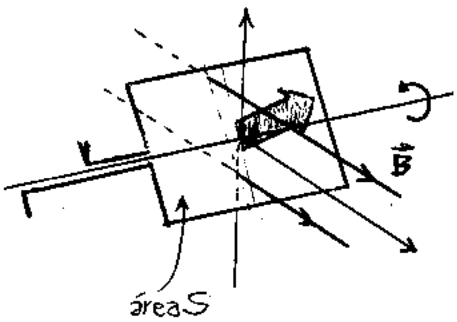
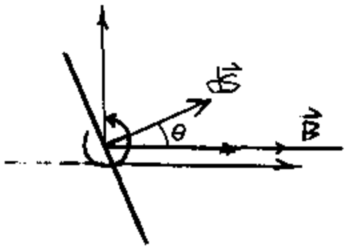
• Corriente Alterna





← Espira que gira en el seno de un campo $\vec{B}$ uniforme.

$$\phi_B = \int \vec{B} \cdot d\vec{S} = B \cdot \cos\theta \cdot S$$

$$-\frac{\partial \phi_B}{\partial t} = B.S.\operatorname{sen}\theta.\dot{\theta} = \varepsilon_{ind}$$

frecuencias
$\omega \equiv$ frec. angular
$\nu = \frac{\omega}{2\pi}$ frecuencia

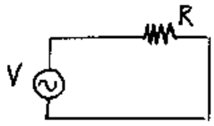
si $\dot{\theta} = \omega \rightarrow \theta = \omega t \rightarrow$

$$\varepsilon_{ind} = B.S.\operatorname{sen}(\omega t).\omega \qquad \text{con } B.S.\omega = \varepsilon_0$$

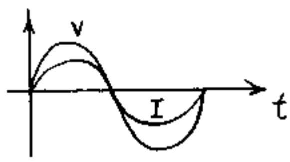
La fem inducida es alterna ⟶ $\boxed{\varepsilon(t) = \underbrace{\varepsilon_0}_{B.S.\omega} . \operatorname{sen}(\omega t)}$

• Resistencia, Condensador e Inductancia a fuente de CA

* Resistencia

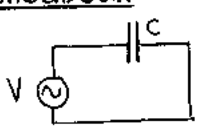


$$I = \frac{V}{R} = \frac{\varepsilon_0}{R} \operatorname{sen}(\omega t)$$



V, I están en fase si el circuito es resistivo

* Condensador

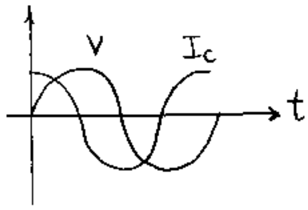


$$V = \frac{Q}{C}$$

$$Q = \varepsilon_0.\operatorname{sen}(\omega t).C$$

$$I = \varepsilon_0.C.\cos(\omega t)\omega = \varepsilon_0\omega.C.\operatorname{sen}\left(\omega t + \frac{\pi}{2}\right)$$

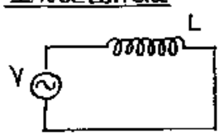
V, I están desfasados ahora

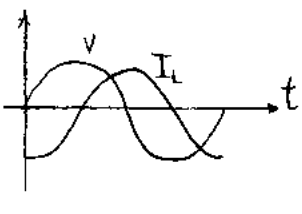


$$I = \frac{\varepsilon_0}{\frac{1}{\omega.C}} . \operatorname{sen}\left(\omega t + \frac{\pi}{2}\right)$$

$\boxed{\varphi_C = \frac{\pi}{2}}$ ← fase del condensador

* Inductancia





$$V = -L.\frac{dI}{dt} \rightarrow \qquad V.dt = +L.dI$$

$$\int \varepsilon_0.\operatorname{sen}(\omega t).dt = +L\int dI$$

$$-\frac{\varepsilon_0}{\omega}\cos(\omega t) = +L.I$$

$$-\frac{\varepsilon_0}{\omega L}.\cos(\omega t) = I$$

Fase de la inductancia $\boxed{\varphi_L = -\frac{\pi}{2}}$

$$I = \frac{\varepsilon_0}{\omega L}.\operatorname{sen}\left(\omega t - \frac{\pi}{2}\right) \quad \leftarrow \quad -\frac{\varepsilon_0}{\omega L}.\operatorname{sen}\left(\omega t + \frac{\pi}{2}\right) = I$$

Las corrientes por el capacitor y por la inductancia están desfasadas

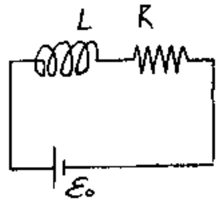
$R_R = R$ $\quad$ $R_C = \dfrac{1}{\omega C}$ $\quad$ $R_L = \omega L$

$U_L = \frac{1}{2} L i^2$	$U_C = \frac{1}{2} C V^2$	$U = i.V$

energía almacenada $\qquad$ energía transferida

$$\boxed{P = -\frac{dU}{dt}}$$

potencia refiere al ritmo de entrega o almacenamiento de energía en los componentes de un circuito.



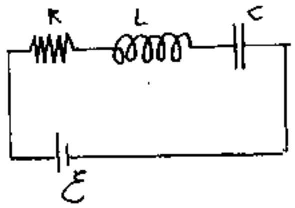
En un circuito LC la energía se transforma entre eléctrica (en el capacitor) y magnética (en la inductancia) [se conserva]
Hay oscilaciones.

$$\mathcal{E}_0 - L\frac{di}{dt} - i.R = 0$$

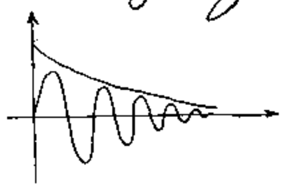
$$i.\mathcal{E}_0 - L.i.\frac{di}{dt} - i^2 R = 0$$

entrega de energía $\qquad$ almac. de energía $\qquad$ disipación de energía

* Circuito RLC



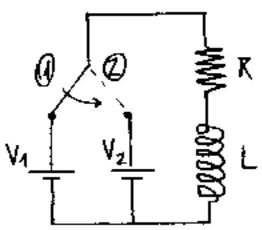
Puede producir oscilaciones amortiguadas o subamortiguadas según el valor de las constantes.



$$\mathcal{E} = i.R + \frac{Q}{C} + L.\frac{di}{dt}$$

$$\mathcal{E} = \frac{Q}{C} + \frac{dQ}{dt}.R + L.\frac{d^2Q}{dt^2}$$

* Nota sobre transitorios



① A tiempo largo la L desaparece (su efecto)
$i_1 = \dfrac{V_1}{R}$

② $i_2(t_0) \equiv \dfrac{V_1}{R} = i_1$; pero ahora hay que resolver el nuevo circuito 2 teniendo en cuenta la inductancia.

• Circuitos de Corriente Alterna

R	L	C
$R,\ \varphi_R = 0$	$\omega L,\ \varphi_L = -\frac{\pi}{2}$	$\frac{1}{\omega C},\ \varphi_C = \frac{\pi}{2}$

* El formalismo Complejo

Se pueden resolver más fácil circuitos alternos con números complejos.

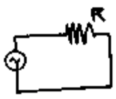
$$e^{i\varphi} = \cos\varphi + i \cdot \operatorname{sen}\varphi \qquad \text{(pasamos la unidad imaginaria a } i = j\text{)}$$

fuente $\boxed{V \equiv V_0 \cdot \cos(\omega t)}$ $\rightarrow$ $V = V_0\left[\cos(\omega t) + j\,\operatorname{sen}(\omega t)\right]$

definición ▲

$\boxed{\mathbb{V} = V_0 \cdot e^{j\omega t}}$ recordando que $V = \operatorname{Re}\{\mathbb{V}\}$

Resistencia



$i = \frac{V}{R} \rightarrow i = \frac{V_0}{R}\cos(\omega t) \Rightarrow I = \frac{V_0}{R} e^{j\omega t} \Rightarrow$

$\boxed{Z_R = R}$ $R = \operatorname{Re}\{Z_R\}$

(en realidad no hay cambio)

Condensador:

$i = \frac{d}{dt}(V C) = \frac{d}{dt}\left[V_0 \cdot \cos(\omega t) \cdot C\right] = -V_0 \cdot C \cdot \omega \cdot \operatorname{sen}(\omega t) \Rightarrow$

$= -\frac{V_0}{\frac{j}{\omega C}} \cdot j \cdot \operatorname{sen}(\omega t) \Rightarrow \frac{V_0}{\frac{-j}{\omega C}}\left(j \cdot \operatorname{sen}(\omega t) + \cos(\omega t)\right)$

$= \frac{V_0}{\frac{1}{\omega C}}\left[-\operatorname{sen}\omega t + j \cdot \cos(\omega t)\right]$

$\operatorname{Re}(I)$

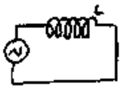
Se busca que la corriente tenga la forma:

$I = \frac{\mathbb{V}}{Z}$

metiendo lo necesario en Z (reactancia)

$\boxed{Z_C = \frac{-j}{\omega C}} \longleftarrow I = \frac{V_0}{\frac{-j}{\omega C}}\left(\cos[\omega t] + j \cdot \operatorname{sen}[\omega t]\right)$

Inductancia:



$i = \int \frac{V_0}{L} dt \rightarrow i = \frac{V_0}{\omega L} \operatorname{sen}(\omega t) \rightarrow$

$I = \frac{V_0}{+\omega L j}\left(\cos[\omega t] + j \cdot \operatorname{sen}[\omega t]\right)$

$\operatorname{Re}(I) = \frac{V_0}{\omega L} \cdot \operatorname{sen}\omega t$

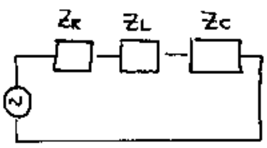
$\boxed{Z_L = j\omega L}$

Definiendo así las Z_R, Z_C, Z_L las caídas de potencial tendrán la forma:

$V = I \cdot Z$ y al final $V = \operatorname{Re}\{I \cdot Z\}$

Los complejos se pueden arrastrar en operaciones lineales

* Serie



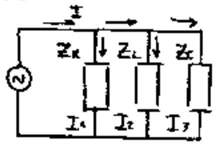
$$V_0 . e^{j\omega t} = I(Z_R + Z_L + Z_C)$$

$$I = \frac{V_0 . e^{j\omega t}}{|Z| . e^{j\varphi}} = \frac{V_0}{|Z|} . e^{j[\omega t - \varphi]}$$

impedancia — fase

$$\varphi = \frac{Im\,|Z|}{Re\,|Z|}$$

* Paralelo



$$I = I_1 + I_2 + I_3$$

$$I = V_0 . e^{j\omega t}\left(\frac{1}{Z_R} + \frac{1}{Z_L} + \frac{1}{Z_C}\right)$$

$$V_0 . e^{j\omega t} = I_1 . Z_R$$
$$'' \quad = I_2 . Z_L$$
$$'' \quad = I_3 . Z_C$$

$$Z = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_L} + \frac{1}{Z_C}} = |Z| . e^{j\varphi}$$

$$\boxed{I = \frac{V_0}{|Z|} . e^{j[\omega t - \varphi]}}$$

* Potencia

$$P = V . i = V_0 \cos(\omega t) . \frac{V_0}{|Z|} . \cos(\omega t - \varphi) = \frac{V_0^2}{|Z|} . \cos(\omega t) . \cos(\omega t - \varphi)$$

$$P = \frac{V_0^2}{|Z|} . \cos(\omega t) . [\cos \omega t . \cos\varphi + \operatorname{sen} \omega t . \operatorname{sen}\varphi]$$

$$P = \frac{V_0^2}{|Z|} . \left[\cos^2(\omega t) . \cos\varphi + \cos(\omega t) . \operatorname{sen}(\omega t) . \operatorname{sen}\varphi\right]$$

$$P = \frac{V_0^2}{|Z|} . \left(\left[\frac{1}{2} + \frac{\cos(2\omega t)}{2}\right] . \cos\varphi + \frac{\operatorname{sen}(2\omega t)}{2} . \operatorname{sen}\varphi\right)$$

$$P = \frac{V_0^2}{2|Z|} . \left(\cos\varphi + \cos(2\omega t) . \cos\varphi + \operatorname{sen}(2\omega t) . \operatorname{sen}\varphi\right)$$

tomando valor medio

$$\boxed{\bar{P} = \frac{V_0^2}{2|Z|} . \cos\varphi} \rightarrow \bar{P} = \frac{V_0 . i_0}{2} . \cos\varphi$$

potencia promedio

$$\bar{V}^2 = \frac{1}{\tau}\int_0^\tau V^2 . dt = \frac{1}{\tau} . \int_0^\tau V_0^2 . \cos^2(\omega t) . dt = \frac{V_0^2}{\tau} . \int_0^{2\pi} \cos^2 U . dU . \frac{1}{\omega} = \frac{V_0^2 . \pi}{\tau . \omega} = \frac{V_0^2}{2}$$

$$\omega = \frac{2\pi}{\tau}$$

$$\omega t = U$$
$$\omega . dt = dU$$

$$\bar{V}^2 = \frac{V_0^2}{2} \rightarrow \boxed{\bar{V} = \frac{V_0}{\sqrt{2}}} \rightarrow \text{Potencial eficaz}$$

$$\bar{V} \equiv V_e$$

$$\bar{i}^2 = \frac{1}{\tau}\int_0^\tau i^2 . dt = \frac{1}{\tau} . \frac{1}{|Z|^2} . V_0^2 \int_0^\tau \cos^2(\omega t) . dt = \frac{1}{\tau . \omega |Z|^2} . V_0^2 . \pi = \frac{V_0^2}{2|Z|^2}$$

$$\bar{i} = \frac{V_0}{\sqrt{2} . |Z|} \Rightarrow$$

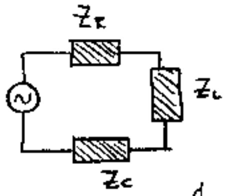
$$\boxed{\bar{i} = \frac{i_0}{\sqrt{2}}} \rightarrow \text{intensidad eficaz}$$

$$\bar{i} \equiv i_e$$

$$\bar{P} = V . i = \frac{V_0}{\sqrt{2}} . \frac{i_0}{\sqrt{2}} . \cos\varphi = V_e . i_e . \cos\varphi$$

$$\boxed{\bar{P} = V_e . i_e . \underbrace{\cos\varphi}_{\text{factor de potencia}} = \bar{V} . \bar{i} . \cos\varphi}$$

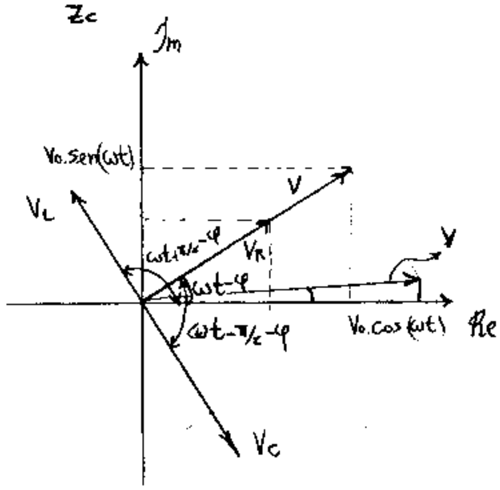
• Fasores



$$\boxed{V = Z \cdot I} \qquad \boxed{\frac{V}{Z} = I}$$

$$V_R = R.I = R.\frac{V_0}{|Z|}.e^{j[\omega t - \varphi]}$$

$$V_L = Z_L.I = j.\omega L.\frac{V_0}{|Z|}.e^{j[\omega t-\varphi]} = \frac{\omega L V_0}{|Z|} e^{j[\omega t + \pi/2 - \varphi]}$$

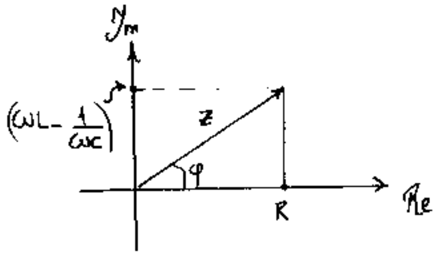


$$Z = Z_R + Z_L + Z_C \qquad V_C = Z_C.I = \frac{-j\,V_0}{|Z|.\omega C}\,e^{j[\omega t - \varphi]}$$

$$V = V_R + V_L + V_C \qquad V_C = \frac{V_0}{\omega C|Z|}.e^{j(\omega t - \varphi - \pi/2)}$$

$$\boxed{I = \frac{V_0}{|Z|}.e^{j(\omega t - \varphi)}}$$

* Triángulo de Impedancia



$$Z = |Z|.e^{i\varphi}$$

impedancia

$$\boxed{\begin{aligned} j &= e^{j\pi/2} \\ -j &= e^{-j\pi/2} \end{aligned}}$$

• Potencia

$$\boxed{\overline{P} = \overline{V.i} = V_e.I_e.\cos\varphi = \overline{V}.\overline{I}.\cos\varphi = \frac{V_0}{\sqrt{2}}.\frac{i_0}{\sqrt{2}}.\cos\varphi}$$

$$\cos\varphi = \frac{\mathcal{R}e\, Z}{|Z|} \longrightarrow \overline{P} = i_0.V_0.\frac{R}{|Z|} = i_0^2.R$$

$$|Z| = \sqrt{R^2 + \left[\omega L - \frac{1}{\omega C}\right]^2}$$

Toda la potencia promedio está disipada por la R

$$\overline{P} = \overline{\mathcal{R}e(V).\mathcal{R}e(I)} = \overline{\frac{R\,V_0}{|Z|}.\cos(\omega t).\frac{V_0}{|Z|}.\cos(\omega t)} = \left(\frac{V_0}{|Z|}\right)^2 R.\cos^2(\omega t) \xrightarrow{\text{promediando}} \overline{P} = \left(\frac{V_0}{|Z|}\right)^2.R$$

$$\overline{P} = \frac{V_0}{\sqrt{2}}.\frac{i_0}{\sqrt{2}}.\cos\varphi = V_0.\frac{V_0}{|Z|}.\frac{R}{|Z|} = \left(\frac{V_0}{|Z|}\right)^2.R = \boxed{i_0^2.R = \overline{P}_R} \Rightarrow \overline{P}_L = \overline{P}_C = 0$$

$$P = V.i \quad \text{pero} \quad \overline{P} \neq \overline{V}.\overline{i}$$

$$\overline{P} = \overline{V}.\overline{i}.\cos\varphi$$

donde $\overline{V} = \frac{V_0}{\sqrt{2}}$; $\overline{i} = \frac{i_0}{\sqrt{2}}$

Importante

La potencia se calcula de dos modos

$$\overline{P} = \overline{\mathcal{R}e(V).\mathcal{R}e(I)}$$

ó bien

$$\overline{P} = \overline{\mathcal{R}e(V)}.\overline{\mathcal{R}e(I)}.\cos\varphi$$

$$\overline{P_L} = \overline{Re(V_L) \cdot Re(I)} = \overline{Re(V_L)}\ \overline{Re(I)} \cdot \cos\varphi$$

$$\overline{P_L} = -\omega L \cdot \overline{Im(I)} \cdot \overline{Re(I)} \cdot \frac{R}{|Z|}$$

$$V_L = \frac{V_0 \cdot \omega L}{|Z|}\, e^{j(\omega t + \pi/2 - \varphi)}$$

$$V_L = \frac{V_0}{|Z|} \cdot e^{j(\omega t - \varphi)} \cdot \omega L \cdot e^{j\pi/2}$$

$$V_L = I \cdot \omega L \cdot e^{j\pi/2} = I \omega L j$$

$$Re(V_L) = -Im(I)\, \omega L$$

$$A + iB = i(C + iD)$$

$$Re(A+iB) = Re\ i(C+iD)$$

$$A = -D$$

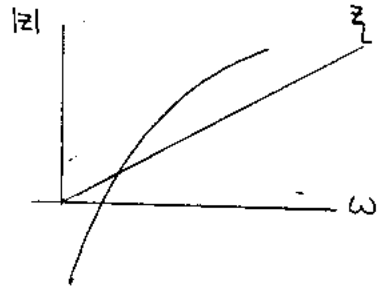
$$Re(A+iB) = -Im(C+iD)$$

* <u>Resonancia</u> (RLC Serie)

$$|Z| = \sqrt{R^2 + \left[\omega L - \frac{1}{\omega C}\right]^2}$$

si $\omega L = \frac{1}{\omega C}$ $\rightarrow$ $|Z| = R$ es mínimo

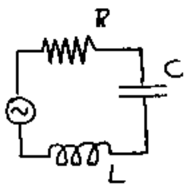
$\omega_0 = \frac{1}{\sqrt{LC}}$ $\quad \omega_0 \equiv$ frec. de resonancia



$$I = \frac{V_0}{|Z|} \cdot e^{i(\omega t - \varphi)}$$

corriente máxima si $|Z|$ es mínimo

* <u>Factor de Mérito</u>



$$V_C = Z_C \cdot I = Z_C \cdot \frac{V_0}{|Z|} \cdot e^{j(\omega t - \varphi)} = \frac{V_0}{\omega C |Z|} e^{j\pi/2} e^{j(\omega t - \varphi)}$$

$$|V_C| = \frac{V_0 / \omega \cdot C}{\sqrt{R^2 + \left[\omega L - \frac{1}{\omega C}\right]^2}}$$

defino $\left\{ \boxed{\begin{array}{l} \alpha = \frac{\omega - \omega_0}{\omega_0} \rightarrow \alpha \ll 1 \\ \text{cerca de la resonancia} \quad \alpha \rightarrow 0 \\ \omega = \alpha \cdot \omega_0 + \omega_0 = (1+\alpha)\,\omega_0 \end{array}} \right.$

es ω_0: $\quad \left.\omega L - \frac{1}{\omega C}\right|_{\omega = \omega_0} = 0$

$$\frac{1}{1+\alpha} \approx 1 - \alpha$$

$$\frac{|V_C|}{V_0} = \frac{1}{C(1+\alpha)\omega_0 \sqrt{R^2 + \underbrace{\left[(1+\alpha)\omega_0 L - \frac{1}{(1+\alpha)\omega_0 C}\right]^2}}}$$

$$\left(\omega_0 L + \alpha \omega_0 L - \frac{1}{\omega_0 C} + \frac{\alpha}{\omega_0 C}\right)^2$$

$$\left[\underbrace{\omega_0 L - \frac{1}{\omega_0 C}}_{=0} + \alpha\left(\omega_0 L + \frac{1}{\omega_0 C}\right)\right]^2$$

$$\frac{V_C}{V_0} = \frac{1-\alpha}{C \cdot \omega_0 \cdot \sqrt{R^2 + \alpha^2\left(\omega_0 L + \frac{1}{\omega_0 C}\right)^2}} = \frac{1-\alpha}{\omega_0 C R \cdot \sqrt{1 + \alpha^2\left(\frac{\omega_0 L}{R} + \frac{1}{\omega_0 C R}\right)^2}}$$

se definen $\left\{ \begin{array}{l} \boxed{\frac{1}{\omega_0 C R} = Q_C} \\ \boxed{\frac{\omega_0 L}{R} = Q_L} \end{array} \right.$

si $\omega = \omega_0$ (resonancia) $\rightarrow Q_C = Q_L$

$$\frac{V_C}{V_0} = \frac{Q}{\sqrt{1 + \alpha^2 4 Q^2}} - \underbrace{\frac{\alpha \cdot Q}{\sqrt{1 + \alpha^2 4 Q^2}}}_{\rightarrow 0 \ \Rightarrow \text{ lo tiro}}$$

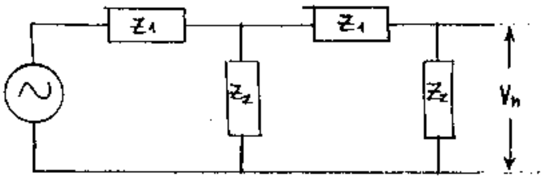
$$\frac{V_C}{V_0} = Q \quad \text{(en resonancia)}$$

Q expresa la amplificación de la diferencia de potencial del circuito

$$\frac{V_c}{V_0} = \frac{Q}{\sqrt{1+\alpha^2 4Q^2}} \xrightarrow[\text{si } \alpha \to 0]{} Q$$

- **Filtros**

$\frac{V_n}{V_0} = f(\omega)$ → el factor de amplificación en un circuito será función de la frecuencia



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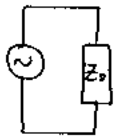
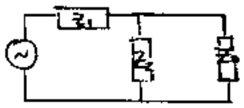


Para resolver se plantea



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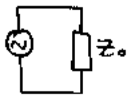
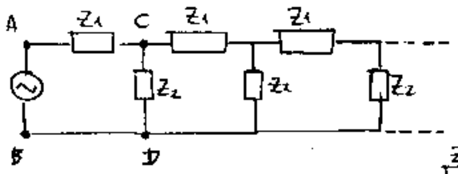
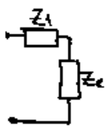


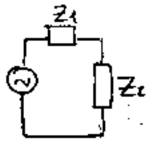


⇒ red infinita (si le agrego un



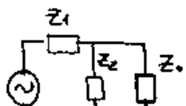
no varío su Z)

Sea $Z_{AB} = Z_0$ →



$Z_0 = Z_1 + Z_2$

Luego la impedancia entre C y D será Z_0 ⇒ red infinita ≡



≡

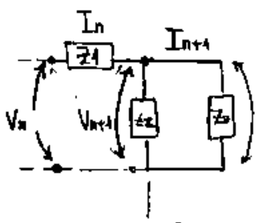
$$\Rightarrow \quad Z_0 = Z_1 + \frac{1}{\frac{1}{Z_2}+\frac{1}{Z_0}} = Z_1 + \frac{1}{\frac{Z_0+Z_2}{Z_2 . Z_0}} = Z_1 + \frac{Z_2 . Z_0}{Z_2+Z_0} = \frac{Z_1 Z_0 + Z_1 Z_2 + Z_0 Z_2}{Z_0+Z_2}$$

$$Z_0^2 + \cancel{Z_0 Z_2} = Z_1 Z_0 + Z_1 Z_2 + \cancel{Z_0 Z_2}$$

$$Z_0^2 - Z_1 Z_0 - Z_1 Z_2 = 0$$

Le corresponde el signo +

$$\boxed{Z_0 = \frac{Z_1 \pm \sqrt{Z_1^2 + 4 Z_1 Z_2}}{2}}$$



sección - n

Luego de la sección n puedo reemplazar lo que queda por Z_0

→ $V_n = I_n . Z_0 \quad \forall n$ → si resto dos dif. de V contiguas es:

$$V_n - V_{n+1} = I_n . Z_1 = \frac{V_n}{Z_0} . Z_1$$

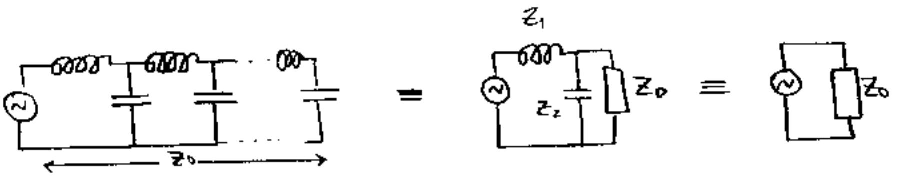
$$V_n \left(1 - \frac{Z_1}{Z_0}\right) = V_{n+1}$$

$$\boxed{\frac{V_{n+1}}{V_n} = 1 - \frac{Z_1}{Z_0}}$$

$$\frac{V_{n+1}}{V_n} = \frac{Z_0 - Z_1}{Z_0} \equiv \alpha \longleftarrow \text{factor de propagación}$$

$$V_{n+1} = \alpha \cdot V_n \quad \rightarrow \quad \boxed{V_n = \alpha^n V_0} \quad \text{con} \quad \alpha \equiv \frac{Z_0 - Z_1}{Z_0}$$

- **Filtros LC**



$$Z_0 = \frac{Z_1}{2} + \frac{\sqrt{Z_1^2 + Z_1 Z_2 4}}{2} \quad \rightarrow \quad Z_0 = \frac{j\omega L + \sqrt{-\omega^2 L^2 + j\omega L \frac{4}{j\omega C}}}{2}$$

$$Z_0 = j\frac{\omega L}{2} + \frac{\sqrt{4L/C - \omega^2 L^2}}{2}$$

$$\alpha = \frac{Z_0 - Z_1}{Z_0} \quad \rightarrow \quad \boxed{\alpha = \frac{\sqrt{L/C - \frac{L^2\omega^2}{4}} - j\frac{\omega L}{2}}{\sqrt{L/C - \frac{L^2\omega^2}{4}} + j\frac{\omega L}{2}}}$$

si $\frac{L}{C} - \frac{L^2\omega^2}{4} > 0$

$$Z_0 = j\frac{\omega L}{2} + \sqrt{\frac{L j^2}{C j^2} + j^2\frac{L^2\omega^2}{4}}$$

$$Z_0 = j\left(\frac{\omega L}{2} + \sqrt{\frac{L}{C j^2} + \frac{L^2\omega^2}{4}}\right) = j\left(\frac{\omega L}{2} + \sqrt{\frac{L^2\omega^2}{4} - \frac{L}{C}}\right)$$

Un circuito con $Z \in \Im m$ tendrá:
$Z_T \in \Im m$

si $\frac{L^2\omega^2}{4} - \frac{L}{C} < 0$
$\omega^2 < \frac{4}{LC}$
$\downarrow$
$Z_0 \in \mathbb{C}$ (tiene parte $\Re e$)

① $\omega^2 < \frac{4}{LC} \rightarrow Z_0 \in \mathbb{C}$ (tiene $\Re e$)

② $\omega^2 > \frac{4}{LC} \rightarrow Z_0 \in \Im m$

① frec. bajas → Hay efecto resistivo (se absorbe energía en el circuito [que se almacena en las L y C])

② frec. altas → No hay efecto resistivo (la corriente no "avanza" mucho por la línea)

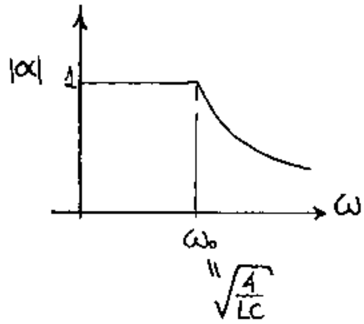
* sea $\omega < \omega_0 \rightarrow \sqrt{\frac{L^2\omega^2}{4} - \frac{L}{C}} \in \Im m \Rightarrow \sqrt{\frac{L}{C} - \frac{L^2\omega^2}{4}} \in \Re e$

$$\alpha = \frac{-j\frac{\omega L}{2} + \sqrt{\quad}}{+j\frac{\omega L}{2} + \sqrt{\quad}} \Rightarrow |\alpha| = 1 \Rightarrow \alpha = |\alpha| \cdot e^{j\varphi}, \quad \alpha = e^{j\varphi}$$

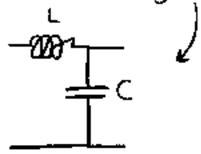
$$V_n = e^{j\varphi} \cdot V_0$$ (solo varía la fase del voltaje)

* sea $\omega > \omega_0$

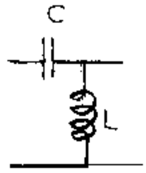
$$\alpha = \frac{\cancel{j}\left(\frac{\omega L}{2} + \sqrt{\frac{\omega^2 L^2}{4} - \frac{L}{C}}\right)}{\cancel{j}\left(\frac{\omega L}{2} + \sqrt{\frac{\omega^2 L^2}{4} - \frac{L}{C}}\right)} \Rightarrow \left.\begin{matrix}\alpha \in \mathbb{R} \\ \alpha < 1\end{matrix}\right\} \rightarrow V_n = \alpha \cdot V_0$$ (el voltaje se va extinguiendo)



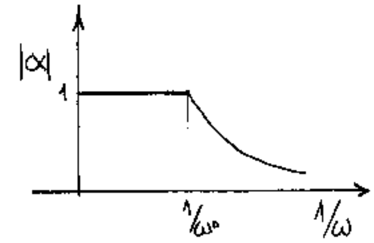
filtro pasabajos



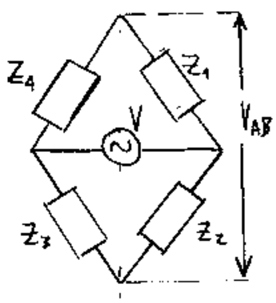
Intercambiando L por C se obtiene un filtro pasaaltos



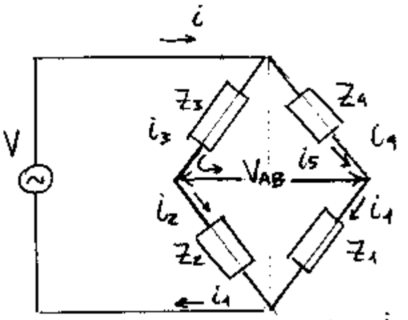
$$\alpha = \frac{+\frac{j}{2\omega C} + \sqrt{\frac{1}{4\omega^2 C} + \frac{\cancel{4}L}{\cancel{4}C}}}{\frac{-j}{2\omega C} + \sqrt{\frac{-1}{4\omega^2 C^2} + \frac{\cancel{4}\cancel{\omega}L}{\cancel{4}\cancel{\omega}C}}}$$



- **Circuito Puente**



≡



Puente en equilibrio $\Rightarrow V_{AB} = 0$

$i = i_3 + i_4 = i_2 + i_1$

$i_3 = i_5 + i_2$

$i_4 = -i_5 + i_1$

$-i_3 Z_3 + V_{AB} + i_4 Z_4 = 0$

$-i_2 Z_2 + i_1 Z_1 - V_{AB} = 0$

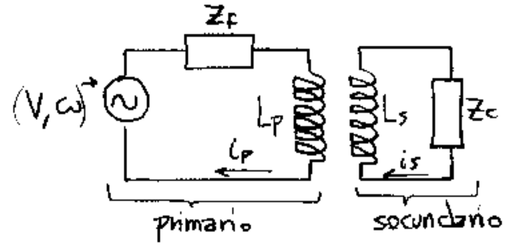
si $V_{AB} = 0 \rightarrow$ $i_5 = 0$

$i_3 Z_3 = i_4 Z_4 = i_1 Z_4$

$i_2 Z_2 = i_1 Z_1 = i_3 Z_2$

Puente en equilibrio ▶ $\boxed{Z_4 \cdot Z_2 = Z_1 \cdot Z_3}$ $\Leftarrow$ $\frac{i_1}{i_3} = \frac{Z_2}{Z_1} = \frac{Z_3}{Z_4}$

- **Transformador**



el potencial alterno provoca en la bobina un $\vec{B}=\vec{B}(t)$
La ΔB produce un $\phi_B=\phi_B(t)$ el cual inducirá sobre la bobina del secundario una fem.

$$L_{ps} = L_{sp} \equiv M$$

coeficientes de inducción mutua

$$V - I_p . Z_f - I_p . Z_p = 0 = V - I_p(Z_f + j\omega L_p)$$
$$-I_s . Z_c - I_s . Z_s = 0 = I_s(Z_c + j\omega L_s)$$

introducimos las variaciones cruzadas

$$(Z_f + j\omega L_p) I_p = V - \overbrace{L_{ps}\omega j}^{Z_{ps}} I_s = V - j\omega \overset{M_{ps}}{M} . I_s$$
$$(Z_c + j\omega L_s) I_s = -L_{sp}\omega j I_p = -j\omega \underset{M_{sp}}{M} I_p$$

$$\begin{aligned} Z_p &= Z_f + j\omega L_p \\ Z_s &= Z_c + j\omega L_s \end{aligned} \Rightarrow \begin{cases} Z_p I_p + j\omega M I_s = V \\ Z_s I_s + j\omega M I_p = 0 \end{cases}$$

$$I_p = \frac{-Z_s I_s}{j\omega M} \rightarrow$$

$$I_s\left(\frac{-Z_s . Z_p}{j\omega M} + j\omega M\right) = V$$

$$I_s = \frac{V}{\frac{j^2\omega^2 M^2 - Z_s . Z_p}{j\omega M}}$$

$$I_p = \frac{-Z_s . V}{j^2\omega^2 M^2 - Z_s . Z_p}$$

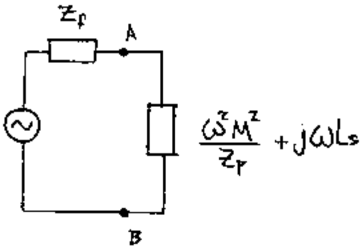
$$I_p = \frac{V . Z_s}{\omega^2 M^2 + Z_s . Z_p} \qquad I_s = \frac{-j\omega M . V}{\omega^2 M^2 + Z_s . Z_p}$$

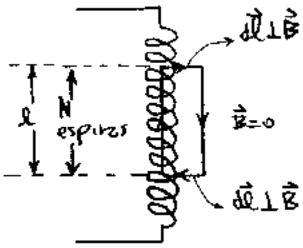
$$Z'_p = \frac{V}{I_p} = \frac{\omega^2 M^2}{Z_s} + Z_p = \frac{\omega^2 M^2}{Z_s} + Z_c + j\omega L_p$$

$$V_{sec} = Z_s . I_s = -j\omega M I_p = \frac{-j\omega M V}{Z_p} = V_{ca} = V_{eq}$$

$$I_s = \frac{\omega M V}{j\omega^2 M^2 + j\underbrace{(Z_c + j\omega L_s)}_{Z_c \to 0} Z_p} = \frac{\omega M V}{j\omega^2 M^2 - \omega L_s Z_p} = \frac{V}{j\omega M - \frac{L_s Z_p}{M}}$$

$$Z_{eq} = \frac{V_{ca}}{I_s} = \frac{-j\omega M \cancel{V}}{Z_p} \cdot \frac{(j\omega M - L_s Z_p / M)}{\cancel{V}} = \frac{\omega^2 M^2}{Z_p} + j\omega L_s$$





autoflujo $\phi_B = \int \vec{B} \cdot d\vec{S} = L . I$

espiras x unidad de Longitud

$$\mu_0 . I_c = \oint B \, dl = B . l \rightarrow \quad B = \frac{\mu_0 . i . n \cancel{l}}{\cancel{l}}$$

$$\phi_{B\,1\,espira} = \mu_0 . i . n \int dS = \mu_0 . i . n . S$$

$$\phi_B^{TOTAL} = \mu_0 \, i . n . N . S = \frac{\mu_0 . i . N^2 . S}{l}$$

$$\phi_B^T = \underbrace{\frac{\mu_0 . N^2 . S}{l}}_{L} . I$$

$$L_p = \frac{\mu_0 . N_p^2 . S}{l}$$

$$L_s = \frac{\mu_0 . N_s^2 . S}{l}$$

$$M = L_{ps} = L_{sp} = \frac{\mu_0 . S}{l} . N_s . N_p$$

$$V_{CD} = V_{eq} = \frac{-\cancel{j\omega} . M . V}{\cancel{j\omega} L_s} = -\frac{\cancel{\mu_0 S} \cancel{N_s} N_p \, V \, \cancel{l}}{\cancel{l} . \cancel{\mu_0} . N_s^2 . \cancel{S}} = \boxed{-\left(\frac{N_p}{N_s}\right) . V}$$

$(Z_c \rightarrow 0)$

$$Z_{eq} = j\omega L_s + \frac{\omega^2 M^2}{j\omega L_p} = j\frac{\omega \mu_0 N_s^2 S}{l} + \omega . \frac{\mu_0^2 S^2 N_p^2 N_s^2 \cancel{l}}{j \, l^2 \cancel{\mu_0 N_p^2 S}} = j\frac{\omega \mu_0 N_s^2 S - j\omega \mu_0 N_s^2 S}{l} = 0$$

$$Z_p' = \frac{\omega^2 M^2}{Z_c} + Z_F + j\omega L_p = \frac{\omega^2 M^2}{R + j\omega L_s} + j\omega L_p = \frac{\cancel{\omega^2 M^2} + j\omega L_p R - \cancel{\omega^2 L_s L_p}}{R + j\omega L_s} = \frac{j\omega L_p . R}{R + j\omega L_s} \cong \frac{L_p}{L_s}$$

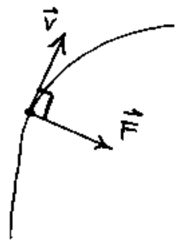
sea $Z_c = R$
$Z_F \rightarrow 0$

si $R \lll \omega L_s$

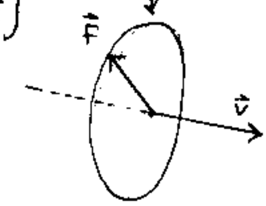
$M = \sqrt{L_s L_p}$
se demuestra

• Movimiento de cargas en un campo $\vec{B}$

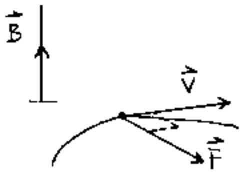
$\vec{F} = q\,\vec{V}\times\vec{B}$



$\left.\begin{array}{l}\vec{F}\perp\vec{V}\\ \vec{F}\perp\vec{B}\end{array}\right\}\Rightarrow$ $\vec{F}$ estará en algún lugar de este disco



si $\vec{B}$ es uniforme y $\vec{B}\perp\vec{V}\Rightarrow$



$\vec{F}\perp\vec{V}\Rightarrow$ $\vec{a}\perp\vec{V}$ $\Rightarrow$ la velocidad solo varía dirección

$\therefore$ hay mov. circular ($\vec{F}^m$ es f. centrípeta)

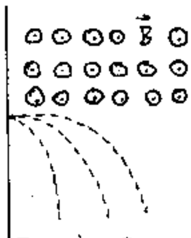
$$m\,\frac{v^2}{r} = |q\,\vec{V}\times\vec{B}| = q\,V B \operatorname{sen}\theta = q\,V B$$

$$V = \frac{q.B.r}{m}$$

$$\omega = \frac{V}{r} \rightarrow \boxed{\omega = \frac{q.B}{m}}$$

Frec. angular de ciclotrón ▲

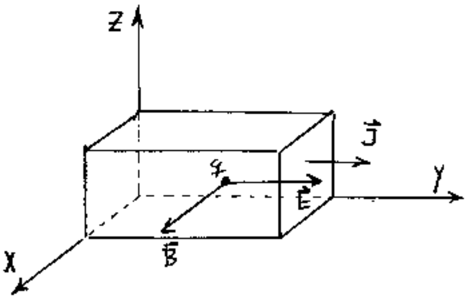
Radio de la órbita ▶ $\boxed{r_o = \frac{m.V}{q.B}}$



Zona de campo $\vec{B}$

se puede usar para construir un separador de velocidades

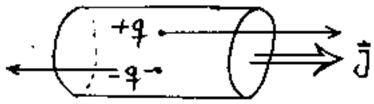
• El Efecto Hall



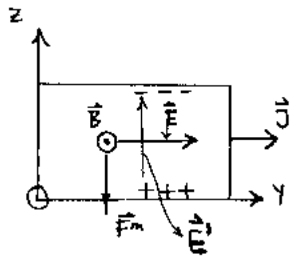
$\hat{y}\times\hat{x}$

$\vec{J} = \sigma\vec{E}$

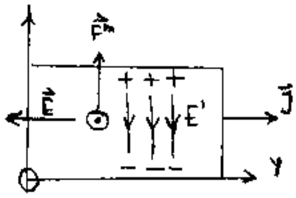
$\delta i = \vec{J}.\delta S$



$q>0$



$q<0$



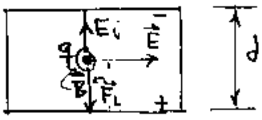
$\vec{F}^m = q\,\vec{v}\times\vec{B} = -q.v.B\,\hat{z}$

$\vec{F}^e = q.E\,\hat{y}$

si meto un campo $E_i = v.B\hat{z} \rightarrow$

$\vec{F}_L = q.v.B\,\hat{z} - q.v.B\,\hat{z} = 0 \longrightarrow$ partículas en línea recta

En este caso : $q>0$



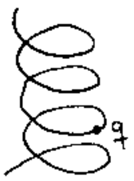
$$\Delta V = E_i \cdot d = v \cdot B \cdot d$$

$$\boxed{v = \frac{\Delta V}{B \cdot d}}$$

velocidad medible con la ΔV

Asimismo viendo el signo de la ΔV se puede saber que cargas se están moviendo.

NOTA



mov. general de una partícula en un campo $\vec{B}$ y dotada de $\vec{v}_0$
(B uniforme)
[es un hélice]