# PRACTICA 1

## 1.

i)

$$\left(\begin{array}{cccc|c} 1 & 1 & -2 & 1 & 0 \\ 3 & -2 & 1 & 5 & 0 \\ 1 & -1 & 1 & 2 & 0 \end{array}\right) \quad \left(\begin{array}{cccc|c} 1 & 1 & -2 & 1 & 0 \\ 3 & -2 & 1 & 5 & 0 \\ 0 & 2 & -3 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 1 & -2 & 1 & 0 \\ 0 & 5 & -7 & -2 & 0 \\ 0 & 2 & -3 & -1 & 0 \end{array}\right) \quad \left(\begin{array}{cccc|c} 1 & 1 & -2 & 1 & 0 \\ 0 & 10 & -14 & -4 & 0 \\ 0 & 10 & -15 & -5 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 1 & -2 & 1 & 0 \\ 0 & 10 & -14 & -4 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right) \quad \left(\begin{array}{cccc|c} 1 & 1 & -2 & 1 & 0 \\ 0 & 5 & -7 & -2 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right)$$

$x_3 + x_4 = 0$

$x_3 = -x_4$

$5x_2 - 7x_3 - 2x_4 = 0$

$5x_2 + 7x_4 - 2x_4 = 0$

$5x_2 = -5x_4$

$x_2 = -x_4$

$x_1 + x_2 - 2x_3 + x_4 = 0$

$x_1 \cancel{-x_4} + 2x_4 \cancel{+x_4} = 0$

$x_1 = -2x_4$

$x_1 = -2x_4$

$X = \lambda(-2x_4, -x_4, -x_4, x_4)$

$\boxed{X = x_4(-2, -1, -1, 1)}$

ii)

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & -3 & 1 & 0 \\ 3 & -5 & 3 & 0 \\ 1 & -1 & 1 & 0 \end{array}\right) \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 4 & 0 & 0 \\ 3 & -5 & 3 & 0 \\ 0 & 2 & 0 & 0 \end{array}\right) \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$4x_2 = 0$

$x_2 = 0$

$x_1 + x_2 + x_3 = 0$

$x_1 = -x_3$

$\tilde{X} = (-x_3, 0, x_3)$

$\boxed{\tilde{X} = x_3(-1, 0, 1)}$

iii)

$$\left(\begin{array}{cccc|c} 1 & 1 & -2 & 1 & -2 \\ 3 & -2 & 1 & 5 & 3 \\ 1 & -1 & 1 & 2 & 2 \end{array}\right) \quad \left(\begin{array}{cccc|c} 1 & 1 & -2 & 1 & -2 \\ 0 & 5 & -7 & -2 & -9 \\ 0 & 2 & -3 & -1 & -4 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 1 & -2 & 1 & -2 \\ 0 & 10 & -14 & -4 & -18 \\ 0 & 10 & -15 & -5 & -20 \end{array}\right) \quad \left(\begin{array}{cccc|c} 1 & 1 & -2 & 1 & -2 \\ 0 & 10 & -14 & -4 & -18 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right) \quad \left(\begin{array}{cccc|c} 1 & 1 & -2 & 1 & -2 \\ 0 & 5 & -7 & -2 & -9 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right)$$

$x_3 + x_4 = 2$

$x_4 = 2 - x_3$

$5x_2 - 7x_3 - 2x_4 = -9$

$5x_2 - 7x_3 - 4 + 2x_3 = -9$

$5x_2 - 5x_3 = -5$

$x_2 - x_3 = -1$

$x_2 = x_3 - 1$

$x_1 + x_2 - 2x_3 + x_4 = -2$

$x_1 + x_3 - 1 - 2x_3 + 2 - x_3 = -2$

$x_1 - 2x_3 = -3$

$x_1 = 2x_3 - 3$

$X = (2x_3 - 3,\ x_3 - 1,\ x_3,\ 2 - x_3)$

$(2x_3, x_3, x_3, -x_3) + (-3, -1, 0, 2)$

$\boxed{X = x_3(2, 1, 1, -1) + (-3, -1, 0, 2)}$

iv) $\left(\begin{array}{ccccc|c} 1 & 1 & 1 & -2 & 1 & 1 \\ 1 & -3 & 1 & 1 & 1 & 0 \\ 3 & -5 & 3 & 0 & 3 & 0 \end{array}\right)$ $\left(\begin{array}{ccccc|c} 1 & 1 & 1 & -2 & 1 & 1 \\ 0 & 4 & 0 & -3 & 0 & 1 \\ 0 & 8 & 0 & -6 & 0 & 3 \end{array}\right)$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 1 & -2 & 1 & 1 \\ 0 & 4 & 0 & -3 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{array}\right)$$

¿incompatible

v) $\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 1 & 3 & 2 & 4 & 0 \\ 2 & 0 & 1 & -1 & 6 \end{array}\right)$ $\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -2 & -1 & -3 & 2 \\ 0 & 2 & 1 & 3 & -2 \end{array}\right)$ $\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -2 & -1 & -3 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

$$x_1 + x_2 + x_3 + x_4 = 2$$
$$-2x_2 - x_3 - 3x_4 = 2$$

$$x_1 = 2 - x_2 - x_3 - x_4 = 2 + 1 + \tfrac{3}{2}x_4 + \tfrac{1}{2}x_3 - x_3 - x_4$$
$$x_1 = 3 + \tfrac{1}{2}x_4 - \tfrac{1}{2}x_3$$
$$x_2 = \frac{2}{-2} + \frac{3x_4}{-2} + \frac{x_3}{-2} = -1 - \tfrac{3}{2}x_4 - \tfrac{1}{2}x_3$$

$$\bar{X} = \left(3 + \frac{1}{2}x_4 - \frac{1}{2}x_3, -1 - \frac{1}{2}x_3 - \frac{3}{2}x_4, x_3, x_4\right)$$

$$\bar{X} = \left(\frac{1}{2}x_4, -\frac{3}{2}x_4, 0, x_4\right) + \left(-\frac{1}{2}x_3, -\frac{1}{2}x_3, x_3, 0\right) + (3, -1, 0, 0)$$

$$\boxed{\bar{X} = x_4\left(\tfrac{1}{2}, -\tfrac{3}{2}, 0, 1\right) + x_3\left(-\tfrac{1}{2}, -\tfrac{1}{2}, 1, 0\right) + (3, -1, 0, 0)}$$

**3.**

i) $\begin{cases} x_1 + kx_2 + x_3 = 0 \\ (k+1)x_2 + x_3 = 0 \\ (k^2-4)x_3 = 0 \end{cases}$

$$\underbrace{(k^2-4)}_{=0}x_3 = 0 \longrightarrow k = \begin{cases} 2 \\ -2 \end{cases}$$

$$\begin{pmatrix} 1 & k & 1 & 0 \\ 0 & k+1 & 1 & 0 \\ 0 & 0 & k^2-4 & 0 \end{pmatrix}$$

$\boxed{k^2-4 \neq 0}$ $k \neq 2, -2$

$x_3 = 0$

$x_1 + kx_2 = 0$

$(k+1)x_2 = 0$

$\underline{k=-1}$

$x_1 = x_2$

$x = (x_1, x_1, 0) = x_1(1,1,0) \rightarrow$ soluc. no trivial ∞ soluc.

$k \neq -1$

$x_2 = 0$

$x_1 + kx_2 = 0$

$x = (0,0,0) \rightarrow$ soluc. trivial

$\boxed{\forall k \in \mathbb{R} : k \neq 2, -2, -1 \text{ se tiene única solución la trivial}}$

$\boxed{k^2-4=0}$ $\underline{k=2}$

$3x_2 + x_3 = 0$

$x_3 = -3x_2$

$x_1 + 2x_2 - 3x_2 = 0$

$x_1 = x_2$

$x = (x_2, x_2, -3x_2)$

$x = x_2(1,1,-3)$

$\underline{k=-2}$

$-x_2 + x_3 = 0$

$x_3 = x_2$

$x_1 - 2x_2 + x_3 = 0$

$x_1 - 2x_2 + x_2 = 0$

$x_1 = x_2$

$x = (x_3, x_3, x_3)$

$x = x_3(1,1,1)$

$k = 2, -2 \rightarrow$ soluc. no trivial ∞ soluc.

ii) $\begin{cases} x_1 + kx_2 + x_3 = 0 \\ 2x_1 + x_3 = 0 \\ 2x_1 + kx_2 + kx_3 = 0 \end{cases}$

$$\begin{pmatrix} 1 & k & 1 & 0 \\ 2 & 0 & 1 & 0 \\ 2 & k & k & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & k & 1 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & k & k-1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & k & 1 & 0 \\ 0 & -2k & -1 & 0 \\ 0 & k & k-1 & 0 \end{pmatrix}$$

$-1+(2).(k-1)$ $$\begin{pmatrix} 1 & k & 1 & 0 \\ 0 & -2k & -1 & 0 \\ 0 & 0 & 2k-3 & 0 \end{pmatrix}$$

$2k-3=0$

(A) $\boxed{k = \frac{3}{2}}$

$\boxed{k=\frac{3}{2} \to \infty \text{ soluciones SCI}}$

$-3x^2 - x^3 = 0$

$-3x^2 = x^3$

$x_1 + \frac{3}{2}x_2 - 3x_2 = 0$

$x_1 = \frac{3}{2}x_2$

$X = \left(\frac{3}{2}x_2, x_2, -3x_2\right) = x_2\left(\frac{3}{2}, 1, -3\right)$

(B) $\boxed{k \neq \frac{3}{2}}$

$\to \quad x_3 = 0$

$-2k\,x_2 = 0$

(1) $k=0$

$x_1 = 0$

$X = (0, x_2, 0) = x_2(0,1,0)$

$\boxed{k=0 \to \infty \text{ soluciones SCI}}$

(2) $k \neq 0$

$x_2 = 0$

$x_1 = 0$

$X = 0,0,0$

Solución trivial

**4.**

i) $$\begin{pmatrix} 1 & -1 & 1 & 2 \\ -1 & 2 & 1 & -1 \\ -1 & 4 & 5 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 3 & 6 & 3 \end{pmatrix} \begin{matrix} \\ F_2+F_1 \\ F_3+F_1 \end{matrix}$$

$$\begin{pmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} \\ \\ F_3-F_2.3 \end{matrix}$$

$x_2 + 2x_3 = 1$

$x_2 = 1 - 2x_3$

$x_1 - x_2 + x_3 = 2$

$x_1 - 1 + 2x_3 + x_3 = 2$

$x_1 = 3 - 3x_3$

$X = (3-3x_3, 1-2x_3, x_3)$

$\boxed{X = x_3(-3,-2,1) + (3,1,0)}$

$$\begin{pmatrix} 1 & -1 & 1 & 0 \\ -1 & 2 & 1 & 0 \\ -1 & 4 & 5 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 3 & 6 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$x_2 + 2x_3 = 0$

$x_2 = -2x_3$

$x_1 + 2x_3 + x_3 = 0$

$x_1 = -3x_3$

$X = (-3x_3, -2x_3, x_3)$

$\boxed{X = x_3(-3,-2,1)}$

ii)

$$\begin{pmatrix} 1 & -1 & 1 & 1 \\ -1 & 2 & 1 & 1 \\ -1 & 4 & 5 & 4 \end{pmatrix} \qquad \begin{pmatrix} 1 & -1 & 1 & 0 \\ -1 & 2 & 1 & 0 \\ -1 & 4 & 5 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 3 & 6 & 5 \end{pmatrix} \qquad \boxed{X = x_3(-3,-2,1)}$$

$$\begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & -1 \end{pmatrix} \leftarrow \text{SI}$$

$$\begin{pmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

iii)

$$\begin{pmatrix} 1 & -1 & -1 & 2 \\ 2 & 1 & -2 & 1 \\ 1 & 4 & 1 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & -1 & 2 \\ 0 & 3 & 0 & -3 \\ 0 & 5 & 2 & -1 \end{pmatrix} \begin{matrix} \\ F_2-2F_1 \\ F_3-F_1 \end{matrix}$$

$$x_1 = 0 \qquad x_2 = 0$$

$$x_1 - x_2 - x_3 = 0$$
$$x_3 = 0$$

$$\begin{pmatrix} 1 & -1 & -1 & 2 \\ 0 & 3 & 0 & -3 \\ 0 & 0 & 2 & 4 \end{pmatrix} F_3 - \tfrac{5}{3}F_2$$

$$X = (0,0,0)$$

solución trivial

$$\begin{pmatrix} 1 & -1 & -1 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

$$x_3 = 2 \qquad x_2 = -1$$

$$x_1 + 1 - 2 = 2$$
$$x_1 = 3$$

$$\boxed{X = (3,-1,2)}$$

iv)

$$\begin{pmatrix} 1 & -1 & -1 & \alpha \\ 2 & 1 & -2 & \beta \\ 1 & 4 & 1 & \gamma \end{pmatrix} \qquad \alpha,\beta,\gamma \in \mathbb{R} \qquad \begin{pmatrix} 1 & -1 & -1 & 0 \\ 2 & 1 & -2 & 0 \\ 1 & 4 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & -1 & \alpha \\ 0 & 3 & 0 & \beta-2\alpha \\ 0 & 5 & 2 & \gamma-\alpha \end{pmatrix} \begin{matrix} \\ F_2-2F_1 \\ F_3-F_1 \end{matrix} \qquad \begin{pmatrix} 1 & -1 & -1 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{pmatrix}$$

$$X = (0,0,0)$$

$$\begin{pmatrix} 1 & -1 & -1 & \alpha \\ 0 & 3 & 0 & \beta-2\alpha \\ 0 & 0 & 2 & (\gamma-\alpha) - 5/3(\beta-2\alpha) \end{pmatrix} F_3 - 5/3\, F_2$$

$$x_1 - \frac{\beta}{3} + \frac{2}{3}\alpha - \frac{7}{6}\alpha + \frac{5}{6}\beta - \frac{\gamma}{2} = \alpha$$

$$x_1 - \frac{3}{2}\alpha + \frac{1}{2}\beta - \frac{\gamma}{2} = 0$$

$$x_3 = \gamma - \alpha - \frac{5}{3}\beta + \frac{10}{3}\alpha = \left(\frac{7}{3}\alpha - \frac{5}{3}\beta + \gamma\right)\cdot\frac{1}{2}$$

$$x_1 = \frac{1}{2}\left(-3\alpha + \beta - \gamma\right)$$

$$3x_2 = \beta - 2\alpha \Rightarrow x_2 = \frac{\beta}{3} - \frac{2}{3}\alpha = \frac{1}{3}(\beta - 2\alpha)$$

sea $X_3 = 0 \Rightarrow$

$$\frac{7}{3}\alpha - \frac{5}{3}\beta + \gamma = 0$$

$$\gamma = \frac{5}{3}\beta - \frac{7}{3}\alpha$$

$$X_1 = \frac{1}{2}\left(-3\alpha + \beta - \frac{5}{3}\beta + \frac{7}{3}\alpha\right) = \frac{1}{2}\left(-\frac{2}{3}\alpha - \frac{2}{3}\beta\right) = -\frac{1}{3}(\alpha+\beta)$$

$$X_2 = \frac{1}{3}\left(\beta - 3\alpha + \alpha\right) = \frac{1}{3}\left(-3\alpha - 3X_1\right)$$

**5.** $p_1$ solución de H (no homogéneo)

$$A.x = b$$
$$\mathbb{R}^{m\times n} \cdot \mathbb{R}^{n\times 1} = \mathbb{R}^{m\times 1}$$
solución

$$A.p_1 = b \qquad p_1 \in S$$
$$A.p_0 = 0 \qquad p_0 \in S_0$$
$$S = \{ p \in \mathbb{R}^{n\times 1} : A.p = b \}$$
$$S_0 = \{ p' \in \mathbb{R}^{n\times 1} : A.p' = 0 \}$$

NB

todas las soluciones del sistema pueden obtenerse sumando a una solución particular $p_1$ todas las del sistema homogéneo asociado

$$A.p_1 = b$$
$$A.p_1 + 0 = b + 0$$
$$A.p_1 + A.p' = b \qquad \forall p' \in S_0$$
$$A(p_1 + p') = b \Rightarrow \quad p_1 + p' \text{ es solución de H } \forall p' \in S_0$$
$$S = \{ p_1 + p' \in \mathbb{R}^{n\times 1} : A(p_1+p') = b \}$$
$$S = S_0 + p_1$$

**6.**

$$\begin{pmatrix} 2 & -1 & 1 & \alpha_1 \\ 3 & 1 & 4 & \alpha_2 \\ -1 & 3 & 2 & \alpha_3 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 1 & \alpha_1 \\ 2 & 2/3 & 8/3 & 2\alpha_2/3 \\ -2 & 6 & 4 & 2\alpha_3 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 1 & \alpha_1 \\ 0 & 5/3 & 5/3 & 2/3\,\alpha_2 - \alpha_1 \\ 0 & 5 & 5 & 2\alpha_3 + \alpha_1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 1 & \alpha_1 \\ 0 & 5/3 & 5/3 & 2/3\,\alpha_2 - \alpha_1 \\ 0 & 5/3 & 5/3 & 2/3\,\alpha_3 + 1/3\,\alpha_1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 1 & \alpha_1 \\ 0 & 5 & 5 & 2\alpha_3 - 3\alpha_1 \\ 0 & 0 & 0 & \end{pmatrix}$$

$$\frac{2}{3}\alpha_3 + \frac{1}{3}\alpha_1 - \frac{2}{3}\alpha_2 + \alpha_1$$

si admite solución admitirá $\infty$

$$\frac{2}{3}\alpha_3 + \frac{4}{3}\alpha_1 - \frac{2}{3}\alpha_2 = 0$$

si esto es $\neq 0$ tenemos SI

$$(2\alpha_3 + 4\alpha_1 - 2\alpha_2)\frac{1}{3} = 0$$

$$\alpha_1 = (2\alpha_2 - 2\alpha_3)\,1/4 = \boxed{\frac{1}{2}(\alpha_2 - \alpha_3) = \alpha_1}$$

$$5x_2 + 5x_3 = 2\alpha_3 - \frac{3}{2}(\alpha_2 - \alpha_3) = -\frac{3}{2}\alpha_2 + \frac{7}{2}\ldots$$

$$\boxed{X_3 = -X_2 - \frac{3}{10}\alpha_2 + \frac{7}{10}\alpha_3}$$

$$2X_1 - X_2 - X_2 - \frac{3}{10}\alpha_2 + \frac{7}{10}\alpha_3 = \frac{\alpha_2}{2} - \frac{\alpha_3}{2}$$

$$2X_1 - 2X_2 = \frac{4}{5}\alpha_2 - \frac{6}{5}\alpha_3$$

$$X_1 - X_2 = \frac{2}{5}\alpha_2 - \frac{3}{5}\alpha_3$$

$$\boxed{X_1 = X_2 + \frac{2}{5}\alpha_2 - \frac{3}{5}\alpha_3}$$

$$X = X_2(1, 1, -1) + \left(\frac{4}{10}\alpha_2 - \frac{6}{10}\alpha_3, 0, -\frac{3}{10}\alpha_2 + \frac{7}{10}\ldots\right)$$

$$+ \alpha_2\left(\frac{4}{10}, 0, -\frac{3}{10}\right) + \alpha_3\left(-\frac{6}{10}, 0, \frac{7}{10}\ldots\right)$$

$$\begin{pmatrix} 2 & -1 & 1 & 0 \\ 0 & 5 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$5x_2 = -5x_3$
$x_2 = -x_3$

$2x_1 - x_2 - x_2 = 0$
$x_1 = \frac{2x_2}{2}$
$x_1 = x_2$

Solución homogeneo $\rightarrow X = (x_2, x_2, -x_2) = \boxed{x_2(1,1,-1)}$

elijo una solución particular:
$\alpha_1 = 1 \Rightarrow \alpha_2 = 4 \wedge \alpha_3 = 2$

$2\alpha_3 - 3\alpha_1$

$$\begin{pmatrix} 2 & -1 & 1 & 1 \\ 0 & 5 & 5 & 1 \end{pmatrix}$$

$5x_2 + 5x_3 = 1$
$x_2 = 1/5 - x_3$ $\quad x_3 = 1/5 - x_2$

$2x_1 - x_2 + 1/5 - x_2 = 0$
$2x_1 = -\frac{1}{5} + 2x_2$
$x_1 = -\frac{1}{10} + x_2$

$X = x_2(1,1,-1) + \left(-\frac{1}{10}, 0, \frac{1}{5}\right)$

**7.**

i)
$$\begin{pmatrix} a & 1 & 1 & b \\ 1 & a & 1 & 1 \\ 1 & 1 & a & -1 \end{pmatrix}$$
$$\begin{pmatrix} a & 1 & 1 & b \\ 0 & a^2-1 & a-1 & a-b \\ 0 & 1-a & a-1 & -2 \end{pmatrix} \begin{matrix} \\ aF_2 - F_1 \\ F_3 - F_2 \end{matrix}$$
$$\begin{pmatrix} a & 1 & 1 & b \\ 0 & a^2-1 & a-1 & a-b \\ 0 & \begin{smallmatrix}1-a\\-a^2+1\end{smallmatrix} & 0 & \begin{smallmatrix}-2\\-a\\+b\end{smallmatrix} \end{pmatrix} F_3 - F_2$$

$(2-a-a^2)x_2 = (-2-a+b)$

$(a^2-1)x_2 + (a-1)x_3 = a-b$

$-a^2 - a + 2$

$\frac{+1 \pm \sqrt{1-4(-1)\cdot 2}}{-2}$

$\frac{+1 \pm 3}{-2} \nearrow -2 \searrow 1$

$\underline{|a=1|} \wedge \underline{|b=1|}$ no tiene solución (falta en fila 3)
$0 + 0 = 1 - b$
$b = 1$

$\underline{|a=-1|}$ solución única (SCD)
$(2+1-1)x_2 = -2+1+b$
$2x_2 = -1+b \Rightarrow x_2 = -\frac{1}{2} + \frac{b}{2}$
$0x_2 - 2x_3 = -1 - b$
$x_3 = +\frac{1}{2} + \frac{b}{2}$

$-x_1 + x_2 + x_3 = b$
$-b + \frac{1}{2} + \frac{b}{2} - \frac{1}{2} + \frac{b}{2} = x_1$

$0 = x_1$

$X = (0, -1/2 + b/2, 1/2 + b/2)$

$\underline{|a=1|} \wedge \underline{|b \neq 3|}$ no tiene solución (SI)
$0 \cdot x_2 = -2-1+b = -3+b$
$3 = b$

$\underline{|a=-2|} \wedge \underline{|b \neq 0|}$ no tiene solución (SI)
$0 = -2+2+b$
$0 = b$

$\underline{|a=1|} \wedge \underline{|b=3|}$ no tiene solución (SI)
$0 + 0 = -2$

$\underline{|a=-2|} \wedge \underline{b=0}$, $\infty$ soluciones (SCI)
$3x_2 - 3x_3 = -2 \longrightarrow x_2 = -2/3 + x_3$
$-2x_1 + x_2 + x_3 = 0$
$-2x_1 + x_3 - 2/3 + x_3 = 0$
$-2x_1 + 2x_3 = \frac{2}{3}$
$x_1 = -\frac{2}{3 \cdot 2} + \frac{2x_3}{2}$
$x_1 = -\frac{1}{3} + x_3$

$X = x_3(1,1,1) + \left(-\frac{1}{3}, -2/3, 0\right)$

ii)
$$\begin{pmatrix} a & 2 & a & 1 \\ a & a+4 & 3a & -2 \\ -a & -2 & 1 & 1 \\ 0 & a+2 & 3a+1 & b \end{pmatrix}$$
$$\begin{pmatrix} a & 2 & a & 1 \\ 0 & a+2 & 2a & -3 \\ 0 & a+2 & 1+3a & -1 \\ 0 & 0 & 0 & b+1 \end{pmatrix} \begin{matrix} \\ F_2-F_1 \\ F_3+F_2 \\ F_4-F_3 \end{matrix}$$
$$\begin{pmatrix} a & 2 & a & 1 \\ 0 & a+2 & 2a & -3 \\ 0 & 0 & 1+a & 2 \\ 0 & 0 & 0 & b+1 \end{pmatrix} \begin{matrix} \\ \\ F_3-F_2 \\ \\ \end{matrix}$$

$\boxed{\forall\, b \neq -1}$ no tiene solución

$b = -1$

$\boxed{a = -1}$ no tiene solución

$\boxed{a \neq -1}$

$\boxed{a = -2}$ no tiene solución

$\boxed{a = 0}$ no tiene solución

$$0 \neq b+1$$
$$-1 \neq b \Rightarrow$$

$$(a+1) \cdot x_3 = 2$$
$a = -1$, $a \neq -1$
$$x_3 = \frac{2}{a+1}$$
$$(a+2)x_2 + 2a x_3 = -3$$
$$(a+2)x_2 + \frac{2a \cdot 2}{a+1} = -3$$

$a = -2$, $a \neq -2$

$$\frac{4a}{a+1} = -3$$
$$x_2 = \left(-3 - \frac{4 \cdot a}{a+1}\right) \cdot \frac{1}{(a+2)}$$
$$\frac{-8}{-1} = -3$$
Abs

$$a x_1 + \left(-6 - \frac{8a}{a+1}\right)\frac{1}{(a+2)} + \frac{a}{a+1} = 1$$
$$a x_1 - \frac{6}{a+2} - \frac{8a}{(a+1)(a+2)} = 1 - \frac{a}{a+1}$$
$$a x_1 = 1 - \frac{a}{a+1} + \frac{6}{a+2} - \frac{8a}{(a+1)(a+2)}$$
$$a = 0 \Rightarrow 0 = 1 - 0 + 3 - 0 \quad \text{abs}$$
$$x_1 = \frac{1}{a} - \frac{1}{a+1} + \frac{6}{(a+2)a} - \frac{8}{(a+1)(a+2)}$$

$$X = \left(\frac{1}{a} - \frac{1}{a+1} + \frac{6}{(a+2)a} - \frac{8}{(a+1)(a+2)}, -\frac{3}{a+2} - \frac{4a}{(a+1)(a+2)}, \frac{2}{a+1}\right)$$

$\boxed{\text{solución única si } b = -1 \wedge a \neq -1, -2, 0}$

$\boxed{\text{no tiene solución si } b \neq -1 \wedge a \in \mathbb{R}}$

$\boxed{\text{no tiene solución si } b = -1 \wedge a = -1, -2, 0}$

**8.**

$(a_{ij}) \in \mathbb{R}^{3\times 3}$

fila columna

$$\begin{array}{c} \begin{matrix} \text{bact. } 1 & \text{bact. } 2 & \text{bact. } 3 \end{matrix} \\ \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{pmatrix} \end{array} \begin{matrix} 1^{er} \text{ alimento} \\ 2^{do} \text{ alimento} \\ 3^{er} \text{ alimento} \end{matrix}$$

alimento consumido diariamente por las bacterias

$\frac{\text{unid. alim}}{\text{bacteria } 1}$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 15000 \\ 30000 \\ 60000 \end{pmatrix}$$

i) No, no se puede porque hay 3 incógnitas ;y 3 ecuaciones pero hay dos L.d.

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{pmatrix} \cdot \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} 15000 \\ 30000 \\ 60000 \end{pmatrix}$$

$X_1 =$ bacterias clase 1
$X_2 =$ bacterias clase 2
$X_3 =$ bacterias clase 3

$$\begin{pmatrix} 1 & 1 & 1 & 15000 \\ 1 & 2 & 3 & 30000 \\ 3 & 4 & 5 & 60.000 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 & 15.000 \\ 0 & 1 & 2 & 15.000 \\ 0 & 1 & 2 & 15.000 \end{pmatrix} \begin{matrix} \\ F_2 - F_1 \\ F_3 - 3F_1 \end{matrix}$$

$$\begin{pmatrix} 1 & 1 & 1 & 15.000 \\ 0 & 1 & 2 & 15.000 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} \\ \\ F_3 - F_2 \end{matrix}$$

$X_2 + 2X_3 = 15000$
$X_2 = -2X_3 + 15000$

$X_1 + 15000 - 2X_3 + X_3 = 15.000$
$X_1 = X_3$

$X = X_3(1,-2,1) + (0,15.000,0)$

ii) $X_1 + X_2 + X_3 = X_3 + (-2X_3) + 15000 + X_3 = \boxed{15000}$

sí, puede

iii) Sea $X_3 = 10.000 \Rightarrow$

$X_2 = -20.000 + 15.000$
$X_2 = -5.000$

no es posible pues no hay bacterias negativas

Sea $X_1 = 2000 \Rightarrow$

$2000 + X_2 + X_3 = 15000$
$X_2 + X_3 = 13000$

$X_2 = -2X_3 + 15000$

$-2X_3 + 15000 + X_3 = 13000$
$\boxed{2000 = X_3}$

$X_2 = -4000 + 15.000$
$\boxed{X_2 = 11.000}$

**9.**

$$\begin{array}{cccc} (j)\ \text{Prod. } 1 & (j)\ \text{Prod. } 2 & (j)\ \text{Prod. } 3 & \\ \begin{pmatrix} 0,1 & 0,2 & 0,3 \\ 0,25 & 0,3 & 0,1 \\ 0,3 & 0,3 & 0,6 \end{pmatrix} & & & \begin{matrix} \text{industria } 1\ (i) \\ \text{"}\ 2\ (i) \\ \text{"}\ 3\ (i) \end{matrix} \end{array}$$

para producir el prod 1. la industria requiere 0,1 unid prod 1

$a_{ij} =$ unid. de producto de la industria $i$ / unidad de producto $j$ para producir

$X_i$ : productos elaborados (j)

$$\underset{\mathbb{R}^{3\times3}}{\begin{pmatrix} 0,1 & 0,2 & 0,3 \\ 0,25 & 0,3 & 0,1 \\ 0,3 & 0,3 & 0,6 \end{pmatrix}} \cdot \underset{\mathbb{R}^{3\times1}}{\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}} = \underset{\mathbb{R}^{3\times1}}{\begin{pmatrix} 10 \\ 25 \\ 20 \end{pmatrix}}$$

$$\begin{pmatrix} 0,1 & 0,2 & 0,3 & 10 \\ 0,25 & 0,3 & 0,1 & 25 \\ 0,3 & 0,3 & 0,6 & 20 \end{pmatrix}$$

$$\begin{pmatrix} 0,1 & 0,2 & 0,3 & 10 \\ 0 & -0,2 & -0,65 & 0 \\ 0 & -0,3 & -0,3 & -10 \end{pmatrix} \begin{matrix} \\ F_2 - 5/2 F_1 \\ F_3 - 3.F_1 \end{matrix}$$

$$\begin{pmatrix} 0,1 & 0,2 & 0,3 & 10 \\ 0 & -0,6 & -1,95 & 0 \\ 0 & -0,6 & -0,6 & -20 \end{pmatrix} \begin{matrix} \\ 3F_2 \\ 2F_3 \end{matrix}$$

$$\begin{pmatrix} 0,1 & 0,2 & 0,3 & 10 \\ 0 & -0,6 & -1,95 & 0 \\ 0 & 0 & 1,35 & -20 \end{pmatrix} \begin{matrix} \\ \\ F_3 - F_2 \end{matrix}$$

$1,35 X_3 = -20$
$X_3 = -14,814$

$-0,6X_2 - 1,95X_3 = 0$
$-0,6X_2 = 1,95X_3$
$X_2 = 48,1455$

$0,1X_1 + 0,2 . 48,1455 + 0,3.(-14,814) = 10$
$X_1 = 48,151$

**10.** i) $\mathbb{R}^{n\times n}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a.e+b.g & a.f+b.h \\ c.e+d.g & c.f+d.h \end{pmatrix}$$

no es conmutativo en $\mathbb{R}^{2\times 2}$

$$\begin{pmatrix} e & f \\ g & h \end{pmatrix}.\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} e.a+f.c & e.b+f.d \\ g.a+h.c & g.b+h.d \end{pmatrix}$$

ii) $\{A \in \mathbb{K}^{3\times 3} : \; A.B = B.A \quad \forall B \in \mathbb{K}^{3\times 3}\}$

Como sucede $\forall B$ debe suceder en particular que:

- para la identidad
- para $B_1$ inversible

$$A.I_n = I_n.A = A$$

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}\cdot\begin{pmatrix} j & k & l \\ m & n & o \\ p & q & r \end{pmatrix} = \begin{pmatrix} aj+bm+cp & ak+bn+cq & al+bo+cr \\ j.d+em+fp & dk+en+fq & dl+eo+fr \\ j.g+hm+i.p & gk+hn+iq & gl+ho+ir \end{pmatrix}$$

CONTINÚA $\Rightarrow$ $\Psi$

**11.**

i) $$A^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^2+bc & ab+bd \\ a.c+dc & c.b+d^2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$ac+dc=0$ — $ac=-dc$ — sea $c\neq 0$ — $a=-d$ — $a^2=d^2$

$ab+bd=0$ — $ab=-bd$ — sea $b\neq 0$ — $a=-d$ — $a^2=d^2$

$$\begin{pmatrix} a & b \\ c & -a \end{pmatrix}.\begin{pmatrix} a & b \\ c & -a \end{pmatrix} \Rightarrow \begin{array}{l} a^2+c.b=-1 \\ c.b+d^2=-1 \\ a^2-d^2=0 \end{array}$$

$a^2+bc-cb-d^2=0$

$bc=cb$

$$= \begin{pmatrix} a^2+bc & a.b-ba \\ ac-ac & b.c+a^2 \end{pmatrix}$$

$ac+dc+ab+bd=0$

$a(c+b)+d(c+b)=0$

$(a+d)(c+b)=0$

Sean $\left.\begin{array}{l} a=1 \\ d=-1 \end{array}\right\} \Rightarrow \quad c.b=-2 \quad \begin{array}{l} b=-2 \\ c=1 \end{array}$

$$A = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix} \Rightarrow A^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

ii) a) $(A.B)^2 \neq A^2.B^2$

$$\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\right]^2 = \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right].\left[\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\right]$$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Rightarrow \text{no vale } \forall n \geq 2$$

b) $A.B = 0$

$$\underbrace{\begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix}}_{\equiv A} \cdot \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}}_{\equiv B} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{pero} \quad A \neq 0 \ \text{ y } \ B \neq 0 \quad \Rightarrow \text{no vale } \forall n \geq 2$$

c) $A.B = A.C \ \text{ y } \ A \neq 0 \ \Rightarrow \ B = C$

Si $A$ es inversible $(|A| \neq 0) \Rightarrow$

$$A.B = A.C$$
$$A^{-1}A.B = A^{-1}A\,C$$
$$I.B = I.C$$
$$B = C$$

si $A$ no es inversible $\Rightarrow$

$$\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}_{\equiv A} \cdot \underbrace{\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}}_{\equiv B} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \cdot \underbrace{\begin{pmatrix} 0 & 1 \\ 4 & 3 \end{pmatrix}}_{\equiv C}$$

$$\underbrace{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}_{A.B} = \underbrace{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}_{A.C} \quad \Rightarrow \text{no vale } \forall n \geq 2$$

$\Rightarrow$ con $AB = A.C$ y $A \neq 0$ $\quad B \neq C$

d) $A.B = 0 \ \Rightarrow \ B.A = 0$

$$\underbrace{\begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}}_{\equiv A} \cdot \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 6 \end{pmatrix}}_{\equiv B} = \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}}_{0} \ \wedge \ \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 6 \end{pmatrix}}_{\equiv B} \cdot \underbrace{\begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}}_{\equiv A} = \begin{pmatrix} 0 & 0 \\ 12 & 0 \end{pmatrix} \neq 0$$

$\Rightarrow$ no vale $\forall n \geq 2$

e) $A^j = 0 \ \Rightarrow \ A = 0$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{pero} \quad \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \neq \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow \text{no vale } \forall n \geq 2$$

f) $A^2 = A \ \Rightarrow \ A = 0 \ \text{ ó } \ A = I$

$$\underbrace{\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}}_{\equiv A} \cdot \underbrace{\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}}_{\equiv A} = \underbrace{\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}}_{\equiv A} \quad \text{pero} \quad \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \neq \left\{ \begin{matrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{matrix} \right. \Rightarrow \text{no vale } \forall n \geq 2$$

iii) $A, B \in \mathbb{K}^{n \times n}$

a) $(A+B)^2 = A^2 + 2AB + B^2$

$$(A+B).(A+B) = A.A + A.B + B.A + B.B = A^2 + AB + BA + B^2 \Rightarrow$$

$\boxed{\text{Sea } A.B = B.A} \Rightarrow \quad = A^2 + 2AB + B^2$

b) $(A^2 - B^2) = (A - B).(A + B)$

$$A.A - B.B = A.A - B.A + A.B - B.B = A^2 - B.A + A.B - B^2$$

$\boxed{\text{Sea } A.B = B.A} \Rightarrow -B.A = -AB \quad \Rightarrow \ = A^2 \cancel{- A.B} \cancel{+ A.B} - B^2$

(Ψ) viene del 10 ii)

$$\begin{pmatrix} j & k & l \\ m & n & o \\ p & q & r \end{pmatrix} \cdot \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} ja+kd+lg & jb+ke+lh & jc+kf+li \\ ma+nd+og & mb+ne+oh & mc+nf+oi \\ pa+qd+rg & pb+qe+rh & pc+qf+ri \end{pmatrix} = \begin{pmatrix} aj+bm+cp & ak+bn+cq & al+bo+cr \\ dj+em+fp & dk+en+fq & dl+eo+fr \\ gj+hm+ip & gk+hn+iq & gl+ho+ir \end{pmatrix}$$

$\cancel{ja} + kd + lg = \cancel{aj} + bm + cp \qquad jb + ke + lh = ak + bn + cq \qquad jc + kf + li = al + bo + cr$

$mb + \cancel{ne} + oh = dk + \cancel{en} + fq$

$pc + qf \cancel{+ ri} = gl + ho \cancel{+ ir} \qquad ma + nd + og = dj + em + fp \qquad mc + nf + oi = dl + eo + fr$

$pa + qd + rg = gj + hm + ip \qquad pb + qe + rh = gk + hn + iq$

**13.**

i)
$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \equiv A$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} F_1-F_2 \\ F_2-F_3 \\ \phantom{0} \end{array}$$

$\Rightarrow$ es inversible $\wedge$

$$\boxed{A^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}}$$

ii)
$$\begin{pmatrix} \cos\theta & -\text{sen}\,\theta & 0 \\ \text{sen}\,\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \equiv A$$

$$\left(\begin{array}{ccc|ccc} \cos\theta & -\text{sen}\,\theta & 0 & 1 & 0 & 0 \\ \text{sen}\,\theta & \cos\theta & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} \cos^2\theta & -\text{sen}\,\theta.\cos\theta & 0 & \cos\theta & 0 & 0 \\ \text{sen}^2\theta & \text{sen}\,\theta.\cos\theta & 0 & 0 & \text{sen}\,\theta & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \times\cos\theta \\ \times\text{sen}\,\theta \\ \phantom{0} \end{array}$$

sean $\cos\theta \neq 0$
$\text{sen}\,\theta \neq 0$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \cos\theta & \text{sen}\,\theta & 0 \\ \text{sen}^2\theta & \text{sen}\,\theta.\cos\theta & 0 & 0 & \text{sen}\,\theta & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} F_1+F_2 \\ (1/\text{sen}^2\theta)\,F_2 \\ \phantom{0} \end{array}$$

$\Rightarrow$ es inversible y

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \cos\theta & \text{sen}\,\theta & 0 \\ 1 & \cos\theta/\text{sen}\,\theta & 0 & 0 & 1/\text{sen}\,\theta & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\boxed{A^{-1} = \begin{pmatrix} \cos\theta & \text{sen}\,\theta & 0 \\ -\text{sen}\,\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \cos\theta & \text{sen}\,\theta & 0 \\ 0 & \cos\theta/\text{sen}\,\theta & 0 & -\cos\theta & \frac{1}{\text{sen}\,\theta}-\text{sen}\,\theta & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \phantom{0} \\ F_2-F_1 \\ \phantom{0} \end{array}$$

$$\frac{1}{\cos\theta} - \frac{\text{sen}^2\theta}{\cos\theta} = \frac{\cos^{\cancel{2}}\theta}{\cancel{\cos\theta}}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \cos\theta & \text{sen}\,\theta & 0 \\ 0 & 1 & 0 & -\text{sen}\,\theta & \frac{1}{\cos\theta}-\frac{\text{sen}^2\theta}{\cos\theta} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \phantom{0} \\ \left(\frac{\text{sen}\,\theta}{\cos\theta}\right)\cdot F_2 \\ \phantom{0} \end{array}$$

iii)
$$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 1 & -2 & 3 \\ 3 & 1 & -1 & 3 \end{pmatrix} \equiv A$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & -2 & 3 & 0 & 0 & 1 & 0 \\ 3 & 1 & -1 & 3 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 3 & 0 & -3 & 0 & 3 & 0 & 0 & 0 \\ & & & & & & & \\ & & & & & & & \\ 0 & 1 & 2 & 3 & -3 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} 3F_1 \\ \\ \\ F_4-F_1 \end{array}$$

$$\left(\begin{array}{cccc|cccc} 2 & 0 & -2 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 3 & -2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 3 & -3 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \frac{2}{3}F_1 \\ \\ F_3-F_1 \\ \phantom{0} \end{array}$$

A no es inversible $\Rightarrow$ el sistema

$$A.X = I$$

no es resoluble

$$\left(\begin{array}{cccc|cccc} 1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 3 & -2 & 1 & 1 & 0 \\ 0 & 0 & -2 & 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & -1 & -1 & -1 & 1 \end{array}\right) \begin{array}{l} F_1/2 \\ F_2+F_3 \\ F_3-F_4 \\ F_4-F_2 \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & -1 & -1 & 1 \\ 0 & 1 & 0 & 3 & -1 & 2 & 2 & -1 \\ 0 & 0 & 1 & 0 & -1/2 & 0 & -1/2 & 1/2 \\ 0 & 0 & 1 & 0 & -1 & -1 & -1 & 1 \end{array}\right) \begin{array}{l} F_1+F_4 \\ F_2-F_4 \\ F_3/-1 \\ \phantom{0} \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & -1 & -1 & 1 \\ 0 & 1 & 0 & 3 & -1 & 2 & 2 & -1 \\ 0 & 0 & 1 & 0 & -1/2 & 0 & -1/2 & 1/2 \\ 0 & 0 & 0 & 0 & -1/2 & -1 & -1/2 & 1/2 \end{array}\right) \begin{array}{l} \\ \\ \\ F_4-F_3 \end{array}$$

$\rightarrow$ incompatible

iv) $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv A$

$\begin{pmatrix} a & b & | & 1 & 0 \\ c & d & | & 0 & 1 \end{pmatrix}$ $a \neq 0$ $a.d \neq b.c$

$c \neq 0$ $\frac{d}{c} \neq \frac{b}{a}$

$\begin{pmatrix} 1 & b/a & | & 1/a & 0 \\ 1 & d/c & | & 0 & 1/c \end{pmatrix} \begin{matrix} F_1/a \\ F_2/c \end{matrix}$

$\Rightarrow$ existe inversa y

$\frac{ad-bc}{ca}$ $\begin{pmatrix} 1 & b/a & | & 1/a & 0 \\ 0 & \frac{d}{c}-\frac{b}{a} & | & -\frac{1}{a} & \frac{1}{c} \end{pmatrix} F_2 - F_1$ $\frac{ad-bc}{ca}$

$\begin{pmatrix} 1 & 0 & | & m & -K/c \\ 0 & \frac{ad-bc}{ca} & | & -\frac{1}{a} & \frac{1}{c} \end{pmatrix} F_1 - \underbrace{\left(\frac{c.a}{ad-bc}\right).\frac{b}{a}}_{\equiv K}.F_2$

$\frac{b}{a}\left(\frac{d}{c}-\frac{b}{a}\right)\left(\frac{ca}{ad-bc}\right)$

$\frac{1}{a}\left(\frac{ad-\cancel{cb}+cb}{ad-cb}\right)$

$F_2.\frac{ca}{ad-bc}$ $\begin{pmatrix} 1 & 0 & | & m & -K/c \\ 0 & 1 & | & -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{pmatrix}$ $m = \frac{1}{a} + \left(\frac{c.a}{ad-bc}\right).\frac{b}{a}.\frac{1}{a}$

$$\boxed{A^{-1} = \begin{pmatrix} \frac{1}{a}\left(1+\frac{c.b}{ad-bc}\right) & -\frac{b}{ad-bc} \\ -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{pmatrix}}$$

$$A^{-1} = \begin{pmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{pmatrix}$$

v) $\begin{pmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & & 0 \\ \dots & \dots & & \dots \\ 0 & 0 & \dots & a_{nn} \end{pmatrix} \equiv A$

$\begin{pmatrix} a_{11} & 0 & \dots & 0 & | & 1 & | & 0 & | \dots | & 0 \\ 0 & a_{22} & \dots & 0 & | & 0 & | & 1 & | \dots | & 0 \\ \dots & \dots & \dots & \dots & | & \dots & | & \dots & | \dots | & \dots \\ 0 & 0 & \dots & a_{nn} & | & 0 & | & 0 & | & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 & \dots & 0 & | & 1/a_{11} & | & 0 & | \cdot | & 0 \\ 0 & 1 & \dots & 0 & | & 0 & | & 1/a_{22} & | \cdot | & 0 \\ \cdot & \cdot & \cdot & \cdot & | & \cdot & | & \cdot & | \cdot | & \cdot \\ 0 & 0 & \dots & 1 & | & 0 & | & 0 & | \cdot | & 1/a_{nn} \end{pmatrix} \begin{matrix} F_1 \\ F_2 \\ F/ \\ F_n/a_{nn} \end{matrix}$

$\Rightarrow$ es inversible y

$$\boxed{A^{-1} = \begin{pmatrix} 1/a_{11} & 0 & \dots & 0 \\ 0 & 1/a_{22} & \dots & 0 \\ \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1/a_{nn} \end{pmatrix}}$$

$\forall \; a_{ii} \neq 0$ con $i = 1, \dots, n$ $\downarrow$ nro. de columnas y filas

vi) $\begin{pmatrix} 2 & 1 & 3 & 1 & 2 \\ 0 & 5 & -1 & 8 & 2 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix} \equiv A$

$\begin{pmatrix} 2 & 1 & 3 & 1 & 2 & | & 1 & 0 & 0 & 0 & 0 \\ 0 & 5 & -1 & 8 & 2 & | & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & | & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & | & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & | & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

$\Rightarrow$ A no es inversible

$\boxed{\nexists \; A^{-1}}$

$\begin{pmatrix} 2 & 1 & 3 & 1 & 2 & | & 1 & 0 & 0 & 0 & 0 \\ 0 & 5 & -1 & 8 & 2 & | & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & | & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & | & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & | & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \rightarrow$ absurdo

**14.** a) $A \in \mathbb{K}^{n\times n}$ es inversible

b) $B, C \in \mathbb{K}^{n\times m}$

i) $(A.B) = (A.C) \in \mathbb{K}^{n\times m}$ Por a) $\exists \; A^{-1}: \quad A.A^{-1} = A^{-1}.A = I$

$\underbrace{A^{-1}.A}_{n\times n}.\underset{n\times m}{B} = \underbrace{A^{-1}.A}_{n\times n}.\underset{n\times m}{C}$

$I_n.B = I_n.C$

$\boxed{B = C}$

ii) $A.B = 0$

$A^{-1}.A.B = A^{-1}.0$

$I.B = 0$

$\boxed{B = 0}$

**15.**

i) $A, B \in \mathbb{K}^{n\times n}$ inversible

si $A+B$ es inversible $\Rightarrow \exists\ M$ :

Sean:

$M.(A+B) = (A+B).M = I$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \Rightarrow \exists \quad A^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \ \wedge \ B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

pero $A+B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \rightarrow (A+B)$ no es inversible

$\boxed{\text{La afirmación es falsa}}$

ii) Sea $A$ inversible

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & \dots \\ \vdots & & & \\ a_{m1} & \dots & \dots & a_{mn} \end{pmatrix} \quad \Rightarrow \quad A^t = \begin{pmatrix} a_{11} & a_{21} & \dots & a_{m1} \\ a_{12} & a_{22} & \dots & \dots \\ \vdots & & & \\ a_{1n} & \dots & \dots & a_{mn} \end{pmatrix}$$

Si $A$ es inversible $\Rightarrow |A| \neq 0$

pero $|A| = |A^t| \Rightarrow |A^t| \neq 0 \Rightarrow A^t$ es inversible $\wedge$

Sea $A^t$ inversible $|A^t| \neq 0 \Rightarrow |A| \neq 0 \Rightarrow A$ es inversible

$\boxed{\text{La afirmación es verdadera}}$

$$|A| = \left|\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & a_{22} & \dots & \\ a_{m1} & \dots & & a_{mn} \end{pmatrix}\right| = \left|\begin{pmatrix} a & b & \dots & c \\ 0 & d & \dots & e \\ 0 & \dots & & f \end{pmatrix}\right| = a.d.\dots f$$

triangular inferior

(la diagonal mayor $\Rightarrow$ no se modifica si intercambiamos filas con columnas)

$\Rightarrow |A^t| = |A|$

iv) $A$ nilpotente pues $\exists\, j \in \mathbb{N} : A^j = 0$

$$\underbrace{A \,.\, A \,.\, A \dots A}_{j \text{ veces}} = 0$$

$|A^j| = |A|^j = 0 \Rightarrow |A| = 0 \Rightarrow A$ no es inversible

$\boxed{\text{La afirmación es verdadera}}$

**16.** $A \in \mathbb{K}^{n\times n}$
$b \in \mathbb{K}^{n\times 1}$ $\qquad Ax = b$

$Ax = b \qquad$ y $\exists\, A^{-1} : A^{-1}.A = I_n$

$\Leftarrow$)) $A^{-1}.Ax = A^{-1}.b$
$I_n.x = A^{-1}.b$
$\boxed{x = A^{-1}.b}$ tiene solución

$A.x = A.(A^{-1}.b) = (A.A^{-1}).b$
$= I.b$
$A.x = b$
la solución tiene sólo la forma $A^{-1}.b$

$\Rightarrow$)) $Ax = b$ tiene solución única
$\exists\, s \in \mathbb{K}^{n\times 1} : A.s = b$

supongamos $A$ no inversible $\Rightarrow |A| = 0 \Rightarrow$ el sistema no tiene solución única {absurdo}
$\Rightarrow A$ es inversible

**12.** $A, B \in \mathbb{K}^{m \times n}$

① $\underset{m \times n}{A} . x = B . x$ ② $\forall x \in \mathbb{K}^{n \times 1}$

Sean: $A x - B x = 0$

$(A - B) x = 0$ y sea $x \in \mathbb{K}^{n \times 1}$: $x_1, x_2, \dots x_n \neq 0$ $\left(\begin{array}{c}\exists \text{ este } x \text{ por} \\ \text{la condición ②}\end{array}\right)$ $\Rightarrow$

$A - B = 0 \Rightarrow \boxed{A = B}$