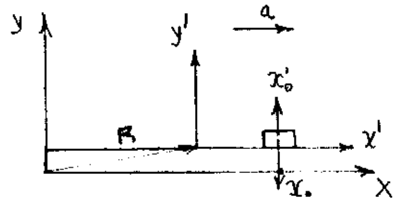
Sistemas No Inerciales

①

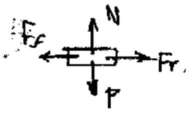


S → inercial S' → No inercial (acelerado)

$$A = A' + \ddot{\vec{R}}$$

A = aceleración del Colectivo (entre sistemas)

a)



(calle) S (inercial)

$\hat{x})$ $\ddot{a} = a' + \underbrace{\ddot{A}}_{\text{colectivo (entre sistemas)}}$

$$m.a = m.a' + m.A$$
$$\Sigma F_r = \Sigma F_r$$

$\hat{x})$ $m.\underbrace{a}_{A} = -F_f + Fre + m.A$

$\hat{y})$ $N - m.g = 0$ con MRUV

si está en reposo.

$F_f = Fre$

$F_f \leq \mu e.m.g$

(colectivo) S'

$\hat{x})$ $0 = m.A - m.A$

$\hat{y})$ $N - m.g = 0$

en reposo

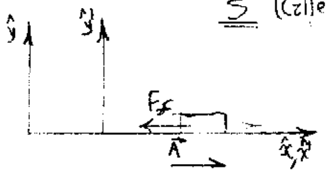
$m A \leq \mu e.M.g$

$A \leq \mu e\, g$

$$\boxed{A_{max} = \mu e.g}$$

b)

S (calle)



$\hat{x})$ $m.a = F_f + m.A$

$0 = -m.A + m.A$

EN reposo

$$\underbrace{a'}_{\text{aceleración [illegible] el no inercial}} = \underbrace{a}_{\text{[illegible] inercial}} - \underbrace{A}_{\text{aceleración del colectivo}}$$

$$a = a' + A$$
$$m a = \Sigma F_f + m A$$

S' (colectivo)

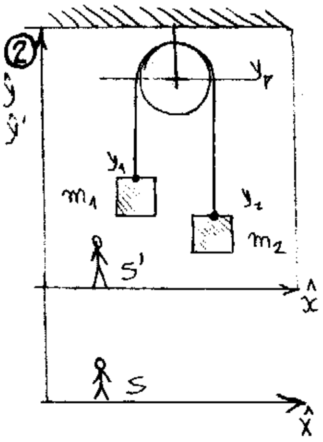
$\hat{x})$ $m.a' = \underbrace{m.a}_{0} - m.A$

$F_f = 0 - m.A$

$m.A = -F_f \Rightarrow a' = \underbrace{-A}_{\text{contrario a la del colectivo}}$

en MRUV

②



$$L = y_p - y_1 + y_p - y_2 + \pi R$$
$$0 = -\ddot{y}_1 - \ddot{y}_2 \Rightarrow \ddot{y}_1 = -\ddot{y}_2$$

$T_1 = T_2 = T$

soga despreciable

a) $V = k$ ↑

S' $a' = a - \underbrace{A}_{0} \Rightarrow a' = a \Rightarrow \underline{\underline{S' = S}}$

m_1

$\hat{y})$ $m_1.\ddot{y}_1 = T - m_1.g$

m_2

$\hat{y})$ $m_2.\ddot{y}_2 = T - m_2.g$

$$(m_1 + m_2)\ddot{y}_1 = (-m_1 + m_2)g$$

$$\boxed{\begin{array}{l} \ddot{y}_1 = \dfrac{(m_2 - m_1)}{(m_2 + m_1)}.g \\ \\ \ddot{y}_2 = \dfrac{(m_1 - m_2)}{(m_2 + m_1)}.g \end{array}}$$

Para ambos observadores

b) $\Uparrow A$ $a = a' + A$ $a' = a - A$

$\underline{\Sigma}'$

$\underline{m_1}$: $\hat{y})$ $m_1 \cdot \ddot{y}_1' = T - m_1 \cdot g - m_1 \cdot a$ $\qquad \ddot{y}_1' = -\ddot{y}_2'$

$\underline{m_2}$: $\hat{y})$ $m_2 \cdot \ddot{y}_2' = T - m_2 \cdot g - m_2 \cdot a$

$$(m_1+m_2)\cdot\ddot{y}_1' = (-m_1+m_2)\cdot g + (-m_1+m_2)\cdot a$$

obs.
no inercial
(ascensor)

$$\boxed{\begin{gathered}\ddot{y}_1' = \frac{(m_2-m_1)}{(m_1+m_2)}\cdot(a+g) \\ \ddot{y}_2' = \frac{(m_1-m_2)}{(m_1+m_2)}\cdot(a+g)\end{gathered}}$$

$\underline{\Sigma}$ $\quad m_1$: $\hat{y})$ $\quad m_1 \cdot \ddot{y}_1 = m_1 \cdot \ddot{y}_1' + m_1 \cdot a$

$$\ddot{y}_1 = \frac{(m_2-m_1)}{(m_1+m_2)}(a+g) + a$$

$$\ddot{y}_1 = \frac{(m_2 \cdot a - m_1 a + m_2 g - m_1 g + m_1 a + m_2 a)}{(m_1+m_2)}$$

obs.
inercial

$$\boxed{\begin{gathered}\ddot{y}_1 = \frac{2m_2 a + g(m_2-m_1)}{m_1+m_2} \\ \ddot{y}_2 = -\frac{2m_2 a - g(m_2-m_1)}{m_1+m_2}\end{gathered}}$$

c) $\quad \downarrow A \quad$ $\underline{\Sigma}'$ $\quad \underline{m_1}$: $\hat{y})$ $m_1 \ddot{y}_1' = T - m_1 \cdot g + m_1 \cdot A$

$\underline{m_2}$: $\hat{y})$ $m_2 \ddot{y}_2' = T - m_2 g + m_2 A$

$$(m_1+m_2)\cdot\ddot{y}_1' = (-m_1+m_2)\cdot g + (m_1-m_2)\cdot A$$

obs.
no inercial

$$\boxed{\begin{gathered}\ddot{y}_1' = \frac{(m_2-m_1)}{(m_1+m_2)}(g-A) \\ \ddot{y}_2' = \frac{(m_1-m_2)}{(m_1+m_2)}(g-A)\end{gathered}}$$

$\underline{\Sigma}$

$$-m_2 A + \cancel{m_1 A} - \cancel{m_1 A} - m_2 A$$

$$\ddot{y}_1 = \frac{(m_2-m_1)g - (m_2-m_1)\cdot A - (m_1+m_2) A}{(m_1+m_2)}$$

obs.
inercial

$$\boxed{\begin{gathered}\ddot{y}_1 = \frac{-2m_2 A + (m_2-m_1)g}{(m_1+m_2)} \\ \ddot{y}_2 = \frac{2m_2 A - g(m_2-m_1)}{(m_1+m_2)}\end{gathered}}$$

d) Cae con $\vec{A} = \vec{g}$ $\quad \downarrow \quad \therefore$

$\underline{\Sigma}'$ obs. no inercial $\boxed{\ddot{y}_1' = 0 \qquad \ddot{y}_2 = 0}$ $\qquad$ las masas flotan

$\underline{\Sigma}$

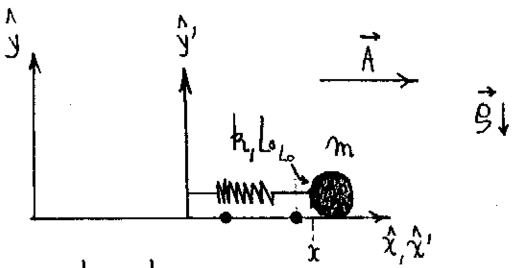
$$-m_2 g - m_1 g$$

$$\ddot{y}_1 = \frac{-2m_2 g + m_2 g - m_1 g}{m_1+m_2} = \boxed{-g}\,\cancel{\frac{(m_2+m_1)}{(m_1+m_2)}}$$

obs.
inercial

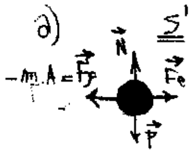
$$\boxed{\ddot{y}_2 = g}$$

③



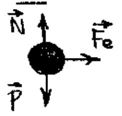
$$a = a' + A$$
$$a' = a - A$$

a)



$\underline{\underline{S'}}$ → al moverse plataforma (no inercial)

$\underline{\underline{S}}$ (inercial)



b) m es empujada hacia adentro por la F_f (a la vez que la Fe aumenta)

$\hat{x}$) $m.\ddot{x}' = k(x - L_0) - m.A$

$\underbrace{m.a'}_{0} = k(x-L_0) - \overbrace{m.A}^{F_f}$

$\Rightarrow \quad k(x-L_0) = m.A$

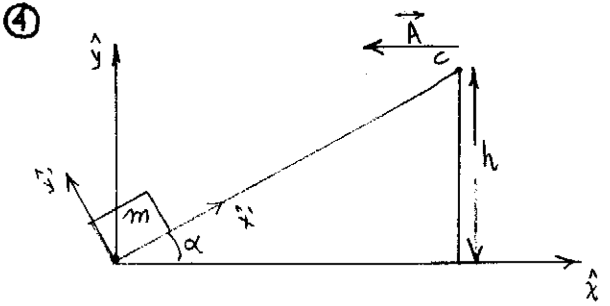
$m.a = 0 + m.A$

$m.a = k(x - L_0)$

c)

$$\overbrace{m_t}.a = M.A + m.A$$
$$-\underbrace{(M+m).a} = (M+m).A$$
$$F = M.A + k(x - L_0)$$

④

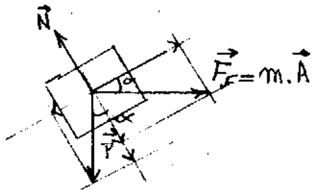


$$a' = a - A$$
$$m.a' =$$

$\text{sen}\,\alpha = \frac{h}{C}$ $\quad tg\,\alpha = \frac{h}{D}$

$\cos\alpha = \frac{D}{C}$ $\quad D = \frac{h}{tg\,\alpha}$

$D = C.\cos\alpha$



$\vec{F_f} = m.\vec{A}$ $\quad \underline{\underline{S'}}$ Plano (No inercial - acelerado)

$\hat{x}'$) $-m.g.\text{sen}\,\alpha + F_f.\cos\alpha = m.a'$

$\hat{y}'$) $-m.g.\cos\alpha + N - F_f.\text{sen}\,\alpha = 0$

$$\cancel{m}A.\cos\alpha - \cancel{m}g\,\text{sen}\,\alpha = \cancel{m}.a'$$
$$A.\cos\alpha - g.\text{sen}\,\alpha = a'$$
$$A = \frac{a' + g.\frac{h}{C}}{\cos\alpha}$$

$$A = \frac{2V_1^2\,\text{sen}\,\alpha}{h\cos\alpha} + \frac{g\cancel{h}\,\text{sen}\,\alpha}{\cancel{h}\,\cos\alpha}$$

$$\boxed{A = tg\,\alpha\left(\frac{2V_1^2}{h} + g\right)}$$

$$A = \frac{a'.C}{D} + \frac{g.h}{D}$$
$$\frac{2C.C}{t^2.D} + \frac{g\,h}{D}$$
$$A = \frac{2\cancel{C^2}V_1^2}{4\cancel{C^2}.D} + \frac{gh}{D}$$

$$A = \frac{2V_1^2 + gh}{\frac{h.\cos\alpha}{\text{sen}\,\alpha}} = \frac{2V_1^2 + gh}{C.\cos\alpha} = \frac{8V_1^2 + 4gh}{4D}$$

$$C = 0 + 0.t + \frac{1}{2}.a'.t^2$$

$$C = \frac{1}{2}a't^2$$
$$\frac{2C}{t^2} = a'$$

$V_{x_c} = 0 + a'.t$

$V_1 = a'.t$

$V_1^2 = 0 + 2.a'(C - 0)$

$V_1^2 = 2.a'.C$

$C = \frac{h}{\text{sen}\,\alpha} = \frac{a'}{2}t^2$

$C = \frac{V_1.t}{2}$

$t = \frac{2C}{V_1}$

$V_1 = a'.t$

$V_1^2 = a'^2.t^2 = 2.a'.C$

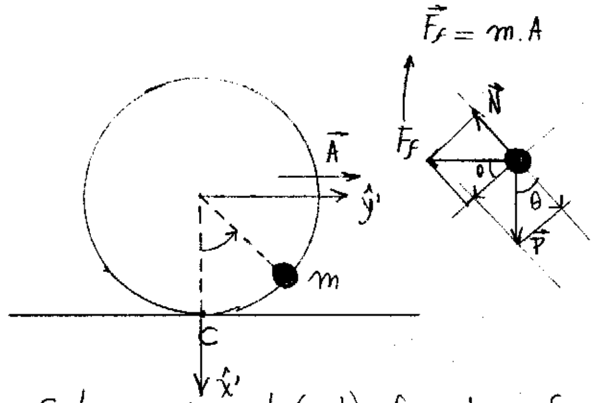
$a'.t^2 = 2C$

$\frac{V_1}{a'} = t$

$\frac{V_1}{a'} = \frac{2C}{V_1}$

$C = \frac{V_1^2}{a'2}$ $\quad t^2 = \frac{V_1^2}{a'^2} \Rightarrow \frac{2C\,a'^2}{V_1^2} = a'$

⑦



$$\vec{a}' = \vec{a} + \vec{A}$$

a) Sistema no inercial (S') fijo a la esfera

$$\hat{r}')\quad -m.R.\dot{\theta}'^2 = m.g.\cos\theta - N - m.A.\operatorname{sen}\theta$$

$$\hat{\theta}')\quad m.R.\ddot{\theta}' = -m.g.\operatorname{sen}\theta - m.A.\cos\theta$$

$$\boxed{R.\ddot{\theta}' = -g.\operatorname{sen}\theta - A.\cos\theta}$$

b)

$$N = mR\dot{\theta}'^2 + m.g.\cos\theta - m.A.\operatorname{sen}\theta$$

$$N = mR\left(\frac{2g\cos\theta}{R} - \frac{2g}{R} - \frac{2A\operatorname{sen}\theta}{R}\right) + mg\cos\theta - mA\operatorname{sen}\theta$$

$$N = 2mg\cos\theta - 2mg - 2mA\operatorname{sen}\theta + mg\cos\theta - mA\operatorname{sen}\theta$$

$$\boxed{N = 3mg\cos\theta - 2mg - mA\operatorname{sen}\theta}$$

$$\dot{\theta}R.d\dot{\theta} = -g.\operatorname{sen}\theta.d\theta - A.\cos\theta.d\theta$$

$$R\left(\frac{\dot{\theta}'^2}{2} - \underbrace{\frac{\dot{\theta}_0^2}{2}}\right) = -g.(-\cos\theta + 1) - A.(\operatorname{sen}\theta)$$

$$R.\frac{\dot{\theta}'^2}{2} = g(\cos\theta - 1) - A\operatorname{sen}\theta$$

c)

$$0 = -g.\operatorname{sen}\theta_e - A.\cos\theta_e$$

$$g.\operatorname{sen}\theta_e = -A\cos\theta_e$$

$$\boxed{tg\,\theta_e = -\frac{A}{g} = \frac{\operatorname{sen}\theta_e}{\cos\theta_e}}\quad \theta_{equilibrio}$$

$$\left.\frac{\partial\ddot{\theta}}{\partial\theta}\right|_{\theta_{eq}} = -\frac{g}{R}.\cos\theta_e + \frac{A}{R}\operatorname{sen}\theta_e$$

será estable si

$$-\frac{g}{R}.\cos\theta_e + \frac{A}{R}\operatorname{sen}\theta_e < 0$$

$$-\frac{g}{R}\cos\theta_e + \frac{A}{R}.-\frac{A}{g}\cos\theta_e < 0 \Rightarrow -g.\cos\theta_e - \frac{A^2}{g}\cos\theta_e < 0$$

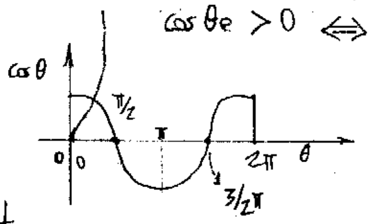
$$\underbrace{-\cos\theta_e}\underbrace{\left[g + \frac{A^2}{g}\right]}_{>0} < 0$$

si $\theta_e \neq 0, \pi$ $\quad \frac{A^2}{g^2} < \frac{-\operatorname{sen}\theta_e}{\operatorname{sen}\theta_e}$

$$\frac{A^2}{g^2} < -1$$

$-\cos\theta_e < 0 \Leftrightarrow$

$\cos\theta_e > 0 \Leftrightarrow$



θ_e ↓

θ_{eq} es : $-tg\theta_e = \frac{A}{g}$ y satisface [2]

$\boxed{\Rightarrow -tg\theta_e = \frac{A}{g} \text{ es estable}}$ y está en el 4to cuadrante

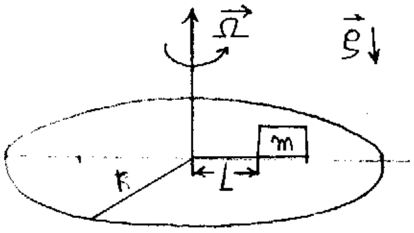
[1] $0 > -tg\theta_e > -\infty$

[2] $+\infty > -tg\theta_e > 0$

$$\Leftarrow \begin{cases} 0 < tg\,\theta_e < +\infty \\ -\infty < tg\,\theta_e < 0 \end{cases} \qquad \Leftarrow \begin{cases} 0 < \theta_e < \pi/2 \\ 3/2\pi < \theta_e < 2\pi \end{cases}$$

⑧

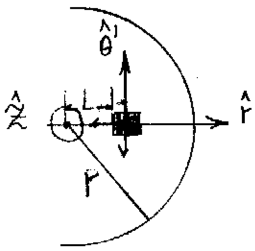
$t=0=t_0$ m en reposo a distancia L

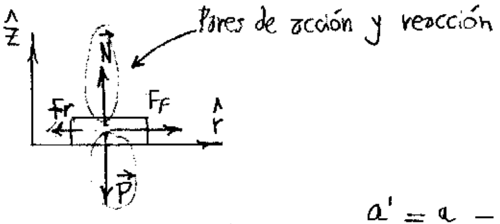


$\Omega = k \Rightarrow \dot{\Omega}=0$

$R = k \Rightarrow \dot{R}=\ddot{R}=0$

a)





$a' = a - \underbrace{\text{CoR}}_{0} - \text{centrif} - \underbrace{\ddot{R}}_{0} - \underbrace{\dot{\vec{\Omega}}\times\vec{r}'}_{0}$

$\hat{z})\quad \underbrace{m.\overset{=\ddot{z}}{\ddot{z}'}}_{0} = N - m.g$

$\hat{r}')\quad \underbrace{-m.r.\dot{\theta}'^2}_{0} = -F_r + m.\Omega^2.L$

$\hat{\theta}')\quad \underbrace{m.r.\ddot{\theta}'}_{0} = \underbrace{-F_r}_{=0}$

El paquete no se mueve en ninguna dirección

coriolis

$2\vec{\Omega}\times\vec{V}_r = 2\Omega\hat{z}\times 0 = 0$

centrífuga

$\vec{\Omega}\times(\vec{\Omega}\times\vec{r}) = \Omega\hat{z}\times(\Omega\hat{z}\times L.\hat{r})$

$\Omega\hat{z}\times(\Omega L\hat{\theta}) = -\Omega^2 L\hat{r}$

$\hat{r}\ \hat{\theta}\ \hat{z}\ \underrightarrow{\hat{r}}$

b)

$F_r = m.\Omega^2.L$

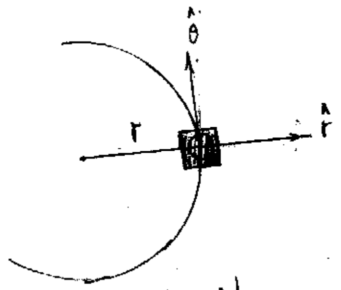
$m.\Omega^2.L \le \mu_e.m.g$

$$\boxed{\Omega_{max} = \sqrt{\frac{g}{L}\mu_e}}$$

c)

$\vec{V} = \vec{V}' + \vec{\Omega}\times\vec{r}' + \dot{\vec{r}}$

$\vec{V}' = \vec{V} - \vec{\Omega}\times\vec{r}' - \underbrace{\dot{\vec{r}}}_{0}$



$\hat{r})\quad \ddot{r}' - r'\dot{\theta}'^2 =$

$\hat{\theta})\quad r'\ddot{\theta}' + 2\dot{r}'\dot{\theta}' =$

coriolis

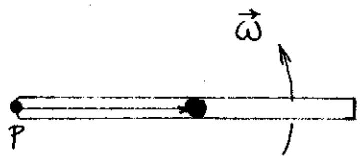
$2\vec{\Omega}\times\dot{\vec{r}}' = 2\Omega\hat{z}\times r'$

centrífuga

$\vec{\Omega}\times(\vec{\Omega}\times\vec{r}') = \Omega\hat{z}\times(\Omega\hat{z}\times r.\hat{r}) =$

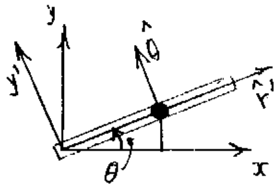
$\Omega\hat{z}\times\Omega.r.\hat{\theta} = -\Omega^2.r.\hat{r}$

⑨



$\Omega = k \Rightarrow \dot\Omega = 0$

a) S' (fijo al tubo – no inercial)



$$a' = a - \text{cor} - \text{cen} - \underbrace{\ddot{R}}_{0} - \underbrace{\dot{\vec\Omega}\times\vec r'}_{0}$$

Coriolis

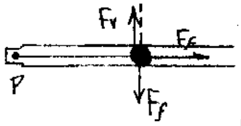
$$2\vec\Omega \times \vec v' = 2\Omega\hat z \times v'.\hat r = 2\Omega v'\hat\theta$$

Centríf

$\hat r\,\hat\theta\,\hat z$ $\quad \vec\Omega\times(\vec\Omega\times\vec r') = \Omega\hat z\times(\Omega\hat z\times r'.\hat r) = \Omega\hat z\times\Omega.r'.\hat\theta = -\Omega^2 r'.\hat r$

S' (el tubo no se mueve) (La bola no se mueve en $\hat\theta$ solo en $\hat r$)

$\theta = k \Rightarrow \dot\theta = \ddot\theta = 0$



$\cos\theta = \frac{x}{x'}$

$x' = \frac{x}{\cos\theta}$

$y' = y$

$\hat r'$) $m.\ddot r = +m\Omega^2 r'$

$\hat\theta'$) $0 = F_v - 2m\Omega.v'$

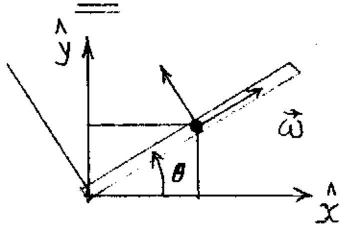
$\hat\theta'$) $F_v = 2m\Omega.\dot r'$ → variable hay que calcularlo

$\vec v' = (\dot r').\hat r + (r'.\underset{0}{\dot\theta'})\hat\theta$

$\vec v' = \dot r'.\hat r$

$\boxed{\ddot r' = \Omega^2 r'.\hat r}$ aceleración respecto un sistema fijo al tubo

S



$a = a' + \text{cor} + \text{cen}$

simplificamos notación

$\ddot x = \omega^2 x$

$\ddot x - \omega^2 x = 0$

$A.e^{\alpha t}(\alpha^2 - \omega^2) = 0$

$\left.\begin{matrix}\alpha^2 = \omega^2 \\ \alpha = +\omega \\ -\omega\end{matrix}\right\} \Rightarrow$

$r(t) = A_1 e^{\omega t} + A_2 . e^{-\omega t}$

Supongamos

$t = 0 \Rightarrow t_0 \Rightarrow r'(t=0) = r'_0 \quad \therefore \quad r(t=0) = A_1 + A_2 = r_0 \ldots$

$t = 0 = t_0 \Rightarrow \dot r'(t=0) = 0 \quad \therefore \quad \dot r(t) = A_1.\omega.e^{\omega t} - A_2.\omega.e^{-\omega t}$

$\dot r(t=0) = 0 = A_1\omega - A_2\omega$

$A_1\cancel{\omega} = A_2\cancel{\omega} \Rightarrow \underline{A_1 = A_2,}$

$r_0 = 2A_1 = 2A_2$

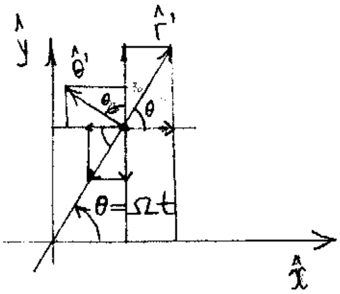
$$r(t) = \frac{r_0}{2}.e^{\omega t} + \frac{r_0}{2}.e^{-\omega t} = r'_0\left(\frac{e^{\omega t} + e^{-\omega t}}{2}\right)$$

$$\boxed{r(t) = r'_0 . \cos h(\omega t)}$$

$$\boxed{r'(t) = r'_0 . \cosh(\Omega t)}$$

Luego $\ddot r' = \Omega^2 . r'_0 . \cosh(\Omega . t)\,\hat r$

$$\vec{a} = \vec{a}' + 2\vec{\Omega}\times\vec{v}' + \vec{\Omega}\times(\vec{\Omega}\times\vec{r}')$$



$$\ddot{\vec{r}} = \Omega^2 . r_0' . \cosh(\Omega t)\,\hat{r}$$

$$\ddot{\vec{r}} \begin{cases} \hat{x}) & |\ddot{\vec{r}}| . \cos(\Omega t) . \hat{x} \\ \hat{y}) & |\ddot{\vec{r}}| . \mathrm{sen}(\Omega t)\,\hat{y} \end{cases}$$

$$2\vec{\Omega}\times\vec{v}' = \begin{cases} \hat{x}) & 2\Omega^2 . r_0' . \mathrm{senh}(\Omega t) . \mathrm{sen}(\Omega t)\,\hat{x} \\ \hat{y}) & 2\Omega^2 . r_0' . \mathrm{senh}(\Omega t) . \cos(\Omega t)\,\hat{y} \end{cases}$$

$$-\Omega^2 . r' . \hat{r} \begin{cases} \hat{x}) & -\Omega^2 . r' . \cos(\Omega t)\,\hat{x} \\ \hat{y}) & -\Omega^2 . r' . \mathrm{sen}(\Omega t)\,\hat{y} \end{cases}$$

$$\dot{\theta} = k = |\vec{\omega}|$$

$$\frac{d\theta}{dt} = k = |\vec{\omega}|$$

$$\int_{\theta_0=0}^{\theta} d\theta = |\vec{\omega}| \int_{t_0=0}^{t} dt$$

$$\theta = |\vec{\omega}|\, t = \Omega t$$

$$a_x = \Omega^2 . r_0' . \cosh(\Omega t) . \cos(\Omega t) + 2\Omega v' . \mathrm{sen}(\Omega t) - \Omega^2 . r' . \cos(\Omega t)$$

Coriolis

$$2\vec{\Omega}\times\vec{v}' = 2\Omega\hat{z}\times\underbrace{\dot{r}'}\,.\hat{r} \qquad = 2\Omega^2 . r_0'\, \mathrm{senh}(\Omega t)\,\hat{\theta}$$

$\hat{r}\,\hat{\theta}\,\hat{z}$

$$\dot{\vec{r}}' = (r_0' . \mathrm{senh}(\Omega t) . \Omega)\,\hat{r}$$

Centrífugo

$$\vec{\Omega}\times(\vec{\Omega}\times\vec{r}') = \Omega\hat{z}\times(\Omega\hat{z}\times r_0' . \cosh(\Omega t)\,\hat{r}) = -\Omega^2 r_0' . \cosh(\Omega t) . \hat{r}$$

$$\vec{a} = \Omega^2 . r_0' . \cosh(\Omega t)\,\hat{r} + 2\Omega^2 r_0' . \mathrm{senh}(\Omega t)\,\hat{\theta} - \Omega^2 r_0' \cosh(\Omega t)\,\hat{r}$$

$$\boxed{\vec{a} = 2 . \Omega^2 . r_0' . \mathrm{senh}(\Omega t)}$$

b)

$$\left((\Omega\hat{z})\underset{0}{.}(r'\hat{r})\right).\underset{0}{\Omega}\hat{z} - \Omega\hat{z} \ldots$$
$$- \Omega^2 \ldots r'\hat{r}$$

$$\vec{f}_{cent} = +m . \Omega^2 . r_0' . \cosh(\Omega t)\,\hat{r}$$

$$\vec{f}_{cor} = -2m\,\Omega^2 r_0' . \mathrm{senh}(\Omega t)\,\hat{\theta}$$

$\hat{\theta}$) $\vec{F}_V = 2m\,\Omega^2 . r_0'\, \mathrm{senh}(\Omega t) . \hat{\theta}$

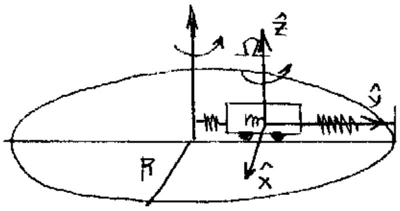
$\hat{r}$) $m . \Omega^2 . r_0' \cosh(\Omega t) = \Omega^2 . r_0' . \cosh(\Omega t)$

$\hat{r}$) $\ddot{r} = \Omega^2 . r'(t)$

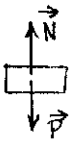
$\hat{\theta}$) $F_{V_\theta} = 2m\Omega^2 . \dot{r}'$

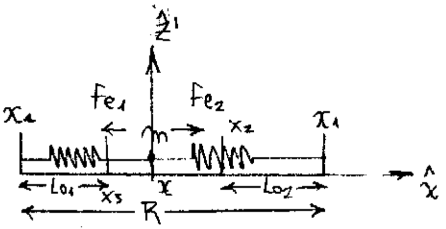
$\hat{z}$) $0 = -m.g + F_{V_{\hat{z}}}$

⑩



$\vec{g}\downarrow$



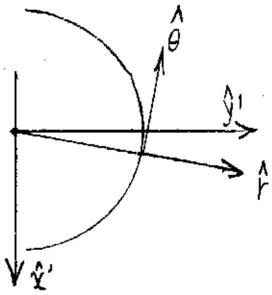


$L_{01} = x_3 - x_4 \qquad L_{02} = x_1 - x_2$

$$\hat{y})\quad m.\ddot{r} = -k_1(x_3 - [x_3 - x_4]) + k_2(x_2 - [x_1 x_2]) + m.\Omega^2.r$$

$$\hat{z})\quad 0 = N - m.g$$

$$\hat{x})\quad 0 = 2m\Omega\dot{r} - F_r$$



$\hat{r}\,\hat{\theta}\,\hat{z}$
$\hat{x}\,\hat{y}\,\hat{z}$

Coriolis

$$2\vec{\Omega}\times\vec{V}' = 2\Omega\hat{z}\times\dot{r}.\hat{r} = 2\Omega\hat{z}\times\dot{r}.\hat{y} = -2\Omega\dot{r}\,\hat{x}$$

centrífuga

$$\vec{\Omega}\times(\vec{\Omega}\times\vec{r}') = \Omega\hat{z}\times(\Omega\hat{z}\times\underset{r.\hat{y}}{r.\hat{r}}) = \Omega\hat{z}\times -\Omega r\hat{x} = -\Omega^2 r.\hat{y}$$