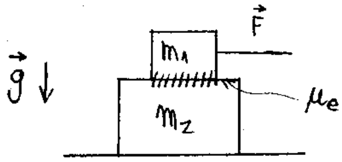
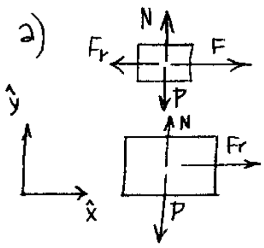
INTERACCION DE ROZAMIENTO

1.



a)



$\underline{\underline{1}}$ $\hat{x}$) $F - F_r = m_1 \cdot a_{1x}$

$\hat{y}$) $N_1 - m_1 \cdot g = m_1 \cdot \underbrace{a_{1y_1}}_{0}$

$N = m_1 \cdot g$

$\underline{\underline{2}}$ $\hat{x}$) $F_r = m_2 \cdot a_{2x}$

$\hat{y}$) $N_2 - m_2 g = m_2 \cdot \underbrace{a_{2y_1}}_{0}$

$N_2 = m_2 \cdot g$

NB: La fuerza de roz. F_r solo es $\mu_e \cdot N$ antes del deslizamiento

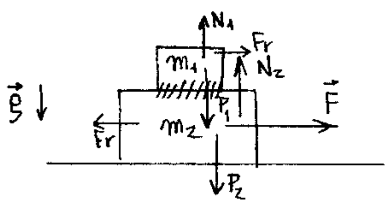
si no se deslizan $a_{1x} = a_{2x} = a_x$ ∴

$-\mu_e \cdot N_1 = a_x (m_1 - m_2)$ ⇒

$\boxed{F_m = (m_1 + m_2) \cdot a_x}$

b) $\boxed{a_x = \frac{\mu_e \cdot N_1}{m_2} = \frac{\mu_e \cdot m_1 \cdot g}{m_2}}$ ∴ $\boxed{F = (m_1 + m_2)\, \mu_e \cdot \frac{m_1}{m_2} \cdot g}$

c)



$\underline{\underline{1}}$ $\hat{x}$) $F_r = m_1 \cdot a_{1x}$

$\hat{y}$) $N_1 = m_1 \cdot g$

$\underline{\underline{2}}$ $\hat{x}$) $F - F_r = m_2 \cdot a_{2x}$

Las aceleraciones deben ser iguales ∴

$\boxed{F_m = a_x (m_1 + m_2)}$

$a_x = \frac{\mu_e \cdot m_1 \cdot g}{m_1} = \boxed{\mu_e \cdot g = a}$

$\boxed{F_m = \mu_e \cdot g\, (m_1 + m_2)}$

d) $2 F_m = 2 \mu_e \cdot g \cdot (m_1 + m_2)$ ⇒

NB: si se deslizan el uno sobre el otro sus aceleraciones ya no son las mismas

$2\mu_e \cdot g\,(m_1 + m_2) - \mu_d \cdot m_1 g = m_2 \cdot a_2$

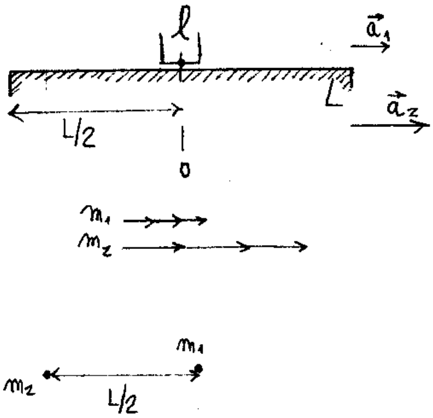
$\frac{2\mu_e \cdot g\, m_1}{m_2} + 2\mu_e \cdot g - \mu_d \cdot \frac{m_1}{m_2} \cdot g = a_2$

$\frac{m_1}{m_2} g \left(2\mu_e - \mu_d\right) + 2\mu_e \cdot g = a_2$

$\frac{\mu_d \cdot \cancel{m_1} g}{\cancel{m_1}} = \boxed{a_1 = \mu_d \cdot g}$

$\boxed{a_2 = 2\mu_e \cdot g + \frac{m_1 \cdot g\,(2\mu_e - \mu_d)}{m_2}}$

e)



$\underline{\underline{m_1}}$ $\quad x = 0 + 0 + \frac{a_1}{2} \cdot t^2$

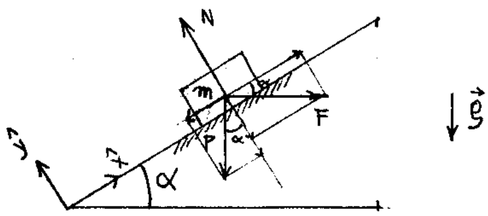
$\underline{\underline{m_2}}$ $\quad x = -L/2 + 0 + \frac{a_2}{2} \cdot t^2$

$$\frac{a_1}{2} \cdot t^2 = -\frac{L}{2} + \frac{a_2}{2} t^2$$

$$\frac{1}{\cancel{2}}(a_1 - a_2) \cdot t^2 = -\frac{L}{\cancel{2}}$$

$$\boxed{t = \sqrt{\frac{-L}{(a_1 - a_2)}}}$$

2.

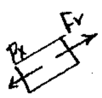


a) $\quad \hat{x})$ $\quad F_r - \text{sen}\,\alpha \cdot m \cdot g = m \cdot a_x$ $\quad$ en reposo

$\mu_e \cdot \cos\alpha \cdot \cancel{m \cdot g} \geq \text{sen}\,\alpha \cdot \cancel{m \cdot g}$

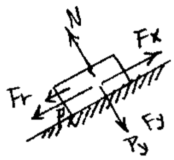
$\mu_e \geq tg\,\alpha$

$\boxed{\text{ARCO } tg\,(\mu_e) \geq \alpha}$



$\hat{y})$ $\quad -\cos\alpha \cdot m \cdot g = -N$

b)



$\hat{x})$ $\quad F_x - F_r - P_x = m \cdot a_x$

$\cos\alpha \cdot F - \mu_e \cdot N - \text{sen}\,\alpha \cdot m \cdot g = 0$

$\cos\alpha \cdot F - \mu_e \cdot m \cdot g \cos\alpha - \text{sen}\,\alpha \cdot m \cdot g = 0$

Está bien hecho en la carpeta

$$F \leq \frac{\text{sen}\,\alpha \cdot m \cdot g + \mu_e \cdot m \cdot g \cdot \cos\alpha}{\cos\alpha}$$

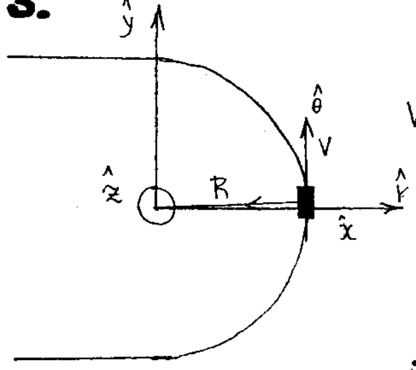
$$\boxed{F \leq (tg\,\alpha + \mu_e) \cdot m \cdot g}$$

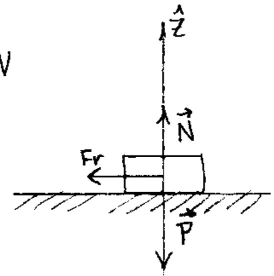
c) $\quad m = 2\,kg$

$\mu_e = tg\,\alpha = 0.3$

$$F_{max} = (2 \cdot 0{,}3) \cdot 2 \cdot 9{,}8 = \boxed{11{,}76\ N}$$

3.





$\hat{z})\quad N = m.g$

$\hat{r})\quad -F_r = m.(-R.\dot{\theta}^2)$

$-F_r = m.R.\omega^2$

$F_r = m.\frac{V^2}{R}$

$\hat{\theta})\quad V = k \Rightarrow a = 0$

$0 = 0$

$\left.\begin{array}{l}\omega = k \\ V = k\end{array}\right\} \Rightarrow$ M.C.U $\qquad \ddot{\theta} = 0$

$V = R.\omega$

a)

$F_r \leq \mu_e . N$

$m.\frac{V^2}{R} \leq \mu_e . m.g$

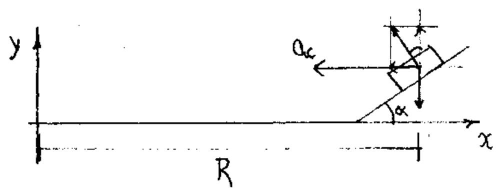
$\frac{V^2}{gR} \leq \mu_e$

$\boxed{\mu_{e_{min}} = \frac{V^2}{gR}}$

La F_R debe generar una a_c que impulse al auto en la curva.
EL rozamiento mantiene al auto en la trayectoria.

EL rozamiento es estático porque el auto debe evitar deslizarse en $\hat{r}$

b)



$\boxed{a_c \text{ siempre se dirige al eje de giro}}$

$\hat{x})\quad -N_x = -m.\frac{V^2}{R}$

$-N.\text{sen}\,\alpha = -m.\frac{V^2}{R}$

$\hat{y})\quad N_y - m.g = 0$

$N.\cos\alpha = m.g$

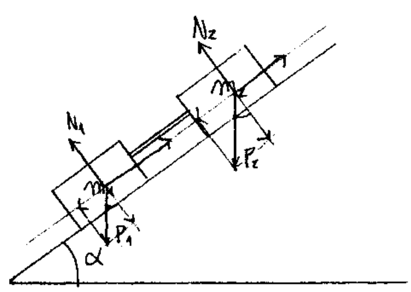
$-m.g.\frac{1}{\cos\alpha}.\text{sen}\,\alpha = -m.\frac{V^2}{R}$

$\boxed{\alpha = \text{Arco tg}\left(\frac{V^2}{gR}\right)} \Leftarrow \qquad \text{tg}\,\alpha = \frac{V^2}{gR}$

4. Si está en reposo $\sum F = 0$; la fuerza de rozamiento la obtengo de las ecuaciones de Newton y de vinculo , porque

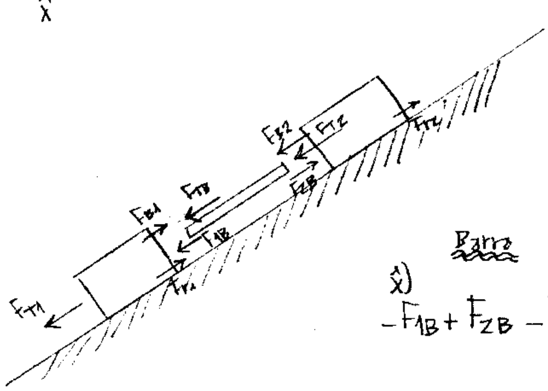
$F_{re} = \mu_e . N$ solo es válida un instante antes de comenzar el movimiento

5.



$\underline{m_1}$

$\hat{x}$



<u>vinculos</u>

barra hace que

$a_1 = a_2 = a_x$

<u>Barra</u>

$\hat{x})$

$-F_{1B} + F_{2B} - F_{TB} = \overset{0}{m}.a_b$

$\underset{m.g.\text{sen}\alpha}{}$

${}^{0}F_{2B} = F_{1B}$

Por pares acción reacción $\quad F_{B1} = F_{B2} \Leftarrow$

m_1

$\hat{x})\quad F_{r1} - P_{1x} + F_{B1} = m_1 \cdot a_1$

$\hat{y})\quad N_1 = m_1 \cdot g \cdot \cos\alpha$

m_2

$\hat{x})\quad F_{r2} - P_{2x} - F_{B2} = m_2 \cdot a_2$

$\hat{y})\quad N_2 = m_2 \cdot g \cdot \cos\alpha$

$$F_{r1} - F_{r2} - m_1 \cdot g \cdot \operatorname{sen}\alpha + m_2 \cdot g \cdot \operatorname{sen}\alpha = 0$$

En reposo $\quad F_{r1} = m_1 \cdot g \cdot \operatorname{sen}\alpha \qquad F_{r2} = m_2 \cdot g \cdot \operatorname{sen}\alpha$

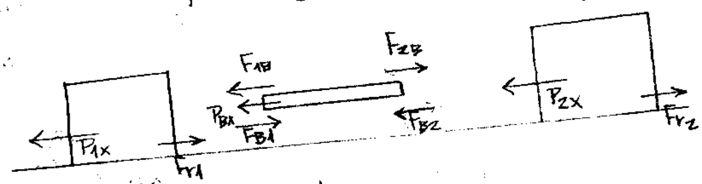
$$F_{r1} + F_{r2} = (m_1 + m_2) \cdot g \cdot \operatorname{sen}\alpha$$

$$(m_1 + m_2) \cdot g \cdot \operatorname{sen}\alpha \leqslant \mu_{e1} \cdot m_1 \cdot g \cdot \cos\alpha + \mu_{e2} \cdot m_2 \cdot g \cdot \cos\alpha$$

$$\operatorname{tg}\alpha \leqslant \frac{\mu_{e1} \cdot m_1 + \mu_{e2} \cdot m_2}{(m_1 + m_2)}$$

eje $\hat{x}$

movimento (si es que lo hay tiene esta dirección) $\leftarrow$



barra

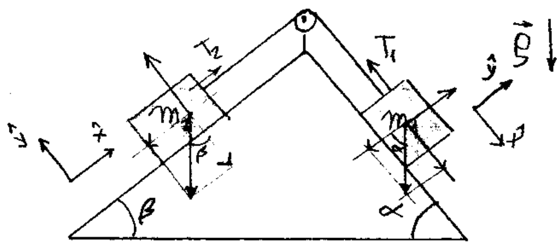
$$-F_{1B} + F_{2B} - P_{Bx} = \overset{0}{m} \cdot a_b$$

$$-0 = 0$$

$$F_{1B} = F_{2B}$$

F_{B2}

6.



$\mu_d = 0.25$
$\mu_e = 0.3$

supongo mov $\vec{V} \rightarrow$

NB
tomo
Fr
positiva
por
default

a) $\underline{m_2}$

$\hat{x})\quad T_2 + |Fr_2| - m_2 \cdot g \cdot \operatorname{sen}\beta = m_2 \cdot a_{2x}$

$\hat{y})\quad N_2 - m_2 \cdot g \cdot \cos\beta = 0$

$N_2 = m_2 \cdot g \cdot \cos\beta$

$\underline{m_1}$

$\hat{x})\quad -T_1 + m_1 \cdot g \cdot \operatorname{sen}\alpha + |Fr_1| = m_1 \cdot a_1$

$\hat{y})\quad N_1 = m_1 \cdot g \cdot \cos\alpha$

La soga tiene masa despreciable y es inextensible $\Rightarrow$ $a_{2x} = a_{1x} = a$
$T_2 = -T_1$

En reposo $a = 0$ $\therefore$

$$|Fr_2| + |Fr_1| - m_2 g \operatorname{sen}\beta + m_1 \cdot g \cdot \operatorname{sen}\alpha = 0$$

$$|Fr_2| + |Fr_1| - g\,(m_2 \cdot \operatorname{sen}\beta - m_1 \cdot \operatorname{sen}\alpha) = 0$$

$|A+B| = |A| + |B| \Leftrightarrow$
$A, B > 0 \ \vee\ A, B < 0$

$$|Fr_2| + |Fr_1| = g\,(m_2 \cdot \operatorname{sen}\beta - m_1 \cdot \operatorname{sen}\alpha)$$

$$|Fr_2 + Fr_1| = g\,(m_2 \cdot \operatorname{sen}\beta - m_1 \cdot \operatorname{sen}\alpha)$$

$$g\,(m_2 \cdot \operatorname{sen}\beta - m_1 \cdot \operatorname{sen}\alpha)| \leq \mu_e \cdot g\,(m_2 \cdot \cos\beta + m_1 \cos\alpha)$$

$$\boxed{-\mu_e\,(m_2 \cdot \cos\beta + m_1 \cdot \cos\alpha) \leq \cancel{g}\,(m_2 \operatorname{sen}\beta - m_1 \operatorname{sen}\alpha) \leq \mu_e \cdot \cancel{g}\,(m_2 \cdot \cos\beta + m_1 \cdot \cos\alpha)}$$

hay reposo si se cumple

b)
$\alpha = 60^\circ$
$\beta = 30^\circ$

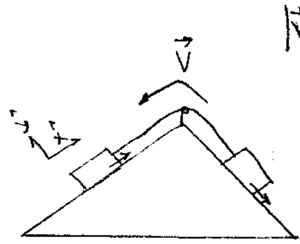
$$\leq -[2 \cdot 0{,}5 - 1 \cdot 0{,}866] \leq 0{,}3\,[2 \cdot 0{,}866 + 1 \cdot 0{,}5$$

$$-0{,}669 \leq -0{,}134 \leq 0{,}669$$

$\boxed{\text{El sistema no se pondrá en movimiento}}$

c) Supongamos que se le da al sistema cierta velocidad inicial $\vec{V}$ y $\vec{V}'$

$|\vec{V}| < 0 \Rightarrow$ Fr_2 y Fr_1 son positivas $\Rightarrow$



$$Fr_2 + Fr_1 - m_2 g \operatorname{sen}\beta + m_1 g \operatorname{sen}\alpha = (m_1 + m_2) \cdot a$$

$$Fr_2 + Fr_1 - 9{,}8 + 8{,}487 = 3a$$

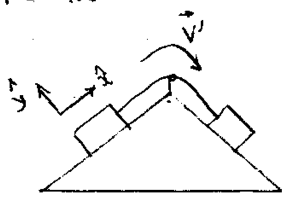
$$\mu_d\,(16{,}97 + 4{,}9) - 1{,}313 = 3a$$

$$5{,}4675 - 1{,}313 = 3a$$

$$\boxed{1{,}38\ \frac{m}{s^2} = a}$$

Con $\vec{V}$ inicial imprimiéndole un mov. der-izq. tenemos una $a > 0$ que tenderá a frenar el avance

Fr_2 y Fr_1 son negativas $\Rightarrow$



$$Fr_2 + Fr_1 - 9{,}8 + 8{,}487 = 3a$$

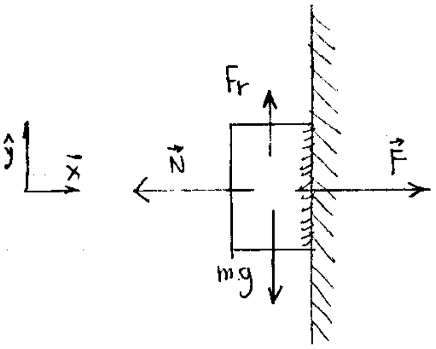
$$\mu_d\,(-16{,}97 - 4{,}9) - 1{,}313 = 3a$$

$$-5{,}4675 - 1{,}313 = 3a$$

$$\boxed{-2{,}26\ \frac{m}{s^2} = a}$$

con $\vec{V}'$ inicial imprimiéndole un movimiento de izq-der tenemos una $a<0$ que tenderá a frenar el avance más rápidamente que en el otro caso.

7.



$\hat{y})\quad Fr - m.g = m.a_y$

$\hat{x})\quad F - N = m.a_x = 0$

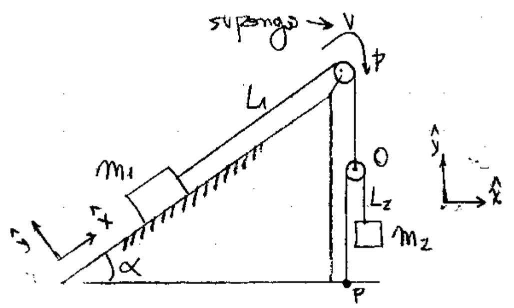
$F = N$

$Fr = m.(a_y + g)$

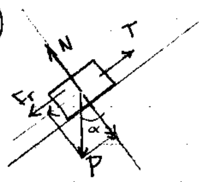
La Fr es contraria al movimiento siempre; si el cuerpo asciende la Fr apunta en la dirección del peso y $\Rightarrow$ la fuerza que tengo que realizar para hacerlo ascender nunca se alcanza porque entre mayor F, mayor Fr y nunca se llega a mover el bloque

$m(a_y + g) \leqslant \mu e.F$

8.



a)

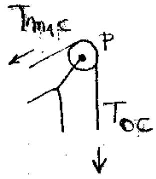


$\underline{m_1}$

$\hat{x})\quad T_{cm_1} - Fr - m_1.g.\text{sen}\,\alpha = m_1.a_x$

$\hat{y})\quad N - m_1.g.\cos\alpha = m_1.a_{1y} = 0$

$N = m_1.g.\cos\alpha$

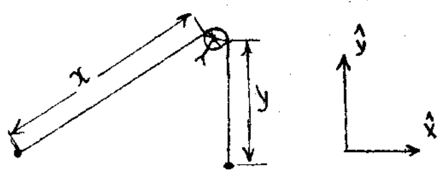


$\underline{L_1}$

$\hat{x})\quad -T_{m_1c} + T_{oc} = \overset{0}{\widehat{m_1}}.a_{L_1}$

$\boxed{T_{oc} = T_{m_1c}} \Rightarrow$ soga masa despreciable

$\therefore\ \boxed{T_{cm_1} = T_{co}}$



$\underline{O}$

$\hat{x})\quad T_{so} + T_{m_2o} - T_{co} = \overset{0}{\widehat{m_o}}.a_{ox}$

$T_{so} + T_{m_2o} = T_{co} = T_{cm_1}$

$\boxed{2T_{m_2o} = T_{cm_1}}$ por

$\underline{\text{vínculos}}$

por par acción-reacción

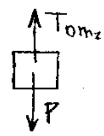
$L = x + y \overset{+(y_2 - y_0)}{} - \pi.R$

$0 = \dot{x} + \dot{y} + 0$

$0 = \ddot{x} + \ddot{y} + 0$

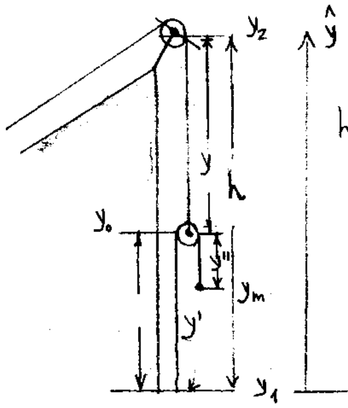
$0 = \ddot{x} - \ddot{y}_0$

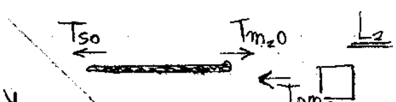
$\ddot{x} = \ddot{y}_0$



$\underline{m_2}$

$T_{om2} - m_2.g = m_2.a_{2y}$





$\underline{L_2}$

$T_{so} - T_{m_2o} = \overset{0}{\widehat{m}}.a_{L_2}$

$T_{so} = T_{m_2o}$

$h = y + y' = y_2 - y_1$

$L_2 = \overset{y'}{(y_0 - y_1)} + \overset{y''}{(y_0 - y_m)}$

$L_2 = 2y_0 - y_1 - y_m =$

$0 = 2\ddot{y}_0 - 0 - \ddot{y}_m$

$\ddot{y}_m = 2\ddot{y}_0$

$\boxed{\ddot{y}_0 = \dfrac{\ddot{y}_m}{2} = \ddot{x}}$

b) <u>m_1</u>

$$T_{cm_1} - Fr - m_1.g.\operatorname{sen}\alpha = m_1.a_{1x} = m_1.\ddot{x}$$
$$T - Fr - m_1.g.\operatorname{sen}\alpha = m_1.\ddot{x}$$

<u>vínculos</u>

$$\ddot{x} = \ddot{y} = +\frac{1}{2}.\ddot{y}_m$$

<u>m_2</u>

$$T_{om_2} - m_2 g = m_2.a_{2y} = m_2.\ddot{y}_m = +2.m_2.\ddot{x}$$
$$2T - m_2.g = +m_2.2.\ddot{x}$$

$$T_{cm_1} = 2T_{om_2}$$
$$T = 2T$$

$$T - \frac{1}{2}m_2.g = +m_2.\ddot{x}$$

$$\boxed{\ddot{x} = \frac{\ddot{y}_m}{2}}$$

acel. m_1 — acel m_2

$$-Fr - m_1.g.\operatorname{sen}\alpha + \frac{1}{2}m_2.g = (m_1 - m_2).\ddot{x}$$
$$-Fr + g\left(\frac{m_2}{2} - m_1.\operatorname{sen}\alpha\right) = (m_1 - m_2).\ddot{x}$$

c)

$$T - Fr - m_1 g \operatorname{sen}\alpha = 0$$

$$2T - m_2 g = 0$$
$$T = \frac{m_2 g}{2}$$

$$\frac{m_2 g}{2} - Fr - m_1.g.\operatorname{sen}\alpha = 0$$

<u>si</u>
$T > m_1 g \operatorname{sen}\alpha$

$$\frac{1}{2}m_2 g - m_1 g.\operatorname{sen}\alpha = Fr \leq \mu e.\ m_1.g\ \cos\alpha$$

$$\frac{m_2}{2m_1} - \operatorname{sen}\alpha \leq \mu e.\cos\alpha$$

$$\boxed{m_2 \leq (\mu e.\cos\alpha. + \operatorname{sen}\alpha)\, 2m_1}$$

$T < m_1 g.\operatorname{sen}\alpha$

$$\frac{m_2 g}{2} + Fr - m_1 g \operatorname{sen}\alpha = 0$$

$$Fr = m_1 g \operatorname{sen}\alpha - \frac{m_2 g}{2}$$

$$\mu e.\ m_1.g.\cos\alpha \geq m_1 g\ \operatorname{sen}\alpha - \frac{m_2}{2}.g$$

$$\frac{m_2}{2} \geq m_1.\operatorname{sen}\alpha - \mu e\ m_1 \cos\alpha$$

$$\boxed{m_2 \geq 2m_1\left(\operatorname{sen}\alpha - \mu e.\cos\alpha\right)}$$

d) i) $2\ddot{x} = \ddot{y}_m = A$

$$-\mu d.m_1.g.\cos\alpha + g\left(\frac{m_2}{2} - m_1.\operatorname{sen}\alpha\right) = (m_1 - m_2).\frac{A}{2}$$

$$\frac{\left(\mu d.m_1.\cos\alpha + \frac{m_2}{2} - m_1.\operatorname{sen}\alpha\right)}{m_1 - m_2} = \frac{A}{2g}$$

$$-\mu d.m_1.g.\cos\alpha - \frac{A}{2}m_1 - g\,m_1.\operatorname{sen}\alpha = -m_2\frac{A}{2} - g\frac{m_2}{2} = -(A+g)\frac{m_2}{2}$$

$$\boxed{\frac{2\left(\mu d\, m_1.g.\cos\alpha - A/_2 m_1 - g.m_1 \operatorname{sen}\alpha\right)}{(-A-g)} = m_2}$$

Imposible que $A>g$ porque los hilos laburan a la extensión no a la compresión ii)

$$L_2 = 2y_0 - y_1 - y_m \qquad (y_0 - y_m)$$

$$0 = 2\ddot{y}_0 - 0 - \ddot{y}_m$$

$$0 = 2\ddot{y}_0 - \ddot{y}_m$$

$$\ddot{y}_m = 2\ddot{y}_0$$

$$A = 2\ddot{y}_0$$

$$-Fr - m_1 g\, \text{sen}\,\alpha + \frac{1}{2} m_2 g = (m_1 - m_2)\,\ddot{x}$$

$$-\mu d\, m_1 g \cos\alpha - m_1 g\, \text{sen}\,\alpha + \frac{1}{2} m_2 g = (m_1 - m_2)\,\ddot{y}_0$$

$$(m_1 - m_2)\frac{d\dot{y}}{dt} =$$

$$(m_1 - m_2)\int_{\dot{y}_0=0}^{\dot{y}} d\dot{y} = \frac{1}{2} m_2 g \int_0^t dt - m_1 g\, \text{sen}\,\alpha \int_0^t dt - \mu d\, m_1 g \cos\alpha \int_0^t dt$$

$$(m_1 - m_2)\,\dot{y}_y = \frac{1}{2} m_2 g\, t - m_1 g\, \text{sen}\,\alpha\, t - \mu d\, m_1 g \cos\alpha\, t$$

$$(m_1 - m_2)\int_{y_0=h'} dy = \int_{t_0=0}^{t} t\, dt$$

$$(m_1 - m_2)(y - h') = \left(\frac{1}{2} m_2 g - m_1 g\, \text{sen}\,\alpha - \mu d\, m_1 g \cos\alpha\right) \cdot \frac{t^2}{2}$$

$$\boxed{y = \left(\frac{1}{2} m_2 g - m_1 g\, \text{sen}\,\alpha - \mu d\, m_1 g \cos\alpha\right) \cdot \frac{1}{(m_1 - m_2)} \cdot \frac{t^2}{2} + h'}$$