PRACTICA 3

1. (a) $f(x) = \dfrac{x}{|x|}$

$$f'(x) = \lim_{h\to 0} \frac{\frac{x+h}{|x+h|} - \frac{x}{|x|}}{h} = \lim_{h\to 0} \frac{\frac{(x+h)|x| - |x+h|(x)}{|x+h|\,|x|}}{h} = \lim_{h\to 0} \frac{1}{h} \cdot \frac{x|x| + h|x| - x|x+h|}{|x+h|\,|x|}$$

si $x > 0 \Rightarrow \lim_{h\to 0} \dfrac{x^2 + hx - x^2 - x h}{h(x^2 + x h)} = \lim_{h\to 0} \dfrac{0}{h(x^2+xh)} = \varnothing \Rightarrow$

si $x < 0 \Rightarrow \lim \dfrac{-x^2 - hx + x^2 + hx}{h(x^2+hx)} = \lim_{h\to 0} \dfrac{0}{h(x^2+xh)} = \varnothing$

$$\boxed{\frac{d}{dx}\left(\frac{x}{|x|}\right) = \varnothing}$$

(b) $f(x) = \begin{cases} x & \text{si } x < \varnothing \\ \ln(1+x) & \text{si } x \geq \varnothing \end{cases}$

$f'(x) = \lim_{h\to 0} \dfrac{x+h - x}{h} = 1$ si $x < 0$

$$f'(x) = \lim_{h\to 0} \frac{\ln(1+x+h) - \ln(1+x)}{h} = \lim_{h\to 0} \frac{\ln\left(\frac{1+x+h}{1+x}\right)}{h} = \lim_{h\to 0} \ln\left(1 + \frac{h}{1+x}\right)^{\frac{1}{h}\frac{(1+x)}{(1+x)}}$$

$\to \ln e$

$\ln e^{\left(\frac{1}{1+x}\right)} = \boxed{\dfrac{1}{1+x}}$ si $x \geq \varnothing$

(c) $f(x) = \begin{cases} \operatorname{sen}(x^2-1) & x < -1 \\ \sqrt{x+1} & x \geq -1 \end{cases}$

$(x^2-1) + (2xh + h^2)$

$$F'(x) = \lim_{h\to 0} \frac{\operatorname{sen}\left[\overbrace{(x+h)^2 - 1}^{x^2+2xh+h^2-1}\right] - \operatorname{sen}\left[x^2-1\right]}{h} =$$

$$\lim_{h\to 0} \frac{\operatorname{sen}(x^2-1)\cdot\cos(2xh+h^2) + \operatorname{sen}(2xh+h^2)\cdot\cos(x^2-1) - \operatorname{sen}(x^2-1)}{h}$$

$$\lim_{h\to 0} \operatorname{sen}(x^2-1)\left[\cos(2xh+h^2) - 1\right] + \operatorname{sen}(2xh+h^2)\cdot\cos(x^2-1)$$

$$+ \left[\operatorname{sen}(2xh)\cos(h)^2 + \operatorname{sen}(h)^2\cos(2xh)\right]\cdot\cos(x^2-1)$$

$$\lim_{h\to 0} \frac{\operatorname{sen}(x^2-1)\left(\cos(2xh+h^2) - 1\right) + \operatorname{sen}(2xh)\cdot\cos(h^2)\cdot\cos(x^2-1) + \operatorname{sen}(h^2)\cdot\cos(2xh)\cdot\cos(x^2-1)}{h}$$

$$\lim_{h\to 0} \frac{\operatorname{sen}(x^2-1)\left(\cos(2xh+h^2) - 1\right)\overbrace{(2xh+h^2)}^{2x+h}}{h(2xh+h^2)} + \lim_{h\to 0} \frac{2x\,\operatorname{sen}(2xh)\,\overbrace{\cos(h^2)}^{\to 1}\cos(x^2-1)}{2x\,h} + \lim h\frac{\operatorname{sen}(h^2)\,\overbrace{\cos(2xh)}^{\to 1}\cos(x^2-1)}{h\cdot h}$$

$h\to 0 \Rightarrow$
$2xh + h^2 \to 0$
$2xh \to 0$

$$f'(x) = \varnothing + 2x\cdot\cos(x^2-1) + 0 = \boxed{2x\cdot\cos(x^2-1)}$$

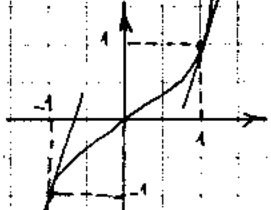
2. $y = |x|$

$$\lim_{h\to 0} \frac{|x+h| - |x|}{h} = \lim_{h\to 0} \frac{(x+h)^2 - (x)^2}{h\,|x+h| + h\,|x|} = \lim_{h\to 0} \frac{\cancel{x^2} + 2x\cancel{h} + \cancel{h^2} - \cancel{x^2}}{\cancel{h}\,(|x+h| + |x|)} = \lim_{h\to 0} \frac{\overbrace{2x+h}^{\to 2x}}{\underbrace{|x+h| + |x|}_{2|x|}}$$

$f'(x) = \dfrac{2x}{2|x|} = \boxed{\dfrac{x}{|x|}}$ es válido lo que se afirmó

3. (a)

$y = x^3$

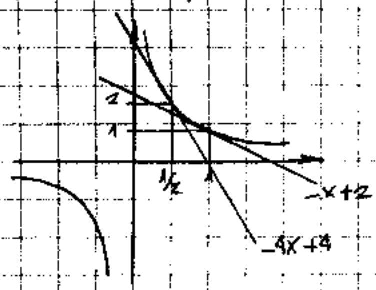


pendiente tg $y = y' = \boxed{3x^2}$

en $x_0 = 1 \Rightarrow y' = 3$ $\quad y_{tg} = 3(x-1) + 1 = \boxed{3x - 2}$

en $x_0 = -1 \Rightarrow y' = 3$ $\quad y_{tg} = 3(x+1) + -1 = \boxed{3x + 2}$

(b) $y = \frac{1}{x} = x^{-1} \Rightarrow y' = -\frac{1}{x^2}$



en $x_0 = 1/2$ $\quad y_{tg} = -4(x - 1/2) + 2 = \boxed{-4x+4}$

en $x_0 = 1$ $\quad y_{tg} = -1(x-1) + 1 = \boxed{-x+2}$

4. (a) $y = x^2 - 3x + 2 \quad [1,2]$ continua en $[1,2]$ $\quad \left.\begin{matrix} F(1) = \emptyset \\ F(2) = \emptyset \end{matrix}\right\} \Rightarrow \exists\, c \in [1,2] : F'(c) = \emptyset$

(b) $y = \overbrace{(x-1)(x+2)}^{(x^2-x+2x-2)(x-3)}(x-3) \quad [1,3]$ continua en $[1,3]$ $\quad \left.\begin{matrix} F(1) = \emptyset \\ F(3) = \emptyset \end{matrix}\right\} \Rightarrow \exists\, c \in [1,3] : F'(c) = \emptyset$

(c) $y = \operatorname{sen}^2(x) \quad [0,\pi]$ continua en $[0,\pi]$ $\quad \left.\begin{matrix} F(0) = 0 \\ F(\pi) = 0 \end{matrix}\right\} \Rightarrow \exists\, c \in [0,\pi] : F'(c) = \emptyset$

(a) $y' = 2x - 3$
$2x - 3 = 0$
$x = 3/2 \in [1,2]$

(b) $y = x^3 - x^2 - 2x^2 + 2x - 3x^2 + 3x + 6x - 6$
$y' = 3x^2 - 2x - 4x + 2 - 6x + 3 + 6$
$y' = 3x^2 - 12x + 11$
$3x^2 - 12x + 11 = 0$
$x = \frac{\sqrt{3}+6}{3}, \frac{-\sqrt{3}+6}{3} \in [1,3]$

(c) $y' = 2 \cdot \operatorname{sen} x \cdot \cos x$
$2 \operatorname{sen} x \cdot \cos x = 0$
$\operatorname{sen} x = 0$
$\cos x = 0$
$x = 0, \pi/2 \in [0,\pi]$

5. $y = \sqrt[3]{x^2 - 5x + 6}$

$\frac{+5 \pm \sqrt{25 - 4 \cdot 1 \cdot 6}}{2} = \begin{matrix} 3 \\ 2 \end{matrix}$

$\sqrt[3]{x^2 - 5x + 6} = 0$
$x^2 - 5x + 6 = 0$
$1(x-3)(x-2) = 0 \Leftrightarrow \begin{matrix} x = 3 \\ x = 2 \end{matrix}$

$(x-3)(x-2) > 0$
$x-3>0 \quad x-2>0 \;\|\; x-3<0 \quad x-2<0$
$x>3 \quad x>2 \;\|\; x<3 \quad x<2$
$(3,+\infty) \cup (-\infty,2)$

$y' = \frac{1}{3} \frac{(2x-5)}{\sqrt[3]{(x^2-5x+6)^2}}$

$\frac{2x-5}{3\sqrt[3]{(x^2-5x+6)^2}} = 0$

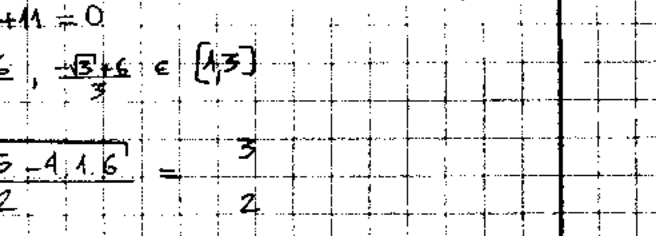
$D(y') = \mathbb{R} - \{2,3\}$

$2x - 5 = \emptyset$
$x = 5/2$

$3\sqrt[3]{(x^2-5x+6)^2} \neq \emptyset$
$x \neq 2$
$x \neq 3$

el cero de y' $(5/2)$ se halla entre los ceros de y $(2,3)$

$5/2 \in [2,3]$



6. $y = 1 - \sqrt[5]{x^4} \quad [-1,1] \quad D(y) = \mathbb{R}$

$y(1) = \emptyset$
$y(-1) = \emptyset$

$y' = -\frac{4}{5} \cdot \frac{1}{\sqrt[5]{x}} = \frac{-4}{5\sqrt[5]{x}} \qquad D(y') = \mathbb{R} \neq 0$

$\forall x \neq 0$

y no cumple el teorema de Rolle
porque pese a ser continua en $[-1,1]$
no es derivable en $x_0 = \emptyset \in [-1,1]$

$\frac{-4}{5\sqrt[5]{x}} \neq \emptyset$

$-4 \neq \emptyset$ nunca se hacerá $\emptyset$ $\forall x \in \mathbb{R} \neq 0$

7. $f(x) = x(x-1)(x-2)(x+5) = x^4+2x^3-13x^2+18x$

$f'(x) = 4x^3+6x^2-26x+18$ → $P_{(x)}$ de orden 3, a lo sumo tiene 3 raíces $\mathbb{R}$

$\left.\begin{array}{l} f(0)=\emptyset \\ f(1)=\emptyset \end{array}\right\} \Rightarrow \exists\ c\in[0,1]: f'(c)=\emptyset$

$\left.\begin{array}{l} f(1)=\emptyset \\ f(2)=\emptyset \end{array}\right\} \Rightarrow \exists\ c\in[1,2]: f'(c)=\emptyset$

$\left.\begin{array}{l} f(-5)=\emptyset \\ f(0)=\emptyset \end{array}\right\} \Rightarrow \exists\ c\in[-5,0]: f'(c)=\emptyset$

$\forall x>2$ $f'(x)$ es $\overset{\text{monótona}}{\text{creciente}}$ $\Rightarrow$ no vuelve a tocar cero

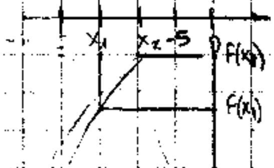
$x_2>x_1>2$ $f(x_2)>f(x_1)$

$\forall x>2$ $4x^3+6x^2-26x+18 > 10 > 0$

$\forall x<-5$ $4x^3+6x^2-26x+18 < -200 < 0$

$\forall x<-5$ $f'(x)$ es $\overset{\text{monótona}}{\text{creciente}}$ $\Rightarrow$ no vuelve a tocar cero

$-5>x_2>x_1 \Rightarrow f(x_2)>f(x_1)$



$x_1<x_2<-5<0$ $\quad 2x_2^3+3x_2^2-13x_2 > 2x$

$\forall x<-5$ $\quad x_2(2x_2^2+3x_2-13) > x_1(2x^2+3x_2-13)$

$35 > 2x^2+3x$

8. $f=3x^5+15x-8=0$ $\quad f, f'$ continuas en $\mathbb{R}$ y derivables en $\mathbb{R}$

$f' = 15x^4+15$ $\quad D(f')=\mathbb{R}$ $\quad$ por teo. Rolle $\nexists$ dos valores $D(f)$ a,b: $f(a)=f(b)$

$15x^4+15=0$ $\quad \nexists\ x\in\mathbb{R}: f'(x)=\emptyset \Rightarrow \nexists\ f(a)=f(b)=\emptyset$ $\forall a\neq b$, $a,b\in D(f)$

$x^4 = -15/15$

$x = \sqrt{-1}$ $\quad \nexists\ x: f'(x)=0$

Luego a lo sumo f tiene una raíz $\mathbb{R}$ (solo un valor $f(c)=\emptyset$)

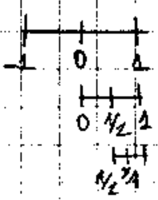
$\left.\begin{array}{l} f(1)=10>\emptyset \\ f(-1)=-26<0 \end{array}\right\} \Rightarrow \exists\ x$ Bolzano $c\in(-1,1): f(c)=0$

$f(0)=-8<0$ $\quad (0,1)$

$f(\tfrac{1}{2})=-0{,}40625<0$ $\quad (\tfrac{1}{2},1)$

$f(\tfrac{3}{4})=3{,}9>0$ $\quad (\tfrac{1}{2},\tfrac{3}{4})$

cero $\in(0{,}5;\ 0{,}75)$



f tiene una raíz en el intervalo $(\tfrac{1}{2},\tfrac{3}{4})$

9. $y=2x-x^2$ en $[0,1]$ $\Rightarrow \exists\ x_0\in[0,1]$

$$f'(x_0) = \frac{f(1)-f(0)}{1-0} = \frac{1-0}{1-0} = 1$$

$y' = 2-2x = -2x+2$

$f'(\tfrac{1}{2})=1$

10. $\underbrace{e^x}_{f(x)} \geq \underbrace{1+x}_{g(x)}$ $\quad f'(x)=e^x>0$ $\forall x\in\mathbb{R}$ $\quad$ defino $H(x)=e^x-1-x$

$g(x)=1>0$ $\forall x\in\mathbb{R}$ $\quad H(x)=f(x)-g(x)$ $\quad H'(x)=(e^x-1)$

$H(x)$ continua en $\mathbb{R}$ y derivable en $\mathbb{R}$ $\Rightarrow \exists\ x_0$:

$e^x-1>0$

$e^x>1$

si $x>0$ $\quad \underbrace{H'(x_0)}_{>0} = \dfrac{H(x)-H(0)}{x-0} = \dfrac{e^x-1-x}{x} > 0 \Rightarrow H(x)-H(0)>\emptyset$

$x>\ln 1$

$x>0$

Luego $H(x)>0 \Rightarrow f(x)-g(x)>0 \Rightarrow f(x)>g(x)$

$e^x>1+x$ si $x>0$

$H'(x)>0$ si $x>0$

$H'(x)<0$ si $x<0$

$H'(x)=0$ si $x=0$

si $x<0$ $\underbrace{H'(x)}_{<0} = \frac{H(x) - H(0)}{x - 0} = \frac{e^x - 1 - x}{x} < 0 \Rightarrow e^x - 1 - x > 0$

$e^x > 1 + x$ si $x<0$

si $x=0$ $e^0 - 1 - 0 = \emptyset \Rightarrow e^x \geq 1 + x \quad \forall x \in \mathbb{R}$

11. $f(x) = x^2$ en $[1,2]$
$g(x) = x^3$

TVM Cauchy dice que $c \in (1,2)$. $\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)} \Rightarrow$

$\frac{f'(c)}{g'(c)} = \frac{2x}{3x^2} = \frac{4-1}{8-1} = \frac{3}{7} \Rightarrow \frac{2x}{3x^2} = \frac{3}{7} \Rightarrow \boxed{\frac{14}{9} = x} \cong 1{,}55\hat{5}$

12. f, g continuas en $[a,b]$ y derivables en (a,b)

(a) $f'(x) = 0 \quad \forall x \in (a,b)$ × TVM Lagrange $\exists c : f'(c) = \frac{f(b) - f(a)}{b - a}$

si $\forall x_1, x_2 \in (a,b)$ se da $\emptyset = \frac{f(x_1) - f(x_2)}{x_1 - x_2} \Rightarrow f(x_1) = f(x_2) \Rightarrow$

$f(x) = K$ en (a,b)

(b) $f'(x) = g'(x) \quad \forall x \in (a,b)$

$H(x) = f(x) - g(x)$
$H'(x) = f'(x) - g'(x)$

si $H'(x) = \emptyset \quad \forall x \in (a,b) \Rightarrow$

$\emptyset = \frac{H(x_1) - H(x_2)}{x_1 - x_2} \quad \forall x_1, x_2 \in (a,b) \Rightarrow$

$H(x_1) = H(x_2) \Rightarrow H(x) = K \quad \forall x \in (a,b) \Rightarrow$

$f(x) - g(x) = K \Rightarrow f(x) = g(x) + K$

(c) $f'(x) > 0 \quad \forall x \in (a,b) \Rightarrow f$ derivable en (a,b) y continua en $[a,b]$

si $f'(x) > 0 \Rightarrow \exists c : f'(c) = \frac{f(b) - f(a)}{b - a}$
$\forall b > a$
$\underline{\underline{b - a > 0}}$

$\underbrace{f'(c)}_{>0} = \frac{f(b) - f(a)}{\underbrace{b - a}_{>0}} > 0 \Rightarrow f(b) > f(a)$ si $b > a$

f estrictamente creciente en (a,b)

(d) $f'(x) < 0 \quad \forall x \in (a,b) \Rightarrow f$ derivable en $(a,b) \Rightarrow f$ continua en $[a,b]$

$f'(x) < 0$
y $b > a$
$b - a > a$

$\Rightarrow \exists c : \underbrace{f(c)}_{<0} = \frac{f(b) - f(a)}{\underbrace{b - a}_{>0}} < 0$

$\Rightarrow f(b) - f(a) < 0 \Rightarrow f(b) < f(a) \quad \forall x \in (a,b)$

f estrictamente decreciente en (a,b)

$x^n-1 \mid x-1$
$-x^n \quad x^{n-1}$

13. L'hôpital

(a) $\lim_{x\to1}\frac{x-1}{x^n-1} \underset{\text{L'hôpital}}{=} \lim_{x\to1}\frac{1}{n\cdot x^{n-1}} = \boxed{\frac{1}{n}}$ $\forall n\in\mathbb{N}$

(b) $\lim_{x\to1}\frac{\operatorname{sen}(x)}{\sqrt{1-\cos x}} = \boxed{\frac{\operatorname{sen}(x)}{\sqrt{1-\cos x}}}$

(c) $\lim_{x\to+\infty} x^n e^{-x} = \lim_{x\to+\infty} x^n\cdot\frac{1}{e^x} \underset{\text{L'hôpital}}{=} \lim_{x\to+\infty}\frac{n\cdot x^{n-1}}{e^x} \underset{\text{sucesivas (n) veces}}{=} \lim_{x\to+\infty}\frac{n(n-1)(n-2)\dots x^0}{e^x} =$

$= \lim_{x\to+\infty}\frac{n!}{e^x} = \boxed{\varnothing}$ pues $n!$ cualquier $n\in\mathbb{N}$ es acotado (por muy grande que sea) y $e^x\to+\infty$ si $x\to+\infty$ y supera con creces a $n!$

(d) $\lim_{x\to0}\frac{e^{-\frac{1}{x^2}}}{x^n} = \lim_{x\to0}\underbrace{\left(\frac{1}{e}\right)^{\frac{1}{x^2}}}_{\to0}\cdot\underbrace{\frac{1}{x^n}}_{\to\infty} = \lim_{x\to0}\frac{(1/e)^{1/x^2}}{x^n} \underset{\text{L'hôpital}}{=} \lim_{x\to0}\frac{e^{-\frac{1}{x^2}}\cdot-(-2)\cdot x^{-3}}{n\cdot x^{n-1}}$

$\lim_{x\to0}\frac{\left(\frac{1}{e}\right)^{\frac{1}{x^2}}\cdot\frac{2}{x^3}}{n\cdot x^{n-1}} = \lim$ continúa abajo

(e) $\lim_{x\to1}\frac{2}{x^2-1}-\frac{1}{x-1} = \lim_{x\to1}\frac{2-x-1}{(x-1)(x+1)} = \lim_{x\to1}\frac{-(x-1)}{(x-1)(x+1)} = \lim_{x\to1}\frac{-1}{x+1} = \boxed{-\frac{1}{2}}$

(f) $\lim_{x\to1} x^{\frac{1}{1-x}} = e^{\lim_{x\to1}\frac{1}{1-x}\cdot\ln x} = e^{\lim_{x\to1}\frac{\ln x}{1-x}} \underset{\text{L'hôpital}}{=} e^{\lim_{x\to1}\frac{1/x}{-1}} = e^{-1} = \boxed{\frac{1}{e}}$

del (d)

$\lim_{x\to0}\frac{e^{-\frac{1}{x^2}}\cdot2}{n\cdot x^{n+2}} \underset{\text{L'hôpital}}{=} \lim_{x\to0}\frac{2\cdot e^{-\frac{1}{x^2}}\cdot2}{n\cdot(n+2)\cdot x^{\underbrace{n+2-1+3}_{n+4}}} = \lim_{x\to0}\frac{4\,e^{-1/x^2}\cdot2}{n(n+2)(n+4)\cdot x^{(n+6)}}$

$\underset{\text{n veces L'hôpital}}{=} \frac{\overset{\text{acotado}}{2^n}\cdot\left(e^{-1/x^2}\right)\to0}{\underbrace{n(n+2)(n+4)\dots(n+(n-2))\cdot x^{(n+2n)}}_{\to\text{acotado}}} = \boxed{\varnothing}$

$\frac{2^n}{n!}\to\varnothing$ $\forall n>3$

(b) $\lim_{x\to0}\frac{\operatorname{sen}(x)}{\sqrt{1-\cos x}} = \lim_{x\to0}\frac{\operatorname{sen} x\cdot\sqrt{1+\cos x}}{\sqrt{1-\cos^2 x}} = \lim_{x\to0}\frac{\cancel{\operatorname{sen}(x)}\cdot\sqrt{1+\cos x}}{\cancel{\operatorname{sen} x}} = \boxed{\sqrt{2}}$

14.

(a) $y^2 = 4px$

$\frac{d}{dx}y^2 = \frac{d}{dx}4px$

$2y\cdot\frac{dy}{dx} = 4p$

$\boxed{\frac{dy}{dx} = \frac{2p}{y}}$

(b) $x^2+y^2 = a^2$

$\frac{d}{dx}x^2 + \frac{d}{dx}y^2 = \frac{d}{dx}a^2$

$2x + 2y\cdot\frac{dy}{dx} = 0$

$\boxed{\frac{dy}{dx} = \frac{-2x}{2y}} = -\frac{x}{y}$

(c) $x^3+y^3-3xy = \varnothing$

$\frac{d}{dx}x^3 + \frac{d}{dx}y^3 + \frac{d}{dx}3xy = \varnothing$

$$3x^2 + 3y^2\frac{dy}{dx} + 3\left(1.y + x.1\frac{dy}{dx}\right) = 0$$

$$3x^2 + 3y^2\frac{dy}{dx} + 3y + x.\frac{dy}{dx}3 = 0$$

$$\frac{dy}{dx}\left(3y^2 + 3y\right) = -3x^2 - 3y$$

$$\frac{dy}{dx} = \frac{-(x^2+y)\cancel{3}}{(y^2+y)\cancel{3}} \Rightarrow \boxed{\frac{dy}{dx} = \frac{-(x^2+y)}{y^2+y}}$$

15. (a) $x = a.\cos(t)$ $\quad y = b.\text{sen}(t)$ $\qquad \cos(t) = \frac{x}{a}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{b.\cos(t)}{a.(-\text{sen}(t))} = \boxed{-\frac{b}{a}\cdot\frac{\cos(t)}{\text{sen}(t)}} \qquad \text{sen}(t) = \frac{y}{b}$$

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

(b) $x = a.\cos^3(t)$ $\quad y = b.\text{sen}^3(t)$ $\qquad \frac{2}{a^2}x + \frac{1}{b^2}2y\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{b.\cancel{3}\,\text{sen}^{\cancel{2}}(t).\cancel{\cos(t)}}{a.\cancel{3}\cos^{\cancel{2}}(t).(-\cancel{\text{sen}(t)})} = \boxed{-\frac{b}{a}\cdot\frac{\text{sen}\ t}{\cos\ t}} \qquad \frac{dy}{dx} = -\frac{2}{a^2}x$$

$$\frac{dy}{dx} = \frac{-\frac{2}{a^2}x}{\frac{2}{b^2}y}$$

(c) $x = \frac{3at}{1+t^2}$ $\quad y = \frac{3at^2}{1+t^2}$ $\qquad \frac{dy}{dx} = -\frac{b^2}{a^2}\cdot\frac{x}{y}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{6at(1+t^2) - 3at^2(2t)}{\cancel{(1+t^2)^2}}}{\frac{3a.(1+t^2) - 3at.(2t)}{\cancel{(1+t^2)^2}}} = \frac{6at + \cancel{6at^3} - \cancel{6at^3}}{3a + 3at^2 - 6at^2} = \frac{6at}{3a - 3at^2} = \boxed{\frac{2t}{1-t^2}}$$

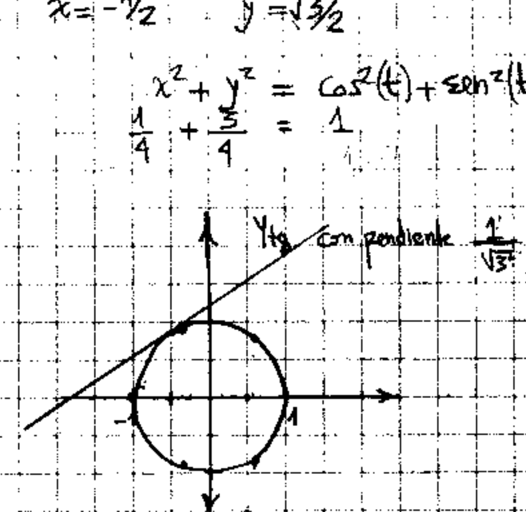
16. (a) $x = \cos(t)$ $\quad y = \text{sen}(t)$ $\quad$ en $\quad x = -1/2$ $\quad y = \sqrt{3}/2$

$$\text{pendiente tg} = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos(t)}{-\text{sen}(t)} \qquad x^2 + y^2 = \cos^2(t) + \text{sen}^2(t)$$

$$\frac{1}{4} + \frac{3}{4} = 1$$

en $x = -1/2$, $y = \sqrt{3}/2$ $\qquad$ pendiente $= \frac{-1/2}{-\sqrt{3}/2} = \frac{1}{\sqrt{3}}$

t	x	y
-2	-0,41	-0,9
-1	0,54	-0,84
0	1	0
1	0,54	0,84
2	-0,41	0,90
π	-1	0

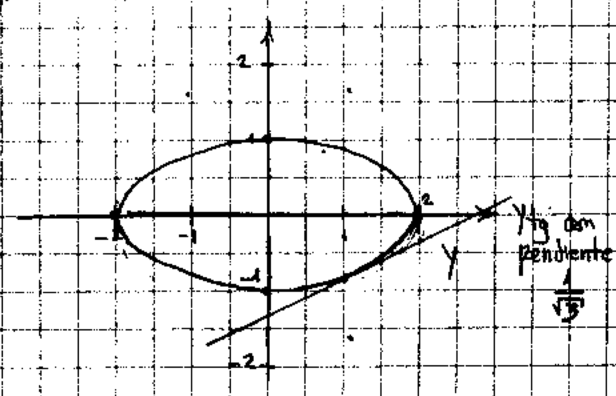


(b) $x = 2\cos(t)$ $\quad y = \text{sen}(t)$ $\quad$ en $\quad x = 1$ $\quad y = -\frac{\sqrt{3}}{2}$

$$\text{pendiente Tg} = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos(t)}{-2.\text{sen}(t)} \qquad \frac{1}{2} = \cos(t) \qquad -\frac{\sqrt{3}}{2} = \text{sen}(t)$$

$$\frac{1}{-\cancel{2}.\frac{-\sqrt{3}}{\cancel{2}}} = \frac{1}{\sqrt{3}} \qquad 1{,}0472 = t \qquad -1{,}0472 = t$$

t	x	y
-π	-2	0
-1	1,08	-0,84
0	2	0
1	1,08	0,84
π	-2	0
π/2	0	1
3/2 π	0	-1
2π	2	0
π/3	1	0,866



21. (a) $f(x)=x^4-5x^3+5x^2+x+2$ $(x-2)$

$f'(x)=4x^3-15x^2+10x+1$
$f''(x)=12x^2-30x+10$
$f'''(x)=24x-30$
$f^{IV}(x)=24$
$f^{V}(x)=\emptyset$

$$P_4(x)=\overset{f(2)}{0}+\overset{f'(2)}{(-7)}.(x-2)+\frac{(-2).(x-2)^2}{2}+\frac{18.(x-2)^3}{6}+\frac{24.(x-2)^4}{24}$$

punto $a=2$

$$\boxed{P_4(x)=-7(x-2)-(x-2)^2+3(x-2)^3+(x-2)^4}$$

(b) $f(x)=\sqrt{x}$ $n=3$ $(x-1)$

$f'(x)=\frac{1}{2\sqrt{x}}=(2^{-1}).x^{-\frac{1}{2}}$

$f''(x)=\frac{1}{2}.-\frac{1}{2}.x^{-3/2}=\frac{-1}{4\sqrt{x^3}}$

$f'''(x)=-\frac{1}{4}.-\frac{3}{2}.x^{-5/2}=\frac{3}{8\sqrt{x^5}}$

$$P_3(x)=\overset{f(1)}{1}+\overset{f'(1)}{\frac{1}{2}}.(x-1)+\left(-\frac{1}{4}\right)\frac{(x-1)^2}{2}+\left(\frac{3}{8}\right)\frac{(x-1)^3}{6}$$

$$=\boxed{1+\frac{1}{2}(x-1)-\frac{1}{8}(x-1)^2+\frac{1}{16}(x-1)^3}$$

22. (a) $f(x)=\sqrt{1+x}$

$f'(x)=\frac{1}{2\sqrt{1+x}}$

$f''(x)=\frac{1}{2}.\frac{-1}{2}.\frac{1}{\sqrt{(1+x)^3}}$

$P_n(x)=1+\frac{1}{2}x+-\frac{1}{4}\frac{x^2}{2}$

$\boxed{P_n(x)=1+\frac{x}{2}-\frac{x^2}{8}}$

(b) $\sqrt{1+x}\cong 1+\frac{1}{2}x-\frac{1}{8}x^2$ cuando $x=0{,}2$

$$R_2(x)=\frac{\frac{3}{8\sqrt{(1+\xi)^5}}.(0{,}2)^3}{6}=\frac{\cancel{3}^{1}\ \cancel{0{,}008}^{1}\ .0{,}0005}{\cancel{8}_1.\cancel{6}_{2}\ \sqrt{(1+\xi)^5}}=$$

$f'''(x)=\frac{3}{8\sqrt{(1+x)^5}}$

si $0<\xi<0{,}2 \Rightarrow$

$1<(1+\xi)^5<1{,}2^5$
$1<\sqrt{(1+\xi)^5}<\sqrt{\left(\frac{12}{10}\right)^5}$

$1>\left|\frac{1}{\sqrt{(1+\xi)^5}}\right|>\frac{1}{\sqrt{\left(\frac{12}{10}\right)^5}}$

$$\left|\frac{1}{2000}\right|>\underbrace{\left|\frac{1}{2000.\sqrt{(1+\xi)^5}}\right|}_{\text{error}}>\left|\frac{1}{2000.\sqrt{\left(\frac{12}{10}\right)^5}}\right|$$

$\boxed{\text{error}<\frac{1}{2000}}$

23. (a) $\lim\limits_{x\to 0}\frac{x-\operatorname{sen}x}{e^x-1-x-\frac{x^2}{2}}=$

$$\lim_{x\to 0}\frac{\cancel{x}-\cancel{x}+\frac{x^3}{6}-\frac{x^5}{120}+\dots+\frac{x^{2n+1}}{(2n+1)!}}{\left(\cancel{1}+\cancel{x}+\cancel{\frac{x^2}{2}}+\frac{x^3}{6}+\frac{x^4}{24}\dots\frac{x^n}{n!}\right)-\cancel{1}-\cancel{x}-\cancel{\frac{x^2}{2}}}=\lim_{x\to 0}\frac{\cancel{\frac{x^3}{6}}\left(1+\frac{x^2}{20}+\dots+\right)}{\cancel{\frac{x^3}{6}}\left(1+\frac{x}{4}+\dots+\right)}$$

(b) $\lim_{x \to 0} \dfrac{\ln^2(1+x) - \text{sen}^2(x)}{1 - e^{-x^2}}$

$\ln^2(1+x)$

$P_n(x) = 0 + 0\,x + \dfrac{\cancel{2}}{\cancel{2}}x^2 - \dfrac{6}{6}x^3 + \dfrac{11}{12}x^4 = x^2 - x^3 + \dfrac{11x^4}{12}$ $\qquad \dfrac{1}{1+x}\ln x + \ln x \cdot \dfrac{1}{1+x}$

$f(x) = \ln^2(1+x)$

$f'(x) = \dfrac{2\ln(1+x)}{1+x}$ $\qquad e^{(x^2)}$ $\qquad \dfrac{\frac{1}{1+x}\cdot(1+x) - \ln(1+x)\cdot 1}{(1+x)^2}$

$P_n(x) = 1 + 0\,x + \dfrac{-2}{2}x^2 + \dfrac{0\,x^3}{6} = 1 - x^2 + \dfrac{x^4}{2}$

$f''(x) = 2 \cdot \dfrac{-\ln(1+x)}{(1+x)^2} + \dfrac{2}{(1+x)^2}$

$\text{sen}^2 x$ $\qquad +$

$f'''(x) = \dfrac{4 \cdot \ln(x+1)}{(1+x)^3} - \dfrac{6}{(1+x)^3}$ $\quad P_n(x) =$

$\left(\dfrac{1}{e}\right)^{x^2} = \dfrac{1}{e^{x^2}}$

$f'(x) = -2x\,e^{-x^2}$

$f''(x) = 4x^2 \cdot e^{-x^2} - 2e^{-x^2}$

$f'''(x) = -8x^3 \cdot e^{-x^2} + 12x\,e^{-x^2}$

$\lim_{x \to 0} \dfrac{\cancel{x^2} - x^3 + \frac{11x^4}{12} - \cancel{x^2} + x^4/3}{\cancel{1} - \cancel{1} + x^2 - \frac{x^4}{2}} = \lim_{x \to 0} \dfrac{\cancel{x^3}\left(-1 + \frac{11x}{12} + \dots + x^2/3\right)}{\cancel{x^3}\left(\frac{1}{x} - \frac{x}{2} + \dots\right)} = \boxed{\emptyset}$

$\to 0 \quad \to 0 \quad \to \infty \quad \to 0$

17.

$$\boxed{(f^{-1})'(x) = \dfrac{1}{f'[f^{-1}(x)]}}$$

$f(x) = \cos(x) \quad f'(x) = -\text{sen}\,x$

$f(x) = \text{tg}(x) \quad f(x) = \dfrac{\text{sen}\,x}{\cos x} \Rightarrow f'(x) = \dfrac{\cos x \cdot \cos x - \text{sen}\,x(-\text{sen}\,x)}{\cos^2 x} = \dfrac{\cos^2 x + \text{sen}^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x}$

* $(\text{arc}\cos(x))' = \dfrac{1}{-\text{sen}(x)} = \dfrac{1}{-\sqrt{1 - \cos^2 x}} = \dfrac{1}{-\sqrt{1 - \cos^2(\text{arc}\cos x)}} = \boxed{\dfrac{1}{-\sqrt{1 - x^2}}}$

* $(\text{arc}\,\text{tg}(x))' = \dfrac{1}{\frac{1}{\cos^2 x}} = \dfrac{1}{\frac{1}{1 - \text{sen}^2 x}} = 1 - \text{sen}^2(x) = 1 - \dfrac{\text{tg}^2(x)}{1 + \text{tg}^2 x}$

$= \dfrac{1 - \text{tg}^2(\text{arc}\,\text{tg}\,x)}{1 + \text{tg}^2(\text{arc}\,\text{tg}\,x)} = 1 - \dfrac{x^2}{1 + x^2}$

$= \dfrac{1 + x^2 - x^2}{1 + x^2}$

$= \boxed{\dfrac{1}{1 + x^2}}$

$\text{tg}\,x = \dfrac{\text{sen}\,x}{\cos x}$

$\text{tg}^2 x = \dfrac{\text{sen}^2 x}{\cos^2 x} = \dfrac{\text{sen}^2 x}{1 - \text{sen}^2 x} \Rightarrow \text{sen}^2 x = \text{tg}^2 x \cdot (1 - \text{sen}^2 x)$

$\text{sen}^2 x = \text{tg}^2 x - \text{tg}^2 x \cdot \text{sen}^2 x$

$\text{sen}^2 x + \text{sen}^2 x \cdot \text{tg}^2 x = \text{tg}^2 x$

$\text{sen}^2 x\,(1 + \text{tg}^2 x) = \text{tg}^2 x$

$\text{sen}^2 x = \dfrac{\text{tg}^2 x}{1 + \text{tg}^2 x}$

* $(\text{arc}\,\text{sen}(x))' = \dfrac{1}{(\text{sen}(x))'} = \dfrac{1}{\cos(x)} = \dfrac{1}{\sqrt{1 - \text{sen}^2 x}} = \dfrac{1}{\sqrt{1 - \text{sen}^2(\text{arc}\,\text{sen}\,x)}} = \boxed{\dfrac{1}{\sqrt{1 - x^2}}}$

18.

$$\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x} \cong f'(x_0) \Rightarrow f(x_0+\Delta x) = f'(x_0)\cdot\Delta x + f(x_0)$$

(a) $f(x) = 2x^2 - x$ $\quad x = 1$, $\Delta x = 0{,}01$

$f'(x) = 4x - 1$

$\Delta y = f(1+0{,}01) - f(1)$

$\Delta y = 1{,}0302 - 1$

$\boxed{\Delta y = 0{,}0302}$

$\frac{dy}{dx} = 4x-1 \Rightarrow$

$\frac{dy}{dx} = \frac{dy}{\Delta x} \Rightarrow dy = f'(x)\cdot\Delta x$

$= (4x-1)(0{,}01)$

$dy = 0{,}04x - 0{,}01 = \boxed{0{,}03 = dy}$

(b) $f(x) = \text{sen}\, x$ $\quad x = \pi/3$

$f'(x) = \cos x$ $\quad \Delta x = \pi/18$

$\Delta y = f(x+\Delta x) - f(x)$

$\Delta y = \text{sen}(\pi/3 + \pi/18) - \text{sen}(\pi/3)$

$\boxed{\Delta y = 0{,}0736672}$

$f'(x) = \frac{dy}{\Delta x}$

$dy = \cos(x)\cdot\Delta x$

$dy = \cos(\pi/3)\cdot\pi/18 \Rightarrow \boxed{dy = 0{,}0872664}$

19. $\text{sen}\, 60^\circ = \sqrt{3}/2$ $\quad$ hallar: $\text{sen}(60^\circ 3')$

$\cos 60^\circ = 1/2$ $\quad$ $\text{sen}(60^\circ 18')$

$3' — 1/20^\circ = 0{,}05^\circ$ $\quad \text{sen}(60{,}05^\circ)$

$60' — 1^\circ$

$18' — 3/10^\circ = 0{,}3^\circ$ $\quad f(x) = \text{sen}(x)$

$f(60^\circ) = \sqrt{3}/2 \Rightarrow f(60^\circ + 0{,}05^\circ) = \cos(60^\circ)\cdot 0{,}05 + \text{sen}(60^\circ)$

$\frac{1}{2}\cdot 0{,}05 + \frac{\sqrt{3}}{2} \cong \boxed{0{,}89102}$

$f(60^\circ + 0{,}3^\circ) = \text{sen}(60^\circ)\cdot 0{,}3 + \text{sen}\, 60^\circ$

$0{,}15 + \sqrt{3}/2 \cong \boxed{1{,}01602}$

20. $\tan(45^\circ 4' 30'') = \frac{\text{sen}\ 45^\circ 4' 30''}{\cos\ 45^\circ 4' 30''}$

$45^\circ 4' 30'' = 45^\circ\, 4{,}5'$

$\text{sen}(45^\circ) = \quad 2700' + 4{,}5'$

$\cos(45^\circ) = \quad 4{,}5' — 0{,}075^\circ$

$\sqrt{2}/2 \quad 60' — 1^\circ$

$\text{sen}(45^\circ\, 4{,}5') = \cos(45^\circ)\cdot(0{,}075) + \text{sen}(45^\circ)$

$\sqrt{2}/2 \cdot 0{,}075 + \sqrt{2}/2$

$\text{sen}(45{,}075^\circ) \cong \boxed{0{,}7601397}$

$\cos(45{,}075^\circ) \cong -\text{sen}(45^\circ)\cdot(0{,}075) + \text{sen}(45^\circ) \Rightarrow \text{tg}(45^\circ 4' 30'') \cong \boxed{1{,}162162}$

$\cos(45{,}075^\circ) \cong \boxed{0{,}6540737}$

21.

(a) $e^x \geq 1 + x \quad \forall x \in \mathbb{R}$

$\downarrow$

$1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots + \frac{x^n}{n!} \geq 1 + x$

$\frac{x^2}{2} + \frac{x^3}{6} + \dots + \frac{x^n}{n!} \geq 0 \quad \forall x \in \mathbb{R}$

(b) $\frac{x}{1+x} < \ln(1+x) \leq x \quad \forall x > 0$

$\frac{x}{1+x} \leq x$

$x \leq x + x^2$

$0 \leq x^2$ $\quad$ válido $\forall x \in \mathbb{R}$

$< x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \leq x$

$\frac{\cancel{x} - \cancel{x} - x^2}{1+x} = \frac{x}{1+x} - x < -\frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \leq 0$

$$\frac{-x^2}{1+x} < -\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\dots \leq 0$$

$$f'(x) = \frac{-x(x+2)}{(x+1)^2}$$
$$f''(x) = \frac{-2}{(x+1)^3}$$
$$f'''(x) = \frac{6}{(x+1)^4}$$

$$\sum_{n=2}^{\infty} (-1)^{n+1} \frac{x^n}{n!}$$ Leibnitz pero

$\frac{x^n}{n!} \to 0$ si $n \to \infty$. a_n decreciente

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right| = 0 < 1 \Rightarrow$$ Converge absolutamente $\Rightarrow$ converge y

$$\frac{-x^2}{1+x} < \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n!} \quad \forall x>0 \Leftarrow \sum (-1)^{n+1} \cdot \frac{x^n}{n!}$$

$\forall x>0$ $\frac{-x^2}{1+x} < 0$

$-x^2 < 0$

$x^2 > 0$ (válido)

$$\frac{-x^2}{1+x} < 0 < \sum (-1)^{n+1} \cdot \frac{x^n}{n!}$$

(c) $|\operatorname{sen} x - \operatorname{sen} y| \leq |x-y| \quad \forall x,y \in \mathbb{R}$

$\operatorname{sen}^2(x) + \operatorname{sen}^2(y) - 2\operatorname{sen}(x)\operatorname{sen}(y) \leq x^2 - 2xy + y^2$

$$\left| x - \frac{x^3}{6} + \frac{x^5}{120} - y + \frac{y^3}{6} - \frac{y^5}{120} + \dots \right|$$

$$x - y + \frac{-x^3+y^3}{6} + \frac{x^5-y^5}{120}$$

continua en $\mathbb{R}$, derivable en $\mathbb{R}$

$H(x) = \operatorname{sen} x - x$

$H'(x) = \cos x - 1 \leq 0 \quad \forall x \in \mathbb{R} \Rightarrow$

$H(x)$ decreciente $\forall x \in \mathbb{R}$

Luego $\exists c: f(c) =$

si $x>0$

$$\underbrace{f'(c)}_{\leq 0} = \frac{H(x) - H(0)}{x - 0} \begin{cases} \leq 0 \\ > 0 \end{cases}$$

$\operatorname{sen} x \leq x \quad \forall x>0$

$\operatorname{sen} x \leq x \quad \forall x > 0 \Rightarrow$

$|\operatorname{sen} x| \leq |x| \wedge |\operatorname{sen} y| \leq |y|$

$|\operatorname{sen} x - \operatorname{sen} y| \geq |\operatorname{sen} x| - |\operatorname{sen} y| \geq |x| - |y|$

25. (a) $\sqrt{3}$ $\qquad x_0 = 2$

$\sqrt{3} = y$

$3 = y^2$

$0 = y^2 - 3 = f(x)$

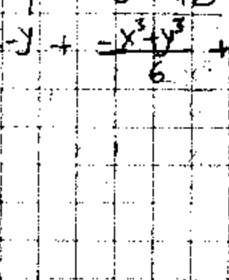
$2x = f'(x)$

$$x_1 = 2 - \frac{1}{4} = \frac{7}{4}$$

$$x_2 = \frac{7}{4} - \frac{1/16}{7/2} = 1{,}7321429$$

$$x_3 = \frac{97}{56} = 1{,}7320508$$

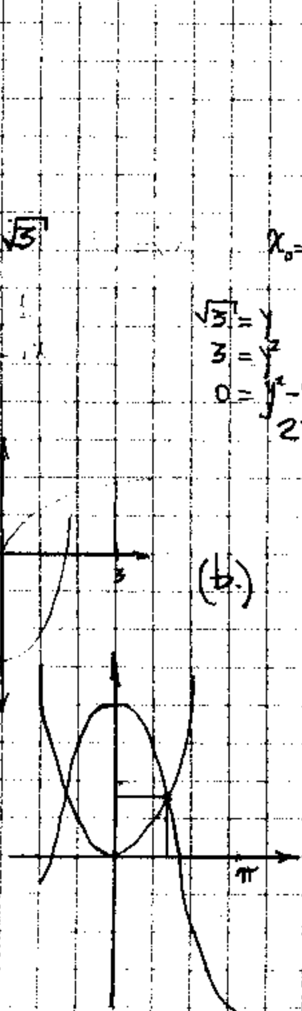
$\boxed{1{,}732}$ 3 cifras significativas



(b) $2\cos(x) - x^2 = 0 \qquad x_0 = 1$

$-2\operatorname{sen}(x) - 2x$

$-2(\operatorname{sen}(x) + x)$



$$x_1 = 1 - \frac{0{,}0806046}{-3{,}682942} = 0{,}978114$$

$$x_2 = \qquad - \frac{0{,}1604686}{-3{,}6151187} = 1{,}0225022$$

$$x_3 = \qquad - \frac{-0{,}0030463}{-3{,}7519342} = 1{,}0216903$$

$$x_4 = \qquad - \frac{-0{,}0000011}{-3{,}7493633} = \boxed{1{,}02169}$$

5 cifras

(c) i. $x^3 + 3x + 1 = \emptyset$ $\qquad -1 < x < 1$

$-1 - 3 + 1 = -3$
$1 + 3 + 1 = 5$

$f'(x) = 3x^2 + 3$

$x_1 = 1 - \frac{5}{6} = \frac{1}{6}$

$x_2 = \frac{1}{6} - \frac{1{,}5046296}{3{,}0833333} = -0{,}3213213$

$x_3 = -\frac{0{,}0028601}{3{,}3097122} = -0{,}3221855$

$x_4 = -\frac{-0{,}0000006}{3{,}3111106} = -0{,}3221853$

$\boxed{x \cong -0{,}322185..}$

ii. $x - \cos(x) = 0$

$-1 < x < 1$

$f'(x) = \operatorname{sen}(x) + 1$

$x_1 = 1 - \frac{0{,}4596976}{1{,}841471} = 0{,}7503639$

$x_2 = -\frac{0{,}0189231}{1{,}681905} = 0{,}7391129$

$x_3 = -\frac{0{,}0000464}{1{,}6736326} = 0{,}7390851$

$x_4 = -\frac{0{,}000000071}{} =$

$\boxed{x \cong 0{,}73905}$

ii $\quad x^3 + 2x^2 - 4 = 0$

$f'(x) = 3x^2 + 4x =$

$1 < x < 2$

$\boxed{x \cong 1{,}1303}$

$x_1 = 2 - \frac{12}{20} = 7/5$

$x_2 = \frac{7}{5} - \frac{3{,}664}{11{,}48} = 1{,}1679443$

$x_3 = -\frac{0{,}321373}{8{,}7641583} = 1{,}1312749$

$x_4 = -\frac{0{,}0073518}{8{,}364418} = 1{,}130396$

$x_5 = -\frac{0{,}0000044}{8{,}354969} = 1{,}1303955$

(d) $\sqrt[3]{x} = 0$

$f'(x) = \frac{1}{3} \frac{1}{\sqrt[3]{x^2}}$

no funciona porque

$F(x_n) \not\to \emptyset$
a medida que aumenta n

$\frac{F(x)}{F'(x)} = \frac{\sqrt[3]{x}}{3\sqrt[3]{x^2}} = \frac{1}{3.x} \xrightarrow[x \to 0]{} \infty$

$x_1 = 1 - \frac{1}{1/3} = -2$

$x_2 = -2 - \frac{\sqrt[3]{-2}}{0{,}2099868} = 4$

$x_3 = 4 - \frac{1{,}5874011}{0{,}1322134} = -8{,}0000004$

$x_4 = -\frac{-2}{0{,}1083333} = 16$

(e) i.
$F(x) = x^3 - 3x^2 + 3x + 2 = \emptyset$

$F'(x) = 3x^2 - 6x + 3$

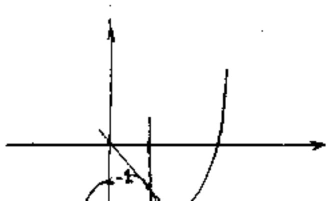
$x_0 = 1$

$x_1 = 1 - \frac{3}{F'(1) = \emptyset} \Rightarrow$ no se puede aplicar con $x_0 = 1$

en $x_0 = 1$ f tiene un punto de inflexión (tal vez extremo local), que no es cero de F

ii. $f(x) = x^3 - 3x^2 + x - 1 = 0$

$f'(x) = 3x^2 - 6x + 1$



$x_0 = 1$

$x_1 = 1 - \frac{-2}{-2} = \emptyset$

$x_2 = 0 - \frac{-1}{1} = 1$

$x_3 = 1 - \frac{-2}{-2} = 0$

$x_4 = 0 - \frac{-2}{2} = 1$

oscila entre 0 y 1

en $x_0=1$ no funciona porque da $X_1=0$ y $x_2=1$ con lo cual vuelve a $x_4=x_3=0$ y la recurrencia no sale de $0;1$. Observar que $F''(1)=\emptyset$