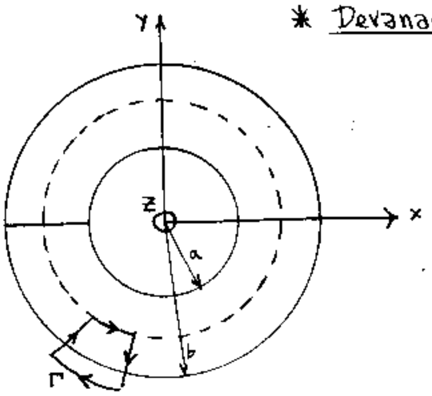
• Cálculo de Flujo Magnético

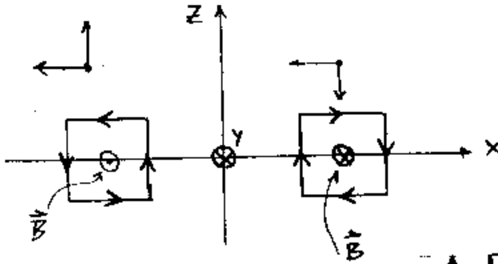
* Devanado toroidal

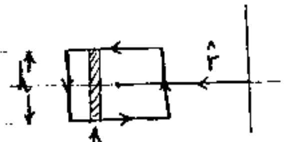
simetría rot. en $\varphi \to B \neq B(\varphi)$

simetría refl. en $\left.\begin{matrix}ZX\\ZY\end{matrix}\right\} \to \begin{matrix}B_r = 0\\ B_z = 0\end{matrix}$

$\vec{B} = B(r,z)\,\hat{\varphi}$







$\vec{B} = B_\varphi(r,z) \;\to\;$ tomo $z=0,\ r=r_0 \to$

$$\oint_\Gamma \vec{B}\cdot d\vec{\ell} = B \cdot r_0\, 2\pi = \mu_0 \cdot i_c \xrightarrow{\text{es la total en el circuito}} \Gamma\quad \vec{B} = \frac{\mu_0\, i N \hat{\varphi}}{2\pi\, r_0} \quad \therefore \quad \vec{B} = \mu_0 \cdot i \cdot n\, \hat{\varphi}$$

i de una espira por N espiras

$n = \frac{N}{2\pi r_0}$ espiras por long.

$$\int_\Gamma B\cdot d\ell = B\int_\Gamma d\ell = B\int_0^{2\pi} r_0\, d\varphi$$

$$\vec{B}\cdot d\vec{\ell} = B_0\,\hat{\varphi}\cdot r_0\, d\varphi\,\hat{\varphi}$$

$B = B(r)$

$$\phi = \int_\Sigma \vec{B}\cdot d\vec{S} = \int_a^b \underbrace{\frac{N\mu_0\, i}{2\pi r}\hat{\varphi}}_{B\ \text{total}}\ \underbrace{h\, dr\, \hat{\varphi}}_{dS\ \text{para cada espira}} = \frac{N\mu_0\, i\, h}{2\pi}\ln r\Big|_a^b = \frac{N\mu_0\, i\, h}{2\pi}\ln\frac{b}{a} \;\to\; \phi_T = N\cdot\phi_{\text{una espira}}$$

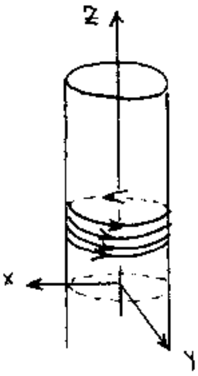
de todo el campo sobre la sup. de 1 espira

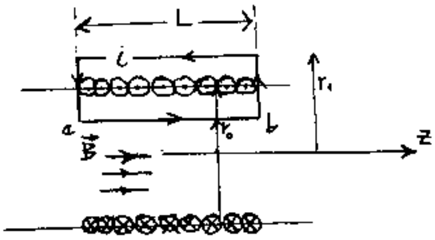
de espiras

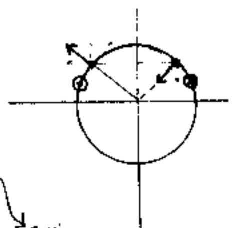
$$\phi_T = \underbrace{\frac{N^2 \mu_0\, h}{2\pi}\ln\left(\frac{b}{a}\right)}_{\equiv L\ \text{autoinductancia}}\ i$$

$\equiv L$ autoinductancia (constante geométrica)

* Devanado Cilíndrico







$\lambda = \frac{N}{L}$ espiras / longitud

$$\oint_\Gamma \vec{B}\cdot d\vec{\ell} = \mu_0 \cdot i$$

$$\int_a^b B\cdot dz - \int_a^b B\cdot dz = \mu_0 \cdot i_c$$

$$B(r_0)\cdot L - \underbrace{B(r_1)}_{=0}\cdot L = \mu_0 \cdot i_c = \mu_0 \cdot i \cdot n \cdot L$$

espiras en L

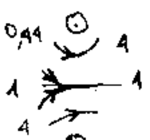
$$B(r_0) = \mu_0\, i\, n$$

simetría rotación en $\varphi \to B \neq B(\varphi)$

simetría reflexión en $\left.\begin{matrix}ZX\\ZY\end{matrix}\right\} \to \begin{matrix}B_r = 0\\ B_z = 0\end{matrix}$

traslación en z (ω en z) $\to B \neq B(z)$
(Siempre me hallo en el 1/2 del cilindro)

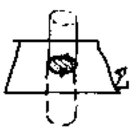
$\vec{B} = B(r)\,\hat{\varphi}$



NOTA

down
up
up
up
down
down
up
down

$\therefore\ \vec{B} = 0$ para un solenoide infinito (afuera)



$$\Sigma\,(\text{una espira})\ \phi_1 = \int_\Sigma \vec{B}\cdot d\vec{S} = \mu_0\, i\, n\, \pi r^2$$

lo tomo uniforme en todo el interior

$$\boxed{\phi_T = \ell\,\mu_0\, n^2\, \pi\, r^2\, i}$$

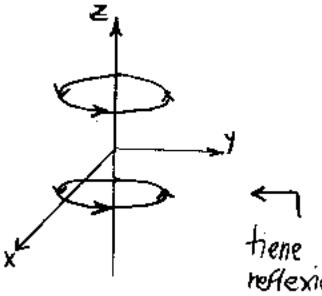
longitud

$\equiv L$ autoinductancia

siempre podemos tomar $\to$ Γ y no concatena corriente alguna

• Magnetización

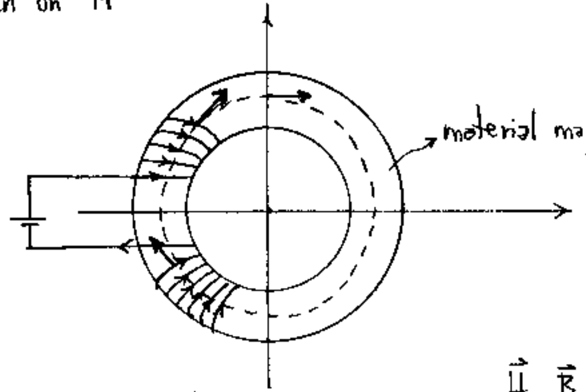
Nota Simetrías



Para $\vec{B}$ se buscan simetrías en las corrientes

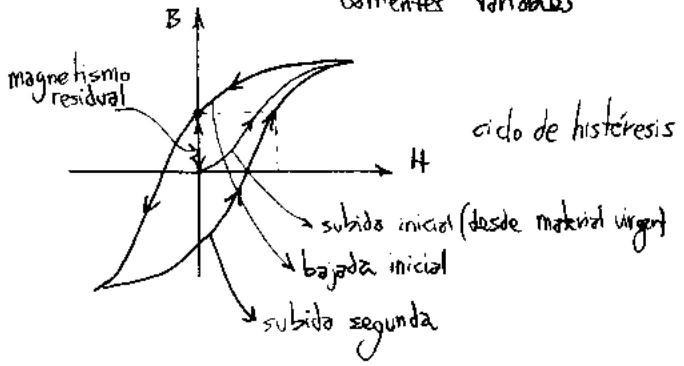
tiene simetría reflexión en ZX, ZY pero NO en XY

La corriente produce un campo $\vec{B}$ y alinea los dipolos produciendo también un $\vec{M}$



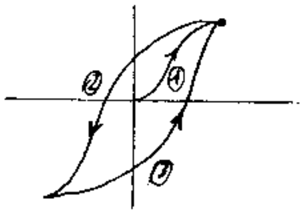
$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

Si se aplica a un hierro virgen corrientes variables

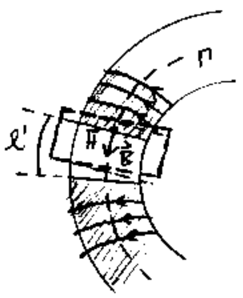


ciclo de histéresis

$$\vec{B} = \mu_0\vec{H} + \mu_0\vec{M}$$

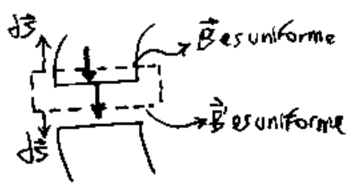


1. Lo llevo al máx.
2. Al disminuir la corriente retorna por arriba
3. A partir del mínimo la subida es por debajo



$$\oint_\Gamma \vec{H}\cdot d\vec{l} = H\cdot l + H'\cdot l' = I_T = N\cdot i$$

$$\int_\Omega \vec{\nabla}\cdot\vec{B}\, d\Omega = \int_{\partial\Omega} \vec{B}\cdot d\vec{S} = 0 \quad \rightarrow \text{ el flujo de } \vec{B} \text{ es cero } \Rightarrow$$



$$\int \vec{B}\cdot d\vec{S} = B(-S) + B'\cdot S = 0 \quad \rightarrow \quad B = B'$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$H' = \frac{B'}{\mu_0} = \frac{B}{\mu_0} \quad \rightarrow \quad \boxed{H\cdot l + \frac{B}{\mu_0} l' = N\cdot i}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

• El Potencial Vector

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{l}' \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}}$$ Ley de Biot-Savart

Pero $$\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} = -\vec{\nabla}_r\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int I\, d\vec{l}' \times -\vec{\nabla}_r\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = \frac{\mu_0}{4\pi}\int I\left[\vec{\nabla}_r\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)\times d\vec{l}'\right]$$

identidad

$$\boxed{\vec{\nabla}\times f\vec{F} = \vec{\nabla}f\times\vec{F} + f\,\vec{\nabla}\times\vec{F}} \Rightarrow$$

$$\vec{\nabla}_r \frac{1}{|\vec{r}-\vec{r}'|}\times d\vec{l}' = \vec{\nabla}_r\times\frac{1}{|\vec{r}-\vec{r}'|}\,d\vec{l}' - \frac{1}{|\vec{r}-\vec{r}'|}\underbrace{\vec{\nabla}_r\times d\vec{l}'}_{=0}$$

Pues $\vec{\nabla}_r$ es grad respecto a $\vec{r}$, no a $\vec{r}'$

$$\vec{B} = \frac{\mu_0}{4\pi}\int I\left(\vec{\nabla}_r\times\frac{d\vec{l}'}{|\vec{r}-\vec{r}'|}\right)$$

$$\vec{B} = \vec{\nabla}_r\times\underbrace{\left(\frac{\mu_0}{4\pi}\int\frac{I\,d\vec{l}'}{|\vec{r}-\vec{r}'|}\right)}_{\equiv\vec{A}(r)}$$

$\equiv \vec{A}(r) \longrightarrow$ el potencial Vector

$$\vec{B}(r) = \vec{\nabla}_r\times\vec{A}(r)$$

$$\boxed{\vec{B} = \vec{\nabla}\times\vec{A}}$$

luego: $\vec{\nabla}\cdot\vec{B} = \vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0 \rightarrow \boxed{\vec{\nabla}\cdot\vec{B} = 0}$

Las lineas de $\vec{B}$ son cerradas

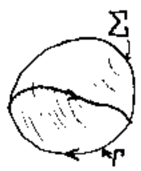
* Gauges (Gosh)

$\vec{\nabla}\times\vec{A} = \vec{B}$; hay libertad de elegir $\vec{\nabla}\cdot\vec{A}$; en general se usa $\vec{\nabla}\cdot\vec{A} = 0$

$$\vec{\nabla}\times\vec{B} = \vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \mu_0\vec{J}$$
$$\vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2 A$$
$$\vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \vec{\nabla}\cdot(\vec{\nabla}A)$$

$$\boxed{-\nabla^2\vec{A} = \mu_0\vec{J}}$$ ← especie de ecuación de Poisson vectorial

$$\int_\Sigma \vec{B}\cdot d\vec{S} = \int_\Sigma \vec{\nabla}\times\vec{A}\cdot d\vec{S} \underset{\text{stokes}}{=} \int_\Gamma \vec{A}\cdot d\vec{l}$$



* Potencial Vector de Circuitos lejanos.

$$\vec{A} = \int \frac{\mu_0}{4\pi} \frac{i \cdot d\vec{l}}{|\vec{r}-\vec{r}'|} \cong \frac{\mu_0}{4\pi} \oint_{\Gamma} i \cdot d\vec{l}' \left[\frac{1}{r}\right] \left(1 + \cos\theta \cdot \frac{r'}{r} + \frac{1}{2}\left(3\cos^2\theta - 1\right)\left(\frac{r'}{r}\right)^2 \right)$$

$$\cong \frac{\mu_0}{4\pi} \cdot \frac{1 i}{r} \left[\underbrace{\oint d\vec{l}'}_{=0} + \oint d\vec{l}' \cdot \cos\theta \frac{r'}{r} + \frac{1}{2} \oint d\vec{l} \left(3\cos^2\theta - 1\right)\left(\frac{r'}{r}\right)^2 \right]$$

$$\cong \frac{\mu_0}{4\pi} \cdot \frac{i}{r} \left[\oint d\vec{l}' \frac{\vec{r} \cdot \vec{r}'}{r^3} + \dots \right] \rightarrow \vec{r} \oint \frac{\vec{r}' \cdot d\vec{l}'}{r^3}$$

identidad

$$\boxed{\vec{A} \times \vec{B} \times \vec{C} = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})}$$

$$\vec{r}' \times d\vec{l}' \times \vec{r} = d\vec{l}'(\vec{r}' \cdot \vec{r}) - \vec{r}\left(\vec{r}' \cdot d\vec{l}'\right)$$

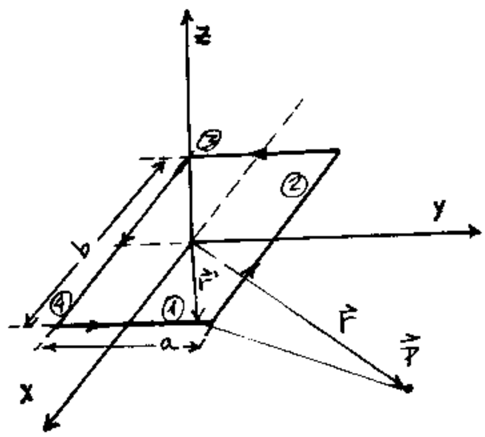
$$\cong \frac{\mu_0 i}{4\pi r} \oint (\vec{r}' \times d\vec{l}') \times \frac{\vec{r}}{r^3}$$

$$\vec{A} \cong \frac{\mu_0}{2\pi r} \cdot \left(\frac{i}{2} \oint \vec{r}' \times d\vec{l}' \times \frac{\vec{r}}{r^3} \right) = \frac{\mu_0}{2\pi r} \cdot \left(\vec{m} \times \frac{\vec{r}}{r^3} \right)$$

$$\vec{A} \approx \frac{\mu_0}{2\pi r} \left(\vec{m} \times \frac{\vec{r}}{r^3} \right)$$

desde lejos

• Potencial Vector



$$\boxed{\vec{A} = \frac{\mu_0}{4\pi}\int \frac{I.\,d\vec{l}}{|\vec{r}-\vec{r}'|}}$$

$*$ ①

$$|\vec{r}-\vec{r}'| = \sqrt{(x-\tfrac{b}{2})^2+(y-y')^2+(z-0)^2}$$

$*$ ③

$$|\vec{r}-\vec{r}'| = \sqrt{(x+\tfrac{b}{2})^2+(y-y')^2+(z-0)^2}$$

aportes de 1,3

$$\vec{A}_{1,3} = \frac{\mu_0}{4\pi}\left(\int \frac{i.\,d\vec{l}_1}{|\ \ |_1} + \int \frac{i.\,d\vec{l}_3}{|\ \ |_3}\right) = \frac{\mu_0}{4\pi}\, i \left[\int_{-a/2}^{a/2} \frac{dy'}{|\ \ |_1} - \int_{-a/2}^{a/2} \frac{dy'}{|\ \ |_3}\right]\hat{y}'$$

me quedo a 1er orden de $x', y', a, b \Rightarrow$

$$A_{1,3} = \frac{\mu_0 i}{4\pi}\int_{-a/2}^{a/2} dy' \left(\frac{1}{|\ \ |_1} - \frac{1}{|\ \ |_3}\right)$$

$$|\ \ |_1 = \sqrt{x^2 - xb + \overbrace{b^2/4}^{\to 0} + y^2 - 2yy' + \overbrace{y'^2}^{\to 0} + z^2} = \sqrt{r^2-(2yy'+xb)} = r\sqrt{1-\frac{(2yy'+xb)}{r^2}}$$

$$|\ \ |_3 = \sqrt{x^2 + xb + \underbrace{b^2/4}_{\to 0} + y^2 - 2yy' + \underbrace{y'^2}_{\to 0} + z^2} = \sqrt{r^2-(2yy'-xb)} = r\sqrt{1-\frac{(2yy'-xb)}{r^2}}$$

$$\frac{1}{|\ \ |_1} - \frac{1}{|\ \ |_3} \cong \frac{1}{r}\left(1 + \frac{1}{2}\left[\frac{2yy'+xb}{r^2}\right] - 1 - \frac{1}{2}\left[\frac{2yy'-xb}{r^2}\right]\right) = \left(\frac{x.b}{2r^2}+\frac{x.b}{2r^2}\right)\frac{1}{r^3}$$

First Order

$$\vec{A}_{1,3} = \frac{\mu_0 . i}{4\pi}\int_{-a/2}^{+a/2} dy' . \frac{x.b}{r^3} = \boxed{\frac{\mu_0 . i}{4\pi} . \frac{x}{r^3} . b . a\ \hat{y}}$$

Analogamente para 2,4 se tendrá

$$\vec{A}_{2,4} = \boxed{-\frac{\mu_0 i}{4\pi} . \frac{y}{r^3} . b . a . \hat{x}}$$

DEFINICIÓN

$i.a.b = m$ momento magnético

$$\vec{m} = i.a.b\,\hat{n}$$

$$\vec{m}\times\vec{r} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & m \\ x & y & z \end{vmatrix} = -m.y.\hat{x} + m.x.\hat{y}$$

$$\boxed{\vec{A} = \frac{\mu_0}{4\pi}\left(\frac{\vec{m}\times\vec{r}}{r^3}\right)}$$

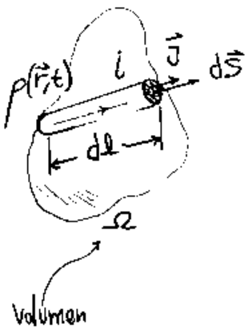
Potencial vector para $r \gg a, b$ (ubicados muy lejos del circuito)

$$\vec{\nabla}\times\vec{A} = \vec{\nabla}.\times\frac{\mu_0}{4\pi}\left[\vec{m}\times\vec{\nabla}_r\frac{1}{r}\right] = \frac{\mu_0}{4\pi}\left(\vec{m}.\vec{\nabla}.\left(\vec{\nabla}_r\frac{1}{r}\right) - \vec{\nabla}_r\frac{1}{r}\left(\vec{\nabla}.\vec{m}\right) + \vec{\nabla}_r\frac{1}{r}.\vec{\nabla}\,\vec{m} - \vec{m}.\vec{\nabla}.\vec{\nabla}_r\frac{1}{r}\right.$$

Corrección a las Ecuaciones de Maxwell

$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$	$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$
$\vec{\nabla}\cdot\vec{B} = 0$	$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$

matemáticamente $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = 0$ pero $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \mu_0\vec{\nabla}\cdot\vec{J} \neq 0$ $\therefore$ falta algo



$$di = \vec{J}\cdot d\vec{S} \rightarrow J = \frac{di}{dS} = \frac{d}{dS}\left(\frac{\delta q}{\delta t}\right) \rightarrow$$

$$i = \int \vec{J}\cdot d\vec{S} = \int \frac{d}{dS}\left(\frac{\delta q}{\delta t}\right) dS = \int \frac{d}{dS\,dl}\left(\frac{\delta q}{\delta t}\right) dS\,dl$$

$$= \int \vec{J}\cdot d\vec{S} = \int \frac{d}{dV}\left(\frac{\delta q}{\delta t}\right) dV = \int \frac{\delta}{\delta t}\left(\frac{\delta q}{\delta V}\right)\delta V$$

$$i = \int \vec{J}\cdot d\vec{S} = \int \frac{\partial}{\partial t}(\rho)\, dV$$

pero ρ refiere a la carga que sale de un volumen (y que constituye i)
$\Rightarrow$ $\rho_{SAL} = -\rho_{\text{dentro del volumen}}$

$$\int_{\partial\Omega} \vec{J}\cdot d\vec{S} = \int_{\Omega} -\frac{\partial\rho}{\partial t}\, dV$$

$$\rightarrow \quad \vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t} \Rightarrow \boxed{\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0}$$

Conservación de la carga

supongamos que

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \mu_0\left(\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t}\right) = 0$$

$\rightarrow$ q.v.q. puedo sacar $\vec{\nabla}\cdot$ afuera del paréntesis $\Rightarrow$

$$\frac{\partial\rho}{\partial t} = \frac{\partial}{\partial t}\left(\vec{\nabla}\cdot\vec{E}\,\varepsilon_0\right) = \varepsilon_0\,\vec{\nabla}\cdot\frac{\partial\vec{E}}{\partial t} \rightarrow$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \mu_0\,\vec{\nabla}\cdot\left(\vec{J} + \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right) = \vec{\nabla}\cdot\left[\mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right] \quad \therefore$$

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}}$$

◀ Ahora las ecuaciones de Maxwell están completas.

● Energía EM / Vector de Poynting

$$U = \int_\Omega \left(\frac{1}{2} \varepsilon_0 \vec{E}^2 + \frac{1}{2} \frac{1}{\mu_0} \vec{B}^2 \right) . d\Omega$$

$$-\frac{\partial U}{\partial t} = -\int_\Omega \left[\frac{1}{\cancel{2}} \varepsilon_0 . \cancel{2}\, \vec{E} . \frac{\partial \vec{E}}{\partial t} + \frac{1}{\cancel{2}} . \frac{1}{\mu_0} . \cancel{2} \vec{B} . \frac{\partial \vec{B}}{\partial t} \right] . d\Omega$$

pero

$$\frac{\partial \vec{E}}{\partial t} = \frac{(\vec{\nabla} \times \vec{B})}{\mu_0 \varepsilon_0} - \frac{\mu_0 \vec{J}}{\mu_0 \varepsilon_0} = \frac{1}{\mu_0 \varepsilon_0} \vec{\nabla} \times \vec{B} - \frac{\vec{J}}{\varepsilon_0}$$

$$-\frac{\partial U}{\partial t} = -\int_\Omega \left[\varepsilon_0 . \vec{E} . \left(\frac{1}{\mu_0 \varepsilon_0} (\vec{\nabla} \times \vec{B}) - \frac{\vec{J}}{\varepsilon_0} \right) + \frac{\vec{B}}{\mu_0} . \left(-\vec{\nabla} \times \vec{E} \right) \right] . d\Omega$$

$$-\frac{\partial U}{\partial t} = \int_\Omega \left(\cancel{\varepsilon_0}\, \vec{E} . \frac{\vec{J}}{\cancel{\varepsilon_0}} - \frac{\vec{E}}{\mu_0} . [\vec{\nabla} \times \vec{B}] + \frac{\vec{B}}{\mu_0} [\vec{\nabla} \times \vec{E}] \right) . d\Omega$$

$$-\frac{\partial U}{\partial t} = \int_\Omega \vec{E} . \vec{J} d\Omega + \frac{1}{\mu_0} \int_\Omega \left[\vec{B} . (\vec{\nabla} \times \vec{E}) - \vec{E} . (\vec{\nabla} \times \vec{B}) \right] . d\Omega$$

$$-\frac{\partial U}{\partial t} = \int_\Omega \vec{E} . \vec{J} . d\Omega + \frac{1}{\mu_0} \int_\Omega \vec{\nabla} . (\vec{E} \times \vec{B}) . d\Omega$$

$$-\frac{\partial U}{\partial t} = \int_\Omega \vec{E} . \vec{J} . d\Omega + \frac{1}{\mu_0} \int_{\partial\Omega} (\vec{E} \times \vec{B}) d\vec{S}$$ → Potencia (Energía por unidad de tiempo)

Con $\boxed{\frac{1}{\mu_0} \vec{E} \times \vec{B} = \text{Vector de Poynting}}$

● Desarrollo de Hertz - Ecuaciones en el vacío

Sea un lugar en el universo donde $\rho = 0 \wedge \mu_0 \vec{J} = 0 \Rightarrow$

$$\boxed{\begin{array}{llr} ① \ \vec{\nabla} . \vec{B} = 0 & \vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t} & ③ \\ ② \ \vec{\nabla} . \vec{E} = 0 & \vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} & ④ \end{array}}$$

No hay fuentes de los campos (ni sumideros)

$$\vec{A} \times \vec{B} \times \vec{C} = (\vec{C} . \vec{A}) \vec{B} - (\vec{B} . \vec{A}) \vec{C}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \underbrace{(\vec{\nabla} . \vec{E})}_{=0 \text{ por la ②}} - \underbrace{(\vec{\nabla} . \vec{\nabla})}_{\text{esto es un escalar}} \vec{E} = -\nabla^2 \vec{E}$$

laplaciano

$$\vec{\nabla} . \underbrace{(\vec{\nabla} \varphi)}_{\vec{E}} = \nabla^2 \varphi$$

$\nabla^2 \vec{E}$ → implica 3 laplacianos de campos escalares

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \times \left(-\frac{\partial \vec{B}}{\partial t} \right) = -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B}) = -\frac{\partial}{\partial t} \left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

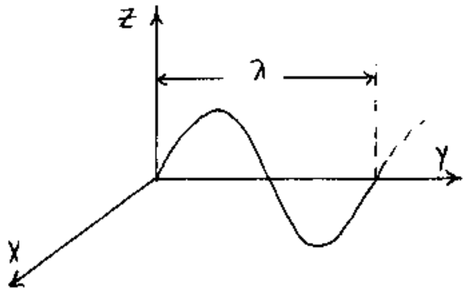
$$= -\mu_0 . \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Ecuación de Ondas → $\boxed{\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$

Propongamos una onda en el plano ZY armónica en el tiempo y en la posición:

$$\vec{E} = E_0 . \operatorname{sen} \left(\frac{2\pi}{\lambda} . y - \frac{2\pi}{\tau} . t \right) \hat{z}$$

$$\vec{E} = E_z(y, t)$$



$$\nabla^2 E_z = \mu_0 \varepsilon_0 \frac{\partial^2 E_z}{\partial t^2}$$

$E_z = E_0 \cdot \operatorname{sen}\left(\frac{2\pi}{\lambda} y - \frac{2\pi}{\tau} t\right)$

$\frac{\partial E_z}{\partial y} = E_0 \cos\left(\frac{2\pi}{\lambda} y - \frac{2\pi}{\tau} t\right) \cdot \frac{2\pi}{\lambda}$

$\frac{\partial^2 E_z}{\partial y^2} = -E_0 \left(\frac{2\pi}{\lambda}\right)^2 \operatorname{sen}\left(\frac{2\pi y}{\lambda} - \frac{2\pi}{\tau} t\right)$

$\frac{\partial E_z}{\partial t} = -E_0 \cos\left(\frac{2\pi}{\lambda} y - \frac{2\pi}{\tau} t\right) \cdot \frac{2\pi}{\tau}$

$\frac{\partial^2 E_z}{\partial t^2} = -E_0 \operatorname{sen}\left(\frac{2\pi}{\lambda} y - \frac{2\pi}{\tau} t\right) \left(\frac{2\pi}{\tau}\right)^2$

$$-\cancel{E_0} \frac{\cancel{(2\pi)^2}}{\lambda^2} \cancel{\operatorname{sen}\left(\frac{2\pi y}{\lambda} - \frac{2\pi t}{\tau}\right)} = -\mu_0 \varepsilon_0 \cancel{E_0} \frac{\cancel{(2\pi)^2}}{\tau^2} \cancel{\operatorname{sen}\left(\frac{2\pi y}{\lambda} - \frac{2\pi t}{\tau}\right)}$$

$$\boxed{\frac{1}{\lambda^2} = \frac{\mu_0 \varepsilon_0}{\tau^2}}$$

$$\frac{\tau^2}{\lambda^2} = \mu_0 \varepsilon_0$$

La velocidad de la onda es

$$v = \frac{\lambda}{\tau} \Rightarrow$$

$$v = \frac{\tau}{\sqrt{\mu_0 \varepsilon_0}} \cdot \frac{1}{\tau} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Haciendo la cuenta se ve que: $v = c \rightarrow$ $\boxed{c^2 = \frac{1}{\mu_0 \varepsilon_0}}$

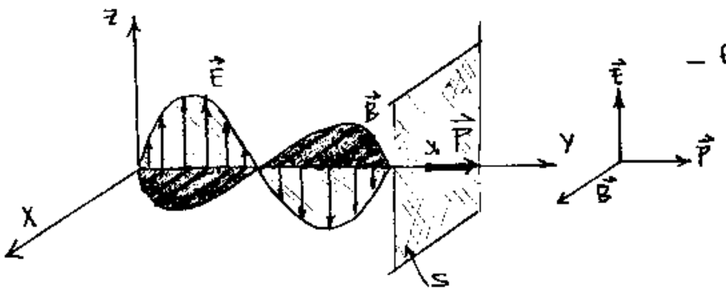
Ahora metemos $\vec{B}$ despejándolo de $\vec{\nabla} \times \vec{E}$

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & E_z \end{vmatrix} = \frac{\partial E_z}{\partial y} \hat{x} - \frac{\partial E_z}{\partial x} \hat{y} = \frac{\partial E_z}{\partial y} \hat{x} = -\frac{\partial \vec{B}}{\partial t}$$

$$E_0 \frac{2\pi}{\lambda} \cos\left(\frac{2\pi}{\lambda} y - \frac{2\pi}{\tau} t\right) = -\frac{\partial B_x}{\partial t}$$

$$-E_0 \frac{2\pi}{\lambda} \operatorname{sen}\left(\frac{2\pi}{\lambda} y - \frac{2\pi}{\tau} t\right) \left(-\frac{\tau}{2\pi}\right) = B_x$$

$$\frac{E_0}{c} \operatorname{sen}\left[\frac{2\pi}{\lambda} y - \frac{2\pi}{\tau} t\right] = B_x$$



$$\underset{\text{Poynting}}{\vec{P}} = \frac{\vec{E} \times \vec{B}}{\mu_0} = \frac{1}{\mu_0} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & E_z \\ B_x & 0 & 0 \end{vmatrix} = \frac{1}{\mu_0} B_x E_z \hat{y} \rightarrow \vec{P} = \frac{E_z B_x}{\mu_0} \hat{y}$$

* Energía transportada por la onda (queremos ver la U que pasa a través de una superficie por unidad de t)

Esto nos da la U por unidad de tiempo

$$\int_{\partial\Omega \equiv S} \frac{1}{\mu_0} (\vec{E} \times \vec{B}) \cdot d\vec{S} = \frac{1}{\mu_0} \int_S E_z B_x \, dx \, dz = \frac{E_z B_x}{\mu_0} \int_S dA = \frac{E_0^2 \operatorname{sen}^2 \phi}{\mu_0 c} S \Rightarrow$$

E_z, B_x son fijos para cierto y

tomando el valor medio para una λ (por área y por unidad de tiempo) se tiene

$$\boxed{\overline{P} = \frac{E_o^2}{2\mu_o . c}}$$

densidad de potencia promedio de una onda EM (en una cierta área)