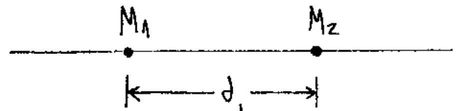
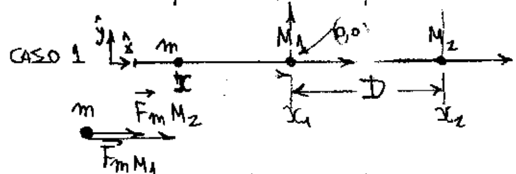
GRAVITACION

1.



a) CASO 1



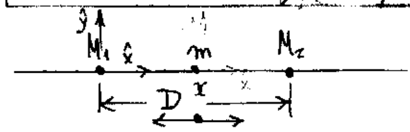
$x_2 - x_1 = D$

$\hat{x})$ $\vec{F}_m = \vec{F}_{GM_1} + \vec{F}_{GM_2}$

$$F_{neta} = \frac{G.M_1.m}{(-x)^2} + \frac{G.M_2.m}{(D-x)^2} = \frac{G.M_1.m}{x^2} + \frac{G M_2.m}{(D-x)^2}$$

$$\boxed{\vec{F}_n = \frac{Gm\left(M_1(D-x)^2 + M_2.x^2\right)}{(x^2).(D-x)^2}\hat{x}}$$

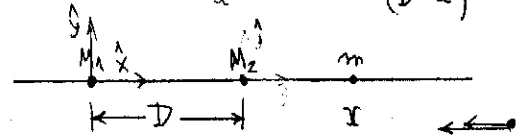
CASO 2



$\hat{x})$
$$\vec{F}_{neta} = -\frac{G.M_1.m}{x^2} + \frac{G.M_2.m}{(D-x)^2} = \boxed{\frac{-Gm\left(M_1.(D-x)^2 - M_2.x^2\right)}{x^2.(D-x)^2}\hat{x}}$$

$x^2 D^2 - 2x^3 D + x^4$

CASO 3



$\hat{x})$
$$\vec{F}_{neta} = -\frac{G M_1 m}{(x^2)} - \frac{G M_2 m}{(x-D)^2} = \boxed{\frac{-Gm\left(M_1(x-D)^2 + M_2.x^2\right)}{(x^2)(x-D)^2}\hat{x}}$$

b) $F_n = -\frac{dU}{dx}$

$$\frac{Gm\left[M_1.(D-x)^2 + M_2 x^2\right]}{x^2.(D-x)^2} = -\frac{dU}{dx}$$

$$\int \frac{Gm\, M_1 \cancel{(D-x)^2}dx}{x^2.\cancel{(D-x)^2}} + \int \frac{Gm\, M_2 \cancel{x^2}}{\cancel{x^2}(D-x)^2}dx = \int -dU$$

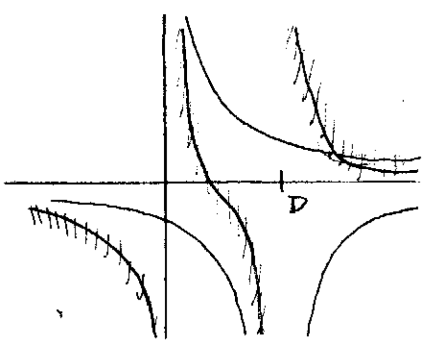
$$G.m.M_1 \int \frac{1}{x^2}dx + GmM_2 \int \frac{1}{(D-x)^2}dx = -U(x) = \overbrace{-V_G(x)}^{\text{Potencial Gravitatorio}}$$

$$-\frac{G.m.M_1}{x} + \frac{G.m.M_2}{D-x} = -V_G \Rightarrow V_G = \frac{GmM_1}{x} - \frac{GmM_2}{D-x}$$

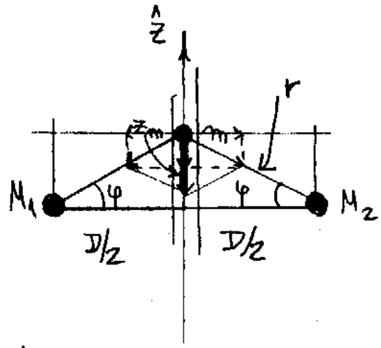
$D-x=z$
$-1dx = dz$

$\int \frac{1}{z^2}dz$

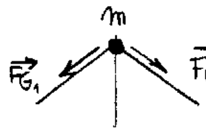
$-\frac{z^{-1}}{-1} \quad \frac{1}{z}$



3.



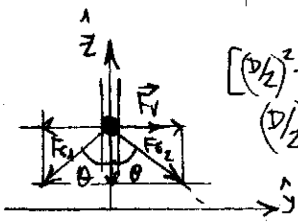
a) $-\dfrac{dE_p}{d\vec{r}} = \vec{F}$



$\text{sen}\,\varphi = \dfrac{z_m}{r}$

$\cos\varphi = \dfrac{D/2}{r}$

$tg\,\varphi = \dfrac{z_m}{D/2} = \dfrac{2z}{D}$



$\left[(D/2)^2+z^2\right]^{3/2} = r^3$

$(D/2)^2 + z^2 = r^2 = \dfrac{z^2}{\text{sen}^2\varphi}$

(*) viene desde abajo:

$(D/2)^2 + z^2 = U$

$2z\,dz = du$

$dz = \dfrac{du}{2z}$ ⇓

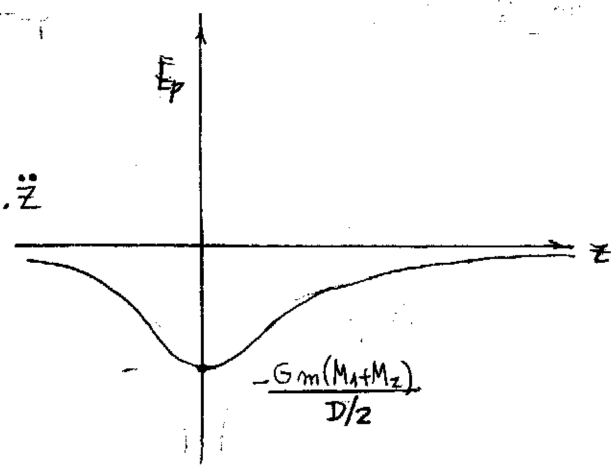
$\displaystyle\int \frac{\cancel{z}\,du}{U^{3/2}\cdot 2\cancel{z}} = -\frac{1}{\sqrt{U}} \Rightarrow$

$E_p = -Gm(M_1+M_2)\cdot\dfrac{1}{\left[(D/2)^2+z^2\right]^{1/2}}$

Ecuaciones Newton para m:

$\hat{y})\quad F_v - F_{G_1}\cdot\text{sen}\,\theta + F_{G_2}\cdot\text{sen}\,\theta = 0$

$\hat{z})\quad -F_{G_1}\cdot\cos\theta - F_{G_2}\cdot\cos\theta = m\cdot\ddot{z}$



$\text{sen}\,\theta = \dfrac{D/2}{r} = \dfrac{D/2}{\sqrt{(D/2)^2+z^2}}$

$\cos\theta = \dfrac{z}{r} = \dfrac{z}{\sqrt{(D/2)^2+z^2}}$

$$-\frac{G\cdot m\cdot M_1}{(D/2)^2+z^2}\cdot\frac{z}{\sqrt{(D/2)^2+z^2}} - \frac{GmM_2}{(D/2)^2+z^2}\cdot\frac{z}{\sqrt{(D/2)^2+z^2}} = m\cdot\ddot{z}$$

$$-\frac{Gm(M_1+M_2)}{\left[(D/2)^2+z^2\right]^{3/2}}\cdot z = -\frac{dE_p}{dz} \Rightarrow E_p = Gm(M_1+M_2)\int\frac{z}{\left[(D/2)^2+z^2\right]^{3/2}}dz \quad (*)$$

b)

$$\ddot{z} = -\frac{G(M_1+M_2)\,z}{\left[(D/2)^2+z^2\right]^{3/2}}$$

$$0 = -\frac{G(M_1+M_2)\,z}{\left[(D/2)^2+z^2\right]^{3/2}}$$

$0 = z$ si $z^2 \neq (D/2)^2$, $|z| \neq D/2$

$$\frac{\partial\ddot{z}}{\partial z} = -G(M_1+M_2)\cdot\left[1\cdot\left[(D/2)^2+z^2\right]^{3/2} - z\cdot\left(\tfrac{3}{\cancel{2}}\left[(D/2)^2+z^2\right]^{1/2}\cdot\cancel{2}z\right)\right]$$

$$\frac{\partial\ddot{z}}{\partial z} = -G(M_1+M_2)\cdot\left[(D/2)^2+z^2\right]^{3/2} - 3\cdot z^2\cdot\left[(D/2)^2+z^2\right]^{1/2}\Big]$$

$$\frac{\partial\ddot{z}}{\partial z}(z=0) = -G(M_1+M_2)\cdot(D/2)^3 < 0 \Rightarrow \boxed{z_{eq}=0 \text{ es estable}}$$

c)

pequeños apartamientos $z \ll d/2 \Rightarrow$

$$\ddot{z} \cong -\frac{G(M_1+M_2)\,z}{\left[(D/2)^2\right]^{3/2}} = -\frac{8\cdot G(M_1+M_2)\cdot z}{D^3}$$

$$\ddot{z} + \underbrace{\frac{8\cdot G\cdot(M_1+M_2)}{D^3}}_{\omega^2} z \cong 0$$

$$[\omega^2] = \left(N\cdot\frac{m^2}{Kg^2}\,\frac{Kg}{m^3}\right)^2 = Kg\cdot\frac{m}{s^2}\cdot\frac{m^2}{kg^2}\cdot\frac{Kg}{m^3}$$

$$\boxed{\omega = \sqrt{\frac{8\cdot G\cdot(M_1+M_2)}{D^3}}}$$

d) $F_V = (F_{G1} - F_{G2}) \cdot \text{sen}\,\theta = \left(\dfrac{G m M_1}{(D/2)^2 + z^2} - \dfrac{G m M_2}{(D/2)^2 + z^2} \right) \cdot \dfrac{D/2}{\sqrt{(D/2)^2 + (z)^2}}$

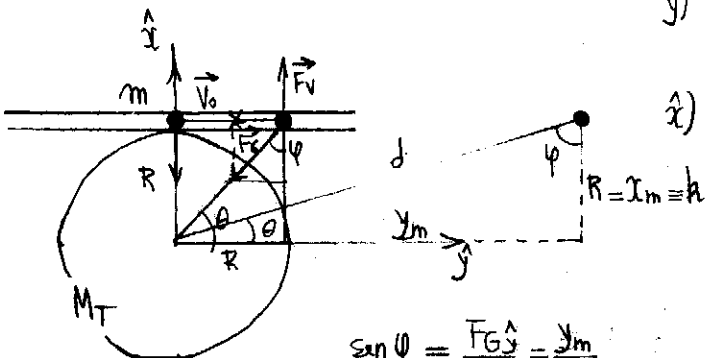
$$\boxed{F_V(z) = \frac{G m \cdot (M_1 - M_2)}{\left[(D/2)^2 + z^2\right]^{3/2}} \cdot (D/2)}$$

4.

a) Newton:

$\hat{y})\quad m \cdot \ddot{y} = -F_G \cdot \text{sen}\,\varphi$

$\hat{x})\quad 0 = F_V - F_G \cdot \cos\varphi$



$m \cdot \ddot{y} = -\dfrac{G \cdot m \cdot M_T}{d^2} \cdot \dfrac{y}{d} = \dfrac{-G \cdot m \cdot M_T \cdot y}{[R^2 + y^2]^{3/2}}$

$\text{sen}\,\varphi = \dfrac{F_G \hat{y}}{F_G} = \dfrac{y_m}{d}$

$\cos\varphi = \dfrac{F_G \hat{x}}{F_G} = \dfrac{R}{d}$

$d^3 = \left(\sqrt{R^2 + y^2}\right)^3$

$\ddot{y} + \left(\dfrac{G\, M_T}{[R^2 + y^2]^{3/2}}\right) \cdot y = 0$

Energía:

$-\dfrac{G m M_T \cdot y}{[R^2 + y^2]^{3/2}} = -\dfrac{dE_p}{dy}$

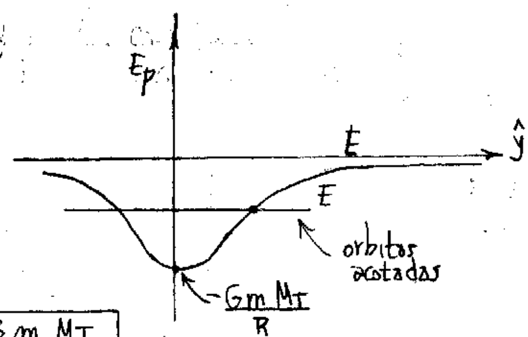
$G m M_T \displaystyle\int^{r} \frac{y}{(R^2 + y^2)^{3/2}}\, dy = \int_0^{E_p} dE_p$

$R^2 + y^2 = z$

$2y \cdot dy = dz$

$G m M_T \displaystyle\int \frac{dz}{2\,(z)^{3/2}} = E_p$

$\dfrac{G M_T m}{2} \cdot \dfrac{-1}{\sqrt{z} \cdot \frac{1}{2}} = \boxed{E_p = -\dfrac{G \cdot m \cdot M_T}{\sqrt{R^2 + y^2}}}$



$\sqrt{R^2 + y^2} = 0$

$R^2 = -y^2$

$\Leftrightarrow$

$R = y = 0$

$F_V \perp$ desplazamiento $\Rightarrow$ no hace W

m se mueve bajo $F_C \Rightarrow E \equiv k$

$E = \dfrac{1}{2} m \cdot V^2 + E_p(r)$

$E_i = \dfrac{1}{2} m V_0^2 +$

Para movimiento ligado necesito energía $E < 0$ pero $R \neq 0$

y el límite es $E = 0$ $\therefore$

en el punto A $\Big\} \Rightarrow 0 = \dfrac{1}{2} m V_0^2 - \dfrac{G m M_T}{\sqrt{R^2 + y^2}}$, $y \to 0$

$\dfrac{1}{2} m V_{0_{max}}^2 = \dfrac{G m M_T}{\sqrt{R^2}}$

$$\boxed{V_{0\,max} = \sqrt{\frac{2 G M_T}{R}}}$$

$[G] = \dfrac{m^3}{s^2 \cdot Kg}$

b) $\ddot{y} + \dfrac{G\, M_T}{(R^2 + y^2)^{3/2}}\, y = 0$

$g(y)_0 \cong g^0(y) \cdot \dfrac{y^0}{0!} + g'(y) \cdot \dfrac{y}{1!}$

$g(y) = G M_T \cdot \dfrac{y}{(R^2 + y^2)^{3/2}}$

Bajo la condición de pequeños apartamientos tendremos MAS:

$g'(y) = G \cdot M_T \left(\dfrac{1 \cdot (R^2 + y^2)^{3/2} - \frac{1}{2}(R^2 + y^2)^{1/2} \cdot 2y \cdot y}{(R^2 + y^2)^3} \right)$

$\dfrac{y^2}{R^2} \ll 1 \qquad \ddot{y} + \dfrac{G\, M_T}{R^3 \left(1 + \frac{y^2}{R^2}\right)^{3/2}}\, y \cong \boxed{\ddot{y} + \dfrac{G \cdot M_T}{R^3} \cdot y = 0}$

mov. armónico simple

$\dfrac{G \cdot M_T \cdot (R^3)}{R^6}$

$\dfrac{G M_T}{R^3}$

c) $\omega = \sqrt{\dfrac{G \cdot M_T}{R^3}}$

$y(t) = A \cdot \cos\left(\sqrt{\dfrac{G \cdot M_T}{R^3}} \cdot t + \varphi_0\right)$

$\dot{y}(t) = -A \cdot \sqrt{\dfrac{G M_T}{R^3}} \cdot \text{sen}\left(\sqrt{\dfrac{G M_T}{R^3}}\, t + \varphi_0\right)$

$g(y) \cong \dfrac{G M_T}{R^3}\, y$

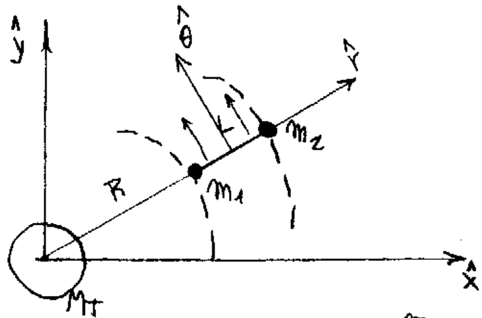
llegamos a la misma aproximación con un Taylor

$V_0 = -A \sqrt{\dfrac{G M_T}{R^3}} \cdot \text{sen}\,(\varphi_0)$, $\varphi_0 = \pi/2$

$-\dfrac{V_0}{\sqrt{\frac{G M_T}{R^3}}} = A$

$$\boxed{y(t) = \frac{V_0}{\sqrt{\frac{G M_T}{R^3}}} \cdot \cos\left(\sqrt{\frac{G M_T}{R^3}} \cdot t + \pi/2\right)}$$

6.



a)

$r \equiv k = R$
$\dot{r} = \ddot{r} = 0$

vínculos

$r_1 + L = r_2$
$\ddot{r}_1 = \ddot{r}_2$

$\theta_1 = \theta_2$
$\dot{\theta}_1 = \dot{\theta}_2$
$\ddot{\theta}_1 = \ddot{\theta}_2$

m_1

$\hat{r}) \quad m_1(-R.\dot{\theta}_1^2) = -\dfrac{G.M_T.m_1}{R^2} + T$

$\hat{\theta}) \quad m_1 R.\ddot{\theta}_1 = 0 \quad \Rightarrow \quad \ddot{\theta}_1 = 0$

m_2

$\hat{r}) \quad m_2[-(R+L).\dot{\theta}_2^2] = -\dfrac{G.M_T.m_2}{(R+L)^2} - T$

$\hat{\theta}) \quad m_2.(R+L).\ddot{\theta}_2 = 0$

$m_2 R \ddot{\theta}_2 = -m_2 L.\ddot{\theta}_2$

b)

$$-m_1 R\dot{\theta}^2 - m_2 R\dot{\theta}^2 - m_2 L\dot{\theta}^2 = -\frac{GM_T m_1}{R^2} - \frac{GM_T m_2}{(R+L)^2}$$

$$\dot{\theta}^2(-m_1R - m_2R - m_2L) = -GM_T\left(\frac{m_1}{R^2} + \frac{m_2}{(R+L)^2}\right)$$

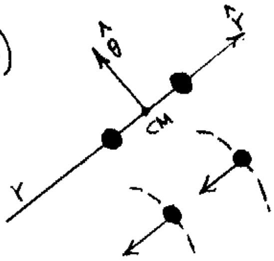
$$\boxed{\dot{\theta}^2 = \frac{-GM_T\left[m_1/R^2 + m_2/(R+L)^2\right]}{\left[-m_1R - m_2(L+R)\right]}}$$

$$T = m_1 R\dot{\theta}^2 + \frac{GM_T m_1}{R^2}$$

$$\boxed{T = -\frac{m_1 R.G.M_T\left[m_1/R^2 + m_2/(R+L)^2\right]}{\left[-m_1R - m_2(L+R)\right]} + \frac{G.M_T.m_1}{R^2}}$$

$$\frac{Kg.m.\dfrac{\cancel{Kg}\,m^{\cancel{3}}}{s^2.Kg}.\dfrac{Kg}{\cancel{m^2}}}{Kg.\cancel{m}}$$

c)



sistema: m_1 y m_2

$$\Sigma\vec{\tau}_{ext} = \vec{r}\times\vec{F}_{ext} = \underset{\hat{r}}{\vec{R}}\times\underset{\hat{r}}{\vec{F_{G_1}}} + \underset{\hat{r}}{(\vec{R}+\vec{L})}\times\underset{\hat{r}}{\vec{F_{G_2}}} = \frac{d\vec{L_o}}{dt} = 0$$

$$\vec{L_o} = \vec{r}\times m.\vec{V}$$
$$= r_1\hat{r}\times m_1 r_1.\dot{\theta}\,\hat{\theta} + r_2\hat{r}\times m_2 r_2.\dot{\theta}\,\hat{\theta}$$
$$L_o = R.m_1.\dot{\theta}\,\hat{z} + (R+L).m_2.\dot{\theta}\,\hat{z}$$

$$\vec{L_{o_i}} = (Rm_1 + Rm_2 + Lm_2).\dot{\theta}_i\,\hat{z} \qquad \dot{\theta}_i = \dot{\theta}_{1i} = \dot{\theta}_{2i}$$

$$\vec{L_{o_f}} = r_1.m_1.\dot{\theta}_1\,\hat{z} + r_2.m_2\,\dot{\theta}_2\,\hat{z}$$

F_G es conservativa $\Rightarrow E \equiv k$

$$E = \frac{1}{2}m_1.R^2\dot{\theta}^2 + \frac{1}{2}m_2(R+L)^2.\dot{\theta}^2 - \frac{GM_Tm_1}{R} - \frac{GM_Tm_1}{R+L}$$

$$-\frac{GM_Tm_1\,dR}{R^2} = -dE_p$$

$$-GM_Tm_1\int\frac{dR}{R^2} = -E_p$$

$$-\frac{GM_Tm_1}{R} = E_p$$

m_1 : solo le actúa $\vec{F}_G$ (Fc) $\Rightarrow$ $E \equiv k$

solo le actúa $\vec{F}_G$ (F central) $\Rightarrow$ $L_0 = k$

$$L_{o_i} = R\hat{r} \times m_1 . R . \dot{\theta}\,\hat{\theta} = m_1 . R^2 . \dot{\theta}\,\hat{z} \nearrow \quad \dot{\theta} = \frac{L_0}{m_1 R^2} \Rightarrow \dot{\theta}^2 = \frac{L_0^2}{m_1^2 R^4}$$

$$E_i = \frac{1}{2} m_1 . R^2 \dot{\theta}^2 + E_p(r) = \frac{1}{2} m_1 R^2 \dot{\theta}_i^2 - \frac{G M_T m_1}{R}$$

$$E_i = \underbrace{\underbrace{\frac{1 . L_0^2}{2 m_1 . R^2}}_{U_{cent}} - \frac{G M_T m_1}{R}}_{U_{eff}} \qquad E_k = 0$$

$$E_i = E = \frac{1}{2} m_1 R^2 . \frac{G . M_T \left(\frac{m_1}{R^2} + \frac{m_2}{(R+L)^2}\right)}{m_1 R + m_2 (R+L)} - \frac{G M_T m_1}{R}$$

$\downarrow$ se mantiene k

El tipo de movimiento que vaya a tener dependerá del signo de su E

$$E = \underbrace{G . M_T . m_1}_{>0} \underbrace{\left[\frac{1}{2} R^2 . \frac{\left(\frac{m_1}{R^2} + \frac{m_2}{(R+L)^2}\right)}{m_1 R + m_2 (R+L)} - \frac{1}{R} \right]}_{<0}$$

$$\frac{R^2}{2} \Phi - \frac{1}{R} > 0$$

Voy a forzar el mov. no ligado

$$\frac{R^2}{2} \text{———} > \frac{1}{R}$$

$$\frac{R^3 \left(\frac{m_1}{R^2} + \frac{m_2}{(R+L)^2}\right)}{2 \left(m_1 R + m_2 (R+L)\right)} > 1$$

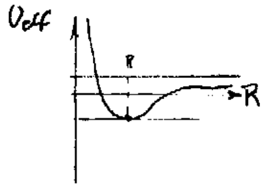
$$\frac{m_1 R + \frac{m_2 R^3}{(R+L)^2}}{2 m_1 R + 2 m_2 (R+L)} > 1$$

$$\frac{m_2 R^3}{(R+L)^2} > 2 m_2 (R+L) \quad \text{NO}$$

$$m_1 R > 2 m_1 R \quad \text{NO}$$

$E < 0$ $\therefore$

m_1 se va a mover en una $\begin{cases} \text{elipse} \\ \text{circunferencia} \end{cases}$



si $E_i = \min U_{eff} \Rightarrow$ se mueve en circunferencia

$$\frac{dU_{eff}}{dR} = \frac{L_0^2}{\cancel{2} m_1} \left(-\frac{\cancel{2}}{R^4}\right) + \frac{G M_T m_1}{R^2}$$

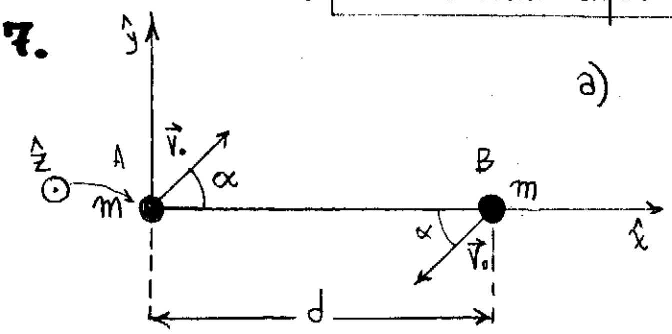
$$\frac{L_0^2}{m_1 R^{\cancel{4}2}} = \frac{G M_T m_1}{\cancel{R^2}}$$

R es mínimo si: $\frac{L_0^2}{m_1 R^2} = G M_T m_1 \wedge R^2 = \frac{L_0^2}{m_1^2 G M_T}$

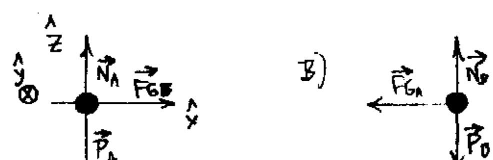
Luego $$U_{eff_{min}} = \frac{\cancel{L_0^2} \cancel{m_1^2} G M_T}{2 \cancel{m_1} \cancel{L_0^2}} - \frac{L_0^2}{m_1 R^3} = -\frac{L_0^2}{m_1 R^3} + \frac{G M_T m_1}{2} \neq E_i$$

$\Rightarrow$ $\boxed{\text{Se mueve en una elipse}}$

7.



a) A)



sistema: A y B

$$\sum \vec{F}e = \underbrace{\vec{N}_A + \vec{P}_A}_{\times \text{Newton} = 0} + \vec{F}_{GB} + \underbrace{\vec{N}_B + \vec{P}_B}_{\times \text{Newton} = 0} + \vec{F}_{GA}$$

se cancelan

$$\vec{P} \equiv k$$

$$\sum \vec{\tau}_{ext} = \overbrace{0 \times F_G \hat{x}}^{A} + \overbrace{d\hat{x} \times -F_G \hat{x}}^{B}$$

$\Rightarrow 0$ $\quad /\!/ \Rightarrow 0$

$$\frac{d\vec{L}_0}{dt} = 0 \quad \therefore \quad \vec{L}_0 \equiv k$$

$$W_T = W_{Fc} + \underbrace{W_{FNC}}_{=0} \Rightarrow E \equiv k$$

$\downarrow$ $\underbrace{\vec{N}, \vec{P}}_{\text{se anulan}}, \vec{F}_G$

b) $$\vec{R}_{cm} = \frac{m.0 + m.d\hat{x}}{2m} = \frac{1}{2}d\hat{x} \quad \therefore \quad \vec{V}_{cm} = \frac{m.\vec{V}_A + m.\vec{V}_B}{2m}$$

$\vec{P} \equiv k$ y $\quad \vec{P} = M.\vec{V}_{CM}$, $\quad 0 = 2m.0$

$\hat{x})\quad V_{cm} = \frac{1}{2}(V_0.\cos\alpha - V_0.\cos\alpha) = 0$

$$\boxed{\vec{V}_{cm_i} = 0}$$

$\hat{y})\quad V_{cm} = \frac{1}{2}(V_0.\operatorname{sen}\alpha - V_0.\operatorname{sen}\alpha) = 0$

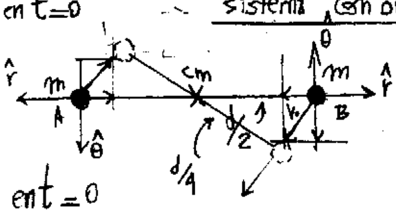
Como $\vec{P} \equiv k$ e inicialmente $\vec{V}_{cm} = 0$ será para $t>0$ $\quad \vec{V}_{cm} = 0$

c) $$0 = m.\vec{V}_A + m.\vec{V}_B$$
$$m.\vec{V}_A = m.\vec{V}_B \Rightarrow -\vec{V}_A = \vec{V}_B$$

$t=0$
$$E_i = \frac{1}{2}m\vec{V}_0^2 + \frac{1}{2}mV_0^2$$

en $t=0$
$$-\frac{Gm^2}{x^2} = -\frac{dF}{dx}$$

en $t=0$ sistema con origen en el CM.



en $t=0$

$$\vec{L}_{cm}(t_0) = d/2\,\hat{r} \times m(\dot{r}\hat{r} + r.\dot{\theta}_0\hat{\theta}) + d/2\,\hat{r} \times m(\dot{r}\hat{r} + r.\dot{\theta}_0\hat{\theta})$$
$$L_{cm} = m.\frac{d^2}{4}.\dot{\theta}_0\hat{z} + m.\frac{d^2}{4}.\dot{\theta}_0\hat{z}$$

$\vec{V}_0$

en t
$$L_{cm} = d/4\,\hat{r} \times m(\dot{r}\hat{r} + d/4\,\dot{\theta}_f\hat{\theta}) + d/4\,\hat{r} \times m(\dot{r}\hat{r} + d/4\,\dot{\theta}_f\hat{\theta})$$
$$L_{cm} = m\frac{d^2}{16}\dot{\theta}_f\hat{z} + m\frac{d^2}{16}\dot{\theta}_f\hat{z}$$

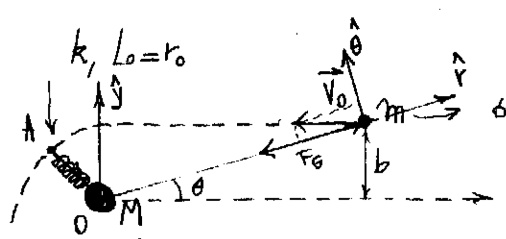
$$\cancel{2}m\frac{d^2}{\cancel{4}}\dot{\theta}_0 = \cancel{2}m\frac{d^2}{\cancel{16}_{4}}\dot{\theta}_f$$

$$\boxed{4\dot{\theta}_0 = \dot{\theta}_f}$$

$$\boxed{\frac{d}{4}.4\dot{\theta}_0 = \dot{r}_f} \rightarrow \vec{V}_f = (V_{f\hat{r}}, V_{f\hat{\theta}})$$

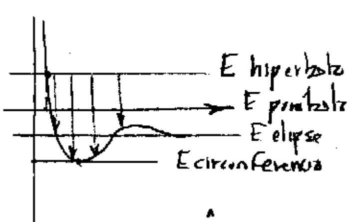
$$\boxed{\vec{V}_f = (d\dot{\theta}_0, 4\dot{\theta}_0)}$$

8.



$U_G = V_G = E_{PG} = 0$ en $x = \infty$

a) órbita hiperbólica $\Rightarrow$ $E>0$



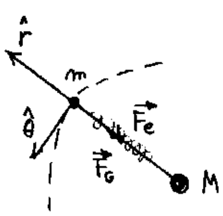
antes de A

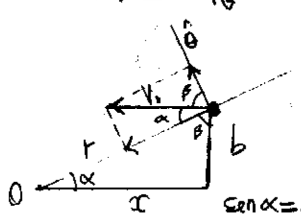
$$\sum \vec{F}_{ext} = \vec{F}_{GM} \Rightarrow \frac{d\vec{P}}{dt} \neq 0 \Rightarrow \boxed{\vec{P} \neq k}$$

$$\sum \vec{\tau}_0^{ext} = \vec{r} \times \vec{F}_{GM} = (r\hat{r}) \times (-F_G\hat{r})$$
son // $\Rightarrow = 0$

$$\Rightarrow \boxed{\vec{L}_0 \equiv k}$$

$$W_T = W_{F_G} \Rightarrow \boxed{E \equiv k}$$





$V_{\hat{\theta}} = r_0\dot{\theta} = V_0.\operatorname{sen}\alpha$

$V_r = \dot{r} = -V_0.\cos\alpha$

$\operatorname{sen}\alpha = \frac{b}{}$ $\quad$ cos después de A

$$\Sigma \vec{F}_{ext} = \vec{F}_{Gn} + \vec{F}_e \Rightarrow \frac{d\vec{P}}{dt} \neq 0 \Rightarrow \boxed{\vec{P} \not\equiv k}$$

$$\Sigma \vec{\tau}^{\,e}_{o} = \underbrace{(r\hat{r}) \times (-F_G \hat{r})}_{son\ //} + \underbrace{(r\hat{r}) \times (-F_e \hat{r})}_{son\ //} = 0 \Rightarrow \boxed{\vec{L}_o \equiv k}$$

$$W_T = \underbrace{W_{FG} + W_{Fe}}_{son\ Fc} \Rightarrow \boxed{E \equiv k}$$

$$\vec{F}_T = -\frac{dE_p}{d\vec{r}} \quad \therefore \quad -\frac{G.M.m}{r^2}.dr = -dE_p$$

$$GMm \int_{\infty}^{r} \frac{1}{r^2} dr = \int_0^{E_p} dE_p$$

$$-\frac{GMm}{r} = E_p = V_G$$

$$E = \frac{1}{2} m V_o^2 - \frac{GMm}{r}$$

$$E = \underbrace{\frac{1}{2} m.\dot{r}^2}_{E_k} + \underbrace{\frac{1}{2} m (r\dot{\theta})^2 - \frac{GMm}{r}}_{U_{eff}}$$

en A
$$L_o = r_o \hat{r} \times m.(\dot{r}\hat{r} + r_o \dot{\theta}_{r_o} \hat{\theta}) = m\, r_o^2 \dot{\theta}_{r_o} \hat{z}$$

r_o se da donde $U_{eff} = E \wedge E_k = 0 \Rightarrow \dot{r}(r_o) = 0$

$$\frac{L_o^2}{m^2 r_o^4} = \dot{\theta}_{r_o}^2$$

en A
$$E(A) = \frac{1}{2} m\, r_o^2 . \dot{\theta}_A^2 - \frac{GMm}{r_o}$$

$$E = \frac{L_o^2}{2 m r_o^2} - \frac{GMm}{r_o}$$

~~antes de A~~ (en $r \to \infty$)

$$\vec{L}_o = r\hat{r} \times m.\vec{V}_o = r.\hat{r} \times m.(\dot{r}\hat{r} + \underbrace{r\dot{\theta}}_{V_o.\frac{b}{r}}\hat{\theta})$$

$$L_o = m.V_o.b\,\hat{z}$$

en $r \to \infty$
$$E(r\to\infty) = \frac{1}{2} m V_o^2 - \frac{GMm}{r} \to 0$$

$$\cancel{m}.V_o b = \cancel{m} r_o^2 \dot{\theta}_A$$

$$V_o = \frac{r_o^2 \dot{\theta}_A}{b}$$

$$V_o = \frac{r_o}{b} . V_A$$

$$V_A = \frac{V_o . b}{r_o}$$

$E_i \qquad E_A$

$$\frac{1}{\cancel{2}} \cancel{m} V_o^2 = \frac{1}{\cancel{2}} \cancel{m} r_o^2 \dot{\theta}_{r_o}^2 - \frac{GM\cancel{m}}{r_o} 2$$

$$V_o^2 = r_o^2 \dot{\theta}_A^2 - \frac{GM}{r_o} 2$$

$$\cancel{V_o^2} = \frac{\cancel{V_o^2}.b^2}{r_o^2} - \frac{GM.2}{r_o.V_o^2}$$

$$1 = \frac{b^2}{r_o^2} - \frac{2MG}{r_o.V_o^2}$$

$$\frac{r_o^4 . \dot{\theta}_A^2}{b^2} - r_o^2 \dot{\theta}_A^2 = -\frac{GM.2}{r_o}$$

$$\left(\frac{r_o^2}{b^2} - 1\right).r_o^2 \dot{\theta}_A^2 = -\frac{GM2}{r_o}$$

$$\boxed{V_{A\hat{\theta}}^2 = -\frac{GM2}{r_o}\left(\frac{b^2}{r_o^2 - b^2}\right)}$$

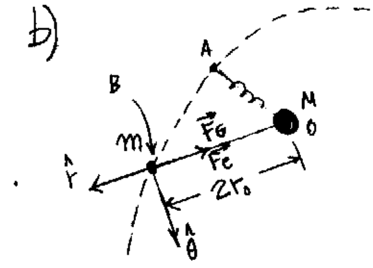
$$V_o^2 - V_A^2 = -\frac{GM2}{r_o}$$

$$\boxed{r_o = \frac{2GM}{V_A^2 - V_o^2}}$$

$$\boxed{V_A = \frac{b}{r_o}.V_o}$$

$$\frac{b.V_o}{V_A} = r_o$$

b)



■ Momento angular

(en $r \to \infty$) $\quad \vec{L}_o = m.V_o.b\,\hat{z}$

en $r = 2r_o$ (B) $\quad \vec{L}_o = m.4r_o^2.\dot{\theta}_B\,\hat{z}$

$$\Rightarrow \cancel{m}\, V_o b = \cancel{m}\, 4 r_o^2 \dot{\theta}_B$$

$$\frac{V_o.b}{2r_o} = 2r_o.\dot{\theta}_B = V_{B\hat{\theta}}$$

■ Energía

$$\vec{F}_t = \vec{F}_G + \vec{F}_e$$

$$\vec{F}_t = -\frac{GmM}{r^2}\hat{r} - k.(r_f - r_o)\hat{r}$$

$$-\frac{dE_p}{dr} = F$$

$$\int E_p = GmM \int \frac{1}{r^2} dr + k \int r_f - r_o\, dr$$

$$E_p = -\frac{GmM}{r} - \frac{k\,(r_f - r_o)^2}{2}$$

(en $r \to \infty$) $\quad E = \frac{1}{2} m V_o^2$

(en B)
$$E = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m (r.\dot{\theta})^2 - \frac{GMm}{r} - \frac{k\,(r_f - r_o)^2}{2}$$

$$E = \frac{1}{2} m V_{B\hat{r}}^2 + \frac{1}{2} m . V_{B\hat{\theta}}^2 - \frac{GMm}{2r_o} - \frac{k\, r_o^2}{2}$$

$$\frac{1}{\cancel{2}}\cancel{m}V_0^2 = \frac{1}{\cancel{2}}\cancel{m}\left(V_{B\hat{r}}^2 + V_{B\hat{\theta}}^2\right) - \frac{GM\cancel{m}}{2r_0} - \frac{k\,r_0^2}{2m}$$

$$\left(V_{B\hat{r}}^2 + V_{B\hat{\theta}}^2\right) = V_0^2 + \frac{GM}{r_0} + \frac{kr_0^2}{m}$$

$$V_{B\hat{r}}^2 + \frac{V_0^2 . b^2}{4r_0^2} = V_0^2 + \frac{GM}{r_0} + \frac{kr_0^2}{m}$$

$$\boxed{V_{B\hat{r}}^2 = V_0^2 - \frac{V_0^2 b^2}{4r_0^2} + \frac{GM}{r_0} + \frac{kr_0^2}{m}} \qquad \boxed{V_{B\hat{\theta}} = \frac{V_0 . b}{2r_0}}$$