# Teoremas de Conservación

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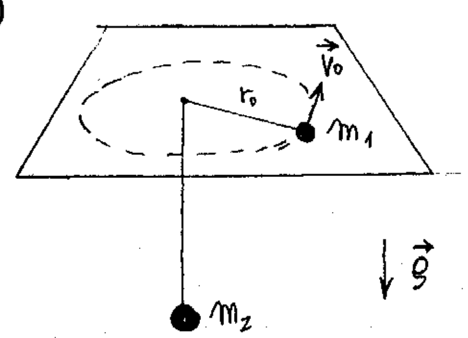
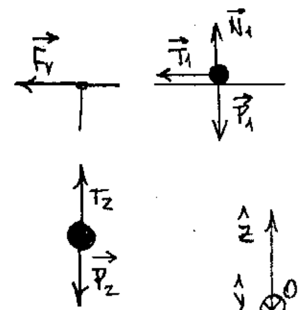



b) * momento lineal — sistema: $m_1, m_2$, hilo

$$\sum \vec{F_e} = \underbrace{\vec{N_1} + \vec{P_1}}_{=0 \text{ por Newton}} + \vec{F_V} + \vec{P_2}$$

$$\sum \vec{F_e} \neq 0 \Rightarrow \boxed{\vec{P} \neq k}$$

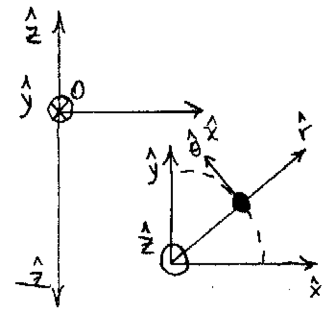
$$\frac{dP}{dt} = -m_2 \cdot g\, \hat{z} + \vec{F_V}$$




* momento angular

$$\sum \vec{\tau}_0^{ext} = \underbrace{\vec{r_1} \times \vec{N_1} + \vec{r_1} \times \vec{P_1}}_{=0 \text{ por ser } \vec{N_1} = -\vec{P_1}} + \underbrace{\vec{0} \times \vec{F_V}}_{=0} + \underbrace{\vec{z_2} \times \vec{P_2}}_{=0 \text{ por } //}$$

$$\sum \vec{\tau}_0^{ext} = \frac{d\vec{L_0}}{dt} = 0 \Rightarrow \boxed{\vec{L_0} \equiv k}$$




$$L_{0i} = L_{0f}$$

$$(r_1 \hat{r} \times m_1 . \vec{V_0}) + \underbrace{(z_2 \hat{z} \times m_2 . \underbrace{\dot{z}}_{0})}_{=0} = L_{0f}$$

(sólo componente tangencial) ← [ $\vec{V_0} = r_0 . \omega_0$

$$(m_1 . r_0^2 . \omega_0)\hat{z} = L_{0f}$$

$$= r_1 . m_1 . r_1 . \omega\, \hat{z} + \underbrace{z_2 \hat{z} \times (m_2 . (\dot{z}_2 \hat{z}))}_{=0 \text{ por ser } //}$$

$\hat{z}$) $\cancel{m_1} . r_0^2 . \omega_0 = r_1^2 . \cancel{m_1} . \omega$

$$\boxed{\frac{r_0 . V_0}{r_1^2} = \omega}$$

vínculos

$$L = r_1 - z_2 \Rightarrow z_2 = r_1 - L$$

$$0 = \dot{r}_1 - \dot{z}_2$$

$$\dot{r}_1 = \dot{z}_2$$

$$\underbrace{dL}_{=0} = dr_1 - dz_2$$

$$dr_1 = dz_2$$

* Energía

$$W_T = \underbrace{W_{T_1} + W_{T_2}}_{F \text{ no cons.}} + \underbrace{W_{F_V}}_{=0} + \underbrace{W_{N_1}}_{=0} + \underbrace{\underbrace{W_{P_1}}_{=0} + W_{P_2}}_{F \text{ cons.}}$$

$W_{F_V} = 0$: $\vec{O}$ no se mueve por $F_V$; $W_{N_1} = 0$: $m_1$ no se mueve por $N_1$; $W_{P_1} = 0$ → no hay desplaz. en $\hat{z}$ para $m_1$

$$W_{NC} = \int \vec{T_1} . dr_1 + \int \vec{T_2} . dz_2 = \int -T_1 . \hat{r} . dr + \int T_2 . \hat{z}\, dz = -\int T_1\, dr + \int T_2\, dr$$

pero $T_1 = T_2$ (soga de masa despreciable) ∴ $W_{NC} = -\int T dr + \int T dr = 0 \Rightarrow \boxed{E_M \equiv k}$

$$E_{Mi} = E_{Mf}$$

$$E_{ki} + E_{pi} = E_{kf} + E_{pf}$$

$$\frac{1}{2} m_1 V_0^2 + \frac{1}{2} m_2 \underbrace{(\dot{z}_{z_i})^2}_{=0} + \underbrace{E_{pi}}_{+ m_2 . g . \underbrace{z_2}_{(r_0 - L)}} = \frac{1}{2} m_1 V_f^2 + \frac{1}{2} m_2 (\dot{z}_{2f})^2 + E_{pf}$$

$$\vec{F_t} = -m_2 \vec{g}\, \hat{z} = -\frac{dE_p}{dz} \Rightarrow E_p = \int m_2 g\, dz = m_2 g z + C$$

$$\frac{1}{2} m_1 V_0^2 + m_2 g (r_0 - L) = \frac{1}{2} m_1 (\dot{r}_1^2 + r_1^2 \dot{\theta}_f^2) + \frac{1}{2} m_2 (\dot{z}_{2f}^2) + m_2 g z_2$$

$$\frac{1}{2}m_1 v_0^2 + m_2 g(r_0-L) = \frac{1}{2}m_1(\dot r_1^2) + \frac{1}{2}m_1(r_1^2 \dot\theta_{1f}^2) + \frac{1}{2}m_2(\dot z_{2f}^2) + m_2 g(z_2)$$

$$\frac{1}{2}m_1 v_0^2 + m_2 g(r_0-L) = \frac{1}{2}m_1 \dot r_1^2 + \frac{1}{2}m_1(v_{\hat\theta_{1f}}^2) + \frac{1}{2}m_2(\dot r_1^2) + m_2 g(r_1-L)$$

$r_0 = r_{1i}$
$r_1 = r_{1f}$

a)

$m_1$

$\hat r)$ $m_1(-r_0 \dot\theta_0^2) = -T$

$\hat\theta)$ $m_1 r_0 \ddot\theta = 0$

$\hat z)$ $0 = N_1 - m_1 g$

$m_2$

$\hat z)$ $0 = T - m_2 g$
$T = m_2 g$

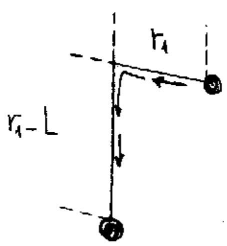
$$-m_1 r_0 \dot\theta_0^2 = -m_2 g$$
$$m_1 \frac{v_0^2}{r_0} = m_2 g$$

Relación $\boxed{\frac{v_0^2}{r_0} = \frac{m_2}{m_1} g}$

$r\dot\theta = v_\theta$
$r^2\dot\theta^2 = v_{\hat\theta}^2$

c)

$$\frac{1}{2}m_1 v_0^2 + m_2 g(r_0-L) = \frac{1}{2}(m_1+m_2)\dot r_1^2 + \frac{1}{2}m_1\left(r_1^2 \frac{r_0^2 v_0^2}{r_1^4}\right) + m_2 g(r_1-L)$$



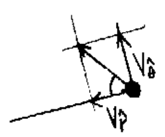

$r_0 - L =$

$r_1 - L = -d$
$r_1 = L-d$

$$\frac{1}{2}m_1 v_0^2 + m_2 g(r_0-L) = \frac{1}{2}m_1\left(\dot r_1^2 + \frac{r_0^2 v_0^2}{(L-d)^2}\right) + \frac{1}{2}m_2(\dot r_1^2) - m_2 g d$$

$$-\frac{1}{2}(m_1+m_2)\dot r_1^2 = -\frac{1}{2}(m_1+m_2)\dot z_2^2 = \frac{m_1}{2}\frac{r_0^2 v_0^2}{(L-d)^2} - m_2 g d - m_2 g(r_0-L) - \frac{1}{2}m_1 v_0^2$$

$$\frac{m_1+m_2}{\cancel{2}}\dot z_2^2 = \frac{m_1}{\cancel{2}} v_0^2\left(\frac{r_0^2}{(L-d)^2} - 1\right) - 2m_2 g(d + r_0 - L)$$




$$\boxed{\dot r_1^2 = \dot z_2^2 = \frac{m_1 v_0^2\left(\frac{r_0^2}{(L-d)^2} - 1\right) - 2m_2 g(d + r_0 - L)}{(m_1+m_2)}}$$

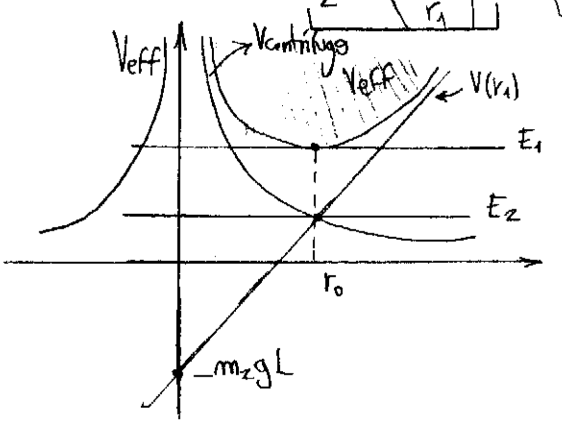
$$\vec V_1 = \left(\dot r_1 \hat r + \overbrace{\frac{r_0 v_0}{r_1}}^{r\dot\theta}\hat\theta\right) \qquad \vec V_2 = \dot z_2$$

$$\operatorname{sen}\alpha = \frac{\frac{r_0 v_0}{r_1}}{|\vec V_1|} \qquad \alpha = \operatorname{asen}\left(\frac{\frac{r_0 v_0}{r_1}}{\sqrt{\dot r_1^2 + \left(\frac{r_0 v_0}{r_1}\right)^2}}\right)$$

d)

$$V_{eff} = \frac{1}{2}m_1\left(\frac{r_0^2 v_0^2}{r_1^2}\right) + m_2 g(r_1 - L)$$
$$\underbrace{\frac{1}{2}m_1\left(\frac{r_0 v_0}{r_1}\right)^2} \quad \underbrace{+ m_2 g r_1 - m_2 g L}_{V(r_1)}$$

$V_{cent} = V(r)$




$m_2$ en reposo
$\Rightarrow$ $m_1$ recorre órbita circular
$E = E_1$

$$\frac{dV_{eff}}{dt} = \frac{1}{2}m_1 r_0^2 v_0^2 \cdot \frac{-2}{r_1^3} + m_2 g$$
$$0 = \frac{1}{\cancel{2}}m_1 r_0^2 v_0^2 \cdot \frac{-\cancel{2}}{r_1^3} + m_2 g$$
$$m_1 \frac{r_0^2 v_0^2}{r_1^3} = m_2 g$$
$$\frac{r_0^2 v_0^2}{r_1^3} = \frac{m_2 g}{m_1} \leftarrow \text{mínimo}$$

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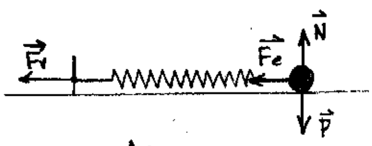
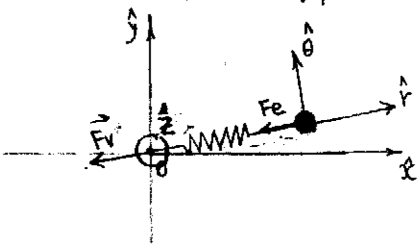



a) * momento lineal

$$\sum \vec{F_e} = \underbrace{\vec{N_1} + \vec{P_1}}_{=0 \text{ por Newton}} + \vec{F_v} \Rightarrow \frac{d\vec{P}}{dt} \neq 0 \Rightarrow \vec{P} \neq k$$

* momento angular

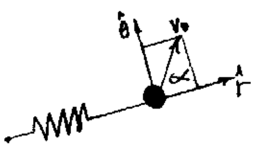
$$\sum \vec{\tau_0^{ext}} = 0 \times \vec{F_v} + \underbrace{r \times \vec{N_1} + r \times \vec{P_1}}_{=0 \text{ por, por } \vec{N} = -\vec{P_1}} = 0 \Rightarrow \vec{L_0} = k$$

* energía

$$W_T = \underbrace{W_{N_1} + W_{P_1}}_{\substack{\perp \text{ despl} \\ \Rightarrow \text{son } 0}} + \underbrace{W_{Fe}}_{F_{cons.}} + \underbrace{W_{Fv}}_{F_{noconv}}$$

$$W_{NC} = \underbrace{W_{Fv}}_{\substack{=0 \text{ porque} \\ \text{no se mueve} \\ \text{el eje } 0}} = 0 \Rightarrow \boxed{E = k}$$

b) $L_0 = r \times m.V$




$$L_0^i = L_0^f$$

$$2l_0 \hat{r} \times m.(V_0.\cos\alpha\, \hat{r} + V_0.\text{sen}\,\alpha\, \hat{\theta}) = \frac{3}{2} l_0 \hat{r} \times m(\dot{r}\hat{r} + r.\dot{\theta}\hat{\theta})$$

$$(2\cancel{l_0}\cancel{m}\, V_0.\text{sen}\,\alpha)\hat{z} = \frac{3}{2}\cancel{l_0}.\cancel{m}\frac{3}{2}l_0\dot{\theta}\,\hat{z}$$

$$2 V_0.\text{sen}\,\alpha = \frac{9}{4} l_0 \dot{\theta} \Rightarrow$$

$$\boxed{\frac{8.V_0.\text{sen}\,\alpha}{9.\ l_0} = \dot{\theta}}$$

$$W_T = W_{Fe} = \Delta E_k \qquad \int -dE_p = \int -kx.dx$$

$$W_{FC_{x_i \to x_f}} = \int_{x_i}^{x_f} -kr.dr = -\frac{k}{2}r^2\Big]_{r_i}^{r_f} = -\frac{k}{2}(r_f^2 - r_i^2)$$

$x_i = r_i = 2l_0$

$x_f = r_f = \frac{3}{2}l_0$

$$= -\frac{k}{2}\left(\frac{9l_0^2}{4} - 4l_0^2\right) = \frac{k}{2}\left(\frac{7}{4}l_0^2\right) = \frac{7}{8}k\,l_0^2$$

$$E_{kf} - E_{ki} = \frac{7}{8}k\,l_0^2 \qquad \frac{Kg}{m}\frac{m^2}{s^2}.m$$

$$\frac{1}{2}m V_0^2 - \frac{1}{2}m V^2 = \frac{7}{8}k\,l_0^2$$

$$\frac{1}{2}m V_0^2 - \frac{7}{8}k\,l_0^2 = \frac{1}{2}m V^2$$

$\frac{kg.m}{m\ s^2}\frac{m}{Kg\,m}$

$$V_0^2 - \frac{7}{4}\frac{k}{m}l_0^2 = V^2$$

$$= \left[\frac{\cancel{8}}{\cancel{9}_3}\frac{V_0.\text{sen}\,\alpha}{\cancel{l_0}}.\frac{\cancel{3}}{2}l_0\hat{\theta} + \dot{r}\hat{r}\right]^2$$

$$V_0^2 - \frac{7}{4}\frac{k}{m}l_0^2 = \frac{V_0^2.\text{sen}^2\alpha}{9} + \dot{r}^2$$

$$\boxed{V_0^2\left(1 - \frac{\text{sen}^2\alpha}{9}\right) - \frac{7}{4}\frac{k}{m}l_0^2 = \dot{r}^2}$$