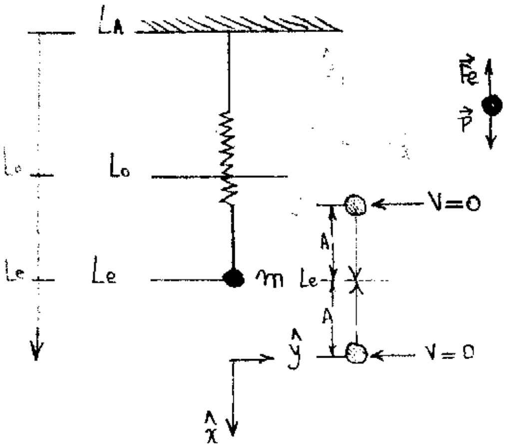
Movimiento Oscilatorio

① $t=0$ $\quad V=0 \longrightarrow$ tiene en 2 puntos



$$-k.x + m.g = m\,\ddot{x}$$

$$0 = -g + \frac{k}{m}x + \ddot{x}$$

$$\ddot{x} + \frac{k}{m}.x - g = 0$$

sol. hom

$$x = A.\cos(\omega t + \varphi)$$

sol. no homog.

$$c = \frac{m.g}{k} \Rightarrow$$

$$\ddot{c} + \frac{k}{m}.c - g = 0$$

$$0 + \frac{k}{m}.\frac{m.g}{k} - g = 0$$

Solución total

$$x(t) = A.\cos(\omega t + \varphi) + \frac{m.g}{k}$$

$$v(t) = -A.\operatorname{sen}(\omega t + \varphi).\omega$$

$$0 = -A.\operatorname{sen}(\varphi).\omega$$

$$0 = \operatorname{sen}(\varphi) \Rightarrow \boxed{\varphi = 0}$$

$$x(t=0) = 2L_0 = A.\cos(\varphi) + \frac{m.g}{k}$$

$$2L_0 = A + \frac{m.g}{k}$$

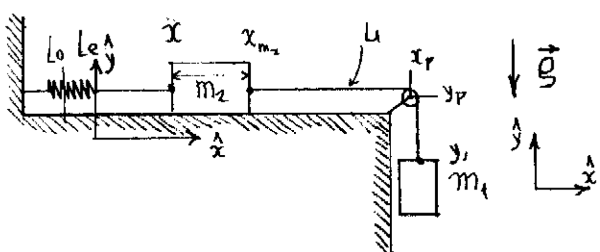
$$\left(2L_0 - \frac{m}{k}.g\right) = A$$

$$x(t) = \left(2L_0 - \frac{m}{k}.g\right).\cos\left(\sqrt{\frac{k}{m}}.t\right) + \frac{m}{k}.g$$

$$Kg.\frac{m}{s^2}.\frac{m}{N}$$

$$\frac{Kg\,m.m}{s^2\,Kg.\frac{m}{s^2}}$$

②



$$L_1 = x_P - x_{m_2} + y_P - y$$

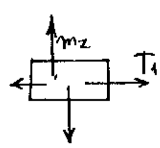
$$0 = -\ddot{x}_{m_2} - \ddot{y}$$

$$\ddot{x}_{m_2} = -\ddot{y}$$

$$\text{Long. } m_2 = x_{m_2} - x$$

$$0 = \ddot{x}_{m_2} - \ddot{x} \Rightarrow \ddot{x} = \ddot{x}_{m_2} = -\ddot{y}$$

a)

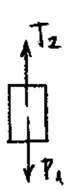


m_2

$\hat{x})\quad T_1 - k.(x\quad) = m_2.\ddot{x}$

$\hat{y})\quad N_2 - m_2.g = 0$

soga sin masa
$T_1 = T_2$



m_1

$\hat{y})\quad T_2 - m_1.g = m_1.\ddot{y} = -m_1.\ddot{x}$

$$T_1 - m_1.g = -m_1.\ddot{x}$$

$$-k.x + m_1.g = m_2\,\ddot{x} + m_1\,\ddot{x}$$

$$-\frac{k}{(m_1+m_2)}.x + \frac{m_1}{(m_1+m_2)}.g = \frac{(m_1+m_2).\ddot{x}}{(m_1+m_2)}$$

$$\ddot{x} + \frac{k}{(m_1+m_2)}x - \frac{m_1}{(m_1+m_2)}g = 0$$

sol. h

$$x(t) = A.\cos\left(\sqrt{\frac{k}{(m_1+m_2)}}t+\varphi\right)$$

$$C = \frac{m_1}{k}.g \Rightarrow$$

$$\ddot{c} + \frac{k}{(m_1+m_2)}.\frac{m_1 g}{k} - \frac{m_1}{(m_1+m_2)}.g = 0$$

$$0 + \frac{m_1 g}{m_1+m_2} - \frac{m_1 g}{m_1+m_2} = 0$$

$$\boxed{x(t) = A.\cos\left(\sqrt{\frac{k}{m_1+m_2}}.t+\varphi\right) + \frac{m_1.g}{k}}$$

$$V(t) = -A.\text{sen}(\omega t+\varphi).\sqrt{\frac{k}{m_1+m_2}}$$

$$x_{eq} = A.\cos(\varphi) + \frac{m_1.g}{k}$$

$$V(t=0) = \underbrace{V_0}_{\neq 0} = -A.\text{sen}(\varphi).\sqrt{\frac{k}{m_1+m_2}}$$

$$x = \frac{-V_0}{\text{sen}\,\varphi.\sqrt{\frac{k}{m_1+m_2}}}$$

$$\boxed{-\frac{V_0}{\text{sen}(\varphi).\sqrt{\frac{k}{m_1+m_2}}} = A}$$

CONDICIÓN de REPOSO INICIAL $(t=0)$ $a=0$

$$a(t=0) = 0 = -A.\cos(\varphi).\frac{k}{m_1+m_2}$$

$$A(t) = -A.\cos(\omega t+\varphi).\frac{k}{m_1+m_2}$$

$$0 = \underbrace{-A}_{\neq 0}.\cos(\varphi)$$

$$A = V_0.\sqrt{\frac{m_1+m_2}{k}}$$

$$0 = \cos(\varphi) \Rightarrow$$

$$\varphi = \frac{3}{2}\pi$$

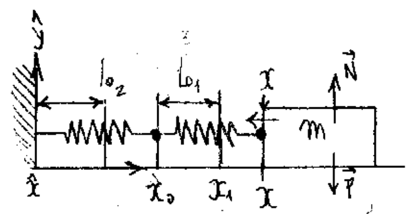
$$\varphi = \frac{\pi}{2}, \frac{3}{2}\pi$$

elijo $\frac{3}{2}\pi$ (Así $\text{sen}\,\varphi$ es -1)

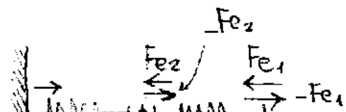
$$\varphi = \frac{\pi}{2}; \frac{3}{2}\pi; \frac{5}{2}\pi; \text{etc}$$

$$\boxed{x(t) = V_0.\sqrt{\frac{m_1+m_2}{k}}.\cos\left(\sqrt{\frac{k}{m_1+m_2}}.t+\frac{3}{2}\pi\right) + \frac{m_1.g}{k}}$$

③



$$f = \frac{1}{T} \qquad T = \frac{2\pi}{\omega}$$



i)

m

$$\hat{x})\quad -k_1.[x - \overbrace{(x_1-x_0)}^{L_{o1}} - x_0] = m.\ddot{x}$$

$$\hat{y})\quad N - m.g = 0$$

$$Fe_2 = Fe_1$$

$$-k_2.(x_0 - L_{o2}) = -k_1(x - x_1 + x_0)$$

$L_{o1} = x_1 - x_0$

$L_{o1} + x_0 = x_1$

$$-k_2(x_0 - L_{o2}) = -k_1(x - x_1)$$

$$-k_2 x_0 + k_2 L_{o2} = -k_1 x + k_1 x_1$$

$L_{o2} = x_0 - (x_0 - L_{o2})$

$$k_2 L_{o2} = k_1 L_{o1} + k_1 x_0 + k_2 x_0$$

$$+k_1 x + k_2 L_{o2} - k_1 L_{o1} = (k_1+k_2).x_0$$

$$-k_1.x + k_1 L_{o1} + k_1 x_0 = m.\ddot{x}$$

$$k_1 x = (k_1+k_2).x_0 - k_2 L_{o2} + k_1 L_{o1}$$

$$~~(k_1+k_2).x_0 + k_2 L_{o2} - k_1 L_{o1} + k_1 L_{o1} + k_1 x_0~~ = m.\ddot{x}$$

$$x_0 = \frac{k_1 x + k_2 L_{o2} - k_1 L_{o1}}{(k_1+k_2)}$$

$$-k_1.x + k_1.L_{o1} + \frac{k_1^2(x - L_{o1}) + k_2 k_1(L_{o2})}{k_1+k_2} = m.\ddot{x}$$

$$x_0 = \frac{k_1(x - L_{o1}) + k_2(L_{o2})}{k_1+k_2}$$

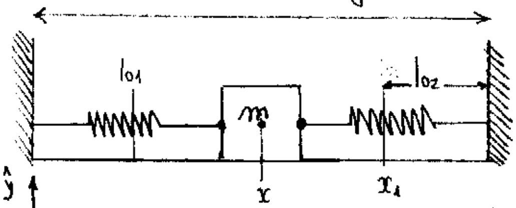
$$\frac{-(k_1+k_2)k_1 x + (k_1+k_2).L_{o1}k_1 + k_1^2(x - L_{o1}) + k_2.k_1.L_{o2}}{k_1+k_2} =$$

$$-k_1^2 x - k_2 k_1 x + k_1^2 L_{o1} + k_2 L_{o1} k_1 + k_1^2 x - k_1^2 L_{o1} + k_2 k_1 L_{o2} = (k_1+k_2).m.\ddot{x}$$

$$m.\ddot{x}(k_1+k_2) + k_2.k_1.x = k_2 k_1 L_{o1} + k_2 k_1 L_{o2} = (k_2.k_1).(L_{o1}+L_{o2})$$

$$\ddot{x} + \frac{k_2.k_1}{m(k_1+k_2)}.x = \left[\frac{k_2.k_1}{(k_1+k_2)}.\frac{(L_{o1}+L_{o2})}{m}\right]$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi} = \boxed{\frac{1}{2\pi}\cdot\sqrt{\frac{k_1 . k_2}{m.(k_1+k_2)}}}$$



$d - l_{02} = x_1$

$Fe_1 \leftarrow \quad \rightarrow Fe_2$

$\hat{x})\quad -Fe_1 + Fe_2 = m.\ddot{x}$

$$-k_1.(x - L_{01}) + k_2.(x_1 - x) = m\ \ddot{x}$$

$$-k_1.(x - L_{01}) + k_2(d - l_{02} - x) = m.\ddot{x}$$

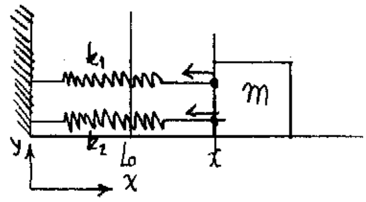
$$-k_1.x - k_2 x + k_1 L_{01} + k_2 d - k_2 L_{02} = m\ddot{x}$$

$$m\ddot{x} + (k_1+k_2).x = -k_1 L_{01} + k_2(d - L_{02})$$

$$\ddot{x} + \frac{(k_1+k_2)}{m}.x = -\frac{k_1.L_{01}}{m} + \frac{k_2(d-L_{02})}{m}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}}$$

$L_{01} = L_{02} = L_0$



$\hat{x})\quad -k_1.(x - L_{01}) - k_2(x - L_{02}) = m.\ddot{x}$

$$-k_1.x + k_1 L_{01} - k_2 x + k_2 L_{01} = m.\ddot{x}$$

$$-(k_1+k_2)x + L_0(k_1+k_2)$$

$$(k_1+k_2)(x - L_0) = m.\ddot{x}$$

$$m.\ddot{x} + (k_1+k_2)x = +(k_1+k_2).L_0$$

$$\ddot{x} + \frac{(k_1+k_2)}{m}.x = +\frac{(k_1+k_2)}{m}.L_0$$

$$f = \frac{\omega}{2\pi} = \boxed{\frac{1}{2\pi}\cdot\sqrt{\frac{k_1+k_2}{m}}}$$

ii) (b)

$$k_1(x - L_{01}) = k_2(d - L_{02} - x)$$

$$(k_1+k_2)x = -k_1 L_{01} + k_2(d - L_{02})$$

$$\boxed{x_{eq} = \frac{-k_1 L_{01} + k_2(d - L_{02})}{(k_1+k_2)}}$$

(a)

$$-k_2 k_1.x + k_2.k_1.L_{01} + k_2.k_1.L_{02} = 0$$

$$k_2.k_1\ (x + L_{01} + L_{02}) = 0$$

$$\boxed{x_{eq} = L_{02} + L_{01}}$$

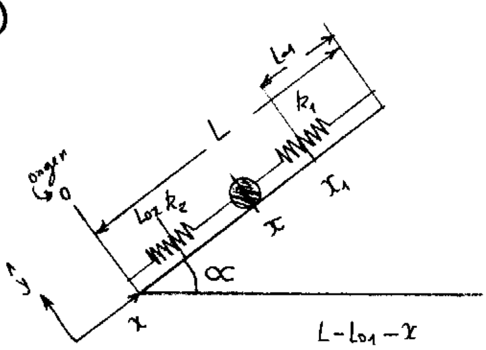
(c)

$$(k_1+k_2).(x - L_0) = 0$$

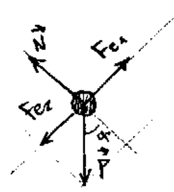
$$x - L_0 = 0$$

$$\boxed{x = L_{01} = L_{02}}$$

④



$\vec{g}\downarrow$



a) $\hat{x})\quad k_1(\overbrace{x_1 - x}^{L - L_{01} - x}) - \text{sen}\,\alpha.m.g - k_2(x - L_{02}) = m.\ddot{x}$

$$m.\ddot{x} + k_1.x + k_2.x = k_1.L - k_1 L_{01} - \text{sen}\,\alpha.m.g + k_2 L_{02}$$

$$\boxed{\ddot{x} + \frac{(k_1+k_2)}{m}.x - \frac{k_1(L - L_{01})}{m} + \text{sen}\,\alpha.g - \frac{k_2.L_{02}}{m} = 0}$$

$$\frac{(k_1+k_2)}{m}.x_e - \frac{k_1(L-L_{01})}{m} = -\operatorname{sen}\alpha . g + \frac{k_2 . L_{02}}{m}$$

$$x_{eq} = \left(\frac{k_1(L-L_{01})}{m} - \operatorname{sen}\alpha . g + \frac{k_2 L_{02}}{m}\right) . \frac{m}{(k_1+k_2)}$$

$$\boxed{x_{eq} = \frac{k_1(L-L_{01}) - \operatorname{sen}\alpha . m . g + k_2 L_{02}}{k_1+k_2}}$$

$$\left.\frac{\partial \ddot{x}}{\partial x}\right|_{x_{eq}} = \frac{k_1+k_2}{m} \not< 0 \Rightarrow x_{eq} \text{ es inestable}$$

c)

$$\ddot{x} + \frac{(k_1+k_2)}{m}.x = \frac{k_1(L-L_{01}) - \operatorname{sen}\alpha . m . g + k_2 . L_{02}}{m} \rightarrow \frac{\phi}{m}$$

$$x = A.\cos(\omega . t + \varphi)$$

$$C = \frac{k_1(L-L_{01}) - \operatorname{sen}\alpha . m . g + k_2 . L_{02}}{(k_1+k_2)}$$

$$\ddot{C} + \frac{k_1+k_2}{m}.C = \frac{\phi}{m}$$

$$0 + \frac{k_1+k_2}{m} . \frac{\phi}{(k_1+k_2)} = \frac{\phi}{m}$$

$$x(t) = A.\cos\left(\sqrt{\frac{k_1+k_2}{m}}.t + \varphi\right) + \frac{\phi}{(k_1+k_2)}$$

$x(t=0) = x_{eq} \Rightarrow$

$V(t=0) = V_0$

hacia arriba (>0)

$$\frac{k_1(L-L_{01}) - \operatorname{sen}\alpha . m g + k_2 L_{02}}{k_1+k_2} = A.\cos(\varphi) + \frac{\phi}{(k_1+k_2)}$$

$$\Rightarrow 0 = A.\cos(\varphi)$$

$$0 = \cos(\varphi)$$

$$\frac{3}{2}\pi , \frac{\pi}{2} = \varphi$$

$$\dot{x}(t) = -A.\operatorname{sen}\left(\sqrt{\frac{k_1+k_2}{m}}.t + \varphi\right).\sqrt{\frac{k_1+k_2}{m}}$$

$$V_0 = -A.\operatorname{sen}(\varphi).\sqrt{\frac{k_1+k_2}{m}}$$

◄ Para que $V_0 > 0$ necesito $\operatorname{sen}(\varphi) = -1$

$$\Rightarrow \varphi = -\pi/2$$

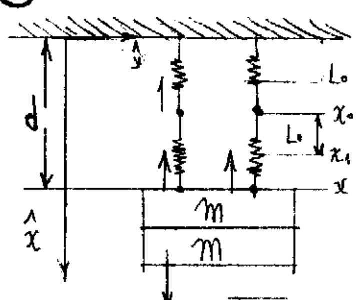
$$\varphi = \frac{3}{2}\pi$$

$$V_0 = A.\sqrt{\frac{k_1+k_2}{m}}$$

$$A = V_0 . \sqrt{\frac{m}{k_1+k_2}}$$

$$\boxed{x(t) = \left(V_0 . \sqrt{\frac{m}{k_1+k_2}}\right).\cos\left(\sqrt{\frac{k_1+k_2}{m}}.t + \frac{3}{2}\pi\right) + \frac{k_1(L-L_{01}) - \operatorname{sen}\alpha . m . g + k_2 . L_{02}}{(k_1+k_2)}}$$

⑤



$\hat{x}$)

$$2m.g - k(x - x_1).2 = 2m.\ddot{x}$$

$$2.m.g - k.(x - x_0 - [x_1 - x_0]).2 = 2.m.\ddot{x}$$

$$m.g - kx + kx_0 + kL_0 = m.\ddot{x}$$

$$m.g - kx + \frac{kx}{2} + kL_0 = m.\ddot{x}$$

$$m.g - \frac{k}{2}x + kL_0 = m.\ddot{x}$$

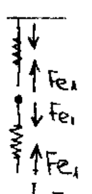
$$m.g + k\left(-\frac{d}{2} + L_0\right) = 0$$

$$k = \frac{-m.g.2}{-d+2L_0}$$

$x - x_0$, $x - x_1$, x, $x_1 - x_0$, x_1

$[x - x_0 - (x_1 - x_0)]$

$$\boxed{k = \frac{2.m.g}{d-2L_0}}$$



• x_0

$$\underbrace{m_{x_0}}_{\to 0} . a_{x_0} = -F_{e1} + F_{e1}$$

$$k(x_0 - L_0) = k(x - x_1) = k(x - [x_1 - x_0] - x_0)$$

$$kx_0 - kL_0 = kx - kx_0 - kL_0$$

$$2x_0 = x \longleftarrow 2kx_0 = kx$$

$kg . \frac{m}{s^2}$ / m

$kg . \frac{m}{s^2} . \frac{1}{m}$

b) i) $m.g - 2.k(x - x_0 - L_{01}) = m.\ddot{x}$

$$\ddot{x} + \frac{2k}{m}(x - x_0 - L_{01}) = g$$

$$\ddot{x} + \frac{2k}{m}x - \frac{2k.x}{m.2} - \frac{2kL_{01}}{m} = g$$

$$\ddot{x} + \frac{k}{m}x = g + \frac{2kL_{01}}{m}$$

$$T = \frac{2\pi}{\omega} \qquad \boxed{T = 2\pi.\sqrt{\frac{m}{k}}}$$

$$x_{eq} = \left(\frac{m.g + 2kL_{01}}{m}\right).\frac{m}{k}$$

$$\boxed{x_{eq} = \frac{m.g}{k} + 2L_{01}}$$

ii)

$$x(t) = A.\cos\left(\sqrt{\frac{k}{m}}.t + \varphi\right) \qquad C = \frac{m.g}{k} + 2L_0$$

$$\ddot{c} + \frac{k}{m}.C = \frac{mg}{} + \frac{2kL_{01}}{m} = \phi$$

$$0 + \frac{k}{m}\left(\frac{mg}{k} + 2L_0\right) = \phi$$

$$\frac{dx(t)}{dt} = \dot{x} = V = -A.\text{sen}\left(\sqrt{\frac{k}{m}}t + \varphi\right).\sqrt{\frac{k}{m}}$$

$$x(t=0) = d = A.\cos(\varphi) + \frac{m.g}{k} + 2L_0$$

$$V(t=0) = 0 = -A.\text{sen}(\varphi).\sqrt{\frac{k}{m}}$$

$$0 = \text{sen}(\varphi)$$

$$\pi, 0 = \varphi$$

$$d - 2L_0 = \frac{2m}{k}g$$

$$d = \frac{2mg}{k} + 2L_0$$

La velocidad posterior a $t=0$ será $<0 \Rightarrow$ necesito $\cos(\varphi) = 1$

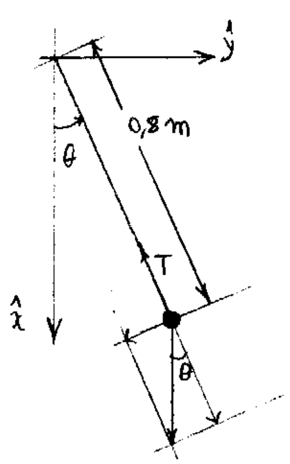
$$\frac{2mg}{k} + 2L_0 = A.\cos(\varphi) + \frac{mg}{k} + 2L_0$$

$$\frac{m.g}{k} = A.\cos(\varphi)$$

$$\frac{1}{\cos(\varphi)}.\frac{m.g}{k} = A \Rightarrow \frac{m.g}{k} = A$$

$$\boxed{x(t) = \left(\frac{m.g}{k}\right).\cos\left(\sqrt{\frac{k}{m}}.t\right) + \frac{m.g}{k} + 2L_0}$$

⑥



a)

$$\hat{r})\ -m.R.\dot{\theta}^2 = -T + m.g.\cos\theta$$

$$\hat{\theta})\ m.R.\ddot{\theta} = -m.g.\text{sen}\,\theta$$

$$R.\ddot{\theta} = -g\,\text{sen}\,\theta$$

$$\ddot{\theta} = -g.\frac{\text{sen}\,\theta}{R}$$

b) Es armónico si θ es pequeño, entonces:

$$\text{sen}\,\theta \cong \theta \quad \therefore$$

$$\ddot{\theta} + \frac{g}{R}.\theta = 0$$

$$T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{R}{g}} = T}$$

c) $t=0 \quad \theta=0 \quad \dot{\theta}= 0.2 \, \frac{1}{seg}$

Aproximacion:

$$\theta(t) = A.\cos(\omega t+\varphi)$$

$$\theta(t=0) = 0 = A.\cos\varphi$$
$$0 = \cos\varphi$$
$$\frac{3}{2}\pi , \frac{\pi}{2} = \varphi$$

$$\dot{\theta}(t) = -A.\omega.\,\text{sen}(\omega t+\varphi)$$
$$0.2 = -A.\sqrt{\frac{g}{R}}.1$$
$$-0.2\sqrt{\frac{R}{g}} = A \quad \Rightarrow \quad \dot{\theta}(t) = \underbrace{+0{,}7.\frac{2}{7}}_{0{,}2}.\text{sen}\left(\frac{2}{7}t+\frac{\pi}{2}\right)$$

Sin Aproximación:

$$\ddot{\theta} = \frac{d\dot{\theta}}{dt} = \frac{d\dot{\theta}}{d\theta}.\frac{d\theta}{dt} \Rightarrow \quad \dot{\theta}.d\dot{\theta} = -g.\frac{1}{R}.\text{sen}\theta.d\theta$$

$$\int_{0.2}^{\dot{\theta}} \dot{\theta}.d\dot{\theta} = -g.\frac{1}{R}\int_0^{\theta}\text{sen}\theta.d\theta$$

$$\frac{\dot{\theta}^2}{2} - 0{,}02 = g.\frac{1}{R}\cos\theta - g.\frac{1}{R}$$

$$\frac{\dot{\theta}^2}{2} = 0{,}02 - 12{,}25 + 12{,}25.\cos\theta$$

$$\dot{\theta}^2 = 0{,}04 - 24{,}5 + 24{,}5.\cos\theta$$

se cumple la aproximación de b $\forall t$

si $t=0$, $\theta=0$ $\Rightarrow \begin{cases} (0{,}2)^2 = 0{,}04 - 24{,}5 + 24{,}5 = 0{,}04 & \text{(exacto)} \\ 0{,}2 = 0{,}2.\,\text{sen}\frac{\pi}{2} = 0{,}2 & \text{(aproximado)} \end{cases}$

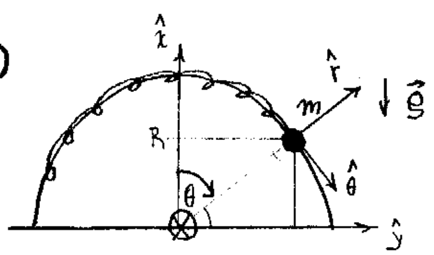
d) $0 = -g.\frac{1}{R}.\text{sen}\,\theta_{eq} \Rightarrow \theta_{eq} = \text{asen } 0$

$\theta_{eq} = 0, \pi$ — el único que tiene sentido (0)

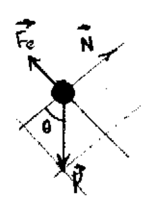
$$\left.\frac{\partial\ddot{\theta}}{\partial\theta}\right|_{\theta=0} = -g.\frac{1}{R}.\cos\theta = -g.\frac{1}{R} < 0$$

Es estable

⑦



$L_0 = \frac{\pi R}{2}$



$S = R.\theta$

a)

$\hat{r})\; -m.R.\dot{\theta}^2 = N - m.g.\cos\theta$

$\hat{\theta})\; m.R.\ddot{\theta} = -k.R.\theta + m.g.\,\text{sen}\theta$

$$\boxed{\ddot{\theta} = -\frac{k}{m}.\theta + \frac{1}{R}.g.\,\text{sen}\theta}$$

b) $\frac{k}{m}.\theta_{eq} = \frac{g}{R}.\text{sen}\,\theta_{eq}$

$$\boxed{\theta_{eq} = 0 \,,\; \frac{\text{sen}\,\theta_{eq}}{\theta_{eq}} = \frac{k.R}{m.g}} \; [2]$$

$$\frac{k.R}{m.g} = \frac{\text{sen}\,\theta_{eq}}{\theta_{eq}} < 1$$

$\text{sen}\,\theta < \theta < tg\,\theta$

si $\theta \neq 0$: $\frac{\text{sen}\,\theta}{\theta} < 1 < \frac{tg\,\theta}{\theta}$

$\Rightarrow$ necesito $k.R < m.g$ condición para [2] c)

$$\left.\frac{\partial\ddot\theta}{\partial\theta}\right|_{\theta_{eq}} = -\frac{k}{m} + \frac{g}{R}\cos\theta_{eq}$$

$\theta_{eq}=0$: $-\frac{k}{m}+\frac{g}{R}<0$

$\Leftrightarrow \quad m.g<k.R$

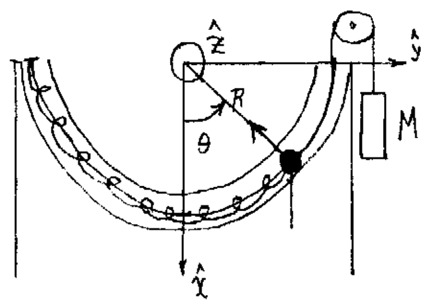
$\theta_{eq} = \theta_{eq}=\frac{mg}{kR}\,sen\,\theta_{eq}$: $-\frac{k}{m}+\frac{g}{R}\cdot\cos\left(\frac{m.g}{kR}\,sen\,\theta_{eq}\right)<0$

$\cos\left(\frac{m.g}{kR}\,sen\,\theta_{eq}\right)<\frac{k.R}{mg}$

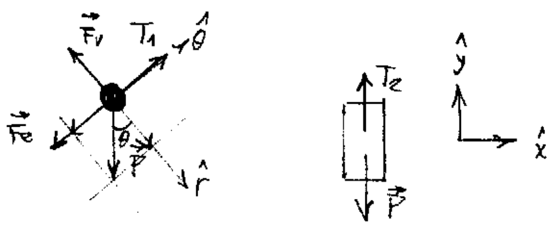
$\Leftrightarrow \quad \frac{kR}{m.g}<1 \Rightarrow k.R<m.g$

$\theta_{eq}=0$ es estable si $m.g<k.R$
θ_{eq} : $\theta_{eq}=\frac{m.g}{k.R}\,sen\,\theta_{eq}$ es estable si $k.R<m.g$

⑧



$\vec g\downarrow$

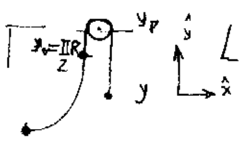


a)

$-\frac{s}{R}=\theta$

vinculo

$R=k \Rightarrow \dot r=\ddot r=0$



bola

r) $-m.R.\dot\theta^2 = -F_r + m.g.\cos\theta$

θ) $m.R.\ddot\theta = T_1 - k.\theta.R - m.g.sen\,\theta$

masa

ŷ) $M.a = T_2 - M.g$

$L=\frac{\pi}{2}R-R.\theta+y_p-y_0+y_p-y+\pi.r_p$

$0=-R.\ddot\theta-\ddot y \Rightarrow R.\ddot\theta=-\ddot y$

$0=-(-T_1)+(-T_2)=T_1-T_2 \Rightarrow T_1=T_2$

$m.R.\ddot\theta-M(-R.\ddot\theta)=T_1-T_2-k\theta R-mg\,sen\,\theta+M.g$

$(m+M).R.\ddot\theta = -k.\theta R+(M-m.sen\,\theta).g$

$$\boxed{\ddot\theta = -\frac{k.\theta}{(m+M)}+\frac{(M-m.sen\,\theta).g}{(m+M).R}}$$

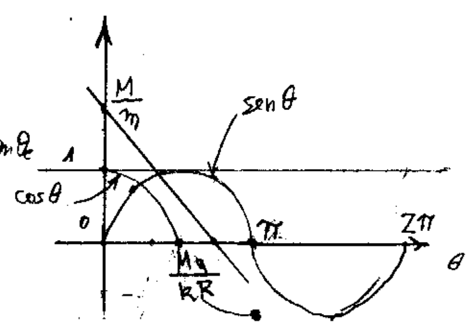
b)

$\frac{k.\theta}{(m+M)} = \frac{(M-m.sen\,\theta)}{(m+M).R}\cdot g$

$k.\theta_e.R = M-m.sen\,\theta_e$

$\frac{k.\theta_e.R}{mg}-\frac{M}{m}=-sen(\theta_{eq}) \Rightarrow$

$-A\theta_e+B=sen\,\theta_e$



$$\left.\frac{\partial\ddot\theta}{\partial\theta}\right|_{\theta_{eq}} = -\frac{k}{(m+M)}-\frac{m.g}{(m+M).R}\cos(\theta_e)$$

$-\frac{m.g}{(m+M).R}\cos(\theta_e)<\frac{k}{(m+M)}$

$\cos(\theta_e)>-\frac{k.R}{m.g}$

Necesito

$-\frac{k\theta_e R}{mg}+\frac{M}{m}<1$

$\frac{M}{m}-1<\frac{k\theta_e R}{\ldots}$

θ_{eq} : $-\frac{kR\theta_{eq}}{mg} + \frac{M}{m} = \text{sen}(\theta_{eq})$ como $0 \leq \theta_{eq} \leq \pi/2$

$\Downarrow$

$\text{sen}\,\theta_{eq} < \theta_{eq} < \text{tang}\,\theta_{eq}$ $0 \leq \text{sen}\,\theta_{eq} \leq 1$

Para estabilidad de θ_{eq} necesito:

$$1 > -\frac{k.R}{m.g} \cdot \frac{1}{\cos(\theta_{eq})}$$

$$1 \geq \text{sen}\,\theta_{eq} > -\frac{k.R}{m.g}\,\text{tang}\,\theta_{eq}$$

$$m.g.1 \geq \frac{-kR\theta_{eq} + Mg}{m.g} > -\frac{kR}{mg}.tg(\theta_{eq})$$

$$m.g - Mg \geq -kR\theta_{eq} > -kR.tg\,\theta_e - Mg$$

$$k.R.\theta_{eq} \geq Mg - mg$$

$$\theta_{eq} \geq \frac{Mg - mg}{kR} \quad [1]$$

necesito

$$0 \leq -\frac{kR\theta_e}{mg} + \frac{M}{m} \leq 1$$

$$0 \leq \frac{-kR\theta_e + Mg}{m.g} \leq 1$$

$$0 \leq -kR\theta_e + Mg \leq m.g$$

$$-M.g \leq -kR\theta_e \leq m.g - Mg$$

$$M.g \geq kR\theta_e \geq -mg + Mg$$

$$\frac{Mg}{kR} \geq \theta_e \geq \frac{Mg - mg}{kR}$$

$$[2] \quad \frac{kR}{Mg} \leq \theta_e \leq \frac{kR}{Mg - mg}$$

Como se cumple [1] gracias a [2] el θ_{eq} es estable

c)

$$F_v = m.g.\cos\theta + m.R.\dot\theta^2$$

$$\dot\theta.d\dot\theta = \left(-\frac{k}{(m+M)}\theta + \frac{Mg}{(m+M).R} - \frac{m.g}{(m+M).R}\,\text{sen}\,\theta\right)d\theta$$

$$\int_{\dot\theta_0}^{\dot\theta}\dot\theta.d\dot\theta = -\frac{k}{(m+M)}\int_{\theta_0}^{\theta}\theta.d\theta + \frac{Mg}{(m+M).R}\int_{\theta_0}^{\theta}d\theta - \frac{mg}{(m+M)/R}\int_{\theta_0}^{\theta}\text{sen}\,\theta.d\theta$$

$$V_0 = R.\dot\theta_0$$

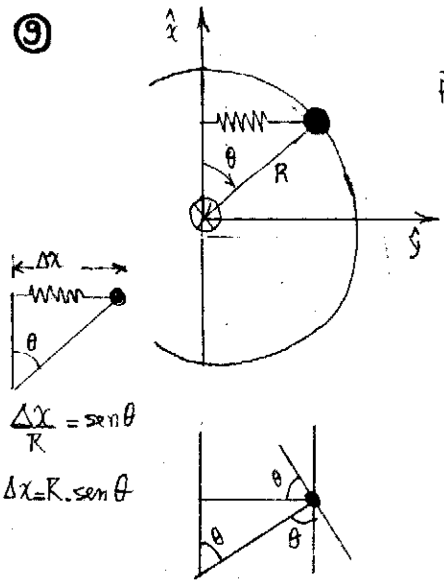
$$\frac{\dot\theta^2}{2} - \frac{\dot\theta_0}{2} = -\frac{k}{(m+M)}\frac{\theta^2}{2} + \frac{Mg}{(m+M).R}.\theta + \frac{mg.\cos\theta}{(m+M).R} - \frac{mg}{(m+M).R}$$

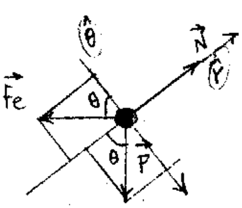
$$\dot\theta^2 = +\frac{\dot\theta_0 2}{2}. -\frac{k\theta^2}{(m+M)} + 2Mg\,\theta + 2mg\cos\theta - 2mg$$

$$F_v = m.g.\cos\theta + m.R\,\dot\theta_0 - \frac{mRk\theta^2}{(m+M)} + \frac{2Mg\,m\theta}{(m+M)} + \frac{2m^2g.\cos\theta}{(m+M)} - \frac{2Mg\,m}{(m+M)}$$

$$\boxed{F_v = m.g.\cos\theta\left(1 + \frac{2m}{m+M}\right) + \frac{2Mg\,m}{(m+M)}(\theta - 1) + mR.\left(\dot\theta_0 - \frac{k\theta^2}{m+M}\right)}$$

⑨





$R = k \Rightarrow \dot r = \ddot r = 0$

a) $\hat r)$ $-m.R.\dot\theta^2 = N - m.g.\cos\theta - k.R.\text{sen}\,\theta.\text{sen}\,\theta$

$\hat\theta)$ $m.R.\ddot\theta = m.g.\text{sen}\,\theta - k.R.\text{sen}\,\theta.\cos\theta$

b) $\theta(t=0) = \theta_0 = \pi/2$ $\dot\theta(t=0) = \dot\theta_0 = 0$

$$\ddot\theta = \frac{g}{R}.\text{sen}\,\theta - \frac{k}{m}.\text{sen}\,\theta.\cos\theta$$

$$\dot\theta.d\dot\theta = \frac{1}{R}.g.\text{sen}\,\theta.d\theta - \frac{k}{m}.\text{sen}\,\theta.\cos\theta.d\theta$$

$$\int_{\theta_0}^{\dot\theta} \dot\theta \, d\dot\theta = \frac{1}{R} \cdot g \int_{\theta_0}^{\theta} \operatorname{sen}\theta \, d\theta - \frac{k}{m}\int_{\theta_0}^{\theta} \operatorname{sen}\theta \cdot \cos\theta \cdot d\theta$$

$$\int v.du = u.v - \int u.dv$$

$$\frac{\dot\theta^2}{2} = g/_R \left(-\cos\theta\right) - \frac{k}{m}\left(\frac{\operatorname{sen}^2\theta}{2} - \frac{1}{2}\right)$$

CA:

$$\dot\theta^2 = -g/_R \cos\theta . 2 - \frac{k}{m}\operatorname{sen}^2\theta + \frac{k}{m}$$

$$\int \operatorname{sen}\theta . \cos\theta . d\theta$$

$$m R \dot\theta^2 = -mg2\cos\theta - kR \operatorname{sen}^2\theta + kR$$

$\operatorname{sen}\theta = v$ $du = \cos\theta d\theta$

$\cos\theta . d\theta = dv$ $u = \operatorname{sen}\theta$

$$N = -m.R.\dot\theta^2 + m.g.\cos\theta + kR\operatorname{sen}^2\theta$$

$$N = 2mg\cos\theta + kR\operatorname{sen}^2\theta - kR + mg\cos\theta + kR\operatorname{sen}^2\theta$$

$$\boxed{N = 3 m.g.\cos\theta + 2kR\operatorname{sen}^2\theta - kR}$$

$\int$

$2\int$ $= \operatorname{sen}\theta.\operatorname{sen}\theta - \int \operatorname{sen}\theta.\cos\theta.d\theta$

$= \frac{\operatorname{sen}^2\theta}{2}$

$\frac{1}{2}\operatorname{sen}^2\theta$

c) $$\ddot\theta = \frac{g}{R}\operatorname{sen}\theta - \frac{k}{m}.\operatorname{sen}\theta.\cos\theta$$

si $\ddot\theta = 0 \Rightarrow$ $\frac{k}{m}\operatorname{sen}\theta.\cos\theta = \frac{g}{R}\operatorname{sen}\theta$

si $\theta_e \neq 0, \neq \pi \Rightarrow$ $\frac{k}{m}.\cos\theta = \frac{g}{R}$

$$\cos\theta_{eq} = \frac{m.g}{kR}$$

$$\left[\begin{array}{l}\theta_{eq} = 0, \pi \\ \theta_{eq} = a\cos\left(\frac{mg}{kR}\right)\end{array}\right]$$

$$\left.\frac{\partial\ddot\theta}{\partial\theta}\right|_{\theta_{eq}} = \frac{g}{R}\cos\theta - \frac{k}{m}\left(\cos\theta.\cos\theta - \operatorname{sen}\theta.\operatorname{sen}\theta\right)$$

$$= \qquad \cos^2\theta - \operatorname{sen}^2\theta$$

$\cos^2 + \operatorname{sen}^2 = 1$

$\cos^2 - 1 = -\operatorname{sen}^2$

$$= \frac{g}{R}\cos\theta - \frac{k}{m}\left(2\cos^2\theta - 1\right)$$

θ_{eq} : $\cos\theta_{eq} = \frac{m.g}{kR}$ es estable si

$$g.\frac{1}{R}.\frac{m}{kR}.g - \frac{k}{m}\left(2.\frac{m^2g^2}{k^2R^2} - 1\right)$$

$$\frac{m.g^2}{k.R^2} - \frac{k}{m}\frac{2m^2g^2}{k^2R^2} + \frac{k}{m} < 0$$

$$-\frac{m.g^2}{kR^2} + \frac{k}{m} < 0$$

$$\frac{k}{m} < \frac{m.g^2}{R^2.k} \Leftrightarrow 0 < (k.R)^2 < (m.g)^2 \Rightarrow \boxed{k.R < m.g}$$

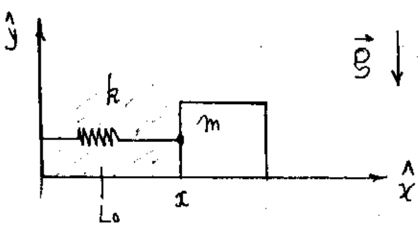
$\theta_{eq} = 0$ es estable si

$$\frac{g}{R} - \frac{k}{m} < 0 \Leftrightarrow \boxed{m.g < kR}$$

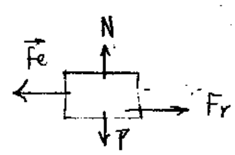
$\theta_{eq} = \pi$ es estable si

$$-\frac{g}{R} - \frac{k}{m} < 0 \Leftrightarrow \boxed{-kR < m.g} \text{ siempre}$$

10



$\overleftarrow{\vec{v}}$ v negativa $\Rightarrow$ $\overrightarrow{\vec{Fr}}$ positiva



$$\boxed{\vec{Fr} = -r.\frac{dx}{dt}.\hat{x}}$$

a) $\hat{x})$ $m.\ddot{x} = -k.(x - L_0) - r.\dot{x}$

$\hat{y})$ $N - m.g = 0$

$$\ddot{x} = -\frac{k}{m}x + \frac{kL_0}{m} - \frac{r.\dot{x}}{m}$$

$$\ddot{x} + \frac{r}{m}\dot{x} + \frac{k}{m}.x = \frac{kL_0}{m}$$

b) $\ddot{x} + \frac{r}{m}\dot{x} + \frac{k}{m}x = \frac{kL_0}{m}$

<u>solución del homogéneo</u>:

$$x = A e^{\alpha t}$$
$$\dot{x} = A\alpha e^{\alpha t}$$
$$\ddot{x} = A\alpha^2 e^{\alpha t}$$

$$A e^{\alpha t}\left(\alpha^2 + \frac{r}{m}\alpha + \frac{k}{m}\right) = 0$$

$$\alpha_1, \alpha_2 = \frac{-\frac{r}{m} \pm \sqrt{\frac{r^2}{m^2} - 4\frac{k}{m}}}{2} = -\frac{r}{2m} \pm \frac{1}{2}\sqrt{4}\cdot\sqrt{\frac{r^2}{4m^2} - \frac{k}{m}}$$

$$\underbrace{-\frac{r}{2m}}_{\beta \to} \pm \sqrt{\beta^2 - \omega^2}$$

$$x_h(t) = A_1 e^{-\beta t + \sqrt{\beta^2-\omega^2}\,t} + A_2 e^{-\beta t - \sqrt{\beta^2-\omega^2}\,t} = A_1 e^{-\beta t} e^{\sqrt{\beta^2-\omega^2}\,t} + A_2 e^{-\beta t} e^{-\sqrt{\beta^2-\omega^2}\,t}$$

$$x_h(t) = \left(A_1 e^{\sqrt{\beta^2-\omega^2}\,t} + A_2 e^{-\sqrt{\beta^2-\omega^2}\,t}\right) e^{-\beta t}$$

$$x_{nh}(t) = L_0 \Rightarrow \ddot{L}_0 + \frac{r}{m}\dot{L}_0 + \frac{k}{m}L_0 = \frac{kL_0}{m}$$
$$0 + 0 + \frac{kL_0}{m} = \frac{kL_0}{m}$$

$$\boxed{x(t) = \left(A_1 e^{\sqrt{\beta^2-\omega^2}\,t} + A_2 e^{-\sqrt{\beta^2-\omega^2}\,t}\right)e^{-\beta t} + L_0}$$

c) i) $\beta^2 > \omega^2 \Rightarrow \underbrace{\beta^2 - \omega^2}_{\phi} > 0$

$$x(t) = \left(A_1 e^{\sqrt{\phi}\,t} + A_2 e^{-\sqrt{\phi}\,t}\right) e^{-\beta t} + L_0$$

ii) $\beta^2 = \omega^2$

$$x(t) = (A_1 + A_2)\, e^{-\beta t} + L_0$$

iii) $\beta^2 < \omega^2 \Rightarrow \beta^2 - \omega^2 < 0$

$$x(t) = \left(A_1 e^{i\sqrt{\omega^2-\beta^2}\,t} + A_2 e^{-i\sqrt{\omega^2-\beta^2}\,t}\right) e^{-\beta t} + L_0$$

$$\left(A_1\left[\cos\sqrt{\omega^2-\beta^2}\,t - i\,\mathrm{sen}\sqrt{\omega^2-\beta^2}\,t\right] + A_2\left[\cos\sqrt{}\,t + i\,\mathrm{sen}\sqrt{}\,t\right]\right.$$

$$A_1\cos - A_1 i\,\mathrm{sen} + A_2\cos + A_2 i\,\mathrm{sen}$$

$e^{i\theta} = \cos\theta - i\,\mathrm{sen}\theta$

$e^{-i\theta} = \cos(-\theta) - i\,\mathrm{sen}(-\theta)$

$\cos\theta + i\,\mathrm{sen}\theta$

$$x(t) = \left[(A_1 + A_2)\cos\sqrt{}\;t + (A_2 - A_1)\,i\,\mathrm{sen}\sqrt{}\;t\right] e^{-\beta t} + L_0$$

$$\left(2a\cos\sqrt{}\;t + 2b\,\mathrm{sen}\sqrt{}\;t\right) e^{-\beta t} + L_0$$

$A_1 + A_2 \in \mathbb{R}$

$A_2 - A_1 \in \mathbb{C}$

$$\left(2\cos\varphi\, C\cos\sqrt{\omega^2-\beta^2}\,t + 2\,\mathrm{sen}\,\varphi\, C\,\mathrm{sen}\sqrt{\omega^2-\beta^2}\,t\right) e^{-\beta t} + L_0$$

$A_1 = a + bi$

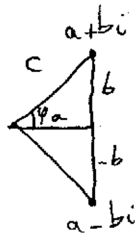
$A_2 = a - bi$

$A_1 + A_2 = 2a$

$A_2 - A_1 = -2bi$

$(A_2 - A_1)\,i = 2b$

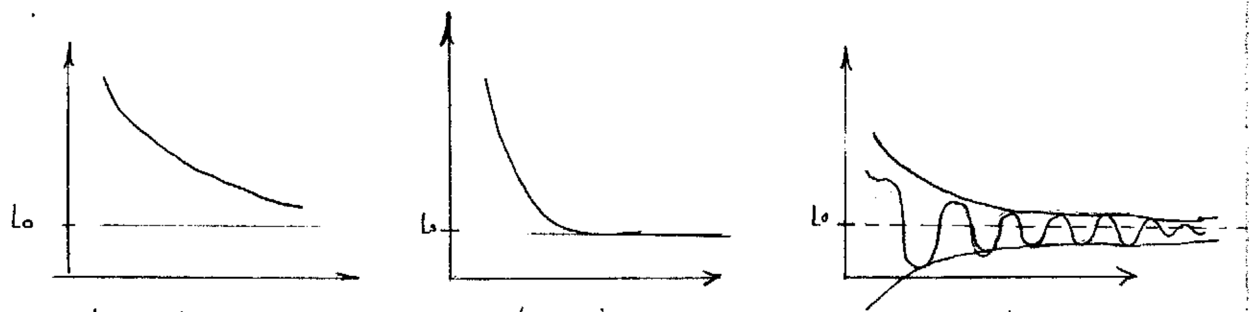
$$\underbrace{2C}_{A_0}\left(\cos\left(\sqrt{\omega^2-\beta^2}\,t + \varphi\right)\right) e^{-\beta t} + L_0 = x(t)$$



$|a+bi| = |a-bi| = c$

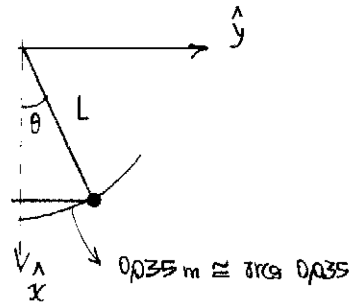
$\cos\varphi = \frac{a}{c}$ $2\cos\varphi\, C = 2a$

d)



$\nexists$ oscilaciones $\qquad$ $\nexists$ oscilaciones $\qquad$ $\exists$ oscilaciones

(11)



$T = 2\,seg$

$m = 0{,}01\ kg$

$A = 2^\circ$

$sen\,\theta \simeq \theta$

$$\ddot{\theta} = -\frac{g}{L}\cdot\theta$$

$$\ddot{\theta} + \frac{g}{L}\theta = 0$$

$\pi' \cong \pi$

$$x(t) = A.\cos\left(\sqrt{\frac{g}{L}}\,t + \varphi\right)$$

$$\dot{x}(t) = -A.\,sen\left(\sqrt{\frac{g}{L}}\,t + \varphi\right).\pi' \qquad x(t=0) = A.\cos(\varphi)$$

$A = 0{,}035$ amplitud

$$T = 2\pi.\sqrt{\frac{L}{g}} = 2$$

$$\frac{L}{g} = \left(\frac{2\ seg}{2\pi}\right)^2$$

$$\ddot{\theta} = -\frac{g}{L}\theta - \frac{r}{L}.\dot{\theta}$$

2° —— $0{,}035$ rad
$0{,}035$

$$L = \frac{4s^2}{4\pi^2}.\,9{,}8\,\frac{m}{s^2} = 0{,}993\ m$$

hay oscilaciones $\Rightarrow$

$\omega^2 > \beta^2$

$$x(t) = 0{,}035.\cos(\pi' t + \varphi)$$

$$x(t) = \underbrace{0{,}035.\,e^{-\beta t}}_{\text{nueva amplitud}}.\cos(\pi' t + \varphi)$$

$f = \frac{1}{T}$ $\qquad T = 2\pi.\sqrt{\frac{L}{g}}$ $\downarrow$ $T \cong 2\ seg$

$1{,}5^\circ$ —— $0{,}026$ rad

$$x(4) = 0{,}035.\,e^{-\beta.4}.\cos(\pi' t + \varphi)$$

2 oscilaciones son
$2T = 4\ seg$

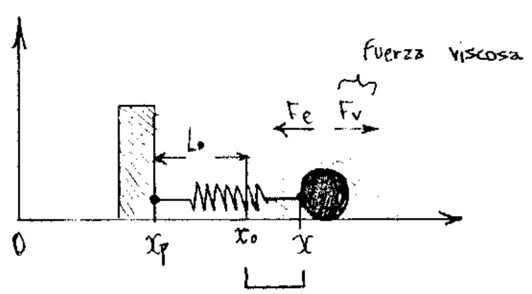
$$0{,}035.\,e^{-\beta.4} = 0{,}026$$
$$e^{-\beta.4} = 0{,}7428571$$
$$-\beta.4 = LN\ 0{,}742851$$
$$\boxed{\beta = 0{,}0743}$$

$$\beta = \frac{r}{2L} = 0{,}0743\,\frac{1}{seg}$$

$$\boxed{r = 0{,}1475}\ \frac{m}{s}$$

$-\frac{r}{L}.\frac{1}{s}$ $\quad$ $\frac{m/s}{m}$ $\quad$ $\frac{m}{s^2}$

(12)



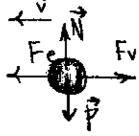
$x_p = L.\cos(\omega t)$ $\qquad$ la pared oscila

$$L_0 = x_0 - x_p$$

$$x - x_p = x - L_0 - x_p$$

$$x(-x_0 + x_p) - x_p$$

a)



$\hat{x})^{m}$ $m.\ddot{x} = -k.(x - L_0 - x_r) - r.\dot{x}$

$\hat{y})$ $0 = N - m.g$

$\hat{x})^{M}$ $M.\ddot{x}_r =$

$$m.\ddot{x} = -k.x + kL_0 + k.L\cos(\omega t) - r.\dot{x}$$

b)

$$\ddot{x} + \frac{k}{m}.x + \frac{r}{m}\dot{x} = +k\left[L_0 + L.\cos(\omega t)\right]$$