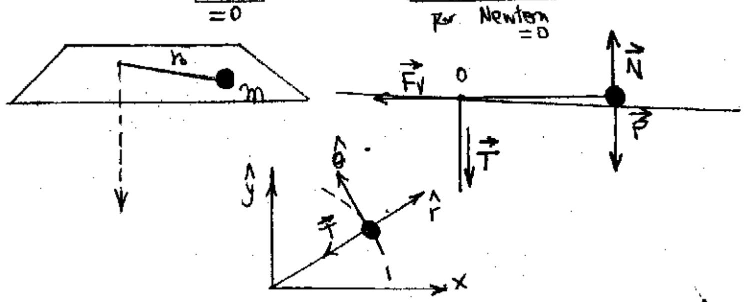
— Impulso Angular

②

$$\sum \vec{\tau}_o^{ext} = \underbrace{0\times\vec{F_V}}_{=0} + \vec{r}\times\vec{T} + \underbrace{\vec{r}\times\vec{N}+\vec{r}\times\vec{P}}_{\text{por Newton } =0} \Rightarrow \frac{d\vec{L_o}}{dt} = r\hat{r}\times -T\hat{r} = 0 \Rightarrow$$

$$\vec{L_o} \equiv k$$



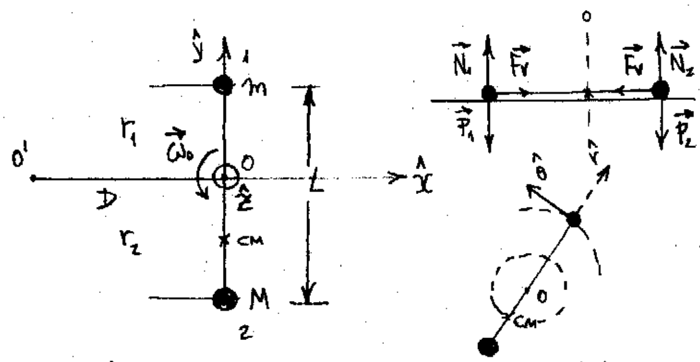
$$L_o = r\hat{r}\times m.(\dot{r}\hat{r} + r.\dot{\theta}\hat{\theta})$$
$$L_o = r^2 m.\dot{\theta}\hat{z}$$

$$L_{o_i} = L_{o_f}$$

$$\cancel{m}.r_o^2.\omega_o = \cancel{m}.r^2\omega$$

$$\boxed{\frac{\omega_o.r_o^2}{r^2} = \omega}$$

①



a)

$$\sum \vec{F}^{ext} = \underbrace{\vec{N_1}+\vec{P_1}}_{=0 \text{ por Newton}} + \underbrace{\vec{N_2}+\vec{P_2}}_{=0 \text{ por Newton}} = 0$$

$$\Rightarrow \frac{d\vec{P}}{dt} = 0 \Rightarrow \vec{P} \equiv k$$

$$\sum \vec{\tau}_o^{ext} = \underbrace{\vec{r_1}\times\vec{N_1}+\vec{r_1}\times\vec{P_1}}_{=0} + \underbrace{\vec{r_2}\times\vec{N_2}+\vec{r_2}\times\vec{P_2}}_{=0}$$

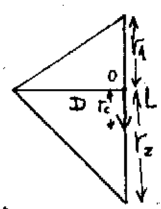
$$\Rightarrow \frac{d\vec{L_o}}{dt} = 0 \Rightarrow \vec{L_o} \equiv k$$

b)

$$\vec{L_o} = m.(r_1\hat{r}\times r_1.\dot{\theta}\hat{\theta}) + M(r_2\hat{r}\times r_2\dot{\theta}\hat{\theta})$$
$$\vec{L_o} = m.r_1^2.\dot{\theta}\hat{z} + M.r_2^2.\dot{\theta}\hat{z}$$

$\hat{z})$ $L_o = \dot{\theta}(mr_1^2 + Mr_2^2)$

$$\omega_o = \frac{L_o}{mr_1^2 + Mr_2^2}$$



c)

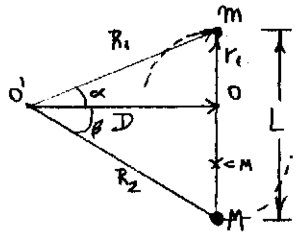
$$\vec{P} = (m+M).\vec{V}_{cm}$$
$$P = (m+M).(\dot{r}_c\hat{r} + r_c.\dot{\theta}\hat{\theta})$$
$$P = m\,r_{cm}\dot{\theta} + M.r_{cm}.\dot{\theta}$$
$$P = (r_{cm}.\dot{\theta}).(m+M)$$

$$\vec{r}_{cm} = \frac{m.r_1 + M.r_2}{(m+M)}\hat{r}$$

$$P = \left(\frac{mr_1 + Mr_2}{\cancel{m+M}}\right)\dot{\theta}\,(\cancel{m+M})$$

$$P_i = P_f$$
$$(mr_1 + Mr_2).\omega_o = (mr_1 + Mr_2).\omega$$
$$\boxed{\omega = \omega_o}$$ la velocidad angular no varía

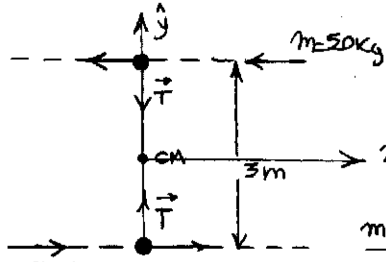
d)



$$\vec{L}_{o'} = \vec{R_1}\times m.\vec{V_1} + \vec{R_2}\times M.\vec{V_2}$$
$$\vec{L}_{o'} = (\vec{D}+\vec{r_1})\times m\vec{V_1} + (\vec{D}+\vec{r_2})\times M.\vec{V_2}$$

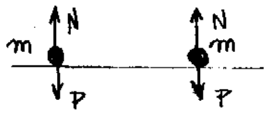
③

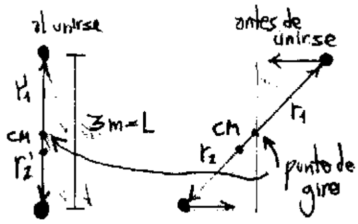
$m = 50\,Kg \quad V = 10\ m/s$



$m = 50\,Kg$
$\vec{V} = 10\ m/s$

ⓐ





Vínculo

$$L = \vec{r}_1{}' - \vec{r}_2{}'$$
$$= (r_1'\hat{y}) - (-r_2'\hat{y})$$
$$L = (r_1' + r_2')\,\hat{y}$$
$$L = r_1' + r_2'$$

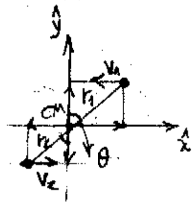
(a partir de que se agarran)

sistema inercial en el cm

$$\vec{R}_{CM} = 0 = \frac{\cancel{m}\vec{r}_1 + \cancel{m}\vec{r}_2}{2m}$$
$$\vec{r}_1 = -\vec{r}_2$$

en todo momento

$$|\vec{r}_1| = |-\vec{r}_2|$$
$$r_1 = r_2$$



Luego $L = r_1' + r_2' = 2r_1' = 2r_2'$

ⓑ

■ momento lineal

$$\sum \vec{F}^{ext} = \vec{N}_1 + \vec{P}_1 + \vec{N}_2 + \vec{P}_2 = \underbrace{2\vec{N} + 2\vec{P}}_{=0 \text{ x Newton}}$$

$$P \equiv k \qquad M.\vec{A}_{cm} = 0$$
$$M.\vec{V}_{CM} = k$$

$$\underset{\text{antes}}{P_i} = \underset{\text{después}}{P_F}$$

$$m.\vec{V}_1 + m.\vec{V}_2 = m.\vec{V}_1' + m.\vec{V}_2'$$
$$m.V_1(-\hat{x}) + m.V_1(\hat{x}) = 0 = m\vec{V}_1' + m\vec{V}_2'$$
$$0 = \cancel{m}\vec{V}_1' + \cancel{m}\vec{V}_2'$$
$$\boxed{\vec{V}_1' = -\vec{V}_2'}$$

el cm permanece siempre en reposo

■ momento angular

$$\sum \vec{\tau}^{\,ext}_{CM} = 0 \Rightarrow \frac{dL_{cm}}{dt} = 0 \Rightarrow L_{cm} \equiv k$$

$$L_{CM}^{i} = L_{CM}^{F}$$

antes de agarrarse

$$\vec{r}_1 \times m\vec{V}_1 + \vec{r}_2 \times m\vec{V}_2 = \vec{r}_1{}' \times m\vec{V}_1' + \vec{r}_2{}' \times m\vec{V}_2'$$
$$\cancel{m}\left(\vec{r}_1 \times (-V_1)\hat{x} + \vec{r}_2 \times (V_1)\hat{x}\right) = \cancel{m}\left(\vec{r}_1{}' \times \vec{V}_1' + \vec{r}_2{}' \times \vec{V}_2'\right)$$
$$-\left([\vec{r}_1 + \vec{r}_2] \times V_1\hat{x}\right) \qquad = (\vec{r}_1{}' \times \vec{V}_1') - (\vec{r}_2{}' \times \vec{V}_1')$$
$$= (\vec{r}_1{}' - \vec{r}_2{}') \times (\vec{V}_1')$$
$$= L\hat{r} \times \vec{V}_1'$$
$$= L\,\dot{r}_1'.\dot{\theta}\,\hat{z}$$
$$= (2r_1'^2.\dot{\theta})\,\hat{z}$$