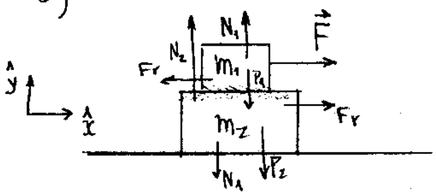
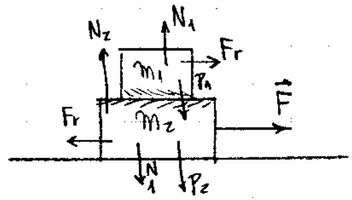
Interacción de Rozamiento

①

a) 

b) 

m_1

$\hat{x})\quad F - F_r = m_1 . a_1$

$\hat{y})\quad N_1 - m_1 . g = 0$

m_2

$\hat{x})\quad F_r = m_2 . a_2$

$\hat{y})\quad N_2 - N_1 - m_2 . g = 0$

m_1

$\hat{x})\quad F_r = m_1 . a_1$

$\hat{y})\quad N_1 - m_1 . g = 0$

m_2

$\hat{x})\quad F - F_r = m_2 . a_2$

$\hat{y})\quad N_2 - N_1 - m_2 . g = 0$

a) $a = a_1 = a_2 \Rightarrow$

$$F = (m_1 + m_2) . a$$

$$F_r \leq \mu_e . m_1 . g \quad \therefore$$

$$F_{max} - \mu_e . m_1 . g = \frac{m_1 . F_{max}}{(m_1 + m_2)}$$

$$F_m - \frac{m_1 F_m}{m_1 + m_2} = \mu_e . m_1 . g$$

$$F_m \left(1 - \frac{m_1}{m_1 + m_2} \right) = \mu_e . m_1 . g$$

$$\left(\frac{m_1 + m_2 - m_1}{m_1 + m_2} \right) =$$

$$\boxed{F_{max} = \frac{(m_1 + m_2) . m_1 . g . \mu_e}{m_2}}$$

b)

$$a = \frac{F}{(m_1 + m_2)}$$

$$\boxed{a = \frac{m_1 . \mu_e . g}{m_2}}$$

c) $F = (m_1 + m_2) . a$

$$F_r \leq \mu_e . m_1 . g$$

$$\boxed{a = \mu_e . g}$$

$$F - \mu_e . m_1 . g = m_2 . a$$

$$F - \mu_e . m_1 . g = m_2 . \frac{F}{(m_1 + m_2)}$$

$$F \left(1 - \frac{m_2}{(m_1 + m_2)} \right) = \mu_e . m_1 . g$$

$$F_{max} = \frac{\mu_e . m_1 . g \, (m_1 + m_2)}{m_1}$$

$$\boxed{F_{max} = \mu_e . g . (m_1 + m_2)}$$

d)

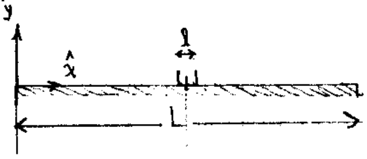
$$\mu_d . \cancel{m_1} . g = \cancel{m_1} . a_1$$

$$\boxed{a_1 = \mu_d . g}$$

$$2 \mu_e . g (m_1 + m_2) - \mu_d . m_1 . g = m_2 . a_2$$

$$\frac{2 \mu_e g m_1}{m_2} + 2 \mu_e g - \frac{\mu_d . m_1}{m_2} g = a_2$$

$$\boxed{g \frac{m_1}{m_2} (2\mu_e - \mu_d) + 2 \mu_e g = a_2}$$

e) 

$l << L$

$$\vec{a}_{m_1} \longrightarrow$$

$$\vec{a}_{m_2} \longrightarrow$$

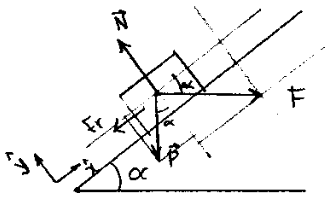
$$\vec{a}_2 - \vec{a}_1 = a_{rel}$$

$$2 \mu_e g + g \frac{m_1}{m_2} (2\mu_e - \mu_d) - \mu_d . g = a_{rel}$$

$$0 = \frac{L}{2} - \frac{1}{2} a_r . t^2$$

$$\frac{L}{\cancel{2}} . \frac{\cancel{2}}{a_r} = t^2 \Rightarrow \sqrt{\frac{L}{2 \mu_e g + g \frac{m_1}{m_2} (2\mu_e - \mu_d) - \mu_d g}} = t$$

②



$\downarrow \vec{g}$

a) $\underline{m}$

$\hat{x})\ F.\cos\alpha - Fr - m.g.\operatorname{sen}\alpha = m.a_x$

$\hat{y})\ N - m.g.\cos\alpha - F.\operatorname{sen}\alpha = 0$

si $F = 0 \Rightarrow$ Fr apunta en $\rightarrow$

$Fr - m.g.\operatorname{sen}\alpha = 0$

$m.g.\operatorname{sen}\alpha = Fr$

$m.g.\operatorname{sen}\alpha \leq \mu e.\, m.g.\cos\alpha$

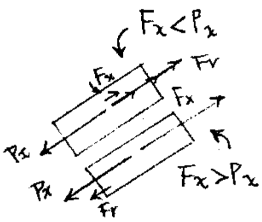
$tg\,\alpha \leq \mu_e$

En reposo si $\boxed{\alpha \leq atang(\mu e)}$

b) SUPONGO $F\hat{x} > P\hat{x}$

$F.\cos\alpha - Fr - m.g.\operatorname{sen}\alpha = 0$

$F.\cos\alpha - Fr = m.g.\operatorname{sen}\alpha$



$F.\cos\alpha - mg\operatorname{sen}\alpha = Fr$

$F.\cos\alpha - m.g.\operatorname{sen}\alpha \leq \mu e.m.g.\cos\alpha + \mu e.F.\operatorname{sen}\alpha$

$F\cos\alpha - \mu e F\operatorname{sen}\alpha \leq \mu e.mg\cos\alpha + mg\operatorname{sen}\alpha$

$$\boxed{F \leq \frac{m.g(\mu e.\cos\alpha + \operatorname{sen}\alpha)}{(\cos\alpha - \mu e.\operatorname{sen}\alpha)}}$$

c)

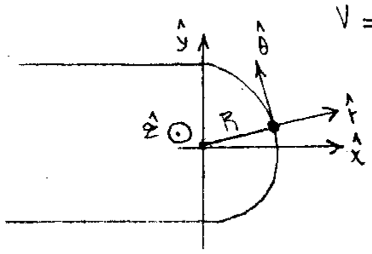
$F = 2kg \cdot 9{,}8\,\frac{m}{s^2} \cdot 0{,}66 = \boxed{12{,}92\ N}$

$tg\,\alpha = 0{,}3$

$\alpha = atan(0{,}3)$

$\alpha = 16°\,41'\,57''$

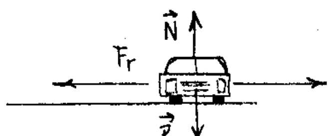
③



$V = k \Rightarrow R.\dot{\theta} = k$

$R = k \Rightarrow \dot{r} = \ddot{r} = 0$

$\dot{\theta} = k \Rightarrow \ddot{\theta} = 0$



$\hat{r})\ \ -Fr = -m.R.\dot{\theta}^2$

$\hat{\theta})\ \ 0 = 0$

$\hat{z})\ \ N - m.g = 0$

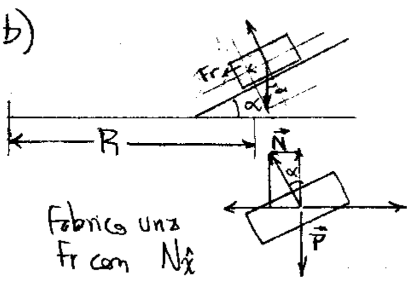
a) $Fr = m.R.\dot{\theta}^2$

El rozamiento es estático para no mover el auto en $\hat{r}$

$m.R.\dot{\theta}^2 \leq \mu e.m.g$

$\frac{R.\dot{\theta}^2}{g} \leq \mu e$ mínimo $\frac{R.\dot{\theta}^2}{g}$

b)



Fabrico una Fr con $N\hat{x}$

$\hat{r})\ \underbrace{-Fr}_{=0} - N.\operatorname{sen}\alpha = -m.R.\dot{\theta}^2$

$\hat{\theta})\ 0 = 0$

$\hat{z})\ N.\cos\alpha - m.g = 0$

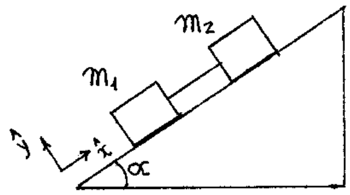
$\frac{m.g}{\cos\alpha}.\operatorname{sen}\alpha = m.R.\dot{\theta}^2$

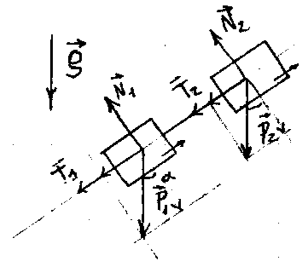
$tg\,\alpha = \frac{R.\dot{\theta}^2}{g}$

$\boxed{\alpha = atan\left(\frac{R.\dot{\theta}^2}{g}\right)}$

④ A partir de las ecuaciones de Newton y de vínculo ($Fr_e = \mu_e N$ en un momento solamente)

⑤





m_2

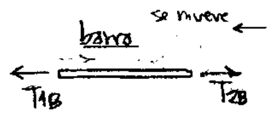
$\hat{x})\ -T_2 - m_2 . g . \operatorname{sen}\alpha + Fr_2 = m_2 . a_2$

$\hat{y})\ N_2 - m_2 . g \cos\alpha = 0$

m_1

$\hat{x})\ -T_1 - m_1 . g . \operatorname{sen}\alpha + Fr_1 = m_1 . a_1$

$\hat{y})\ N_1 - m_1 g \cos\alpha = 0$



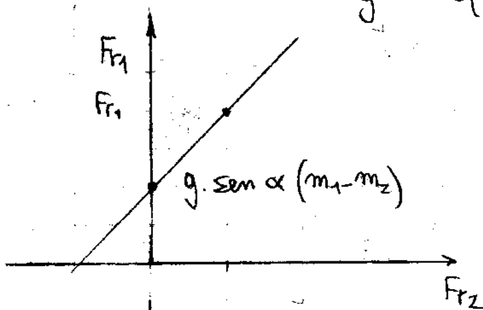
$-T_{1B} = T_{2B} \Rightarrow T_1 = T_2$

$T_{B1} = T_{B2}$

a)

$-m_2 g \operatorname{sen}\alpha + Fr_2 + m_1 g \operatorname{sen}\alpha - Fr_1 = 0$

$g \operatorname{sen}\alpha (m_1 - m_2) \leq Fr_1 - Fr_2 \Rightarrow$ relación $\boxed{Fr_1 \geq Fr_2 + g . \operatorname{sen}\alpha (m_1 - m_2)}$



$g . \operatorname{sen}\alpha (m_1 - m_2) \leq \mu e_1 . m_1 g \cos\alpha - \mu e_2 . m_2 g . \cos\alpha$

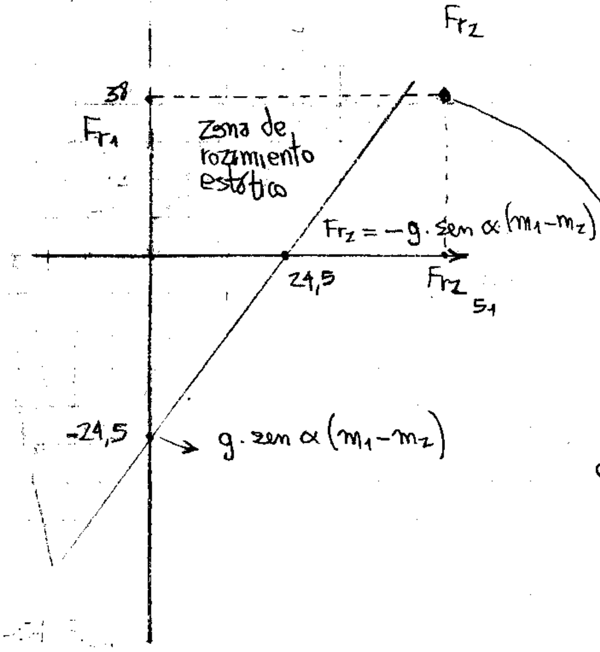
$g . \operatorname{sen}\alpha (m_1 - m_2) \leq g . \cos\alpha (\mu e_1 . m_1 - \mu e_2 m_2)$

$\boxed{tg\alpha (m_1 - m_2) \leq (\mu e_1 . m_1 - \mu e_2 m_2)}$

$y \leq kx + b$

	Fr_1	Fr_2
$-24{,}5 \leq$	$0{,}9 . 5 . 8{,}49$	$-0{,}6 . 10 . 8{,}49$
	$38{,}205$	$-50{,}94$
$-24{,}5 \leq -12{,}735$		ok

b)



$= \quad + 9{,}8 . 0{,}5 (-5)$

$= \quad -24{,}5$

$38{,}205 = +50{,}94 \quad -24{,}5$

$k = 1{,}23$

c) Con estos datos no es posible el reposo

d) Sistema en reposo necesita:

$$\alpha \leq \operatorname{arctang} \frac{(\mu e_1 m_1 - \mu e_2 m_2)}{(m_1 - m_2)}$$

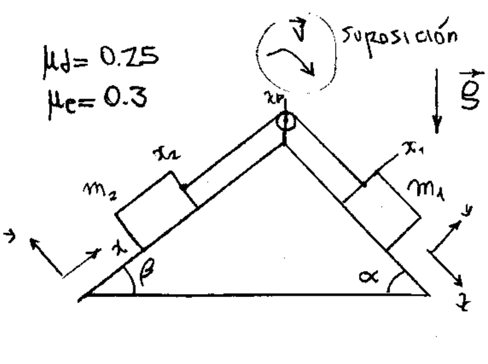
$$\alpha \leq \operatorname{arctang} \left[\frac{(-1{,}5)}{-5} \right]$$

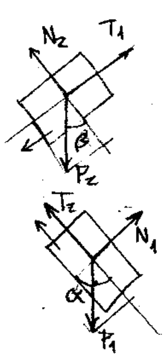
$$\boxed{\alpha \leq 16° 41' 57''}$$

⑥

$\mu d = 0.25$

$\mu e = 0.3$





m_2

$\hat{x})\ T_1 - Fr_2 - m_2 . g . \operatorname{sen}\beta = m_2 . a_2$

$\hat{y})\ N_2 - m_2 . g . \cos\beta = 0$

m_1

$\hat{x})\ m_1 . g . \operatorname{sen}\alpha - Fr_1 - T_2 = m_1 . a_1$

$\hat{y})\ N_1 - m_1 . g . \cos\alpha = 0$

$T_1 = T_2$

a) Necesitamos el reposo $\Rightarrow$ A) si $\vec{v}\rightarrow$

$$-Fr_2 - Fr_1 - m_2 g \operatorname{sen}\beta + m_1 g \operatorname{sen}\alpha = 0$$
$$(m_1 \operatorname{sen}\alpha - m_2 \operatorname{sen}\beta) g = Fr_2 + Fr_1$$

$$\boxed{(m_1 \operatorname{sen}\alpha - m_2 \operatorname{sen}\beta)\cdot \cancel{g} \leq \mu e\, m_2\, \cancel{g}\cdot\cos\beta + \mu e\, m_1 \cdot \cos\alpha}$$

B) si $\overleftarrow{\vec{v}}$

$$Fr_2 + Fr_1 - m_2 g \operatorname{sen}\beta + m_1 g \operatorname{sen}\alpha = 0$$
$$Fr_2 + Fr_1 = m_2 g \operatorname{sen}\beta - m_1 g \operatorname{sen}\alpha$$

$$\cancel{g}(m_2 \operatorname{sen}\beta - m_1 \operatorname{sen}\alpha) \leq \mu e . m_2 \cancel{g}.\cos\beta + \mu e . m_1 . \cancel{g}.\cos\alpha$$
$$\boxed{(m_1 . \operatorname{sen}\alpha - m_2 \operatorname{sen}\beta) \geq -\mu e\, m_2 \cos\beta - \mu e\, m_1 \cos\alpha}$$

b)
$$(-0{,}134) \geq -(0{,}52) - (0{,}15)$$
$$0{,}67 \geq -0{,}134 \geq -0{,}67$$

$\boxed{\text{No estará en reposo}}$

c) V_i $\vec{v_i}\rightarrow$

$L = x_p - x_2 + x_1 - x_p$
$0 = -\ddot{x}_2 + \ddot{x}_1$
$\ddot{x}_1 = \ddot{x}_2 = a$

$$-Fr_2 - Fr_1 - m_2 g \operatorname{sen}\beta + m_1 g \operatorname{sen}\alpha = m_2 . a_2 + m_1 a_1$$
$$-\mu d . m_2 g \cos\beta - \mu d\, m_1 g \cos\alpha - 9{,}8 + 8{,}48 = (m_2 + m_1) . a$$
$$-4{,}24 \quad -1{,}225 \quad -1{,}32 = 3a$$
$$\boxed{-2{,}26 \frac{m}{s^2} = a}$$

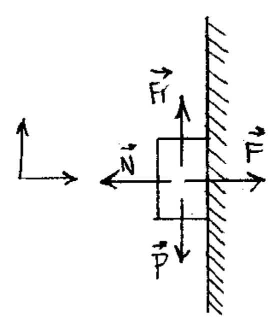
se mueve hacia $\rightarrow$ pero a una vel. decreciente

V_i $\overleftarrow{\vec{v_i}}$

$$Fr_2 + Fr_1 - m_2 g \operatorname{sen}\beta + m_1 g \operatorname{sen}\alpha = (m_2 + m_1)\, a$$
$$1{,}225 + 4{,}24 \quad -9{,}8 + 8{,}48 = 3a$$
$$\boxed{1{,}38 \frac{m}{s^2} = a}$$

se mueve hacia $\leftarrow$ pero a una vel. decreciente (a un ritmo menor de decrecimiento)

⑦



$\hat{x}$) $F - N = 0 \Rightarrow F = N$

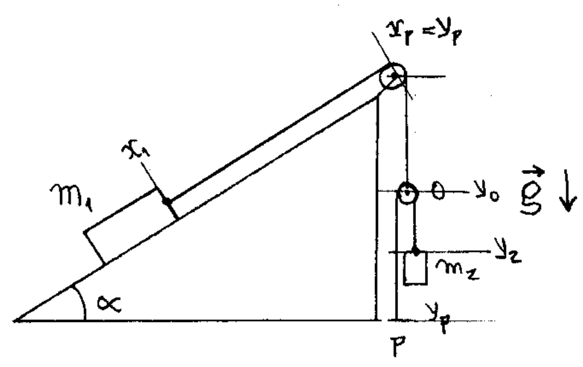
$\hat{y}$) $Fr - m.g = 0$

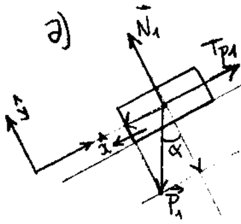
$Fr = m.g$

$Fr \leq \mu_e . F$
$m.g \leq \underbrace{\mu_e . F}_{Fr\, max}$

EL vicio es que si la fr es contraria al movimiento, entonces si existiese alguna tendencia de que el bloque suba ↑ la fr tiraría para abajo ↓.

⑧



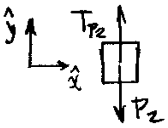
a) 

$\underline{m_1}$

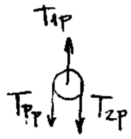
$\hat{x})\quad T_{P1} - F_r - m_1.g.\operatorname{sen}\alpha = m_1.a_1$

$\hat{y})\quad N_1 - m_1.g.\cos\alpha = 0$

$\underline{m_2}$



$\hat{y})\quad T_{P_2} - m_2.g = m_2.a_2$



Polea I

$\hat{y})\quad -T_{2P} - T_{PP} + T_{1P} = \underset{\to 0}{m_{P}}.a_p = 0$

$T_{1P} = -T_{2P} - T_{PP} = -2T_{2P}$

$\boxed{T_{P1} = -2T_{P2}}$

hilos

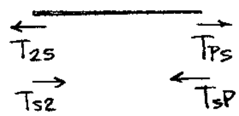
$L_1 = x_p - x_1 + y_p - y_0$

$0 = -\ddot{x}_1 - \ddot{y}_0 \Rightarrow \boxed{\ddot{y}_0 = -\ddot{x}_1}$

$L_2 = y_0 - P + y_0 - y_2$

$0 = 2\ddot{y}_0 - \ddot{y}_2 \Rightarrow \boxed{2\ddot{y}_0 = \ddot{y}_2}$

hilo 2



$\ddot{y}_2 = -2\ddot{x}_1$

$T_{2P} = T_{PP}$

b)

$-2T_{P2} - F_r - m_1 g\operatorname{sen}\alpha = m_1.\ddot{x}_1$

$\boxed{\ddot{x}_1 = -\frac{\ddot{y}_2}{2}}$

$T_{P2} - m_2.g = -2m_2.\ddot{x}_1$

$2T_{P2} - 2m_2 g = -4m_2.\ddot{x}_1$

$-F_r - m_1 g\operatorname{sen}\alpha - 2m_2 g = \ddot{x}_1(m_1 - 4m_2)$

Si, influye es el vínculo

c) En reposo $\Rightarrow$ $T_{P2} = m_2 g$

$\overleftarrow{\vec{F_r}}$

$-F_r = m_1 g\operatorname{sen}\alpha + 2m_2 g$

$m_1.\cancel{g}.\operatorname{sen}\alpha + 2m_2\cancel{g} \leq \mu_e.m_1.\cancel{g}.\cos\alpha$

$2m_2 \geq -\mu_e.m_1.\cos\alpha - m_1.\operatorname{sen}\alpha$

$\boxed{m_2 \geq -\frac{\mu_e.m_1}{2}\left(\cos\alpha + \frac{\operatorname{sen}\alpha}{\mu_e}\right)}$

$\overrightarrow{\vec{F_r}}$

$F_r = m_1 g\operatorname{sen}\alpha + 2m_2 g$

$m_1\cancel{g}\operatorname{sen}\alpha + 2m_2\cancel{g} \leq \mu_e m_1\cancel{g}\cos\alpha$

$m_2 \leq \frac{\mu_e.m_1.\cos\alpha - m_1.\operatorname{sen}\alpha}{2}$

$\boxed{m_2 \leq \frac{\mu_e.m_1}{2}\left(\cos\alpha - \frac{\operatorname{sen}\alpha}{\mu_e}\right)}$

d) i) $\ddot{y}_2 = -A$

$-F_r - m_1.g.\operatorname{sen}\alpha - 2m_2 g = \frac{A}{2}(m_1 - 4m_2)$

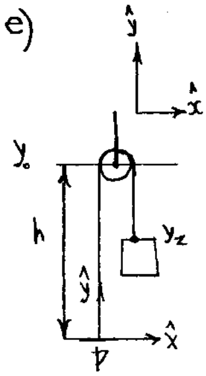
$-\mu_d.m_1.g.\cos\alpha - m_1.g.\operatorname{sen}\alpha - \frac{A}{2}m_1 = -\overset{2}{\cancel{4}}m_2\frac{A}{\cancel{2}} + 2m_2 g$

$m_1.g\left(-\mu_d.\cos\alpha - \operatorname{sen}\alpha - \frac{A}{2g}\right) = 2m_2(g - A)$

$\boxed{\frac{m_1.g.(-\mu_d.\cos\alpha - \operatorname{sen}\alpha - A/2g)}{2(g-A)} = m_2}$

si $A > g \Rightarrow m_2 < 0$
lo cual no es posible

e) $2\ddot{y}_0 = \ddot{y}_2 \Rightarrow \ddot{y}_0 = \frac{\ddot{y}_2}{2}$ y además $\ddot{y}_0 = -\ddot{x}_1$



$$y = y_0 + 0 - \frac{1}{2}\left(\frac{-\mu d. m_1 .g. \cos\alpha - 2m_2 g - m_1 g \operatorname{sen}\alpha}{m_1 - 4m_2}\right).t^2$$

en el sistema con origen en P

$$\boxed{y = h + \frac{1}{2}.\left(\frac{\mu d\, m_1 \cos\alpha + 2m_2 + m_1 \operatorname{sen}\alpha}{(m_1 - 4m_2)}\right) g\, . t^2}$$

$$(-m_1 + 4m_2)\ddot{y} = -\mu d m_1 g \cos\alpha - 2m_2 g - m_1 g \operatorname{sen}\alpha$$

$$(-m_1 + 4m_2)\int_{\dot{y}_0=0}^{\dot{y}} d\dot{y} = -\mu d. m_1 . g. \cos\alpha \int_{t_0=0}^{t} dt - 2m_2 g \int_{t_0=0}^{t} dt - m_1 g \operatorname{sen}\alpha \int_{t_0=0}^{t} dt$$

$$(-m_1 + 4m_2)\,\dot{y} = \left(-\mu d\, m_1 g \cos\alpha - 2m_2 g - m_1 g \operatorname{sen}\alpha\right).t$$

$\frac{d^2 y}{dt^2} = \ddot{y}$

$\frac{dy}{dt} = \dot{y}$

$\frac{d\dot{y}}{dt} = \ddot{y}$

$$\frac{dy}{dt} = \dot{y} = \frac{(\qquad)}{(\qquad)}.t$$

$$\int_{y_0=h}^{y} dy = \frac{(\qquad)}{(\qquad)} \int_{t_0=0}^{t} t\, dt$$

$$\boxed{y - h = \frac{(-\mu d m_1 g \cos\alpha - 2m_2 g - m_1 g \operatorname{sen}\alpha)}{(4m_2 - m_1)} \frac{t^2}{2}}$$