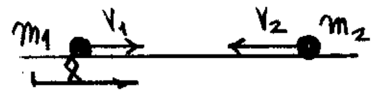
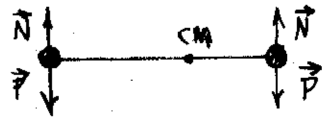
TEOREMAS DE CONSERVACIÓN

1.



choque elástico $(E_K \equiv k)$
E_K se conserva



$$\sum \vec{F}_e = 0 \quad \therefore \quad \frac{d\vec{P}}{dt} = 0 \quad \Rightarrow \quad \vec{P} \equiv k$$

$\hat{x})$

$$\text{I)} \quad m_1 . V_1 - m_2 . V_2 = m_1 . |V_1'| + m_2 . |V_2'|$$

$$E_{K_i} = E_{K_f} \Rightarrow \text{II)} \quad \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 = \frac{1}{2} m_1 V_1'^2 + \frac{1}{2} m_2 V_2'^2$$

$$m_1 V_1 - m_1 |V_1'| = m_2 V_2 + m_2 |V_2'|$$
$$m_1 (V_1 - |V_1'|) = m_2 (V_2 + |V_2'|)$$

$$\frac{1}{2} m_1 (V_1^2 - V_1'^2) = \frac{1}{2} m_2 (V_2'^2 - V_2^2)$$
$$m_1 (V_1^2 - V_1'^2) = (V_2'^2 - V_2^2) . m_2$$
$$m_1 . (V_1 - |V_1'|) . (V_1 + |V_1'|) = (|V_2'| - V_2) . (|V_2'| + V_2) . m_2$$
$$V_1 + |V_1'| = |V_2'| - V_2$$
$$\text{III)} \quad V_1 + V_2 = |V_2'| - |V_1'|$$

a)
$$m_1 V_1 - m_2 V_2 = m_1 |V_1'| + m_2 . |V_2'|$$

$$m_1 . (V_1 + V_2) = (|V_2'| - |V_1'|) . m_1$$
$$m_2 . (V_1 + V_2) = (|V_2'| - |V_1'|) . m_2$$

$$2 m_1 V_1 + (m_1 - m_2) . V_2 = (m_1 + m_2) . |V_2'|$$

$$\boxed{\frac{2 m_1 V_1 + (m_1 - m_2) . V_2}{(m_1 + m_2)} = |V_2'|}$$

$$\boxed{\frac{(m_1 + m_2) . V_1 - 2 m_2 |V_2'|}{(m_1 - m_2)} = |V_1'|}$$

b)
$$\frac{1}{2} m_1 V_1'^2 - \frac{1}{2} m_1 V_1^2 = -\frac{1}{2} m_2 V_2'^2 + \frac{1}{2} m_2 V_2^2$$
$$\frac{1}{2} m_1 . (V_1'^2 - V_1^2) = \frac{1}{2} m_2 (-V_2'^2 + V_2^2)$$
$$= \frac{1}{2} m_2 \left(-\left[\frac{4 m_1^2 V_1^2 + 4 m_1 V_1 (m_1 - m_2) V_2 + V_2^2 (m_1^2 - 2 m_1 m_2 + m_2^2)}{(m_1 + m_2)^2} \right] + V_2^2 \right)$$

$$\Delta E_{K_{1,2}} = \frac{1}{2} m_2 \left(- \frac{\left[4 m_1^2 V_1^2 + 4 m_1^2 V_1 V_2 - 4 m_1 m_2 V_1 V_2 + V_2^2 m_1^2 - 2 m_1 m_2 V_2^2 + V_2^2 m_2^2 \right]}{(m_1 + m_2)^2} + V_2^2 \right)$$

c) si $V_2 = 0$

$$|V_2'| = \frac{2 m_1 . V_1}{(m_1 + m_2)}$$

$$|V_1'| = \frac{(m_1 + m_2) . V_1 - 2 m_2 . \frac{2 m_1 . V_1}{(m_1 + m_2)}}{m_1 - m_2} = \frac{\left[\frac{(m_1 + m_2)^2 - 4 m_1 m_2}{(m_1 + m_2)} \right] V_1}{(m_1 - m_2)}$$

$$\boxed{\Delta E_{K_{1,2}} = \frac{1}{2} m_2 \left[- \frac{4 m_1^2 V_1^2}{(m_1 + m_2)^2} \right]}$$

d) * si $m_1 = m_2 \Rightarrow$ $\boxed{|V_2'| = V_1} \Rightarrow V_2' = V_1$

$$m . V_1 = m |V_1'| + m . V_1 \quad \boxed{|V_1'| = 0} \quad \Rightarrow \quad V_1' = 0$$

$$\Delta E_{K_{1,2}} = \frac{1}{2} m . \frac{(-\cancel{4 m^2})}{\cancel{4 m^2}} . V_1^2 = \boxed{-\frac{1}{2} m V_1^2}$$

* si $m_1 \gg m_2 \Rightarrow \frac{m_2}{m_1} \ll 1$ $\quad |V_2'| = \frac{2\cancel{m_1} V_1}{\cancel{m_1}\left(1+\frac{m_2}{m_1}\right)} \Rightarrow \boxed{V_2' \cong 2V_1}$

$$|V_1'| = \frac{\cancel{m_1}\left(1+\frac{m_2}{m_1}\right)V_1 - \frac{4 m_2 \cancel{m_1} V_1}{\cancel{m_1}(1+m_2/m_1)m_1}}{\cancel{m_1}\left(1-\frac{m_2}{m_1}\right)} \Rightarrow \boxed{V_1' \cong V_1}$$

$$\Delta E_{k_{1,2}} = \frac{1}{2}m_2\left[-\frac{\cancel{(m_1^2)}\left(4V_1^2+4V_1V_2-4\frac{m_2}{m_1}V_1V_2+V_2^2-2\frac{m_2}{m_1}V_2^2+V_2^2\left(\frac{m_2}{m_1}\right)^2\right)}{\cancel{m_1^2}\left(1+\frac{m_2}{m_1}\right)^2}+V_2^2\right]$$

$$\frac{1}{2}m_2 \cdot \frac{-4m_1^2V_1^2}{m_1^2\left(1+\frac{m_2}{m_1}\right)^2} \qquad \boxed{-2m_2V_1^2}$$

$$= \frac{1}{2}m_2\left[-4V_1^2-4V_1V_2-\cancel{V_2^2}+\cancel{V_2^2}\right] = \frac{1}{\cancel{2}}m_2 \cdot -\overset{2}{\cancel{4}}V_1 \cdot (V_1-V_2)$$

$$\boxed{\Delta E_{k_{1,2}} = -m_2 \cdot 2 \cdot V_1 (V_1 - V_2)} \Rightarrow \boxed{-2m_2V_1^2 = E_{k_{1,2}}}$$

* si $m_2 \gg m_1$ $\quad 1 \gg \frac{m_1}{m_2}$

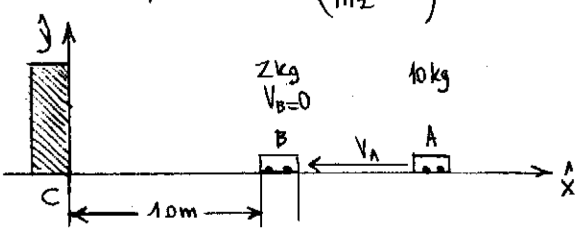
$$|V_2'| = \frac{2m_1 V_1}{m_2\left(\frac{m_1}{m_2}+1\right)} \Rightarrow \boxed{V_2' \cong 0}$$

$$|V_1'| = \frac{\cancel{m_2^2}\left(\frac{m_1}{m_2}+1\right)^2 - 4\cancel{m_2}m_1^2\frac{}{m_2}}{\cancel{m_2}\left(\frac{m_1}{m_2}-1\right)} \cdot V_1 \cdot \frac{1}{\cancel{m_2}\left(\frac{m_1}{m_2}+1\right)} = \frac{V_1}{-1} = -V_1$$

$$\boxed{V_1' \cong -V_1}$$

$$\Delta E_{k_{1,2}} = \frac{1}{\cancel{2}}\cancel{m_2} \cdot \frac{-\overset{2}{\cancel{4}}m_1^2V_1^2}{m_2^2\left(\frac{m_1}{m_2}+1\right)^2} = -2\frac{m_1}{m_2}V_1^2 \qquad \boxed{\Delta E_{k_{1,2}} \cong 0}$$

2.



a)

choques elásticos ($E_k = k$)

hay $\sum \vec{F}e = 0 \Rightarrow \frac{d\vec{P}}{dt} = 0 \Rightarrow \vec{P} = k$

$$-m_A \cdot V_A + m_B \cdot V_B = m_A \cdot V_A' + m_B \cdot V_B'$$
$$-10kg \cdot 10\frac{m}{s} + 0 = 10kg \cdot V_A' + 2kg \cdot V_B'$$
$$-100\ kg\frac{m}{s} = 10kg \cdot V_A' + 2kg \cdot V_B'$$

$$V_A' = \frac{100\ kg\frac{m}{s}}{10\ kg} - \frac{2kg \cdot V_B'}{10kg}$$

$$V_A' = -10\frac{m}{s} - \frac{1}{5}V_B' \qquad E_{k_i} = E_{k_f} \Rightarrow \frac{1}{2}10kg \cdot \left(10\frac{m}{s}\right)^2 = \frac{1}{2}10kg\ V_A'^2 + \frac{1}{2}2kg\ V_B'^2$$

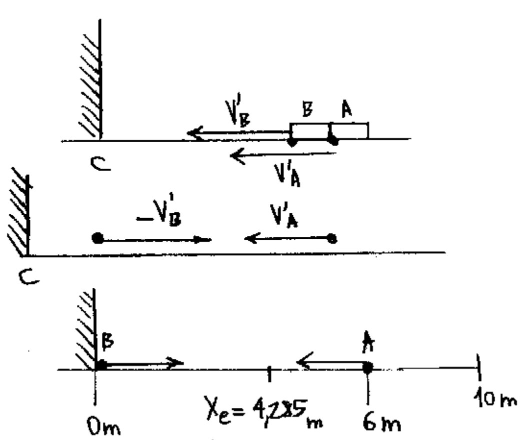
$$V_A'^2 = -100\frac{m^2}{s^2} + 20\frac{m}{s}\cdot\frac{1}{5}V_B' + \frac{1}{25}V_B'^2 \qquad 500\ kg\frac{m^2}{s^2} = 5kg\ V_A'^2 + 1kg \cdot V_B'^2$$

$$\cancel{500\ kg\frac{m^2}{s^2}} = \cancel{500\ kg\frac{m^2}{s^2}} + 20\frac{m}{s}kg\ V_B' + \frac{1}{5}kg\ V_B'^2 + 1kg\ V_B'^2$$

$$0 = +20\frac{m}{s}kg\ V_B' + \frac{6}{5}kg\ V_B'^2$$
$$0 = V_B'\left(+20\frac{m \cdot kg}{s} + \frac{6}{5}kg\ V_B'\right) \longrightarrow V_B' = 0$$

$$V_B' = -20\frac{kg\,m}{s}\cdot\frac{5}{6}\cdot\frac{1}{kg}$$

$$V_A' = -6{,}66\frac{m}{s} \qquad V_B' = -16{,}66\frac{m}{s}$$



m_A

$$X = X_0 + V_0 \cdot t$$
$$X = 10m - 6{,}66\frac{m}{s}\cdot t \quad 0{,}6$$
$$X = 6$$

m_B

$$0 = 10m - 16{,}66\frac{m}{s}t$$
$$t = 0{,}6$$

$$0 + 16{,}66\frac{m}{s}\cdot t = 6m - 6{,}65\frac{m}{s}t$$
$$23{,}33\frac{m}{s}t = 6m$$
$$t_e = 0{,}2571\ seg$$

$$X = 16{,}66 \cdot 0{,}2571$$
$$\boxed{X = 4{,}285\ m}$$

b) $$|V_A''| = \frac{2m_B \cdot V_B' + (m_B - m_A') \cdot V_A'}{(m_B + m_A)}$$

$$|V_A''| = \frac{4\,kg \cdot 16{,}66\,\frac{m}{s} + (-8\,kg)\cdot(-6{,}66\,\frac{m}{s})}{(12\,kg)} = 10\,\frac{m}{s}$$

$$\boxed{V_A'' = 10\,\frac{m}{s}}$$

$$\frac{12\,kg \cdot 16{,}66\,\frac{m}{s} - 2 \cdot 10\,kg \cdot 10\,\frac{m}{s}}{(-8\,kg)} = |V_B''| \rightarrow \boxed{V_B'' = 0}$$

c) El $\vec{P}$ se conserva entre el segundo y el primer choque solo en valor absoluto

$$m_A \cdot V_A + m_B \cdot V_B = m_A \cdot V_A' + m_B \cdot V_B' \neq m_A \cdot V_A'' + m_B\, V_B''$$
$$-100\,kg\,\frac{m}{s} = 10\,kg \cdot -6{,}66\,\frac{m}{s} + 2\,kg \cdot -16{,}66\,\frac{m}{s} \neq 100\,kg\,\frac{m}{s}$$

debido al cambio de dirección de la pared C

d) choque 1

$Ec_i = Ec_f$

$$\frac{1}{2} \cdot 10\,kg \cdot 100\,\frac{m^2}{s^2} = \frac{1}{2}\, 10\,kg \cdot 44{,}44\,\frac{m^2}{s^2} + \frac{1}{2}\, 2\,kg \cdot 277{,}77\,\frac{m^2}{s^2}$$

$$E_{k_{Bf}} - E_{k_{Bi}} = 1\,kg \cdot 277{,}77\,\frac{m^2}{s^2} = \boxed{277{,}77\,kg\,\frac{m^2}{s^2}}$$

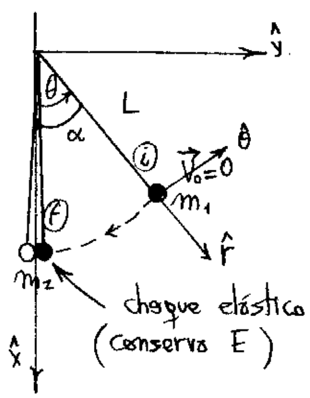
choque 2

$$\frac{1}{2} \cdot 10\,kg \cdot 44{,}4\,\frac{m^2}{s^2} + \frac{1}{2}\, 2\,kg\; 277{,}77\,\frac{m^2}{s^2} = \frac{1}{2} \cdot 10\,kg \cdot 100\,\frac{m^2}{s^2}$$

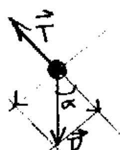
$$E_{k_{Bf}} - E_{k_{Bi}} = \boxed{-277{,}77\,kg\,\frac{m^2}{s^2}}$$

La energía es la misma pero solo en valor absoluto; por el choque con la pared C

3.



a) $\vec{T} \perp$ despl. $\Rightarrow$ no produce W



$$W_{\underset{\alpha}{i} \to \underset{0}{f}} = \int_i^f -m \cdot g \cdot \operatorname{sen}\theta \cdot L \cdot d\theta$$
$$= -\int_0^\alpha -m \cdot g \cdot \operatorname{sen}\theta \cdot L \cdot d\theta$$
$$= m \cdot g \cdot L \cdot \int_0^\alpha \operatorname{sen}\theta \cdot d\theta$$

$$W_{i \to f} = m \cdot g \cdot L\,(-\cos\alpha + \cos 0)$$

$$\frac{1}{2} m\, V_f^2 - \underbrace{\frac{1}{2} m V_i^2}_{0} = -m \cdot g\, L \cdot \cos\alpha + m g L$$

$$V_f^2 = 2 g L\,(\cancel{-\cos\alpha + 1})$$
$$\boxed{V_f^2 = 2 \cdot g \cdot L\,(1 - \cos\alpha)}$$

En el lugar del choque

$$\sum \vec{F}_e = 0 \Rightarrow \vec{P} = k$$

m_1: antes del choque

m_1 $V_1 = \sqrt{2 g L\,(1 - \cos\alpha)}$

m_2: $V_2 = 0$

Antes — después

$$-m_1 \cdot \overbrace{\sqrt{2 g L (1 - \cos\alpha)}}^{V_1} = m_2 \cdot |V_2'| + m_1 \cdot |V_1'| \quad [\text{cons. } \vec{P}]$$

$$\rightarrow -m_1 \left[\sqrt{2gL(1-\cos\alpha)} + |V_1'|\right] = m_2 |V_2'|$$

$$\frac{1}{\cancel{2}} m_1 \cdot \cancel{2} g L (1 - \cos\alpha) = \frac{1}{2} m_2 \cdot V_2'^2 + \frac{1}{2} m_1 \cdot V_1'^2 \quad [W_{cons}\, E_K]$$

$$\Rightarrow m_1 \overbrace{2 g L (1 - \cos\alpha)}^{V_1^2} - m_1 V_1'^2 = m_2 V_2'^2$$

$$m_1 \left[\overbrace{2gL(1-\cos\alpha)}^{V_1^2} - V_1'^2\right] = m_2 V_2'^2$$

$$m_2 V_2'^2 = m_1 \left[\sqrt{2gL(1-\cos\alpha)} - |V_1'|\right] \cdot \left[\sqrt{} + |V_1'|\right]$$

$$\left[\sqrt{} + |V_1'|\right] = -\frac{m_2}{m_1} |V_2'|$$

$$m_1 (-\sqrt{} + |V_1'|) = m_2 |V_2'|$$

$$\cancel{m_2} V_2'^2 = \cancel{m_2} |V_2'| \cdot \left[-\sqrt{2gL(1-\cos\alpha)} + |V_1'|\right]$$

$$|V_2'| = -\sqrt{2gL(1-\cos\alpha)} + |V_1'|$$

$$|V_2'| - |V_1'| = -\sqrt{2gL(1-\cos\alpha)}$$

$[II] \times m_2 \quad m_2|V_2'| - m_2|V_1'| = -m_2\sqrt{2gL(1-\cos\alpha)}$

de $[\text{cons } \vec{P}]$ $\quad m_2|V_2'| + m_1|V_1'| = -m_1\sqrt{2gL(1-\cos\alpha)}$

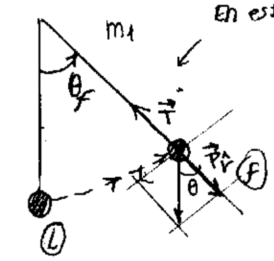
$$-m_2|V_1'| - m_1|V_1'| = (-m_2+m_1)\sqrt{2gL(1-\cos\alpha)}$$

$$\boxed{|V_1'| = -\frac{(m_1-m_2)}{(m_1+m_2)}\sqrt{2gL(1-\cos\alpha)}} = \frac{(m_2-m_1)}{(m_1+m_2)}\sqrt{2gL(1-\cos\alpha)}$$

$$|V_2'| = -\frac{(m_1-m_2)}{(m_1+m_2)}\sqrt{2gL(1-\cos\alpha)} - \sqrt{2gL(1-\cos\alpha)}$$

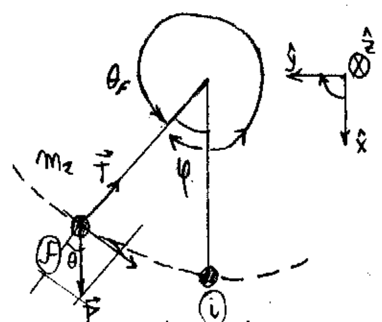
$$\boxed{|V_2'| = \left[-\frac{(m_1-m_2)}{m_1+m_2} - 1\right]\left(\sqrt{2gL(1-\cos\alpha)}\right)} = \left(\frac{-2m_1}{m_1+m_2}\right)\cdot\sqrt{2gL(1-\cos\alpha)}$$

b) en este trayecto solo actúa $\vec{P}$ que afecta al W ; Luego el W es

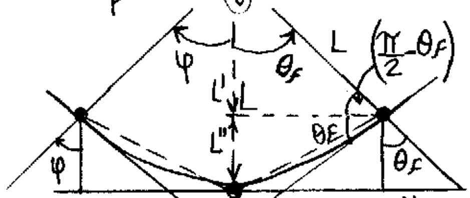


$$W_{i\to f} = \int_{\theta_i}^{\theta_f} -m_1 g\, \text{sen}\,\theta \cdot L\, d\theta = -m_1 g L(-\cos\theta_f + \cos\theta_i)$$
$$m_1 g L(\cos\theta_f - \cos\theta_i)$$
$$0 - \frac{1}{2}\cancel{m_1} V_i^2 = \cancel{m_1} g\cdot L(\cos\theta_f - 1)$$
$$-|V_1'|^2 = -V_i^2 = 2gL(\cos\theta_f - 1)$$
$$-\frac{(m_2-m_1)^2}{(m_1+m_2)^2}\cancel{2gL}(1-\cos\alpha) = \cancel{2gL}(\cos\theta_f - 1)$$
$$\left[\frac{m_2-m_1}{m_1+m_2}\right]^2\cdot(\cos\alpha - 1) = \cos\theta_f - 1$$

m_1: $\quad\boxed{\cos\theta_f = 1 + \left(\frac{m_2-m_1}{m_1+m_2}\right)^2\cdot(\cos\alpha - 1)}$



$$W_{i\to f} = -\int_{\theta_i=0}^{\theta_f=\varphi} m_2 g\,\text{sen}\,\theta\cdot L\,d\theta = -m_2 g\cdot L\cdot[-\cos(\varphi) + \cos 0]$$
$$-\frac{1}{2}\cancel{m_2}V_2'^2 = \cancel{m_2}g\cdot L\cdot[-\cos\varphi + 1]$$
$$V_2'^2 = +2gL(1-\cos\varphi)$$



$$\left(\frac{-m_1+m_2-m_1-m_2}{m_1+m_2}\right)^2 2gL(1-\cos\alpha) = \frac{4m_1^2}{(m_1+m_2)^2}\cancel{2gL}(1-\cos\alpha) = +\cancel{2gL}(1-\cos\varphi)$$

$\text{sen}\left(\frac{\pi}{2}-\theta_f\right) = \frac{L'}{L}$

$L\cdot\cos\theta_f = L'$ $\quad L'' = L - L' = \boxed{L(1-\cos\theta_f)}$

m_2: $\quad\boxed{\cos\varphi = -\frac{4m_1^2}{(m_1+m_2)^2}(1-\cos\alpha) + 1}$

c) ⊛ $m_1 = m_2$ $\quad\boxed{V_1' = 0}$ $\quad\boxed{V_2' = -\sqrt{2gL(1-\cos\alpha)}}$

$\cos\theta_f = 1 \quad\therefore\quad \boxed{\theta_{f_1} = 0}$ $\quad\boxed{\theta_{f_2} = \varphi = \alpha}$

$\boxed{h_1 = 0}$ $\quad\boxed{h_2 = L(1-\cos\alpha)}$

⊛ $m_1 \gg m_2$

$1 \gg \frac{m_2}{m_1}$

$$V_1' = \frac{\cancel{m_1}\left(\frac{m_2}{m_1}-1\right)}{\cancel{m_1}\left(1+\frac{m_2}{m_1}\right)}\sqrt{2gL(1-\cos\alpha)} = \boxed{-\sqrt{2gL(1-\cos\alpha)}}$$

$$V_2' = \frac{-2\cancel{m_1}}{\cancel{m_1}\left(1+{m_2}/{m_1}\right)}\sqrt{2gL(1-\cos\alpha)} = \boxed{-2\sqrt{2gL(1-\cos\alpha)}}$$

$$\cos\theta_f = 1 + \frac{\cancel{m_1^2}(m_2/m_1 - 1)^2}{\cancel{m_1^2}(1+m_2/m_1)^2}(\cos\alpha - 1) \Rightarrow \boxed{\theta_{f_1} = -\alpha}$$

le pongo el signo porque sé que es <0 y no sale pues $\cos(-A) = \cos A$

$$\boxed{h_1 = L\left[1-\cos(-\alpha)\right]}$$

$$\cos\varphi = -\frac{4m_1^2}{m_2^2\left(1+\frac{m_2}{m_1}\right)^2}(1-\cos\alpha)+1 = -4+4\cos\alpha+1 \Rightarrow$$

$$\boxed{h_2 = L(1-\cos\varphi)} \leftarrow \boxed{\varphi = \text{Acos}(-3+4\cos\alpha)}$$

$\circledast$ $m_1 \ll m_2$

$$\frac{m_1}{m_2} \ll 1$$

$$V_1' = \frac{\cancel{m_2}\left(1-{m_1}/{m_2}\right)}{\cancel{m_2}\left(\frac{m_1}{m_2}+1\right)}\cdot\sqrt{2gL(1-\cos\alpha)} \cong \boxed{\sqrt{2gL(1-\cos\alpha)}}$$

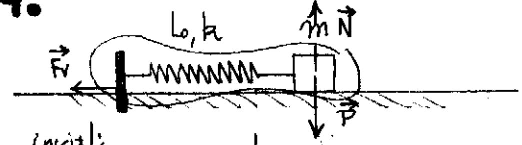
$$V_2' = \left(\frac{-2m_1}{m_2\left({m_1}/{m_2}+1\right)}\right)\cdot\sqrt{2gL(1-\cos\alpha)} \cong \boxed{0}$$

$$\cos\theta_f = 1+\frac{\cancel{m_2^2}\left(1-{m_1}/{m_2}\right)^2}{\cancel{m_2^2}\left({m_1}/{m_2}+1\right)^2}(\cos\alpha-1) = \cos\alpha \quad\Rightarrow\quad \boxed{\theta_f = \alpha} \quad \boxed{h_1 = L(1-\cos\alpha)}$$

$$\cos\varphi = -\frac{4m_1^2}{m_2^2\left(\frac{m_1}{m_2}+1\right)^2}\cdot(1-\cos\alpha)+1 = 1 \quad\Rightarrow\quad \boxed{\varphi = 0}$$

$$\boxed{h_2 = 0}$$

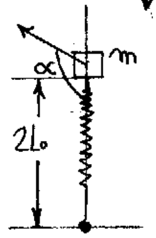
4.

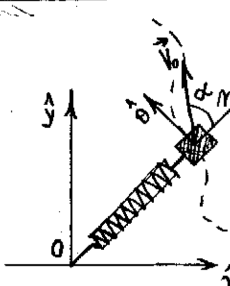


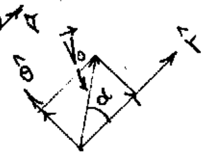
sistema: masa, resorte, eje $\longrightarrow$ masa despreciable

① $\sum\vec{F}_{ext} = \underbrace{\vec{N}+\vec{P}}_{\text{se cancelan por Newton}}+\vec{F_v}$

inicial:







$$\vec{F_v} = \frac{d\vec{P}}{dt} \Rightarrow \boxed{\vec{P}\ \text{no se conserva}}$$

$$\therefore \vec{P} \not\equiv k$$

② $\sum\vec{\tau}_{Fe_0} = \vec{\tau}_{N_0}+\vec{\tau}_{P_0}+\vec{\tau}_{Fv_0} = \vec{r_m}\times\vec{N}+\vec{r_m}\times m\vec{g}+\underbrace{\vec{r_0}\times\vec{F_v}}_{=0 \text{ por estar aplicada en O la } \vec{F_v}}$

$$\begin{vmatrix}\hat{r} & \hat{\theta} & \hat{z}\\ r & 0 & 0\\ 0 & 0 & N\end{vmatrix} \quad m(r\hat{r}\times -g\hat{z})$$

$$-r.N.\hat{\theta}+m.r.g.\hat{\theta} = \underbrace{(mg-N)}_{0}.r.\hat{\theta}$$

$$\Rightarrow \sum\vec{\tau}_{fe_0} = 0 \quad\therefore\quad \frac{d\vec{L_0}}{dt} = 0 \Rightarrow \vec{L_0} \equiv k$$

$$\vec{L_0} = \vec{R}\times m.\vec{V} = r.\hat{r}\times m.(\overset{\dot{r}\hat{r}+r\dot{\theta}\hat{\theta}}{V.\cos\alpha.\hat{r}+V.\text{sen}\,\alpha\,\hat{\theta}}) \quad \boxed{\text{se conserva } \vec{L_0}}$$

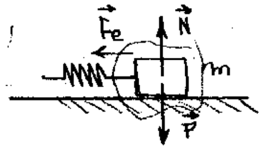
$$\vec{L_0} = (r.m.V.\text{sen}\,\alpha.)\hat{z} \equiv k$$

$$m.r^2.\dot{\theta}.\hat{z}$$

inicial — en otro momento

$$2L_0.m.V_0.\text{sen}\,\alpha = \frac{3}{2}L_0.m.V.\text{sen}\,\alpha$$

③



$$\frac{2.2}{3}.V_0 = V \quad\Rightarrow\quad V = \frac{4}{3}V_0$$

$$V_0 = \frac{3}{4}V$$

sistema: m

$$m.\frac{1}{\cancel{2}}.\frac{\cancel{16}^{8}}{9}V_0^2 - \frac{1}{2}m.V_0^2 =$$

$$\frac{7}{18}m.V_0^2 = \Delta E_k \qquad \boxed{\text{no se conserva } E_k}$$

Energía

$$\Delta E = \underbrace{W^{\vec{N}}}_{=0 \text{ por } \perp}+W^{\vec{Fe}}+\underbrace{W^{\vec{P}}}_{=0 \text{ por } \perp} = \frac{1}{2}mV^2-\frac{1}{2}mV_0^2$$

$$\vec{F_e} = -k.r\,\hat{r} \quad\therefore\quad W^{\vec{Fe}} = \int(-k.r).(dr+r.d\theta) = \int -kr.dr$$

$$-k\frac{r^2}{2}\Big]_{r_0}^{r}$$

$$-\frac{k}{\cancel{2}}r^2+\frac{k}{\cancel{2}}(2L_0)^2 = \frac{1}{\cancel{2}}mV^2-\frac{1}{\cancel{2}}mV_0^2$$

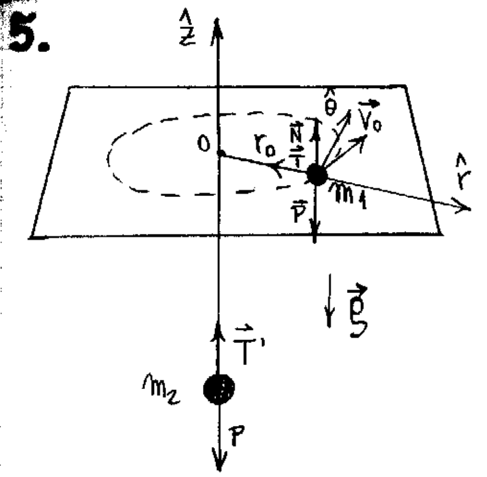
$$-k\frac{9}{4}L_0^2+k4L_0^2 = mV^2-mV_0^2$$

$$\frac{7}{4}kL_0^2 = mV^2-m\frac{9}{16}V^2 = \frac{7}{16}mV^2$$

$$\frac{N}{m}\cdot\frac{1}{kg} = \frac{\cancel{kg}.\cancel{m}}{\cancel{m}\,s^2}\frac{1}{\cancel{kg}} \qquad \boxed{V = 2\sqrt{\frac{k}{m}}.L_0}$$

$$\frac{4k.L_0^2}{m} = V^2 \qquad \frac{16}{\cancel{7}}\frac{\cancel{7}kL_0^2}{\cancel{4}m} = V^2$$

5.



a)

$$\hat{r})\ m_1(\ddot{r} - r\dot{\theta}^2) = -T$$

$$\hat{\theta})\ m_1(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = 0$$

$$\hat{z})\ m_2\ddot{z} = T' - m_2 g \qquad T' = T$$

$$\hat{z})\ m_1\ddot{z} = N - m_1 g$$

$$\cancel{m_1} r\ddot{\theta} = -2\dot{r}\dot{\theta}\cancel{m_1} \qquad T = m_2 g$$

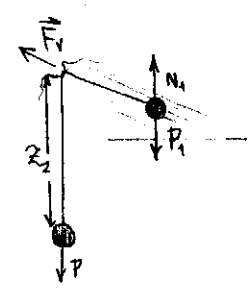
$$m_1\ddot{r} - m_1 r\dot{\theta}^2 = -T \qquad N = m_1 g$$

Fuerzo $\dot{r} = \ddot{r} = 0$ por ser $r = r_0 \Rightarrow$

$$-m_1 r\dot{\theta}^2 = -T$$
$$m_1 r\dot{\theta}^2 = m_2 g$$
$$r_0\dot{\theta}^2 = \frac{m_2}{m_1} g \quad \Rightarrow \quad \boxed{\frac{V_0^2}{r_0} = \frac{m_2}{m_1} g}$$

$V = r\dot{\theta}$
$V^2 = r^2\dot{\theta}^2$
$\frac{V^2}{r} = \dot{\theta}^2 r$

b)



- <u>Momento Lineal</u>

$$\sum \vec{F_e} = \underbrace{\vec{N_1} + \vec{P_1}}_{=0 \text{ por Newton}} + \vec{P_2} + \vec{F_V} \neq 0 \Rightarrow \frac{d\vec{P}}{dt} \neq 0 \Rightarrow \vec{P} \neq k$$

$$-m_2 g\hat{z} + \vec{F_V} = \frac{d\vec{P}}{dt}$$

$$\vec{P} = m_1\vec{V_1} + m_2\vec{V_2}$$
$$\vec{P} = m_1(V_{1x} + V_{1y}) + m_2(V_{2z})$$

- <u>Momento Angular</u>

$$\sum \vec{\tau}_0^{ext} = \sum \vec{M}_0^{ext} = \underbrace{r_0\hat{r} \times N_1\hat{z}}_{\tau_0^{N_1}} + \underbrace{r_0\hat{r} \times -m_1 g\hat{z}}_{\tau_0^{P_1}} + \underbrace{z_2\hat{z} \times -m_2 g\hat{z}}_{\tau_0^{P_2} = 0} + \underbrace{0 \times \vec{F_V}}_{\tau_0^{F_V} = 0}$$

$$\sum \vec{\tau}_0^{ext} = r_0\hat{r} \times \underbrace{(N_1\hat{z} - m_1 g\hat{z})}_{=0 \text{ por leyes de Newton}} = 0 \quad \therefore \quad \vec{L_0} \equiv k$$

$$\vec{L_0} = r\hat{r} \times m_1\vec{V_1} + z_2\hat{z} \times m_2\vec{V_2}$$
$$m_1(r\hat{r} \times r\dot{\theta}\hat{\theta}) + \underbrace{z_2\hat{z} \times m_2\dot{z}\hat{z}}_{=0} = m_1 r^2\dot{\theta}\hat{z} = \vec{L_0} \equiv k$$

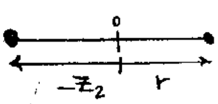
- <u>Energía</u>

$$W_T = \underbrace{W^{N_1}}_{NC} + \underbrace{W^{P_1}}_{\perp \text{ despl} =0} + W^{P_2} + \underbrace{W^{F_V}}_{NC} + \underbrace{W^{T}}_{NC} + \underbrace{W^{T'}}_{NC}$$

$$W^{NC} = \underbrace{W^{N_1}}_{=0\ \perp \text{despl}} + \underbrace{W^{F_V}}_{=0 \text{ porque el origen no se mueve}} + W^T + W^{T'} = \Delta E = \int \vec{T}\cdot d\vec{r}_{m_1} + \int \vec{T}'\cdot d\vec{z}_{2\,m_2} = -\int T\,dr + \int T\,dz_2$$

<u>Vínculo</u>:

$$L = r + z_2 \Rightarrow dL = 0 = dr + dz_2 \Rightarrow dr = dz_2$$



hilo inextensible

$$W^{NC} = -\int T\,dr + \int T\,dr = 0 \Rightarrow E \equiv k \quad \text{pues} \quad W^{NC} = 0$$

$$\frac{1}{2}m_1 V_1^2 + \frac{1}{2}m_2 V_2^2 + V_1 + V_2 \equiv k$$

$$V_1 = m_1 g \overset{k}{z} = 0$$
$$V_2 = m_2 g z$$

$$W_T = W^{P_2} = \int m_2 g\,dz$$

c) $E_i = E_f \Rightarrow$ el signo menos

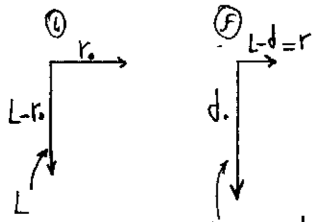
$$\frac{1}{2} m_1 V_0^2 - m_2 \cdot g \cdot (\overbrace{L - r_0}^{>0}) = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 - m_2 g (\overbrace{L-r}^{>0})$$

$L = r - z_2$
$0 = \dot r - \dot z_2$

$$\frac{1}{2} m_1 \cdot r_0^2 \cdot \dot\theta_0^2 - m_2 g L + m_2 g r_0 = \frac{1}{2} m_1 (\dot r \hat r + r \dot\theta \hat\theta)^2 + \frac{1}{2} m_2 \dot z_2^2 - m_2 g (L - r)$$

$V_0 = r_0 \cdot \dot\theta_0$

hilo



$L = r_0 + L - r_0 \qquad L = r + d$

$r - r_0 =$

$$= \frac{1}{2} m_1 \dot r^2 + \frac{1}{2} m_1 r^2 \dot\theta^2 + \frac{1}{2} m_2 \dot r_2^2 + m_2 g (L - r)$$

$$\frac{1}{2} m_1 V_0^2 - m_2 g r_0 = \frac{1}{2} \dot r^2 (m_1 + m_2) + \frac{1}{2} m_1 (r\dot\theta)^2 + m_2 g r$$

$$\dot r^2 (m_1 + m_2) = \cancel{2}\frac{1}{\cancel{2}} m_1 V_0^2 \left(\frac{r_0^2}{r^2} - 1\right) + 2 m_2 g (r + r_0)$$

$\vec{L_i} = \vec{L_f} \Rightarrow$

$$\cancel{m_1} \cdot r_0^2 \cdot \dot\theta_0 = \cancel{m_1} \cdot r^2 \dot\theta$$

$$V_0 \cdot r_0 = r^2 \dot\theta$$

$$\boxed{\dot\theta = \frac{V_0 r_0}{r^2}}$$

$$\boxed{V_1' = r \cdot \dot\theta = \frac{V_0 r_0}{r}}$$

$$\boxed{\dot r^2 = \frac{-m_1 V_0^2 (r_0^2 - 1) - 2 m_2 g (r + r_0)}{(m_1 + m_2)}}$$

$$\boxed{\dot r^2 = \dot z_2^2 = V_2'^2}$$

d)

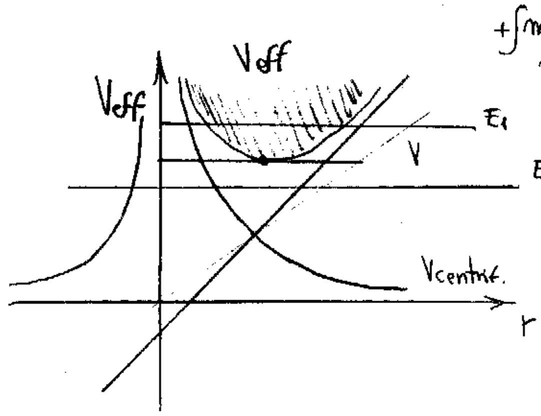
$$V_{eff} = \frac{L^2}{2 m r^2} + U(r) = \frac{(m_1 \cdot r^2 \cdot \dot\theta)^2}{2 m \quad r^2} +$$

$$V_{eff} = \frac{1}{2} m_1 (r\dot\theta)^2 - m_2 g (L - r) = \underbrace{\frac{1}{2} m_1 \frac{V_0^2 r_0^2}{r^2}}_{V_{ce}} - \underbrace{m_2 \cdot g \cdot (L - r)}_{+ m_2 g r - m_2 g L}$$

$$\vec F = -\frac{dE_p}{d\vec r}$$

$$\vec P_2 = -\frac{dE_p}{d\vec r} = -m_2 g \hat z = -\frac{d V_G}{dz} \Rightarrow$$

$$-m_2 g \hat z \cdot dz = -dV_G$$
$$+\int m_2 g dz = V_G$$
$$m_2 g z = V_G$$



a un nivel de Energía E no llegaría
en E_1 tendrá una órbita elíptica

mínima Veff → órbita circular → m_2 en reposo

$$V_{eff}(r) \Rightarrow \frac{dV_{eff}(r)}{dt} = \frac{1}{\cancel{2}} m_1 V_0^2 r_0^2 \left(\frac{-\cancel{2} r}{r^{4\,3}}\right) + m_2 g$$

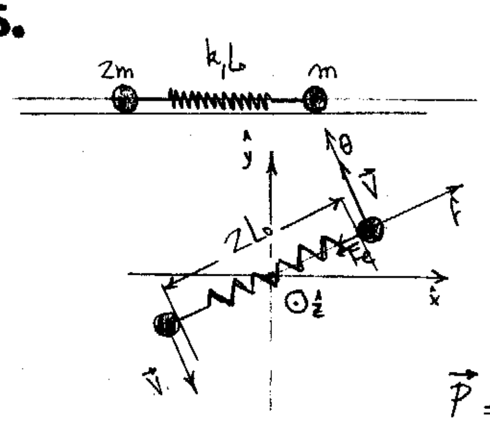
$$-m_2 g = -m_1 \cdot V_0^2 \frac{r_0^2}{r^3}$$

si es órbita circular $r = r_0$ $\Leftarrow$ $\frac{m_2}{m_1} g = \frac{V_0^2 r_0^2}{r^3}$

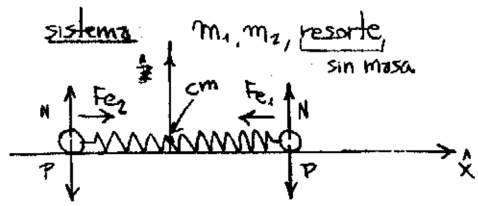
$\Downarrow$

$$\boxed{\frac{m_2}{m_1} g = \frac{V_0^2}{r}}$$

6.



sistema m_1, m_2, resorte, sin masa



① momento lineal

$$\sum \vec F_{ext} = \underbrace{\vec N_1 + \vec P_1}_{=0} + \underbrace{\vec N_2 + \vec P_2}_{=0} = 0 \Rightarrow \vec P = k$$

por Newton

$$\searrow M \cdot \vec A_{cm} = 0$$
$$\vec A_{cm} = 0$$

$$\vec P = m \vec V + 2m \vec V$$

cm se mueve con velocidad constante en línea recta (si curvase debería tener $\vec A$)

② Momento angular

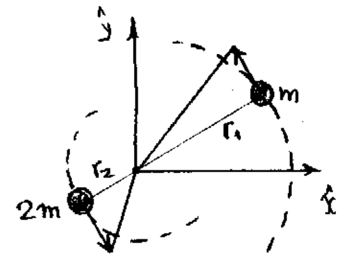
$$\sum \vec{\tau}_{cm}^{ex} = \underbrace{\tau^{N_1} + \tau^{P_1}}_{\substack{r_1\hat{r}\times N_1\hat{z} + r_1\hat{r}\times -mg\hat{z} \\ =0 \text{ por concatenación}}} + \underbrace{\tau^{N_2} + \tau^{P_2}}_{=0} + \underbrace{\tau^{Fe_1} + \tau^{Fe_2}}_{\substack{r\hat{r}\times -kr\hat{r} \\ =0 \text{ por ser } // \ Fe \ y \ \hat{r}}}$$

$$\sum \vec{\tau}_{cm}^{ex} = 0 = \frac{d\vec{L}_{cm}}{dt} \Rightarrow \vec{L}_{cm} \equiv k$$

$$\vec{L}_{CM} = \vec{r}_1 \times m.\vec{V} + \vec{r}_2 \times 2m\vec{V} = r_1\hat{r}\times m.V.\hat{\theta} + r_2\hat{r}\times 2m.V.\hat{\theta} \qquad V = r\dot{\theta}$$

$$m.r_1.V.\hat{z} + 2m.r_2.V.\hat{z} =$$

$$m.r_1.r_1.\dot{\theta}_0\hat{z} + 2m\, r_2.r_2.\dot{\theta}_0\hat{z} = m\dot{\theta}(r_1^2 + 2r_2^2)\hat{z} \qquad \dot{\theta} = \frac{V_t}{r}$$



L_{cm} ⓘ $\qquad L_{cm}$ ⓕ

$$\hat{z}) \quad \cancel{m}.\dot{\theta}_0.(r_1^2 + 2r_2^2) = \cancel{m}.\dot{\theta}.(r_1'^2 + 2r_2'^2)$$

$$4r_2^2.\dot{\theta}_0 = \dot{\theta}.4r_2'^2$$

$$l_0^2.\frac{4}{9}.\dot{\theta}_0 = \frac{1}{4}.l_0^2.\dot{\theta}$$

$$\boxed{\frac{16}{9}\dot{\theta}_0 = \dot{\theta}}$$ velocidad angular del sistema

vínculo: resorte

① $r_1 + r_2 = 2l_0$
$r_1 = 2l_0 - r_2$

ⓕ $r_1' + r_2' = \frac{3}{2}l_0$
$r_1' = 3/2\, l_0 - r_2'$

$$\vec{L}_{CM_i} = \left(m.\dot{\theta}_0.4.\frac{4}{9}l_0^2\right)\hat{z} = \left(m.\dot{\theta}_0.\frac{3}{4}r_1.\frac{16}{9}l_0\right)z$$

$$m.\dot{\theta}_0.r_1.\frac{4}{3}l_0$$

$$\vec{L}_{CM_f} = \left(m.\dot{\theta}.\cancel{4}.\frac{l_0^2}{\cancel{4}}\right)z$$

$$P_i = P_f$$

$$m.r_1.\dot{\theta}_0 + 2m\,r_2.\dot{\theta}_0 = m.r_1'.\dot{\theta} + 2m\,r_2'.\dot{\theta}$$

$$m.\dot{\theta}_0(r_1 + 2r_2) = m.\dot{\theta}(r_1' + 2r_2')$$

CM

$$\vec{R}_{cm} = 0 = \frac{m.\vec{r}_1 + 2m\vec{r}_2}{3m}$$

$$-\vec{r}_1 = 2\vec{r}_2$$

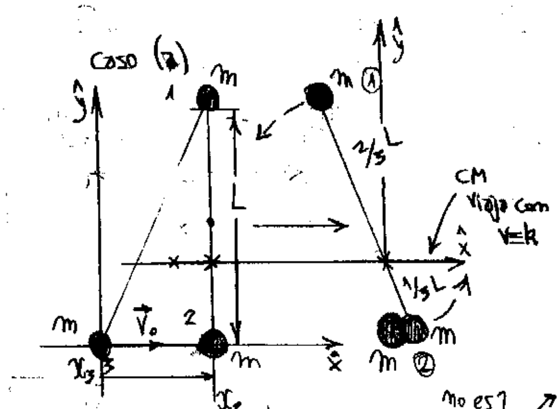
$$\vec{R}_{cm}' = 0 = \frac{m.\vec{r}_1' + 2m\,\vec{r}_2'}{3m}$$

$$-\vec{r}_1' = 2\vec{r}_2'$$

$$r_1 = \frac{4}{3}l_0 \qquad \frac{2}{3}l_0 = r_2 \leftarrow r_1 = 2l_0 - r_2 = 2r_2$$

$$\frac{1}{2}l_0 = r_2' \leftarrow r_1' = 3/2\,l_0 - r_2' = 2r_2'$$

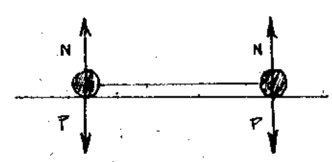
7.



choque plástico: hay ΔE_k

sistema: 3 masas

$$\sum F_{ext} = 3\vec{P} + 3\vec{N} = 0$$



$$P \equiv k$$

$$\overset{P_i}{m.\vec{V}_3} = \overset{P_f}{m\vec{V}_1 + m\vec{V}_2 + m\vec{V}_3} \qquad 3m.\vec{V}_{cm} \equiv k$$

$$m.\vec{V}_3 = m\vec{V}_1 + 2m\vec{V}_2$$

no es muy útil $\} \to m.V_0\hat{x} = m(V_1'\hat{x} + V_1'\hat{y}) + 2m(V_2'\hat{x} + V_2'\hat{y})$

CM se mueve con vel. constante en línea recta

$$\vec{R}_{cm} = \frac{m.\vec{r}_1 + m.\vec{r}_2 + m.\vec{r}_3}{3m}$$

$$\frac{(x_2\hat{x} + L\hat{y}) + (x_2\hat{x})}{3}$$

$$3m\,V_{cm} = m\,\vec{V}_3$$

$$V_{cm} = \frac{\vec{V}_0}{3}$$

antes $\vec{R}_{cm} = \frac{2}{3}x_2\hat{x} + \frac{L}{3}\hat{y}$

después (un instante) $\vec{R}_{cm} = \frac{3}{3}x_2\hat{x} + \frac{L}{3}\hat{y} = x_2\hat{x} + \frac{L}{3}\hat{y}$

viaja con V≡k

$$\text{después } L_{cm} = \vec{r}_1 \times m.(\dot{r}_1\hat{r} + r_1.\dot{\theta}\hat{\theta}) + \vec{r}_2 \times 2m.(\dot{r}_2\hat{r} + r_2.\dot{\theta}\hat{\theta})$$

será la misma

$$\text{antes } L_{cm} = \vec{r}_1 \times m.\vec{V}_1 + \vec{r}_2 \times m.\vec{V}_2 + \vec{r}_3 \times m.\vec{V}_0$$

$$(x_2\hat{x} + L\hat{y}) \times m\left(-\frac{V_0}{3}\right)\hat{x} + (x_2\hat{x}) \times m\left(-\frac{V_0}{3}\hat{x}\right) + 0 \times m.\vec{V}_0$$

antes: $L_{CM} = m.L.\frac{V_0}{3}\hat{z}$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ x_2 & L & 0 \\ -\frac{V_0}{3} & 0 & 0 \end{vmatrix} = +\frac{V_0}{3}L\hat{z}$$

después: $\frac{2}{3}L\hat{r} \times m\left(\frac{2}{3}L.\dot{\theta}\hat{\theta}\right) + \frac{1}{3}L\hat{r}\times 2m\left(\frac{1}{3}L\,\dot{\theta}\hat{\theta}\right)$

$\frac{4}{9}L^2 m.\dot{\theta}\,\hat{z} + \frac{2}{9}L^2 m.\dot{\theta}.\hat{z} = \frac{2}{3}mL^2\dot{\theta}\,\hat{z}$

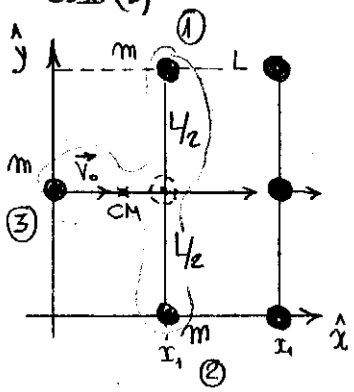
$$E_{k_i} = \frac{1}{2}m\left(\frac{-V_0}{3}\right)^2 + \frac{1}{2}m\left(-\frac{V_0}{3}\right)^2 + \frac{1}{2}m\left(\frac{2V_0}{3}\right)^2$$

$$E_{k_f} = \frac{1}{2}m\left(\frac{2}{3}L\dot{\theta}\right)^2 + \frac{1\,2m}{2}\left(\frac{1}{3}L\,\dot{\theta}\right)^2$$

$$= \frac{1}{2}m\frac{4}{9}L^2\dot{\theta}^2 + \frac{1}{\cancel{2}}\cancel{2}m\frac{1}{9}L^2\dot{\theta}^2 - \frac{1}{2}m\frac{V_0^2}{9} - \frac{1}{2}m\frac{V_0^2}{9} - \frac{4}{2\,9}m V_0^2$$

$$\boxed{\Delta E_k = \frac{1}{1}m\left(\frac{2}{3}L^2\dot{\theta}^2 - \frac{1}{3}V_0^2\right)}$$

Caso (b)



■ Momento lineal 3 masas y barra

$$\sum F_{ext} = \underbrace{3\vec{P} + 3\vec{N}}_{\times\ \text{Newton}} = 0 \quad \Rightarrow \frac{d\vec{P}}{dt} = 0 \Rightarrow \vec{P} \equiv k$$

$$P = 3m.V_{cm} = k$$

antes $\vec{R_{cm}} = \frac{\cancel{m}(x_1\hat{x}+L\hat{y}) + \cancel{m}(x_1\hat{x}) + \cancel{m}(L/2\,\hat{y})}{3\cancel{m}} = \frac{2x_1}{3} + \frac{3/2}{3}L\hat{y}$

después $\vec{R_{cm}} = \frac{\cancel{m}(x_1\hat{x}+L\hat{y}) + \cancel{m}(x_1\hat{x}+L/2\,\hat{y}) + \cancel{m}(x_1\hat{x}+0\hat{y})}{3m} = x_1 + \frac{1}{2}L\hat{y}$

$$\vec{P_i} = \vec{P_f}$$

$$m.V_0\hat{x} = m\vec{V_1} + m\vec{V_2} + m\vec{V_3} \qquad \therefore\ 3m\,\vec{V_{cm}} = m.V_0\hat{x}$$

$$\boxed{\vec{V_{cm}} = \frac{V_0}{3}\hat{x}}$$

$\xrightarrow{V_0}$ x

■ Momento Angular

$$\sum \vec{\tau}_{ext_{CM}} = 0 \Rightarrow \frac{d\vec{L_{CM}}}{dt} = 0 \Rightarrow \vec{L_{cm}} \equiv k$$

Luego antes $\vec{L_{cm}} = (x_1\hat{x}+L\hat{y})\times m(-V_0)\hat{x} + \underbrace{(x_1\hat{x})\times m(-V_0)\hat{x}_1}_{=0} + (L/2\,\hat{y})\times m(V_0)\hat{x}$

$m.L.V_0\hat{z} + -m.L/2.V_0\hat{z} = \left(\frac{1}{2}mLV_0\right)\hat{z}$

después $\vec{L_{cm}} = (x_1\hat{x}+L\hat{y})\times m(V_1)\hat{x} + (x_1\hat{x}+L/2\,\hat{y})\times m\,V_1\hat{x} + \underbrace{(x_1\hat{x})\times m.V_1\hat{x}}_{=0}$

$-LmV_1\hat{z} \qquad -m.\frac{L}{2}V_1.\hat{z}$

$$= \left(-\frac{3}{2}LmV_1\right)\hat{z}$$

$$\frac{1}{\cancel{2}}\cancel{m}\cancel{L}V_0 = -\frac{3}{\cancel{2}}\cancel{L}\cancel{m}V_1$$

$$\boxed{-\frac{V_0}{3} = V_1}$$

■ Energía

$$\Delta E_k = E_{k_f} - E_{k_i}$$

respecto del origen

$$E_{k_f} = \frac{1}{2}m(V_1)^2 + \frac{1}{2}m(V_1)^2 + \frac{1}{2}m(V_1)^2 = \frac{3}{2}mV_1^2 = \frac{\cancel{3}}{2}m\frac{V_0^2}{\cancel{9}} = \frac{1}{6}mV_0^2$$

$$E_{k_i} = \frac{1}{2}m(V_0)^2 \qquad \Delta E_k = \frac{1}{6}mV_0^2 - \frac{1}{2}mV_0^2 = \boxed{-\frac{1}{3}mV_0^2}$$

respecto del cm

$$E_{k_f} = 0$$

$$E_{k_i} = \frac{1}{2}m\left(\frac{-V_0}{3}\right)^2 + \frac{1}{2}m\left(\frac{-V_0}{3}\right)^2 + \frac{1}{2}m\left(V_0 - \frac{V_0}{3}\right)^2 = \frac{1}{9}mV_0^2 + \frac{1}{\cancel{2}}m.V_0^2.\frac{\cancel{4}^2}{9} = \frac{1}{3}mV_0^2$$

NB

No importa como se mida la Energía (respecto a que origen) la ΔE_k es la misma

$\xrightarrow{V_0}$ $\times\xrightarrow{V_{cm}}$

$$\boxed{\Delta E_k = -\frac{1}{3}mV_0^2}$$