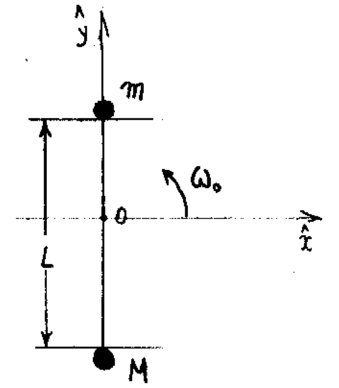
# IMPULSO ANGULAR

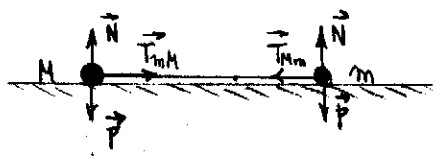
**1.**




$$\omega(t=0) = \omega_0 = \dot{\theta}_0$$

$$\sum \vec{F}_e = 0$$

a)




$$\sum \vec{F}_e = \vec{N}_M + \vec{P}_M + \vec{N}_m + \vec{P}_m = 0 \quad \text{por Newton}$$

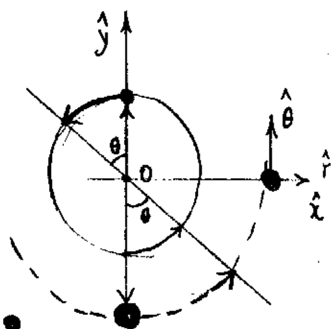
$$\sum \vec{F}_e = \frac{d\vec{P}}{dt} = 0 \Rightarrow \vec{P} = k$$

$$\boxed{\text{se conserva } \vec{P}}$$

$$\frac{d\vec{L}_o}{dt} = \vec{R}_m \times \underbrace{\vec{F}e_m}_{=0} + \vec{R}_M \times \underbrace{\vec{F}e_M}_{=0} = 0 \Rightarrow \vec{L}_o = k$$

$$\boxed{\text{se conserva } \vec{L}_o}$$

b)

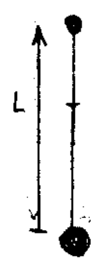



$$\vec{L}_o = \vec{R}_m \times m.\vec{V}_m + \vec{R}_M \times M.\vec{V}_M$$

$$\vec{L}_o = \vec{R}_m \times m.\vec{V}_m + (\vec{R}_m - \vec{L}) \times M.\vec{V}_M$$

$$\vec{L}_o = R_m.\hat{r} \times m.R_m.\dot{\theta}_m.\hat{\theta} + (R_m - L).\hat{r} \times M.(R_m - L).\dot{\theta}_M.\hat{\theta}$$

$$\vec{L}_o = m.R_m^2.\dot{\theta}_m\,\hat{z} + M.(R_m - L)^2.\dot{\theta}_M.\hat{z}$$




$$-\vec{R}_M + \vec{R}_m = \vec{L}$$
$$\vec{R}_m = \vec{L} + \vec{R}_M$$
$$\vec{R}_M = \vec{R}_m - \vec{L}$$

$$\vec{V}_m = R_m.\dot{\theta}_m.\hat{\theta}$$
$$\vec{V}_M = (R_m - L).\dot{\theta}_M.\hat{\theta}$$

$$\dot{\theta}_m = \dot{\theta}_M = \dot{\theta}_0$$

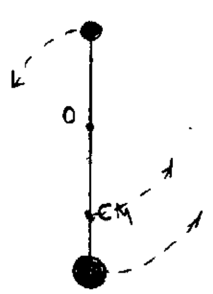
por el vínculo de la barra

$$\boxed{\vec{L}_o = \left[m.R_m^2 + M.(R_m - L)^2\right].\dot{\theta}_0.\hat{z}}$$

$\hat{r}$ $\hat{\theta}$ $\hat{z}$

La $\omega$ de la barra no varía con el tiempo; es constante e igual a $\dot{\theta}_0$

c)




$$\vec{R}_{cm} = \frac{m.\vec{R}_m + M.\vec{R}_M}{m+M}$$

$$\vec{V}_{cm} = \frac{m.\vec{V}_m + M.\vec{V}_M}{m+M}$$

$$\vec{V}_{cm} = \frac{(m.R_m.\omega_0)\hat{\theta} + (M.(R_m - L).\omega_0)\hat{\theta}}{m+M}$$

$$\vec{V}_{cm} = \frac{[m.R_m + M(R_m - L)]}{m+M}\omega_0\,\hat{\theta}$$

$$\boxed{\vec{V}_{cm} = R_{cm}.\omega_0.\hat{\theta}} \quad \text{Velocidad constante}$$

$\hat{\theta}$)

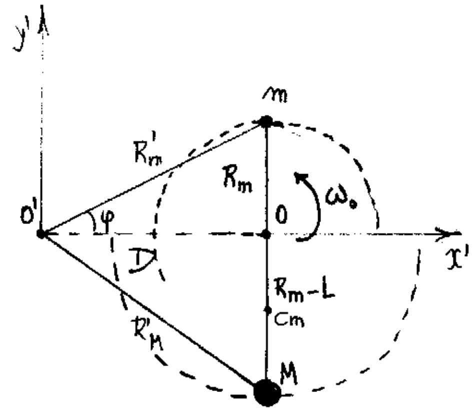
$$\frac{V_{cm}}{R_{cm}} = \omega_0$$

$$\dot{\theta} = \omega_0$$

$$\frac{d\theta}{dt} = \omega_0$$

$$\int_0^\theta d\theta = \int_0^t \omega_0 \, dt \quad \Rightarrow \quad \boxed{\theta_{cm} = \omega_0 t}$$

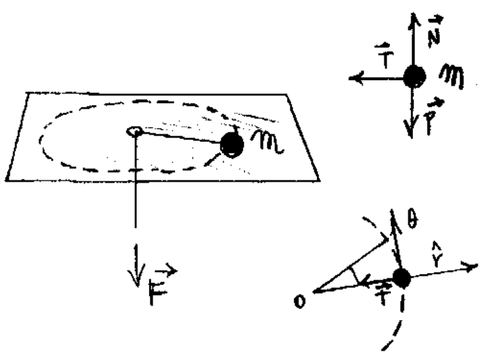
d)




$$\vec{L} = \sum_{i=1}^{n} \vec{R}_i \times m_i \vec{V}_i$$

$$\vec{L}_{o'} = \sqrt{(D^2 + R_m^2)} \times m \,.\, V_m' + \sqrt{D^2 + (R_m - L)^2} \times M \,.\, V_M'$$

**2.**




$$\vec{N} + \vec{P} = 0$$

$$V = \dot{r}\hat{r} + r.\dot{\theta}\hat{\theta}$$

$$\hat{r}) \quad m(\ddot{r} - r.\dot{\theta}^2) = -T$$

$$\vec{F_e} = \vec{T} \qquad \vec{\tau} = \vec{r} \times \vec{T} \quad \therefore \quad \vec{\tau} = \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{z} \\ r & \emptyset & \emptyset \\ -T & \emptyset & \emptyset \end{vmatrix} = (0+0+0) - (0+0+0) = 0$$

$$\Rightarrow \quad \frac{d\vec{L}}{dt} = 0 \quad \therefore \quad \vec{L} \equiv k \quad \text{para } m$$

$$\vec{L}_o = \vec{r} \times m.\vec{v} \quad \therefore \quad \vec{L}_o = r.\hat{r} \times m.(\dot{r}\hat{r} + r.\dot{\theta}.\hat{\theta})$$

$$\vec{L}_o = r.\hat{r} \times m.\dot{r}\hat{r} + r.\hat{r} \times m.r.\dot{\theta}.\hat{\theta}$$

$$\vec{L}_o = 0 + (r.m.r.\dot{\theta})\hat{z} = (m.r^2.\dot{\theta})\hat{z}$$

inicial

$$\hat{z}) \quad L_{oi} = m.r_0^2.\omega_0$$

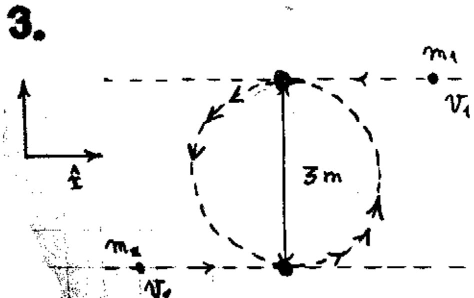
tiempo t

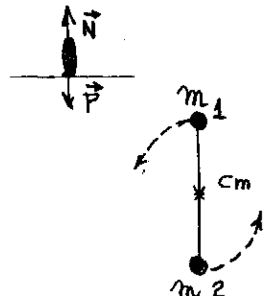
$$L_o = m.r^2.\omega \qquad \text{pero } L_{oi} = L_o \Rightarrow$$

$$m r_0^2 \omega_0 = m r^2 \omega$$

$$\boxed{\omega = \frac{r_0^2}{r^2}.\omega_0}$$

**3.**



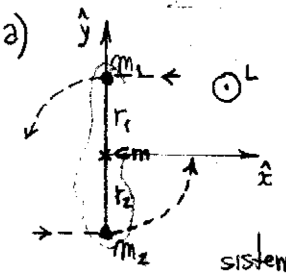




$$\sum \vec{F_e} = 0$$

$$\Downarrow$$

$$\sum \vec{\tau_e} = 0 \Rightarrow \vec{L} \equiv k$$

a)




$\vec{L}_o = \vec{r} \times m.\vec{v}$

antes de agarrarse

$\vec{L}_{cm} = \vec{r}_1 \times m.\vec{V}_1 + \vec{r}_2 \times m.\vec{V}_2$

una vez agarrados

$\vec{L} = r_1 \hat{r} \times m.r_1\dot{\theta}_1\hat{\theta} + r_2\hat{r} \times m.r_2\dot{\theta}_2\hat{\theta}$

sistema: $m_1, m_2$, barra

$\vec{L} = m r_1^2 \dot{\theta}_1 \hat{z} + m r_2^2 \dot{\theta}_2 \hat{z} = m(r_1^2\dot{\theta}_1 + r_2^2\dot{\theta}_2)\hat{z}$

$\hat{z})\quad L = m.2.r^2.\dot{\theta}$

Como L es constante ambas giran con velocidad angular constante

$L = 50kg.2.(1,5m)^2.\dot{\theta}$

$50kg.2.1,5m.\underbrace{1,5m.\dot{\theta}}_{10\frac{m}{s}}$

$\boxed{\dot{\theta} = 6\frac{2}{3}\frac{1}{s}}$ $\quad L = 1500\ kg\frac{m^2}{s}$

b) $1500 kg\frac{m^2}{s} = 2.50kg.0,5m.0,5m.\dot{\theta}$

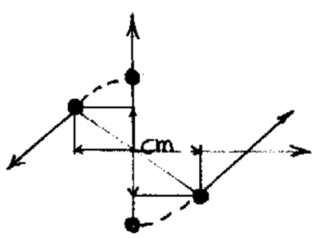
$\frac{1500\ kg.\frac{m^2}{s}}{100\ kg.0,25m^2} = \dot{\theta}$

$\boxed{60\frac{1}{s} = \dot{\theta}}$

Al acortar la varilla no se altera el momento angular (solo hay fuerzas internas de tensión).

Aumenta la velocidad angular

c)

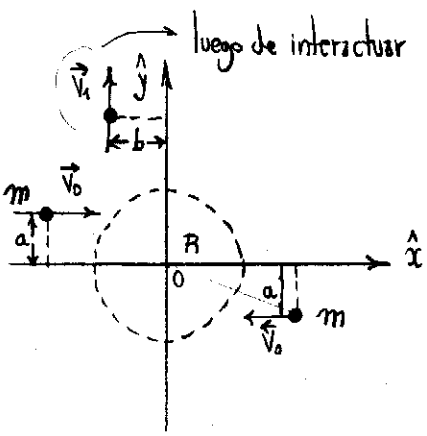



Al soltar la varilla c/u sigue en línea recta porque se desvanecen las T de la misma.
El momento se sigue conservando

caso a)

$1500 = r$

**4.**




a) sistema: 2 átomos

$\sum \vec{F}_{ext} = 0$ (no actúa $\vec{g}$ porque no está en la [figura])

sistema (antes de la interacción)

$\vec{P} = \frac{m.V_0\hat{x} + m.-V_0\hat{x}}{2m} = \frac{(mV_0 - mV_0)}{2m}\hat{x}$

$\vec{P} = 0$ Luego $\frac{d\vec{P}}{dt} = 0$

① $\sum \vec{F}ext = 0 = \frac{d\vec{P}}{dt} = M.\vec{A}_{cm} = 2m.\vec{A}_{cm}$

$\Rightarrow \boxed{P \text{ se conserva}}$

$\vec{L}_0 = \vec{r}_1 \times m.\vec{V}_0 + \vec{r}_2 \times m.\vec{V}_0$

$\vec{L}_0 = m(\vec{r}_1 \times \vec{V}_0 + \vec{r}_2 \times \vec{V}_0)$

② $\sum \vec{\tau}_{ext} = 0 \Rightarrow \frac{d\vec{L}}{dt} = 0 \therefore$

respecto de 0

$\sum_{i=1}^{2} \vec{r}_i \times \underbrace{\vec{F}e_i}_{0} = 0$ $\quad \boxed{L \text{ se conserva}}$

b) Como P se conserva $\frac{d\vec{P}}{dt} = M.\vec{A}_{CM} = 0 \Rightarrow M.\vec{V}_{CM} = k$

Antes || $\hat{x})\ 2m.V_{cm} = mV_0 - mV_0 = 0$ $\quad \hat{y})\ 2m.V_{cm} = m.0 + m.0 = 0$

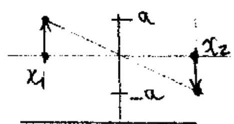
tanto $V_x = 0$, $V_y = 0$ $\Rightarrow$ $\vec{V}_{cm} = \vec{0}$ Luego si $\vec{P}$ se conserva $\boxed{\vec{V}_{cm} = 0}$ antes, durante y después

c) Si

$$\vec{R}_{cm} = \frac{m.\vec{r}_1 + m.\vec{r}_2}{m+m} \quad \therefore \quad \vec{R}_{cm} = \frac{\cancel{m}}{2\cancel{m}}(\vec{r}_1+\vec{r}_2) \quad \therefore \quad \vec{R} = \frac{1}{2}(\vec{r}_1+\vec{r}_2)$$

$\hat{x})$ $R_x = \frac{1}{2}[x_1 + x_2]$

$\hat{y})$ $R_y = \frac{1}{2}[(a)+(-a)] = 0$




$\therefore$ $x_1 = x_2$

$\boxed{\vec{R}_{cm} = (0,0)}$ antes, durante y después

d) $0 = m.\vec{V}_{1i} + m.\underbrace{\vec{V}_{2i}}_{(-\vec{V}_{1i})}$ [antes]

[después] $0 = m.\vec{V}_{1f} + \underbrace{m.\vec{V}_{2f}}_{\text{segundo átomo}}$

$\vec{0} = m.\vec{r}_1 + m.\vec{r}_2$ [antes]

$\hat{x})$ $0 = m.\underbrace{V_{1F}}_{=0} + m.V_{2F}$ $\Rightarrow$ $\hat{x})$ $V_{2F} = 0$ ($\vec{V}_{1F} = -\vec{V}_{2F}$)

$\hat{y})$ $0 = m.V_{1F} + m.V_{2F}$ $\hat{y})$ $V_{2F} = -V_{1F}$

$V_{y_2} = -V_{1f}$

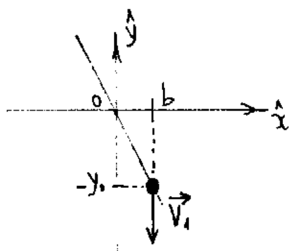
$$\boxed{\vec{V}_{2F} = (0, -V_{1F})}$$

$\hat{x})$ $0 = m(-x_0) + m.x_0$ $\hat{y})$ $0 = m.a + m.(-a)$

$\hat{x})$ $0 = m(-b) + m.x$, $x = b$ $\hat{y})$ [después] $0 = m.y_0 + m.y$, $y = -y_0$

posición átomo 2 : $(b, -y_0)$

e)



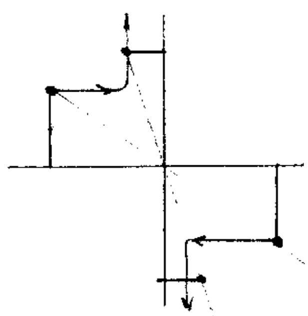

$$\frac{dr}{dt} = -V_1 \quad \text{(velocidad constante)}$$

$$\int dr = \int -V_1 . dt$$

$$r = -V_1 . t + C$$

$$\boxed{\vec{r}_2 = (-V_1 . t + r_{0z})\hat{y}}$$

f)



ver más abajo

d) $L_0 = m.(\vec{r}_1 \times \vec{V}_{1i} - \vec{r}_2 \times \vec{V}_{2i})$

$m.[(-x_1\hat{x} + a\hat{y}) \times (V_{1i}\hat{x}) + (x_1\hat{x} + (-a)\hat{y}) \times (\overbrace{V_{2i}}^{-V_{1i}}.\hat{x})]$ $\hat{x}\,\hat{y}\,\hat{z}$

$L_0 = m[-a.V_{1i}\hat{z} + -a.V_{1i}\hat{z}]$

i) $L_0 = -2.m.a\,V_{1i}\,\hat{z}$

f) $L_0 = m[r_1 \times \vec{V}_{1F} + r_2 \times \vec{V}_{2F}]$

$m[(-b\hat{x} + y_1\hat{y}) \times (V_{1F}\hat{y}) + (b\hat{x} + (-y_1)\hat{y}) \times (V_{2F}\hat{y})]$

$m[-b\,V_{1F}\hat{z} + -b.V_{2F}\hat{z}]$

$\vec{L}_0 = -m.b\,(V_{1F} - V_{2F})\hat{z}$

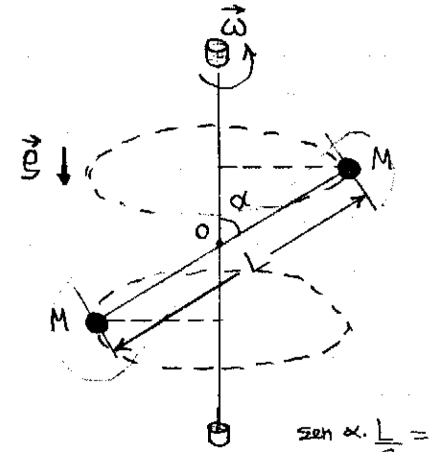
$\therefore \hat{z})$ $\cancel{+}2m.a\,V_{1i} = \cancel{-}m.b(V_{1F} - V_{2F})$

$$\frac{a}{b} = \frac{1}{2}\frac{(V_{1F} - V_{2F})}{V_{1i}} = \frac{1}{2}\cdot\frac{V_{1F}+V_{1F}}{V_{1i}} = \frac{\cancel{2}}{\cancel{2}}\frac{V_{1F}}{V_{1i}} = \frac{V_{2F}}{V_{2i}} = \frac{V_1}{V_0}$$

f) si $a = b \Rightarrow V_1 = V_0$ y si $a < b \Rightarrow V_1 < V_0$

si $a > b \Rightarrow V_1 > V_0$

**5.**



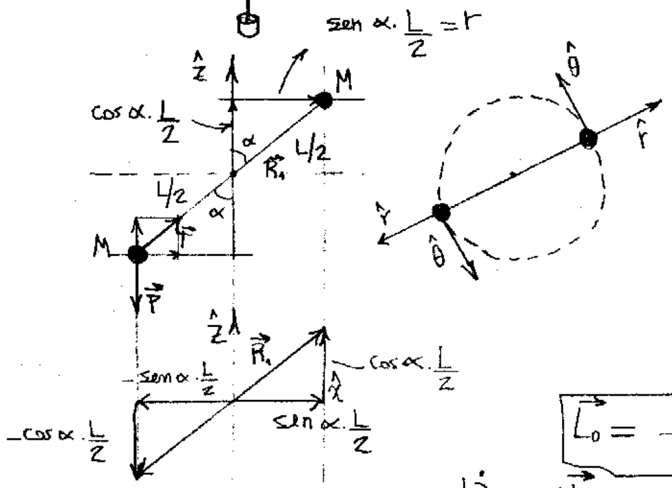
a) sistema: M, M, barra

$$\vec{L}_{cm} = \vec{R}_1 \times m.\vec{V}_1 + \vec{R}_2 \times m.\vec{V}_2$$

$$\vec{L}_{cm} = M.[\vec{R}_1 \times \vec{V}_1 + \vec{R}_2 \times \vec{V}_2]$$

$$L_o = M.\Big[\Big(\text{sen}\,\alpha.\frac{L}{2}\hat{r} + \cos\alpha\frac{L}{2}\hat{z}\Big) \times \Big(\text{sen}\,\alpha.\frac{L}{2}.\dot{\theta}\hat{\theta}\Big) + \Big(-\text{sen}\,\alpha.\frac{L}{2}\hat{r} - \cos\alpha\frac{L}{2}\hat{z}\Big) \times \Big(-\text{sen}\,\alpha.\frac{L}{2}\dot{\theta}\hat{\theta}\Big)\Big]$$

$$L_o = M\Big[\Big(\text{sen}^2\alpha\frac{L^2}{4}\dot{\theta}\hat{z} - \cos\alpha\,\text{sen}\,\alpha\frac{L^2}{4}\dot{\theta}\hat{r}\Big) + \Big(\text{sen}^2\alpha\frac{L^2}{4}\dot{\theta}\hat{z} - \cos\alpha\,\text{sen}\,\alpha\frac{L^2}{4}\dot{\theta}\hat{r}\Big)\Big]$$




$$L_o = M.\Big[2\,\text{sen}^2\alpha\frac{L^2}{4}.\dot{\theta}.\hat{z} - 2\cos\alpha.\text{sen}\,\alpha.\frac{L^2}{4}\dot{\theta}\hat{r}\Big]$$

$$\vec{L}_o = \cancel{2}M\,\text{sen}\,\alpha\,.\frac{L^2}{\cancel{4}_2}\dot{\theta}\,(\text{sen}\,\alpha\,\hat{z} - \cos\alpha\,\hat{r})$$

$$\boxed{\vec{L}_o = -\Big(M.\text{sen}\,\alpha.\cos\alpha\frac{L^2}{2}.\omega\Big)\hat{r} + \Big(M\,\text{sen}^2\alpha\,\frac{L^2}{2}\omega\Big)\hat{z}}$$

$$\hat{r} = r.\cos\varphi.\hat{x} + r.\text{sen}\,\varphi.\hat{y}$$

$$\frac{d\varphi}{dt} = \dot{\varphi}$$

b)

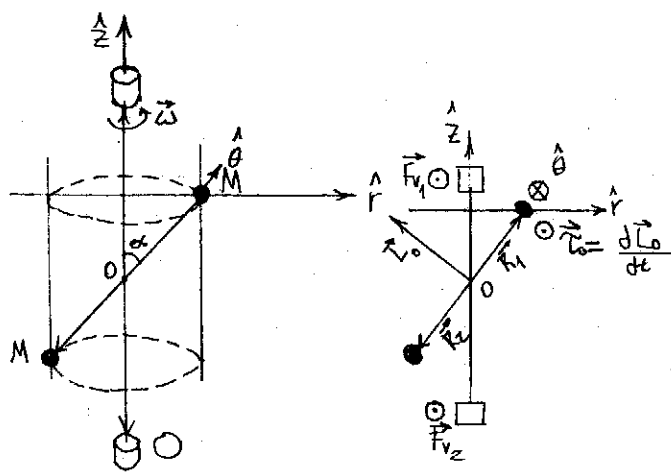
$$\frac{d\vec{L}_o}{dt} = -M.\text{sen}\,\alpha.\cos\alpha.\frac{L^2}{2}.\omega.\frac{d\hat{r}}{dt}$$

$$\frac{d\vec{L}_o}{dt} = -M.\text{sen}\,\alpha.\cos\alpha.\frac{L^2}{2}\omega.\omega.\hat{\theta} = \vec{\tau}_{ext}$$

$$\boxed{\frac{d\vec{L}_o}{dt} = -M.\text{sen}\,\alpha.\cos\alpha.\frac{L^2}{2}\omega^2.\hat{\theta}}$$

$\vec{L}$ no es constante en el tiempo porque cambia su orientación

c)




$\vec{L}_o$, $\vec{\tau}_o$, $\vec{F}_v$ rotan con el sistema manteniendo sus módulos constantes

d)

$$\sum_{i=1} \vec{R}_i \times \vec{F}e_i = \cancel{\vec{R}_1 \times M.\vec{g} + \underbrace{\vec{R}_2}_{-\vec{R}_1} \times M.\vec{g}} + \vec{Z} \times \vec{F}_{v_1} + \vec{Z} \times \vec{F}_{v_2}$$

$$+\; z\hat{z} \times (-F_{v_1})\hat{y} + (-z)\hat{z} \times (-F_{v_2})\hat{y}$$

e) La gravedad no influye porque se ve compensada por fuerzas de vínculo y por ser las masas M iguales