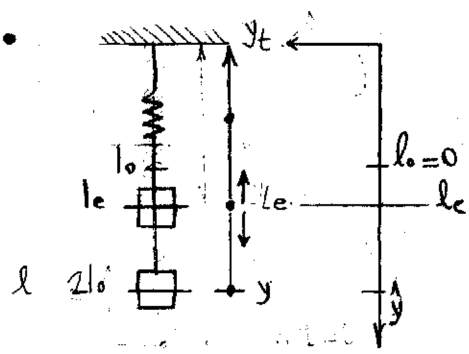
MOVIMIENTO OSCILATORIO

1.



$$\frac{d^2y}{dt^2} = -\frac{k}{m}y$$

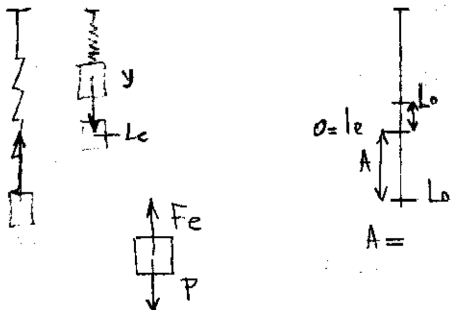
$$m.g - k.x = m.a$$

$$m.g - k.y = m.\frac{d^2y}{dt^2}$$

$$\frac{dv}{dy}\cdot\frac{dy}{dt} = -\frac{k}{m}y + g \qquad g - \frac{k}{m}y = \frac{d^2y}{dt^2}$$

$$\int_{v_0}^{v} v.\,dv = -\frac{k}{m}\int_{y_0}^{y} y.\,dy + g\int_{y_0}^{y} dy$$

$$\frac{v^2}{2} - \frac{v_0^2}{2} = -\frac{k}{m}\left(\frac{y^2}{2} - \frac{y_0^2}{2}\right) + g(y - y_0)$$



$$v^2 = -\frac{k}{m}\left(\frac{y^2}{2} - 2l_0^2\right) + g(y - 2l_0)$$

$$\left(\frac{dy}{dt}\right)^2 =$$

$\Rightarrow$

$$-\frac{k}{m}\left(\frac{y^2}{2} - 2l_0^2\right) + g.y - 2g.l_0$$

En equilibrio

$$m.g - k.\overset{(l_e - L_0)}{y} = 0 \Rightarrow$$

$$m.g = k\,l_e$$

$$\frac{1}{-\sqrt{\frac{k}{m}}}\cdot\left(\text{ARCO sen}\left(\frac{y}{2l_0}\right) - \text{ARCO sen}\left(\frac{2l_0}{2l_0}\right)\right) = t$$

$$\text{ARCO sen}\left(\frac{y}{2l_0}\right) - \frac{\pi}{2} = -\sqrt{\frac{k}{m}}.t$$

$$\boxed{y(t) = \text{sen}\left(-\sqrt{\frac{k}{m}}.t + \frac{\pi}{2}\right).2l_0}$$

$$y(0) = 2l_0$$

$y)$

$$m.g - k.(y - l_e) = m.a$$

$$m.g - k.(y - l_e) = m.\ddot{y}$$

$$\ddot{y} = g - \frac{k}{m}.y + \frac{k\,l_e}{m}$$

$$\ddot{y} + \frac{k}{m}.y = g + \frac{k\,l_e}{m}$$

Sol. h

$$y(t) = A.\cos\left(\sqrt{\frac{k}{m}}.t + \varphi\right)$$

Sol. nh

$$y = \frac{g.m}{k} + \frac{k.l_e.\cancel{m}}{\cancel{m}.k}$$

$$0 + \frac{\cancel{k}}{\cancel{m}}\frac{g\cancel{m}}{\cancel{k}} + \frac{\cancel{k}}{m}.\frac{k.l_e}{\cancel{k}} = g + \frac{k\,l_e}{m}$$

$$V(t) = \dot{y}(t) = -A.\,\text{sen}\left(\sqrt{\frac{k}{m}}.t + \varphi\right).\sqrt{\frac{k}{m}}$$

Sol. Completa

$$y(t) = A.\cos\left(\sqrt{\frac{k}{m}}.t + \varphi\right) + \frac{gm + k.L_e}{k}$$

$$y(t=0) = A.\cos\varphi + \frac{gm}{k} + l_e$$

en $t=0$ está a $(2L_0)$ del techo; es decir L_0 en nuestro sistema

$$V(t=0) = -A.\,\text{sen}\,\varphi.\sqrt{\frac{k}{m}}$$

$$L_0 = A.\overbrace{\cos(\varphi)}^{1} + \frac{gm + k.L_e}{k}$$

$$0 = -A.\,\text{sen}\,\varphi.\sqrt{\frac{k}{m}}$$

$$0 = \varphi \quad \Rightarrow$$

$$\boxed{y(t) = \left(\frac{L_0.k + gm - k.L_e}{k}\right).\cos(\omega.t) + \frac{gm + k.L_e}{k}}$$

$$-2L_0.\text{sen}(\omega t)$$

$$2L_0.\cos(\omega t) +$$

$$\frac{L_0.k}{k} - \frac{gm}{k} - l_e$$

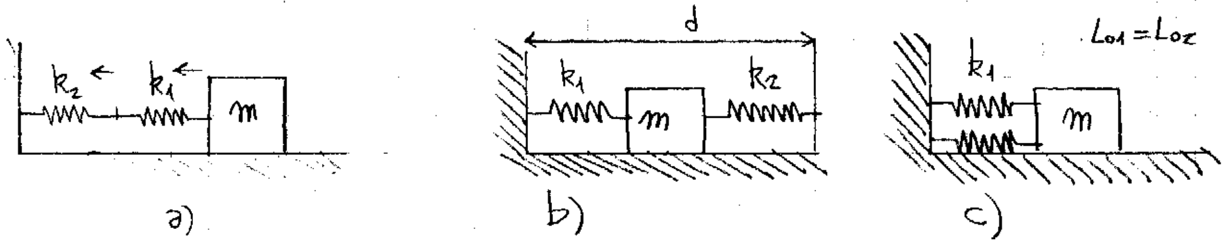
$$L_0 - \frac{m.g}{k} - \frac{m.g}{k}$$

$$L_0 - 2\frac{m.g}{k}$$

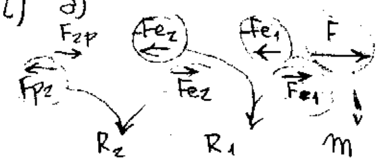
$$L_0 - 2L_e$$

2. Está hecho en la carpeta

3.



i) a)



$$-Fe_2 + Fe_1 = 0$$
$$Fe_1 = Fe_2$$

Luego
$$k_2 . x_2 = k_1 . x_1$$

$$T = \frac{2\pi}{\omega} \qquad f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$k_2 = \frac{k_1 . x_1}{x_2} \Rightarrow \frac{k_2}{k_1} = \frac{x_1}{x_2}$$

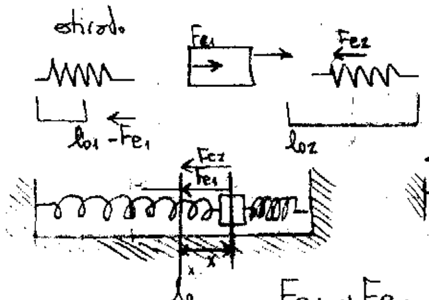
despl. total
$$x = x_1 + x_2$$
$$x = \frac{k_2 . x_2}{k_1} + \frac{k_1 . x_1}{k_2} = \frac{Fe_2}{k_1} + \frac{Fe_1}{k_2}$$
$$x = Fe . \left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$
$$x = Fe . \frac{k_2 + k_1}{k_1 . k_2}$$
$$\left[\frac{k_1 . k_2}{(k_1 + k_2)}\right] x = Fe_{total}$$

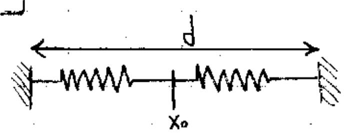
$$m . \ddot{x} = -\left(\frac{k_1 . k_2}{k_1 + k_2}\right) . x$$
$$m . \ddot{x} + \left(\frac{k_1 . k_2}{k_1 + k_2}\right) x = 0 \Rightarrow \ddot{x} + \frac{k_1 . k_2}{(k_1 + k_2) m} . x = 0$$

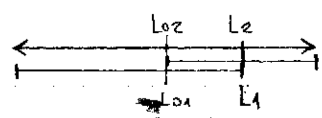
$$x(t) = A . \cos(\omega . t + \varphi) \qquad \text{con } \omega^2 = \frac{k_1 . k_2}{k_1 + k_2}$$

$$T = \frac{2\pi}{\sqrt{\frac{k_1 k_2}{k_1 + k_2}}} \qquad \boxed{f = \frac{1}{2\pi} . \sqrt{\frac{k_1 . k_2}{k_1 + k_2 m}}}$$

b y c)







$$Fe_1 + Fe_2 = m . a$$
$$-k_1 . x_1 - k_2 x_2$$

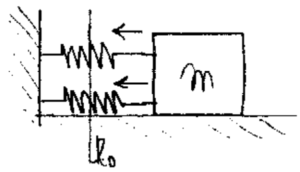
$$\ddot{x} = -\frac{k_1}{m} . x_1 - \frac{k_2}{m} . x_2$$
$$\ddot{x} = -\frac{k_1}{m} . x - \frac{k_2}{m} . x$$
$$\ddot{x} = -x \left(\frac{k_1 + k_2}{m}\right)$$
$$\frac{d^2 x}{dt^2} + \left(\frac{k_1 + k_2}{m}\right) x = 0$$

$$l_{01} + l_{02} = d$$
$$x_1 = L_1 - L_{01}$$
$$x_2 = L_2 - L_{02}$$

donde $L_{02} = L_{01}$ y $L_2 = L_1$
$$\Rightarrow x_1 = x_2 = x$$

$$x(t) = A . \cos\left(\sqrt{\frac{k_1 + k_2}{m}} . t + \varphi\right) \qquad \text{Luego} \quad f = \frac{\omega}{2\pi}$$

$$\boxed{f = \frac{1}{2\pi} \sqrt{\frac{k_1 + k_2}{m}}}$$

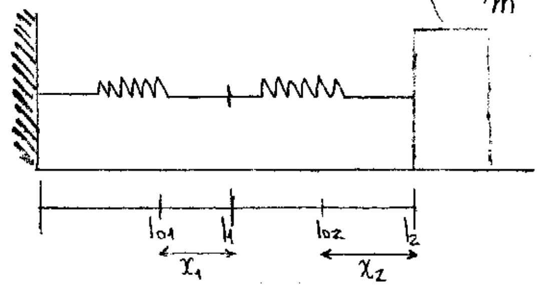


$$Fe_1 + Fe_2 = m.a \qquad x_1 = x_2$$

$$-\frac{k_1}{m}x_1 - \frac{k_2}{m}x_2 = \ddot{x}$$

$$-x\left(\frac{k_1+k_2}{m}\right) = \ddot{x} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$$

ii)



En la carpeta están resueltos 2 i) en forma más elegante

ii) Posición de equilibrio $\ddot{x} = 0$

b)

$$\frac{(k_1+k_2)}{\cancel{m}}x = \frac{k_1.L_{01} + k_2(d-l_{02})}{\cancel{m}}$$

$$\boxed{x_{eq} = \frac{k_1.L_{01} + k_2(d-l_{02})}{k_1+k_2}}$$

a)

$$\frac{k_1.k_2}{\cancel{m(k_1+k_2)}}x = \frac{(L_{01}+L_{02}).k_1.k_2}{\cancel{m(k_1+k_2)}}$$

$$\boxed{x_{eq} = L_{01} + L_{02}}$$

c)

$$-\frac{k_1}{\cancel{m}}.x_1 = \frac{k_2}{\cancel{m}}x_2$$

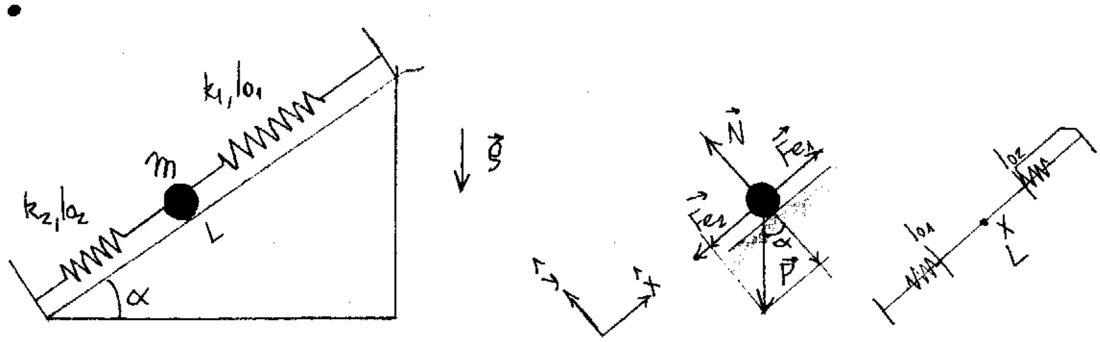
$$-k_1.(x - L_{01}) = k_2(x - L_{02})$$

$$-k_1x - k_2x = -k_1L_{01} - k_2L_{02}$$

$$x(k_1+k_2) = (k_1+k_2).L_{02} = (k_1+k_2).L_{01}$$

$$\boxed{x = L_{02} = L_{01}}$$

4.



a)

$\hat{x}$)

$$Fe_1 - Fe_2 - m.g.\operatorname{sen}\alpha = m.\ddot{x}$$

$$k_1.(x - L_{01}) - k_2.(L - x - L_{02}) - m.g.\operatorname{sen}\alpha = m.\ddot{x}$$

$\hat{y}$)

$$N - m.g.\cos\alpha = 0$$

$$N = m.g.\cos\alpha$$

$$\ddot{x} - \frac{k_1}{m}(x - L_{01}) + \frac{k_2}{m}(L - x - L_{02}) = -g.\operatorname{sen}\alpha$$

$$\ddot{x} - \frac{k_1}{m}x - \frac{k_2}{m}x = -g.\operatorname{sen}\alpha - \frac{k_1 L_{01}}{m} - \frac{k_2 L}{m} + \frac{k_2.L_{02}}{m}$$

$$\boxed{\ddot{x} - \left(\frac{k_1+k_2}{m}\right).x = -\frac{m.g.\operatorname{sen}\alpha - k_1 L_{01} + k_2 L - k_2 L_{02}}{m}}$$

b)

$$\frac{(k_1+k_2)}{\cancel{m}}.x = \frac{m.g.\operatorname{sen}\alpha + k_1 l_{01} - k_2 L + k_2 L_{02}}{\cancel{m}}$$

$$\boxed{x_e = \frac{m.g.\operatorname{sen}\alpha + k_1 L_{01} - k_2 L + k_2 L_{02}}{k_1+k_2}}$$

$$= \frac{k_1+k_2}{m}x_+ - \frac{m.g.\operatorname{sen}\alpha - k_1 l_{01} + k_2 L - k_2 L_{02}}{m}$$

$$p \quad = \frac{(k_1+k_2)x_e}{m} + -\frac{m.g.\operatorname{sen}\alpha - k_1 L_{01} + k_2 L - k_2 L_{02}}{m} + \underbrace{\frac{(k_1+k_2)}{m}}_{>0}.\Delta x$$

todos los x_e (en este caso uno) son inestables

c) $\ddot{x} - b.x = B \quad [1]$

$$\begin{cases} b = \dfrac{k_1+k_2}{m} \\ B = -\dfrac{m g \operatorname{sen}\alpha - k_1 L_{01} + k_2 (L - L_{02})}{m} \end{cases}$$

homogénea

$$x = A.e^{\alpha.t}$$
$$\dot{x} = A.\alpha.e^{\alpha t}$$
$$\ddot{x} = A.\alpha^2.e^{\alpha t}$$

$$A.\alpha^2.e^{\alpha t} - b.Ae^{\alpha t} = 0$$
$$\underbrace{A.e^{\alpha t}}_{>0}(\alpha^2 - b) = 0$$

particular

$$-b.x = B$$
$$x = -\frac{B}{b}$$

$$[B] = \frac{[l]}{[t]^2}$$
$$[b] = \frac{1}{[t]^2}$$

Análisis dimensional

$$\alpha^2 = b$$
$$\alpha = \sqrt{b} \begin{cases} +\sqrt{b} \\ -\sqrt{b} \end{cases}$$

$$x = A_1 e^{\sqrt{b}t} + A_2 e^{-\sqrt{b}t} \Rightarrow x_{nh} = A_1 e^{\sqrt{b}t} + A_2 e^{-\sqrt{b}t} - \frac{B}{b}$$

$$\dot{x}_{nh} = A_1 e^{\sqrt{b}.t}\sqrt{b} - A_2 e^{-\sqrt{b}.t}\sqrt{b}$$
$$\dot{x}_{nh} = \sqrt{b}.\left[A_1 e^{\sqrt{b}.t} - A_2 e^{-\sqrt{b}t}\right]$$

$$x(t=0) = A_1 + A_2 - \frac{B}{b} \Rightarrow A_1 + A_2 = \frac{B}{b}$$

en $x(t=0)$ la posición es $x_0 = x_{eq}$.

$$V(t=0) = V_0 = \sqrt{b}.(A_1 - A_2) \qquad A_1 - A_2 = \frac{V_0}{\sqrt{b}}$$

$$\frac{B}{b} - A_2 - A_2 = \frac{V_0}{\sqrt{b}}$$

$$-2A_2 = \frac{V_0}{\sqrt{b}} - \frac{B}{b} = \frac{\sqrt{b}.V_0 - B}{b}$$

$$A_1 = \frac{B}{b} + \frac{B - \sqrt{b}V_0}{2b} \qquad \boxed{A_2 = \frac{B - \sqrt{b}.V_0}{2b}}$$

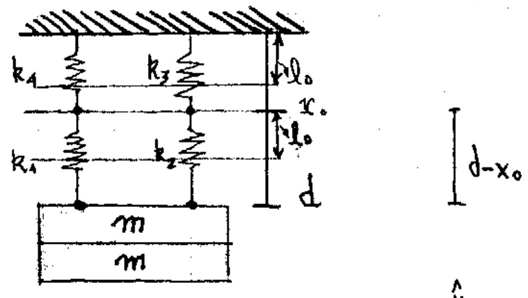
$$A_1 = \frac{2B + B - \sqrt{b}V_0}{2b} = \boxed{\frac{3B - \sqrt{b}.V_0}{2b} = A_1}$$

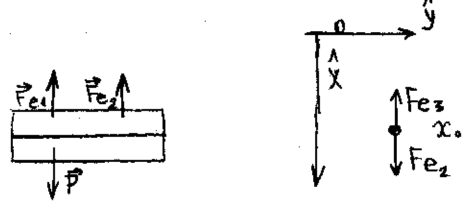
$$\boxed{x(t) = \left(\frac{3B - \sqrt{b}V_0}{2b}\right).e^{\sqrt{b}t} + \left(\frac{B - \sqrt{b}V_0}{2b}\right)e^{-\sqrt{b}.t} - \frac{B}{b}}$$

satisface [1] $\rightarrow$ $\dfrac{(3B - \sqrt{b}V_0)}{2\cancel{b}}.\cancel{b}.e^{\sqrt{b}t} + \dfrac{(B - \sqrt{b}V_0)}{2\cancel{b}}.\cancel{b}.e^{-\sqrt{b}t} - \dfrac{(3B - \sqrt{b}.V_0)}{2}e^{\sqrt{b}t} - \dfrac{(B - \sqrt{b}V_0)}{2}e^{-\sqrt{b}t} + \dfrac{\cancel{b}B}{\cancel{b}} = B$

5.

a)





$$2m.g - k_1(x_0 - l_0) - k_2(x_0 - l_0) = m.\ddot{x}$$
$$2g - \frac{2k}{m}(x_0 - l_0) = \ddot{x}$$
$$m.g = k(x_0 - l_0)$$
$$\boxed{\frac{g.m}{\left(\frac{d}{2} - l_0\right)} = k}$$

$$Fe_2 - Fe_3 = m.\overbrace{\ddot{x}_0}^{\to 0}$$
$$Fe_2 = Fe_3$$
$$k_2(d - x_0 - l_0) = k_3(x_0 - l_0)$$
$$d - x_0 - l_0 = x_0 - l_0$$
$$d = 2x_0$$
$$x_0 = \frac{d}{2}$$

$$Fe_4 - Fe_1 = m.\overbrace{\ddot{x}_0}^{\to 0}$$
$$Fe_4 = Fe_1$$
$$k_4(x_0 - l_0) = k_1(d - x_0 - l_0)$$

b) i)

$$m.g - 2k(x_0 - l_0) = m.\ddot{x}$$
$$g - \frac{2k}{m}(x_0 - l_0) = \ddot{x}$$
$$\ddot{x} + \frac{2k}{m}\left(\frac{x}{2} - l_0\right) = g$$
$$\ddot{x} + \frac{k}{m}x = g + \frac{2k.l_0}{m}$$

$$T = \frac{2\pi}{\omega_0} \qquad T = 2\pi.\sqrt{\frac{m}{k}} \qquad x_e =$$

$$\frac{k}{m}x_{eq} = g + \frac{2k.l_0}{m}$$
$$x_{eq} = \frac{g.m}{k} + 2l_0$$

ii)

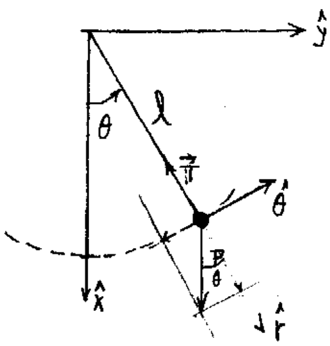
$$x_h = A.\cos(\omega.t + \varphi) \qquad x_{nh}^p = \frac{g.m}{k} + 2l_0$$
$$x(t) = A.\cos(\omega t + \varphi) + \frac{gm}{k} + 2l_0$$
$$x(t=0) = d = A.\cos\varphi + \frac{gm}{k} + 2l_0$$

$$\frac{dx}{dt} = \dot{x} = v(t) = -A.\mathrm{sen}(\omega t + \varphi).\omega$$
$$v(t=0) = 0 = -A.\mathrm{sen}(\varphi).\omega$$
$$\boxed{0 = \varphi} \Rightarrow \boxed{A = d - \frac{gm}{k} - 2l_0}$$

$$\boxed{x(t) = \left(d - \frac{gm}{k} - 2l_0\right).\cos(\omega t) + \frac{m.g}{k} + 2l_0}$$

6.

a)



$R = l = k \Rightarrow$
$\dot{r} = \ddot{r} = 0$

$\hat{r})\ -T + m.g.\cos\theta = m(-l.\dot{\theta}^2)$

$\hat{\theta})\ -m.g.\text{sen}\,\theta = m.l.\ddot{\theta}$

b) Es armónico mientras $\text{sen}\,\theta \cong \theta$. Bajo esta aproximación

$$T = \frac{2\pi}{\omega_0}$$

$$\ddot{\theta} = -g/l.\text{sen}\,\theta \cong -g/l.\theta \qquad \theta(t) = A.\cos\left(\sqrt{l/g}.t + \varphi_0\right)$$

$$\frac{d^2\theta}{dt} + \frac{g}{l}.\theta = 0$$

$$\boxed{T = 2\pi.\sqrt{\frac{l}{g}}}$$

c) en $t=0$, es $\theta=0$, y es $\dot{\theta}_0 = 0.2\ \frac{1}{seg}$

$$T = 2\pi.\sqrt{\frac{0.80\,m}{9.8\,\frac{m}{seg^2}}}$$

COND. INICIALES

$$\theta(t=0) = A.\cos(\varphi_0) = 0$$

$$\varphi_0 = \pi/2$$

$$\theta(t) = \theta_0.\cos\left(\sqrt{g/l}.t + \pi/2\right)$$

$$\int_{\dot{\theta}_0}^{\dot{\theta}} \dot{\theta}.d\dot{\theta} = -g/l\int_{\theta_0=0}^{\theta} \theta.d\theta$$

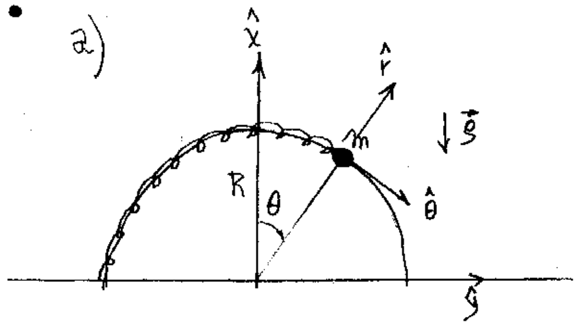
$$\frac{\dot{\theta}^2}{2} - \frac{\dot{\theta}_0^2}{2} = -g/l.\frac{\theta^2}{2}$$

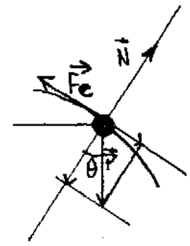
$$\dot{\theta}^2 = \dot{\theta}_0^2 - g/l.\theta^2$$

$$\dot{\theta}(\theta) = \sqrt{\dot{\theta}_0^2 - g/l.\theta^2}$$

7.

a)





$L = 2\pi R$ $\qquad$ $r = R = k \Rightarrow \dot{R} = \ddot{R} = 0$

$\frac{L}{4} = \frac{\pi R}{2} = l_0$

$\hat{r})\quad N - m.g.\cos\theta = m(-R.\dot{\theta}^2)$ $\qquad$ 2 ecuaciones, 2 incógnitas (θ, N)

$\hat{\theta})\quad -k.\underset{\left[\theta.R - \frac{\pi.R}{2}\right]}{\underset{[l-l_0]}{\theta.R}} + m.g.\operatorname{sen}\theta = m.R.\ddot{\theta}$

ARCO $\quad \frac{s}{R} = \theta$ en rad $\quad$ radio

b) Posiciones de equilibrio:

$\ddot{\theta} = 0$

$\ddot{\theta} = -\frac{k}{m}.\theta_e + \frac{g}{R}.\operatorname{sen}\theta_e \qquad$ si $\ddot{\theta} = 0 \Rightarrow$

$\frac{k}{m}.\theta_e = \frac{g}{R}.\operatorname{sen}\theta_e$

$x.b = \operatorname{sen} x$ $\qquad$ $\theta_e\frac{k.R}{m.g} = \operatorname{sen}\theta_e \qquad [1]$

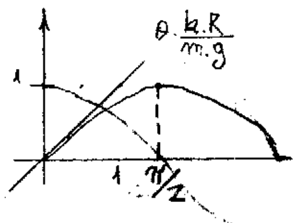
Puntos de equilibrio

$\boxed{\begin{array}{l}\theta = 0 \\ \theta_{eq}:\quad \theta_{eq}.\frac{k.R}{m.g} = \operatorname{sen}\theta_{eq}\end{array}}$

$0 = \operatorname{sen} x - x.b$

$= \cos x - b$

arco cos (b)



si: $0 \leq \theta \leq \pi/2$

si $\frac{k.R}{m.g} < 1 \Rightarrow \exists\ \theta_e$ que satisface [1]

si $\frac{k.R}{m.g} \geq 1 \Rightarrow \nexists\ \theta_e$ que satisface [1]

c) $\quad \ddot{\theta} = -\frac{k}{m}.\theta + \frac{g}{R}.\operatorname{sen}\theta$

Taylor $\ddot{\theta}(\theta)$:

$T = \left(-\frac{k}{m}.\theta + \frac{g}{R}.\operatorname{sen}\theta\right) + \left(-\frac{k}{m} + \frac{g}{R}.\cos\theta\right).\Delta\theta$

$\underline{\underline{\theta = 0}}\qquad T = (-0+0) + \left(-\frac{k}{m} + \frac{g}{R}\right).\Delta\theta = \underbrace{\frac{-k.R + m.g}{m.R}}.\Delta\theta$

$-k.R + mg < 0 \Leftrightarrow mg < kR$ (1)

$\theta = 0$ es estable $\Leftrightarrow$ (1) $\Rightarrow$ es el único θ_{eq} $\quad \theta'_{eq}$

$\underline{\underline{\theta = \theta'_{eq}}}\qquad T = \left(-\frac{\cancel{k}.\operatorname{sen}\theta'_e.\cancel{mg}}{\cancel{m}.\cancel{k}.R} + \frac{g}{\cancel{R}}.\theta_{eq}.\frac{k.R}{m.\cancel{g}}\right) + \left(-\frac{k}{m} + \frac{g}{R}.\cos\left[\frac{mg.\operatorname{sen}\theta'_e}{k.R}\right]\right)\Delta\theta$

$\left(-\frac{g}{R}\operatorname{sen}\theta'_e + \frac{k}{m}.\theta'_e\right) + \left(-\frac{k}{m} + \frac{g}{R}.\cos\theta'_{eq}\right).\Delta\theta$

$\left(-\frac{\cancel{g}}{\cancel{R}}.\theta'_e.\frac{k\cancel{R}}{m\cancel{g}} + \frac{k}{m}\theta'_e\right) + \left(\frac{-kR + gm.\cos\theta'_{eq}}{mR}\right).\Delta\theta$

$(\quad 0 \quad)$

$\frac{kR}{mg} > \cos\theta'_{eq}$ $\qquad$ condición de estabilidad

[Ya lo vimos arriba] $\qquad$ si $k.R > m.g \Rightarrow \nexists\ \theta'_{eq}$ como punto de equilibrio

si $k.R < m.g \Rightarrow \exists\ \theta'_{eq}$ como punto de equilibrio y

$\Rightarrow \frac{\operatorname{sen}\theta_e}{\theta_e} > \cos\theta_e$

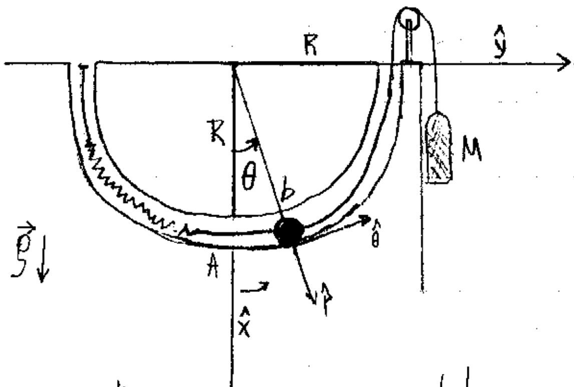
$m.g < k.R \Rightarrow$ Solo $\theta = 0$ es punto de equilibrio

$m.g > k.R \Rightarrow \theta = 0$ es inestable, Además $\exists\ \theta_{eq} : \left\{ \theta_{eq}.\frac{kR}{m.g} = \operatorname{sen}\theta_{eq} \right\}$
que es punto de equilibrio estable

$\rightarrow$ lo cual nos lleva a $\frac{\operatorname{sen}\theta_e}{\cos\theta_e} > \theta_e$

$\operatorname{tang}\theta_e > \theta_e$ lo cual se cumple siempre $\Rightarrow$

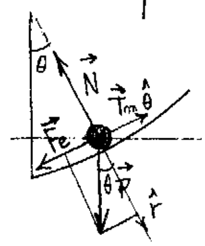
8.



k
$l_0 = \frac{\pi.R}{2}$
soga inextensible, masas soga resorte polea $\rightarrow 0$

$t_0 \Rightarrow (t=0)$: $A\ (\theta=0)$ V_0

a)



bola

$\hat{r})\quad -N + m.g.\cos\theta = m(-R.\dot{\theta}^2)$

$\hat{\theta})\quad \underset{-k.[\theta.R]}{-Fe} + T_m - m.g.\operatorname{sen}\theta = m.R.\ddot{\theta}$

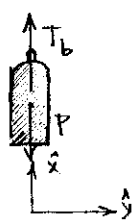
$\frac{\text{arco}}{R} = \theta$
$\theta.R$

VINCULOS
$r = R \Rightarrow \dot{r} = \ddot{r} = 0$

masa

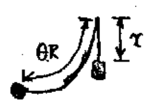
$\hat{x})\quad Tb - M.g = M.a_M$

θ, T_m, N
Tb, a_M



soga

Long constante $\rightarrow L = \left(\frac{\pi}{2} - \theta\right)R + x$



$L = \frac{\pi}{2}R - \theta.R + x$

$\frac{dL}{dt} = \frac{\pi}{2} - \dot{\theta}R + \dot{x}$

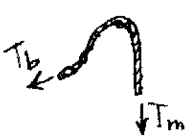
$\frac{d^2L}{dt^2} = 0 - \ddot{\theta}R + \ddot{x} = 0$

$\boxed{\ddot{\theta}R = \ddot{x}}$

$\ddot{\theta}R = a_M$

$Tb - Tm = \underbrace{m_s}_{0}.a_M$

$T_b = T_m = T$



bola $\hat{r})\quad -N + m.g.\cos\theta = -m.R.\dot{\theta}^2$

$\hat{\theta})\quad -k.\theta.R + T - m.g.\operatorname{sen}\theta = m.R.\ddot{\theta}$

masa $\hat{x})\quad T - M.g = m.\ddot{\theta}.R$

$T = M.\ddot{\theta}.R + M.g \Rightarrow$

$-k.\theta.R + M.\ddot{\theta}.R + M.g - m.g.\operatorname{sen}\theta = m.R.\ddot{\theta}$

$-k.\theta.R + Mg - m.g.\operatorname{sen}\theta = \ddot{\theta}.R.[m - M]$

$$\frac{d\dot\theta}{d\theta}\cdot\frac{d\theta}{dt}\cdot R\,[m-M]$$

$$[m-M]\,\dot\theta\cdot R\cdot\frac{d\dot\theta}{d\theta} = M.g - k.R.\theta - m.g.\operatorname{sen}\theta$$

$$(m-M).\dot\theta R\,.\,d\dot\theta = M.g.d\theta - k.R.\theta.d\theta - m.g.\operatorname{sen}\theta.d\theta$$

$$(m-M).R\int_{\dot\theta_0}^{\dot\theta}\dot\theta.d\dot\theta = M.g\int_0^\theta d\theta - k.R\int_0^\theta \theta.d\theta - m.g.\int_0^\theta \operatorname{sen}\theta.d\theta \qquad \theta_0=0 \quad V_0=R.\dot\theta_0$$

$$[m-M].R\left(\frac{\dot\theta^2}{2}-\frac{\dot\theta_0^2}{2}\right) = M.g.\theta - k.R.\frac{\theta^2}{2} + m.g.\cos\theta$$

$$(m-M).R.\frac{\dot\theta^2}{2} = M.g\theta - \frac{k}{2}.R.\theta^2 + m.g.\cos\theta + \frac{\dot\theta_0^2}{2}.R.[m-M]$$

b) $\ddot\theta=0$ $\qquad$ $\ddot\theta = -\frac{k}{m}.\theta + \frac{Mg}{mR} - \frac{g}{R}.\operatorname{sen}\theta$

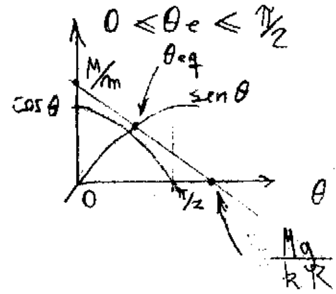
$$-k.\theta_e R + M.g - m.g.\operatorname{sen}\theta_e = 0$$

$$-\frac{k.\theta_e R}{m.g} + \frac{M}{m} - \operatorname{sen}\theta_e = 0$$

$$-\frac{k.\theta_e R}{m.g} + \frac{M}{m} = \operatorname{sen}\theta_e$$

$$\frac{-k\theta_e.R + g.M}{m.g} = \operatorname{sen}\theta_e$$

donde $0 \leq \theta_e \leq \pi/2$



$\frac{g}{k}\cdot\frac{M}{m}\Big/\frac{m}{R} = \theta_{eq}$

$$-\frac{k.R}{m.g}.\theta_{eq} + \frac{M}{m} \leftarrow \text{función lineal}$$

equilibrio $T_{eq} = M.g$ $\qquad$ equilibrio $-k.\theta_e.R + T - m.g.\operatorname{sen}\theta_{eq} = 0$

$0 \leq \theta_{eq} \leq \pi/2 \Rightarrow 0 \leq \operatorname{sen}\theta_{eq} \leq 1 \Rightarrow$ $\qquad 0 \leq \frac{-k.\theta_e R + M.g}{mg} \leq 1$ $\quad$ (1)

$$0 \leq -k.\theta_e R + M.g \leq m.g$$

$$-\frac{Mg}{mg} \leq -\frac{kR\,\theta_e}{mg} \leq 1 - \frac{Mg}{mg}$$

$$\frac{M}{m} \geq \frac{kR}{mg}\theta_e \geq \frac{M}{m} - 1$$

$$\frac{Mg}{k.R} \geq \theta_{eq} \geq \frac{Mg - mg}{kR} \Leftarrow -\frac{M.g}{kR} \leq -\theta_{eq} \leq \frac{m.g - Mg}{k.R}$$

Taylor $\quad P_{(\theta)} = \left(-\frac{k}{m}\theta + \frac{Mg}{mR} - \frac{g}{R}.\operatorname{sen}\theta\right) + \left(-\frac{k}{m} + \frac{g}{R}.\cos\theta\right).\Delta\theta$

$$P_{(\theta_{eq})} = -\frac{k}{m}\theta_e + \frac{Mg}{mR} + \frac{g}{R}\frac{k\theta_e.R}{m.g} - \frac{M}{m}.\frac{g}{R} + \left(-\frac{k}{m} + \frac{g}{R}.\cos\theta_{eq}\right)\Delta\theta$$

$$0 > -\frac{k}{m} + \frac{g}{R}\cos\theta_e$$

$$\frac{k.R}{m.g} > \cos\theta_e$$

Si θ_{eq} satisface (1) $\Rightarrow$ es estable

$$\frac{1}{\cos\theta_e}.\frac{kR}{m.g} > 1$$

$$tg\,\theta_e.\frac{kR}{m.g} > \operatorname{sen}\theta_e$$

$$tg\,\theta_e.\frac{kR}{mg} \geq \frac{kR}{m.g}\theta_e \geq \underbrace{\frac{M}{m}}_{>1} - 1 > \operatorname{sen}\theta_e$$

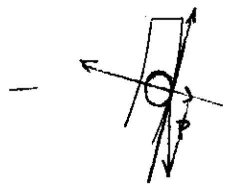
$$\boxed{\theta_{eq}\text{ es estable}}$$

c) $N = m.R.\dot{\theta}^2 + m.g.\cos\theta$

$$N = m.\left[\frac{2.M.g}{[m-M]}.\theta - \frac{k.R.\theta^2}{[m-M]} + \frac{2mg\cos\theta}{[m-M]} + \dot{\theta}_0.R\right] + m.g.\cos\theta$$

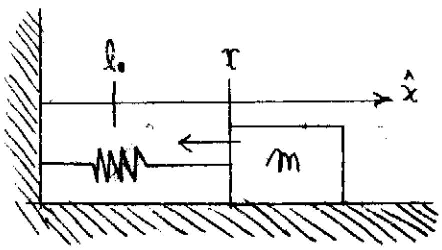
$$\boxed{N = \frac{2m.Mg}{m-M}.\theta - \frac{k.R.\theta^2 m}{m-M} + \frac{2m^2 g\cos\theta}{m-M} + m.\dot{\theta}_0^2.R + m.g.\cos\theta}$$

$\frac{[m].[1].[l]}{[s^2]}$



9. Hecho en la carpeta con lujo de detalle

10.



$F_r = r.\dot{x} = r.\frac{dx}{dt}$

a) $\vec{F_r} = -r.\dot{x}.\hat{x}$

b) $m\ddot{x} = -k.x - r.\dot{x}$

Se trata de un movimiento en un seno resistivo.

$$\ddot{x} + \frac{r}{m}.\dot{x} + \frac{k}{m}.x = 0$$

$$\left.\begin{array}{l} x = A.e^{\alpha t} \\ \dot{x} = A.\alpha.e^{\alpha t} \\ \ddot{x} = A\,\alpha^2.e^{\alpha t} \end{array}\right\} \Rightarrow \underbrace{A.e^{\alpha t}}_{\neq 0}\left(\alpha^2 + \frac{r}{m}\alpha + \frac{k}{m}\right) = 0$$

$$\alpha_{1,2} = \frac{-r/m \pm \sqrt{r^2/m^2 - 4.k/m}}{2}$$

$$-\frac{r}{2m} \pm \frac{\cancel{2}}{\cancel{2}}\sqrt{\frac{r^2}{4m^2} - \frac{k}{m}}$$

Sean $\beta = \frac{r}{2m}$

$\omega = \frac{k}{m}$

$$x = A_1 e^{-\beta t + \sqrt{\beta^2-\omega^2}t} + A_2.e^{-\beta t - \sqrt{\beta^2-\omega^2}t}$$

$$x = A_1.e^{-\beta t}.e^{\sqrt{\beta^2-\omega^2}t} + A_2.e^{-\beta t}.e^{-\sqrt{\beta^2-\omega^2}.t}$$

$$\boxed{x(t) = e^{-\beta t}\left[A_1.e^{\sqrt{\beta^2-\omega^2}t} + A_2.e^{-\sqrt{\beta^2-\omega^2}t}\right]}$$

c) i) $x(t) = e^{-\beta t}\left(A_1.e^{\sqrt{\beta^2-\omega^2}t} + A_2.e^{-\sqrt{\beta^2-\omega^2}t}\right)$ con $\beta^2 > \omega^2$

ii) $x(t) = e^{-\beta t}.\left(A_1 + A_2\right)$

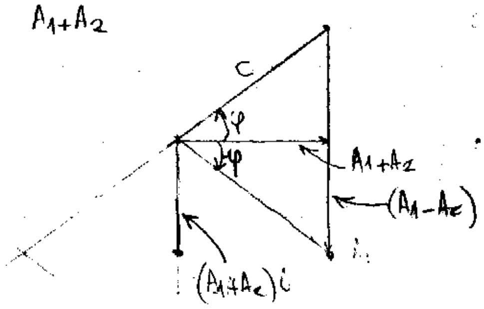
iii) $x(t) = e^{-\beta t}.\left(A_1.e^{i\sqrt{\omega^2-\beta^2}t} + A_2.e^{-i\sqrt{\omega^2-\beta^2}.t}\right)$

$$x(t) = e^{-\beta t}\left(A_1.\left[\cos\sqrt{\omega^2-\beta^2}.t + i.\text{sen}\sqrt{\omega^2-\beta^2}.t\right] + A_2.\left[\cos\left(-\sqrt{\omega^2-\beta^2}\right)t + i.\text{sen}\left(-\sqrt{\omega^2-\beta^2}\right)t\right]\right)$$

$$x(t) = e^{-\beta t}.\left[A_1.\cos\sqrt{\beta^2-\omega^2}t + A_1.i.\text{sen}\sqrt{\beta^2-\omega^2}t + A_2.\cos\sqrt{\beta^2-\omega^2}t - A_2.i.\text{sen}\sqrt{\beta^2-\omega^2}t\right]$$

$$e^{-\beta t}\left[(A_1+A_2).\cos\sqrt{\beta^2-\omega^2}t + (A_1-A_2)\,i.\text{sen}\sqrt{\beta^2-\omega^2}t\right]$$

A_1+A_2



$$\underbrace{A_1+A_2}_{\in \mathbb{R}} = \cos\varphi \cdot \overset{hip}{C}$$

$$(A_1-A_2).i = \operatorname{sen}(-\varphi).C = -\operatorname{sen}(\varphi).C$$

$$\sqrt{(A_1+A_2)^2+(A_1-A_2)i^2} = C$$

$$A_1^2+2A_1A_2+A_2^2 - A_1^2+2A_1A_2-A_2^2 = C$$

$$4A_1A_2 = C$$

$$\varphi = \operatorname{Arcotg}\left(\frac{A_1-A_2}{A_1+A_2}\right)$$

$$x(t) = e^{-\beta t}\left[(A_1+A_2).\cos\sqrt{}\,t + (A_1-A_2).i.\operatorname{sen}\sqrt{}\,t\right]$$

$$\left[C.\cos\varphi.\cos\sqrt{}\,t + (-\operatorname{sen}\varphi).C.\operatorname{sen}\sqrt{}\,t\right]$$

$$x(t) = e^{-\beta t}.C.\cos\left(\varphi+\sqrt{\beta^2-\omega^2}\,t\right)$$

esto está mal

$$Z_1+Z_2 = R \in \mathbb{R} \Leftrightarrow \quad Z_2 = \overline{Z}_1 \Rightarrow$$

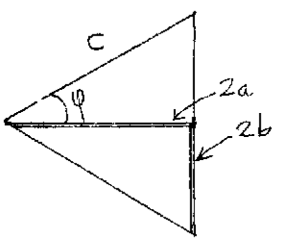
$$Z_1 = a+i.b$$
$$Z_2 = a-i.b$$

$$A_1+A_2 = a_1+a_2 \in \mathbb{R}$$
$$A_1-A_2 = a_1-a_2+b_1i+b_2i \in \mathbb{C}$$

si $\left.\begin{array}{l} a_1=a_2 \\ b_1=b_2 \end{array}\right\} \Rightarrow \quad \begin{array}{l} A_1+A_2 = 2a \\ A_1-A_2 = 2bi \end{array}$

$$x(t) = e^{-\beta t}\left[2a.\cos\sqrt{}\,t + 2bi\,i.\operatorname{sen}\sqrt{}\,t\right]$$
$$x(t) = e^{-\beta t}\left[2a.\cos\sqrt{}\,t - 2b.\operatorname{sen}\sqrt{}\,t\right]$$

$$x(t) = e^{-\beta t}\left[\cos\varphi.C.\cos\sqrt{}\,t - \operatorname{sen}\varphi.C.\operatorname{sen}\sqrt{}\,t\right]$$

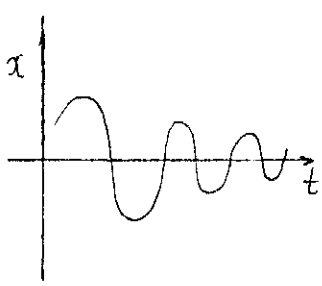


$$\begin{array}{l} 2a = \cos\varphi.C \\ 2b = \operatorname{sen}\varphi.C \end{array}$$

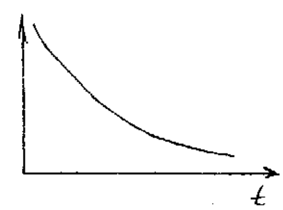
$$\boxed{x(t) = e^{-\beta t}.C.\cos\left(\varphi+\sqrt{}\,t\right)}$$

$$C = \sqrt{4a^2+4b^2}$$
$$C = \sqrt{4(a^2+b^2)} = 2\sqrt{a^2+b^2}$$
$$C = 2\sqrt{\left(\frac{A_1+A_2}{2}\right)^2+\left(\frac{A_1-A_2}{2}\right)^2}$$
$$C = 2\sqrt{A_1.A_2}$$

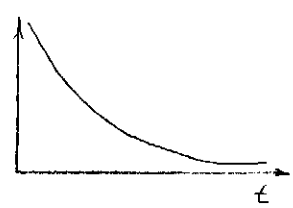
d) i)



ii



iii)



11.

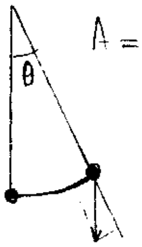
$0{,}01$ kg m

oscilaciones pequeñas

$\tau = 2$ seg

$A = \left(\pi/90\right) \cdot l$

$2° \text{———} \pi/90$
$180° \text{———} \pi$



$A = 0{,}0346$ m

$m \cdot \ddot{\theta} = -\frac{g}{l} m \operatorname{sen}\theta$

$m \cdot \ddot{\theta} \cong -\frac{g}{l} \cdot \theta \cdot m$

$\ddot{\theta} + \frac{g}{l}\theta = 0$

$\theta(t) = A \cdot \cos\left(\sqrt{g/l}\, t + \varphi\right)$

Como el péndulo sigue oscilando luego del rozamiento $\Rightarrow$
$\beta^2 - \omega^2 < 0$

$\tau = \frac{2\pi}{\omega} = 2\pi \cdot \sqrt{\frac{l}{g}}$

$g \cdot \left(\frac{2 \text{ seg}}{2\pi}\right)^2 = l$

$\boxed{0{,}993 m = l}$

$m \cdot \ddot{\theta} l = -m \cdot g \cdot \theta - r \cdot \dot{\theta}$

$\ddot{\theta} = -\frac{g\theta}{l} - \frac{r}{ml}\dot{\theta} \Rightarrow \ddot{\theta} + \frac{r}{m l}\dot{\theta} + \frac{g\theta}{l} = 0$

$\dot{\theta}(t) = -A \cdot \operatorname{sen}\left(\sqrt{g/l} \cdot t + \varphi\right)\sqrt{g/l}$

$0 = \operatorname{sen}(\varphi) \Rightarrow \varphi = 0$

$\theta = A \cdot e^{\alpha t}$

$\underbrace{A \cdot e^{\alpha t}}_{\neq 0}\left(\alpha^2 + \frac{\alpha r}{m l} + \frac{g}{l}\right) = 0$

$\frac{-r/lm \pm \sqrt{\frac{r^2}{m^2 l^2} - 4 \cdot g/L}}{2}$

$\frac{-r/lm \pm \sqrt{4}\sqrt{\frac{r^2}{2^2 m^2 l^2} - g/L}}{2}$

$\theta(t) = A_1 \cdot e^{-\beta t + \sqrt{\beta^2 - \omega^2}\, t} + A_2 \cdot e^{-\beta t - \sqrt{\beta^2 - \omega^2}\, t}$

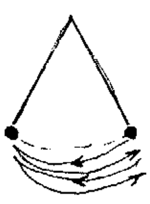
$\theta(t) = e^{-\beta t} \cdot \left[A_1 \cdot e^{i\sqrt{\omega^2 - \beta^2}\, t} + A_2 \cdot e^{-i\sqrt{\omega^2 - \beta^2}\, t}\right]$

$\alpha_1, \alpha_2 = \underbrace{\frac{-r}{2lm}}_{-\beta} \pm \sqrt{\underbrace{\left(\frac{r}{2lm}\right)^2 - g/L}_{\beta^2 - \omega^2}}$

$\theta(t) = C \cdot e^{-\beta t} \cdot \left(\cos \underbrace{\omega' t}_{\sqrt{\omega^2 - \beta^2}} + \varphi\right)$

$[r] = \frac{kg}{seg}$

$\theta(t) = C \cdot e^{-\beta t} \cdot \cos\left(\sqrt{\omega^2 - \beta^2}\, t\right)$



$\beta^2 < \omega^2$

$\frac{r^2}{4 l^2 m^2} < \frac{g}{l}$

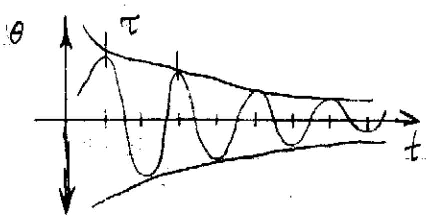
$r^2 < g\, l\, m^2 4$

$r < 0{,}0624$

$\theta(t=0) = C = \frac{\pi}{90}$

$\theta(t) = \frac{\pi}{90} \cdot e^{-\beta t} \cdot \cos\left(\sqrt{\omega^2 - \beta^2}\, t\right)$

$\theta(2\tau) = \frac{\pi}{90} \cdot e^{-\beta \cdot 2\tau} \cdot \cos\left(\sqrt{\omega^2 - \beta^2} \cdot 2\tau\right)$



2 oscilaciones completas ——— 2τ

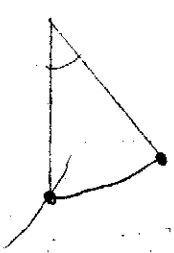
$\frac{\pi}{120} = \frac{\pi}{90} \cdot e^{-\beta \cdot 2\tau} \cdot \cos \cancel{\sqrt{\omega^2 - \beta^2}} \cdot \frac{2\pi \cdot 2}{\cancel{\sqrt{\omega^2 - \beta^2}}}$

$\frac{3}{4} = e^{-\beta \cdot 2\tau}$

$A = 1{,}5°$

$\frac{4}{3} = e^{\beta \cdot 2\tau} \Rightarrow \ln \frac{4}{3} = \beta \cdot 2\tau$

$1{,}5° \text{———} \pi/120$
$180° \text{———} \pi$



$0{,}287 = \frac{r}{2lm} \cdot 2 \cdot \frac{2\pi}{\sqrt{\frac{g}{L} - \left(\frac{r}{2lm}\right)^2}}$

$0{,}287 = \frac{2\pi \cdot r}{m \cdot l \sqrt{\frac{4m^2 gL - r^2}{2^2 l^2 m^2}}}$

arco $= \frac{\pi}{120} \cdot 0{,}993 = \frac{\pi \cdot l}{120}$

$0{,}0004535 = \frac{r}{\sqrt{}}$

resistencia

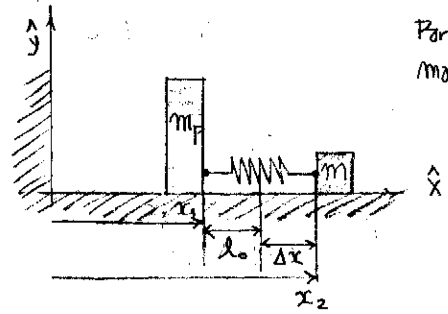
$\frac{4m^2 g l - r^2}{2^2 l^2 m^2} = \left(\frac{1}{0{,}0004535}\right)^2 \cdot r^2$

$$\frac{g}{l} = 4860684 \cdot r^2 + \frac{1}{2lm} r^2$$

$$\frac{g}{l} = 4860735 \cdot r^2$$

$$0{,}001425 = r$$

12.



Pared 1
masa 2

desplazamiento

$$l_0 + x = x_2 - x_1$$
$$x = x_2 - x_1 - l_0$$
$$\dot{x} = \dot{x}_2 - \dot{x}_1$$
$$\ddot{x} = \ddot{x}_2 - \ddot{x}_1$$

a) masa (supongo resorte estirado)

F_e, F_r

$\hat{x}$) $m_2 \ddot{x}_2 = \underbrace{-k \cdot x}_{F_e} \underbrace{- r \cdot \dot{x}_2}_{F_r}$

LA PARED SE MUEVE CON M.A.S.

$$x_p = L \cdot \cos(\omega t)$$
$$\dot{x}_p = -L \cdot \operatorname{sen}(\omega t) \cdot \omega$$
$$\ddot{x}_p = -L \cdot \omega^2 \cdot \cos(\omega t)$$

pared

$\hat{x}$) $m_1 \cdot \ddot{x}_1 = k \cdot x$

b) $\ddot{x}_2 + \frac{r}{m_m}\dot{x}_2 + \frac{k}{m_m} x = 0$

$$x = A \cdot e^{\alpha t}$$
$\dot{x}$
$\ddot{x}$

$$x(t) = A_1 \cdot e^{-\beta t + i\sqrt{\omega^2-\beta^2}\, t} + A_2 \cdot e^{-\beta t - i\sqrt{\omega^2-\beta^2}\, t}$$

$$x(t) = e^{-\beta t} \cdot \left(A_1 \cdot e^{i\sqrt{\ }\, t} + A_2 \cdot e^{-i\sqrt{\ }\, t} \right)$$

$$\alpha^2 + \alpha \cdot \frac{r}{m} + \frac{k}{m} = 0$$

$$x(t) = A_0 \cdot e^{-\beta t} \cdot \cos\left(\sqrt{\omega^2-\beta^2}\, t + \varphi_0\right)$$

$$\alpha_{1,2} = -\frac{r}{m} \pm \sqrt{\frac{r^2}{m^2} - 4 \cdot \frac{k}{m}}$$

$$\alpha_{1,2} = -\underbrace{\frac{r}{2m}}_{\beta} \pm \sqrt{\underbrace{\frac{r^2}{4m^2}}_{\beta^2} - \underbrace{\frac{k}{m}}_{\omega^2}}$$

$$\beta t \gg 1 \Rightarrow e^{-\beta t} \longrightarrow 1$$
$$t \gg \frac{1}{\beta} \Rightarrow \beta \longrightarrow 0$$

$$\omega^2 > \beta^2 \Rightarrow$$
$$\beta^2 - \omega^2 = \sqrt{-1 \cdot \omega^2 - \beta^2} \quad i\underbrace{\sqrt{\omega^2-\beta^2}}_{\in \mathbb{R}}$$

$x(t) \cong A_0 \cdot \cos(\omega \cdot t)$ para $t \gg 1/\beta$

$$m_1 \cdot m_2 \cdot \ddot{x}_2 = -k x m_1 - r \cdot m_1 \dot{x}_2$$

$$m_1 \cdot m_2 \cdot \ddot{x}_1 = k \cdot x\, m_2$$

$$m_1 \cdot m_2 \left(\ddot{x}_2 - \ddot{x}_1\right) = -kx \cdot (m_1 + m_2) - r\, m_1 \dot{x}_2$$

$$\frac{m_1 \cdot m_2}{m_1 + m_2}\left(\ddot{x}_2 - \ddot{x}_1\right) + kx + \frac{m_1}{(m_1+m_2)} r \cdot \dot{x}_2 = 0$$

$$\frac{m_1 \cdot m_2}{m_1 + m_2}\ddot{x} + k \cdot x + \frac{m_1}{m_1+m_2} \cdot r \cdot \dot{x}_2 = 0$$