x^3)}{3} - \frac{(x^2+x^3)}{3}\alpha_4 - \frac{(5+x^2)}{3}\alpha_3 - \frac{(1+x)}{3}\alpha_2$

si $x \neq 1$ $\quad \alpha_2 = \frac{2x^2 - x^3}{1+x} - \frac{(x^2+x^3)}{1+x}\alpha_4 - \frac{(5+x^2)}{1+x}.\alpha_3 - \frac{3\alpha_1}{1+x}$

$\alpha_3 = \frac{2x^2 - x^3}{5+x^2} - \frac{3\alpha_1}{5+x^2} - \frac{\alpha_2(1+x)}{5+x^2} - \frac{\alpha_4(x^2+x^3)}{5+x^2}$

v) $\mathbb{V} = \underbrace{\mathbb{R}_3[X]}_{\text{dim. } 4}$

$B = \{3, 1+X, X^2+5, X^3+X^2\}$

$v = 2X^2 - X^3$

$2x^2 - x^3 = \lambda_1 . 3 + \lambda_2(1+X) + \lambda_3(5+X^2) + \lambda_4(X^2+X^3)$

$$\left(\begin{array}{cccc|c} 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 & 0 \\ 3 & 1 & 5 & 0 & 0 \end{array}\right)$$

$\lambda_4 = -1$

$\lambda_3 = 2 - \lambda_4 = 3$

$\lambda_2 = 0$

$\lambda_1 = -\frac{5}{3}\lambda_3 - \frac{\lambda_2}{3} = -5$

$$\boxed{[v]_B = (-5, 0, 3, -1)}$$

$-\cancel{15} + 3x^2 + \cancel{15} - x^3 - x^2 = 2x^2 - x^3$

vi) $\mathbb{V} = \mathbb{R}^{2\times 2}$ $\quad v = \begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \end{pmatrix}$

$(a_{11}\ a_{12}\ a_{13}\ a_{11}) = \lambda_1(1,3,0,-1) + \lambda_2(1,4,3,2) + \lambda_3(0,2,1,-1) + \lambda_4(1,1,2,5)$

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & 1 & a_{11} \\ 3 & 4 & 2 & 1 & a_{12} \\ 0 & 3 & 1 & 2 & a_{13} \\ -1 & 2 & -1 & 5 & a_{14} \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 1 & 0 & 1 & a_{11} \\ 0 & 1 & 2 & -2 & a_{12} - 3.a_{11} \\ 0 & 3 & 1 & 2 & a_{13} \\ 0 & 3 & -1 & 6 & a_{14} + a_{11} \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & 1 & a_{11} \\ 0 & 1 & 2 & -2 & a_{12} - 3.a_{11} \\ 0 & 0 & -5 & 8 & a_{13} - 3(a_{12} - 3a_{11}) \\ 0 & 0 & -2 & 4 & a_{14} + a_{11} - a_{13} \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 1 & 0 & 1 & a_{11} \\ 0 & 1 & 2 & -2 & a_{12} - 3.a_{11} \\ 0 & 0 & -5 & 8 & a_{13} - 3a_{12} + 3a_{11} \\ 0 & 0 & 5 & \cancel{4}\,\frac{5}{\cancel{8}} & -\frac{5}{2}(a_{14} + a_{11} - a_{13}) \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & 1 & a_{11} \\ 0 & 1 & 2 & -2 & a_{12} - 3a_{11} \\ 0 & 0 & -5 & 8 & a_{13} - 3a_{12} + 3a_{11} \\ 0 & 0 & 0 & -2 & -\frac{5}{2}(a_{14} + a_{11} - a_{13}) + a_{13} - 3a_{12} + 3a_{11} \end{array}\right)$$

$\lambda_4 = \frac{5}{4}a_{14} + \frac{5}{4}a_{11} - \frac{1}{2}a_{13} - \frac{1}{2}a_{13} + \frac{3}{2}a_{12} - \frac{3}{2}a_{11}$

$\lambda_4 = -\frac{1}{4}a_{13} - \frac{1}{4}a_{11} + \frac{3}{2}a_{11} + \frac{5}{4}a_{14}$ $\qquad \frac{5}{4} - \frac{3}{2} = \frac{7}{4}$

$\lambda_3 = \left[a_{13} - 3a_{12} + 3a_{11} - 8\left(-\frac{1}{4}a_{13} - \frac{1}{4}a_{11} + \frac{3}{2}a_{11} + \frac{5}{4}a_{14}\right)\right] \cdot -\frac{1}{5}$

$\lambda_2 = \left[a_{12} - 3a_{11} + 2\left(-\frac{1}{4}a_{13} - \frac{1}{4}a_{11} + \frac{3}{2}a_{11} + \frac{5}{4}a_{14}\right) - 2 . \lambda_3\right]$

$\lambda_1 = a_{11} - \left(-\frac{1}{4}a_{13} - \frac{1}{4}a_{11} + \frac{3}{2}a_{11} + \frac{5}{4}a_{14}\right) - \lambda_2$