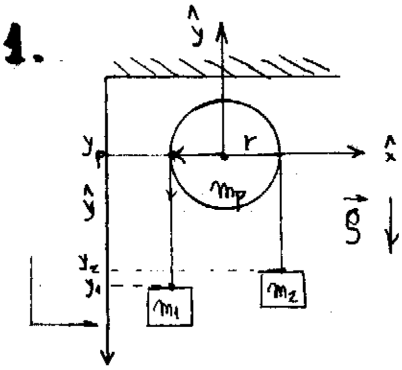
DINÁMICA DEL CUERPO RÍGIDO

1.



vínculo: soga inextensible $\Rightarrow$

$$L = y_p - y_1 + y_p - y_2 + \pi . r$$
$$0 = -\ddot{y}_1 - \ddot{y}_2 \qquad \ddot{y}_1 = -\ddot{y}_2$$

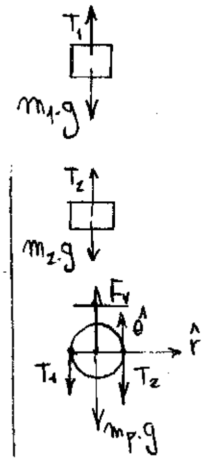
soga No desliza $\Rightarrow \vec{V}_{yp} = \dot{y}_1 \Rightarrow \ddot{y}_p = \ddot{y}_1$ $\qquad \vec{V}_p = \vec{V}_{CM} + \vec{\omega \hat{z}} \times \vec{r}\hat{r}$

$\alpha . r = \ddot{y}_1$ $\qquad \dot{y}_p = \omega . r \hat{\theta}$

$\ddot{y}_p = \alpha . r$

$\ddot{y}_1 = \ddot{y}_p = -\ddot{y}_2$

pero en polares $\ddot{y}_1 = \ddot{y}_2$



$\hat{y})\ m_1 . \ddot{y}_1 = -m_1 . g + T_1$

$-m_1 . \ddot{y}_2$

$\hat{y})\ m_2 . \ddot{y}_2 = -m_2 . g + T_2$

$$(m_2 - m_1) . \ddot{y}_2 = -(m_1 + m_2) . g + (T_1 + T_2)$$
$$(m_2 - m_1)\ddot{y}_2 + (m_2 + m_1) . g = (T_1 + T_2)$$
$$T_2 = (m_1 + m_2) . g - T_1$$
$$T_2 = m_2 (g + \ddot{y}_2)$$
$$T_1 = m_1 (g - \ddot{y}_2)$$

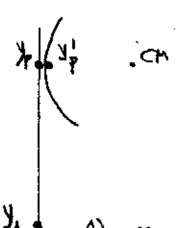
$$I\vec{\alpha} + \underbrace{\vec{R}_{CM} \times M . \vec{A}_{cm}}_{=0} = \vec{\tau}^{\,e}_{CM} = \vec{r}_1 \times \vec{T}_1 + \vec{r}_2 \times \vec{T}_2$$

$$m_p . \frac{R^2}{2} . \vec{\alpha} = \left(-r\hat{x} \times -T_1 . \hat{y} + r\hat{x} \times -T_2 \hat{y}\right)$$
$$T_1 r \hat{z} - r T_2 \hat{z}$$
$$m_p . r^2 \vec{\alpha} = 2 r (T_1 - T_2) \hat{z}$$

$\hat{z})\ \left(r . \frac{m_p}{2}\right)\vec{\alpha} = (T_1 - T_2)$

$$\frac{m_p}{2} . \ddot{y}_p = m_1 g - m_1 \ddot{y}_2 - m_2 g - m_2 \ddot{y}_2$$

$$-\frac{m_p}{2}\ddot{y}_p = m_1 g - m_1 \ddot{y}_2 - m_2 g - m_2 \ddot{y}_2$$
$$\left(-m_1 - m_2 - \frac{m_p}{2}\right)\ddot{y}_p = (m_1 - m_2) g$$
$$\boxed{\ddot{y}_p = \frac{(m_2 - m_1) . g}{\left(m_1 + m_2 + \frac{m_p}{2}\right)}}$$



$$\vec{\dot{y}}'_p = \vec{V}_{CM} + \omega \hat{z} \times r\hat{r}$$
$$\dot{y}'_p = \omega z \hat{\theta} \qquad \text{pero} \qquad \hat{\theta} = -\hat{y}$$

$\boxed{\ddot{y}_1 = \ddot{y}_p}$ $\qquad \boxed{\ddot{y}_2 = -\ddot{y}_p}$

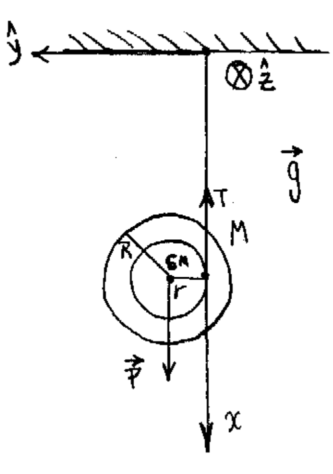
$\hat{y})\ \ddot{y}_1 = \ddot{y}_p \qquad \hat{\theta})\ \ddot{y}'_p = \omega z = -\ddot{y}_p$

$$\vec{\dot{y}}'_p = \omega \hat{z} \times -r\hat{x}$$

$\hat{y})\ \dot{y}'_p = -\omega r \hat{y} = \dot{y}_p \Rightarrow \ddot{y}_1 = -\dot{\omega} r = -\alpha . r$

$$\ddot{y}''_p = \ddot{y}_p = \ddot{y}_1 = -\ddot{y}_2 \qquad \alpha . r = -\ddot{y}_p$$

2.



$R = 10 . r \qquad I_{CM} = \frac{1}{2} M R^2$

a) soga no desliza

$$\sum \vec{F}_{ext} = \vec{P}_{eso} + \vec{T} = \frac{d\vec{P}}{dt} = M . \vec{A}_{cm}$$

$\hat{x})\ M . g - T = M . A_{cm}$

$\vec{V}_{A'} = \vec{V}_{CM} + \vec{\omega} \times \vec{r}$

eje instantáneo de rotación

$$0 = \vec{V}_{CM} + \vec{\omega} \times \vec{r}$$
$$\vec{V}_{CM} = -[\omega \hat{z} \times -r\hat{y}]$$
$$\vec{V}_{CM} = +\omega r \hat{x}$$

$$\vec{\tau}^{\,0}_{CM} = \underbrace{\vec{R}_{Peso} \times \vec{P}}_{=0} + \vec{r} \times \vec{T}$$
$$(-r\hat{y}) \times (-T\hat{x})$$
$$\vec{\tau}^{\,e}_{CM} = \frac{d\vec{L}_{CM}}{dt} = -Tr\hat{z}$$

$$(-Tr)\hat{z} = \underbrace{\vec{R}_{CM} \times M . \vec{A}_{CM}}_{=0} + I . \frac{d\omega}{dt}$$

$\hat{z})\ -Tr = \frac{1}{2} M . R . R . \alpha$

$$-\frac{2T}{10 MR} = \frac{d\omega}{dt}$$

$$-T = 5MR\frac{d\omega}{dt} \qquad \vec{L}_{CM} = \frac{1}{2} M R^2 . \omega \qquad A_{cm} = r . \frac{d\omega}{dt}$$

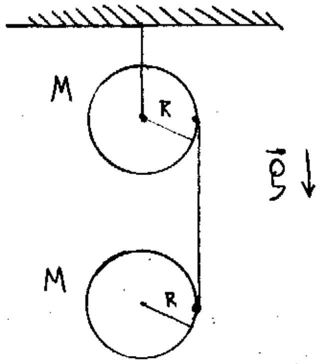
$$Mg + 5MR\frac{d\omega}{dt} \stackrel{\hat{x}}{=} M . A_{cm} \qquad (\hat{x})\ g + 5 A_{cm} 10 = A_{cm}$$

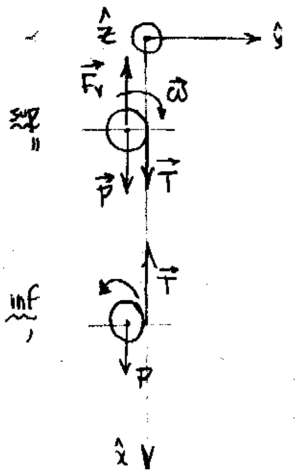
$$g + 49\,A_{cm} = 0$$
$$A_{cM} = -g/49 = \boxed{-0{,}2\,\frac{m}{s^2}\,\hat{x}}$$ es 49 veces menor

b)

$$M_g - M\,A_{cm} = T$$
$$Mg + Mg/49 = T$$
$$\boxed{\frac{50}{49}Mg = T}$$ es Ligeramente mayor

3.





■ Momento Lineal

a) inf $\sum_i \vec{F}_e = \vec{P} - \vec{T} = Mg - T \Rightarrow$

[1] $M.A'_{cm} = M.g - T$

sup $\sum_i \vec{F}_e = \vec{P} + \vec{T} - \vec{F}_V \Rightarrow$

[2] $M.A''_{cM} = M.g + T - F_V$

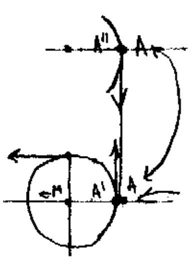
■ Momento Angular

inf $\sum \vec{\tau}^{\,ext}_{cM} = \vec{R} \times \vec{T} = R\hat{y} \times -T\hat{x}$

[3] $\frac{d\vec{L}'_{cm}}{dt} = (R.T)\hat{z} = I.\frac{d\omega'}{dt} = \frac{1}{2}MR^2.\dot{\omega}'$

sup $\sum \vec{\tau}^{\,ext}_{cm} = \vec{R} \times \vec{T} = R\hat{y} \times .T\hat{x}$

[4] $\frac{d\vec{L}''_{cm}}{dt} = -R.T.\hat{z} = I.\frac{d\omega''}{dt} = \frac{1}{2}MR^2.\dot{\omega}''$



igual velocidad (soga inextensible)

No desliza $(A' = A$ y $A'' = A) \Rightarrow V_A = V_{A'} = V_{A''}$

$$\vec{V}_{A''} = \vec{V}''_{cM} + \vec{\omega}'' \times \vec{R}$$
$$\vec{V}_A = \vec{V}_{A''} = \vec{V}_{A'}$$

$$0 + \vec{\omega}'' \times \vec{R} = \vec{V}'_{cM} + \vec{\omega}' \times \vec{R}$$
$$\omega''\hat{z} \times R\hat{y} = \vec{V}'_{cm} + \omega'\hat{z} \times R\hat{y}$$
$$-\omega''.R\hat{x} = -\omega' R\hat{x} + \vec{V}'_{cm}$$

$$\vec{V}'_{cm} = R(\omega' - \omega'')$$
$$\frac{d\vec{V}'_{cm}}{dt} = \vec{A}'_{cm} = R.[\dot{\omega}' - \dot{\omega}'']$$

de [3]

$$R.T = \frac{1}{2}M.R^2\frac{d\omega'}{dt}$$
$$T = \frac{1}{2}M.R.\frac{d\omega'}{dt}$$
$$M\,A_{cm} = Mg - \frac{M}{2}R\frac{d\omega'}{dt}$$
$$A_{cm} = g - \frac{R}{2}\frac{d\omega'}{dt}$$

de [4]

$$-R.T = \frac{1}{2}MR^2\dot{\omega}''$$
$$-\frac{T.2}{M.R} = \dot{\omega}''$$
$$\frac{2T}{MR} = \dot{\omega}'$$
$$\vec{A}'_{cm} = \cancel{R}.\frac{4T}{M\cancel{R}}$$
$$M.\vec{A}'_{cM} = 4T$$

en [1]

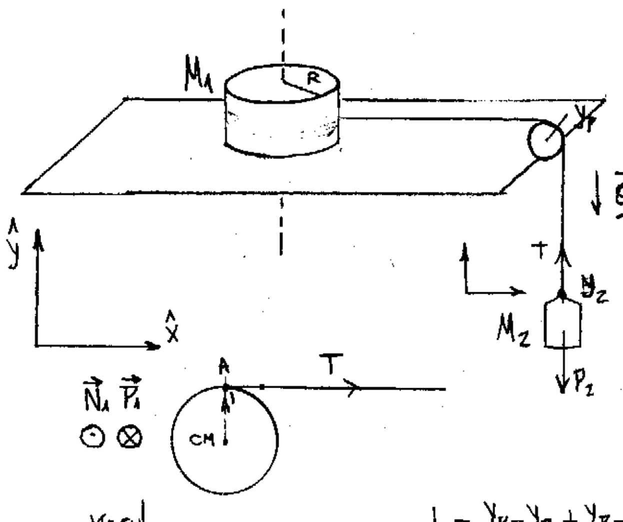
$$M.A_{cm} + \frac{4}{4}M.A_{cm} = M.g$$
$$\frac{5}{4}A_{cm} = g$$
$$\boxed{\vec{A}_{cm} = \frac{4}{5}g}$$

b) $$T = M.g - M.\frac{4}{5}g = \boxed{\frac{1}{5}Mg = T}$$

$$X_F = X_0 + V_0.t + \frac{1}{2}A.t^2$$
$$10R = \frac{1}{2}.\frac{4}{5}.g\,t^2$$
$$\frac{100}{4}R = t^2$$
$$5\sqrt{R} = t$$

$$V_F = V_0 + A.t$$
$$V_F = \frac{4}{\cancel{5}}g\,\cancel{5}\sqrt{R}$$
$$\boxed{V_F = 4g\sqrt{R}}$$

4.



a)

M_2:

[1] $M_2 \cdot \ddot{y}_2 = -M_2 \cdot g + T$

M_1: ■ Momento lineal

$$\sum_i \vec{F}_e = \frac{d\vec{P}}{dt} = \underbrace{\vec{P}_1 + \vec{N}_1}_{=0 \text{ x Newton}} + \vec{T}$$

[2] $\vec{T} = M_1 \vec{A}_{cm}$

■ Momento Angular

$$\sum \vec{\tau}^{\,ext}_{CM} = \underbrace{\vec{R}_{cm} \times \vec{P}}_{=0} + \underbrace{\vec{R}_{cm} \times \vec{N}}_{=0} + R\hat{y} \times T\hat{x}$$

$$\frac{d\vec{L}_{cm}}{dt} = -R.T.\hat{z} = \frac{1}{2} M_1 . R^2 . \dot{\omega}_1$$

[3] $-T = \frac{1}{2} M_1 R \dot{\omega}_1$

[4] $A_{cm} = R . \dot{\omega}_1 - \ddot{y}_2$

Vínculo

$L = y_P - y_2 + y_P - A \qquad \ddot{y}_2 = -\ddot{A}$

soga inextensible →
soga no desliza

$V_{y_2} = V_A = V_{A'}$

$= \vec{V}_{cm} + \vec{\omega}_1 \times \vec{R}$

$\vec{V}_{A'} = \vec{V}_{cm} + \omega_1 \hat{z} \times R\hat{y}$

$\vec{V}_{A'} = \vec{V}_{cm} - \omega R \hat{x}$

$\frac{d}{dt}\left(\vec{V}_{A'}\right) = \frac{d}{dt}\left(\vec{V}_{cm}\right) - R . \dot{\omega} \hat{x}$

$-\ddot{y}_2 = A_A = A_{A'} = A_{cm} - R\dot{\omega}$

$-\frac{2T}{M_1} = R\dot{\omega}_1$

$A_{cm} + \frac{2T}{M_1} = -\ddot{y}_2$

$-M_2 A_{cm} - M_2 \frac{2T}{M_1} = -M_2 g + M_1 A_{cm}$

$-\frac{M_2 2 M_1 A_{cm}}{\cancel{M_1}} = -M_2 g$

$-3M_2 A_{cm} - M_1 A_{cm} =$

$A_{cm} = \frac{-M_2 g}{-3M_2 - M_1}$

$\boxed{A_{cm} = \frac{M_2}{3M_2 + M_1} . g}$

b) $-\frac{2}{\cancel{M_1}} . \cancel{M_1} \frac{M_2}{3M_2 + M_1} . g = R . \dot{\omega}_1$

$$\boxed{-\frac{2 M_2 g}{R(3M_2 + M_1)} = \dot{\omega}_1}$$

c) $\ddot{y}_2 = -A_{cm} + R\dot{\omega}_1$

$$\ddot{y}_2 = -\frac{M_2 . g}{3M_2 + M_1} + \frac{-2M_2 g}{3M_2 + M_1} = \boxed{\frac{-3M_2 g}{3M_2 + M_1}}$$

d) $\boxed{T = \frac{M_1 . M_2}{3M_2 + M_1} . g}$

e) i) $V_{A'} = V_{cm} - \omega . R$

$$2R = \frac{1}{2} . \left(\frac{M_2}{3M_2 + M_1} . g\right) t^2$$

$$\sqrt{\frac{4R . (3M_2 + M_1)}{M_2 . g}} = t$$

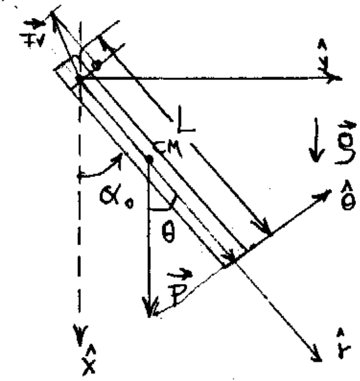
$$V_f = \frac{M_2 . g}{3M_2 + M_1} . \sqrt{\frac{4R(3M_2 + M_1)}{M_2 . g}}$$

$$\boxed{V_f = \sqrt{\frac{M_2 g}{3M_2 + M_1}} . 2\sqrt{R}}$$

f) $V_f = -\frac{3M_2 g}{3M_2 + M_1} . \sqrt{\frac{4R(3M_2 + M_1)}{M_2 g}}$

$$V_f = \frac{-3\sqrt{M_2 g}}{\sqrt{3M_2 + M_1}} \, 2\sqrt{R} \Rightarrow \boxed{V_f = -6\sqrt{\frac{R M_2 g}{3M_2 + M_1}}}$$

5.



- Momento lineal

$$\sum \vec{F_e} = \vec{F_V} + \vec{P} = M.\vec{A_{CM}}$$

- Momento Angular

$$\sum \tau_0^{ext} = 0 \times \vec{F_V} + \vec{r_{0cm}} \times M.\vec{g} = (L/2)\hat{r} \times M.g[\cos\theta \hat{r} - \operatorname{sen}\theta\,\hat{\theta}]$$

$$\sum \tau_0^{ext} = (-L/2 . M.g.\operatorname{sen}\theta)\hat{z}$$

$$\hat{z})\quad \frac{dL_0}{dt} = -L/2 . M.g.\operatorname{sen}\theta = I_0 . \frac{d\omega}{dt}$$

$$\checkmark\quad L_0 = \vec{R_{cm}} \times M.\vec{V_{cm}} + I_0 . \vec{\omega}$$

$$(L/2)\hat{r} \times M.(L/2).\omega\,\hat{\theta} + [I_{cm} + L^2/4 . M]\vec{\omega}$$

$$L_0 = M\,L^2/4 . \omega\,\hat{z} + \left(\frac{ML^2}{12} + \frac{ML^2}{4}\right)\vec{\omega}$$

$$L_0 = \frac{1}{4}M.L^2\,\omega\,\hat{z} + \frac{1}{3}ML^2 . \omega\hat{z}$$

$$L_0 = \left(\frac{7}{12}ML^2\omega\right)\hat{z}$$

$$M.-\frac{L}{2}\omega^2 = Mg\cos\theta - F_{Vr}$$

$$-\frac{L}{2}\cancel{M}.g.\operatorname{sen}\theta = \frac{1}{3}\cancel{M}L^2 . \dot{\omega}$$

$$-\frac{3}{2}g.\frac{\operatorname{sen}\theta}{L} = \dot{\omega}$$

$$-\frac{3}{2}g\frac{1}{L}.\operatorname{sen}\theta = \frac{d\omega}{d\theta}.\frac{d\theta}{dt_\omega} = \frac{d\omega}{dt}$$

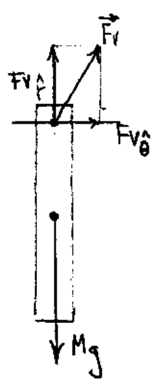
$$-\frac{3}{2}g\frac{1}{L}\int_{\alpha_0}^{\alpha}\operatorname{sen}\theta . d\theta = \int_0 \omega . d\omega$$

$$\frac{3}{2}g.\frac{1}{L}(\cos\alpha - \cos\alpha_0) = \frac{\omega^2}{2}$$

$$\omega = -\sqrt{\frac{3g}{L}(\cos\alpha - \cos\alpha_0)}$$

$$\boxed{\omega(\theta=0) = \sqrt{\frac{3g}{L}(1 - \cos\alpha_0)}}$$

b)



$$\frac{dL_0}{dt} = \vec{R_{cm}} \times M.\vec{A_{cm}} + I_{cm}.\vec{\dot{\omega}} = -\frac{L}{2}M.g.\operatorname{sen}\theta$$

$$\frac{L}{2}\hat{r} \times M.[-L/2\,\omega^2\,\hat{r} + \frac{L}{2}\dot{\omega}\,\hat{\theta}]$$

$$\frac{L^2}{4}M\,\dot{\omega}\,\hat{z} + M.\frac{L^2}{12}.\dot{\omega}\,\hat{z} = -\frac{L}{2}M.g.\operatorname{sen}\theta$$

$$\frac{1}{3}L^2M\dot{\omega} = -\frac{L}{2}Mg\operatorname{sen}\theta$$

$$M.\vec{A_{cm}} = \vec{P} + \vec{F_V}$$

$$M.\left(-\frac{L}{2}\omega^2\right) = +M.g - F_{V\hat{r}}$$

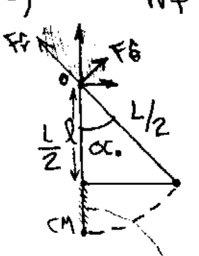
$$\hat{y})\quad M.L.\dot{\omega} =$$

$$-\frac{M.L}{2}\omega^2 - M.g = -F_{V\hat{r}}$$

$$M\left(\frac{L}{2}\omega^2 - g\right) = F_{V\hat{r}}$$

$$M\left[\frac{L}{2}\frac{3g}{\cancel{L}}(1-\cos\alpha_0) - g\right] = \boxed{Mg + M\frac{3}{2}g(1-\cos\alpha_0) = F_{V\hat{r}}}$$

c)



$$W_T = \underbrace{W_{FV}}_{\substack{=0\\ \perp\ \text{desplaz}}} + W_P$$

$$\cos\alpha_0 = \frac{l}{L/2}$$

$$l = \cos\alpha_0 . \frac{L}{2}$$

$$L/2 - l = L/2\,(1 - \cos\alpha_0)$$

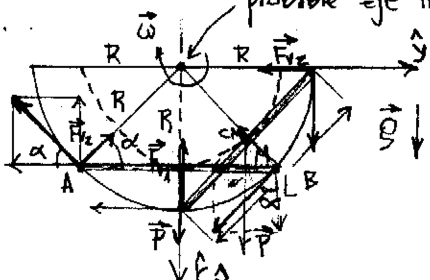
$$E_{ki} = \frac{1}{2}M.0^2 + \frac{1}{2}.\left(\frac{M.L^2}{12}\right).\vec{\omega}^2$$

$$E_{ki} = \frac{ML^2}{24}\vec{\omega}^2$$

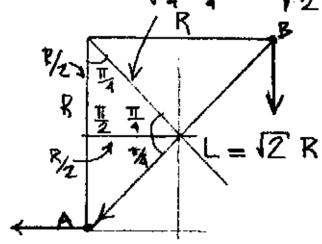
$$E_{kf} =$$

6.

probable eje instantáneo de rotación



$\sqrt{\frac{R^2}{4}+\frac{R^2}{4}} = \sqrt{\frac{R^2}{2}} = \frac{R}{\sqrt{2}}$

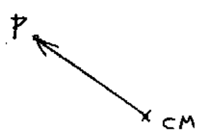


$\frac{L}{R} = \sqrt{2}$

$\mathrm{ACos}\left(\frac{L/2}{R}\right) = \alpha$

$\mathrm{ACos}\ \frac{\sqrt{2}\,R}{2\,R} = \frac{\pi}{4}$

a) si $\bar{P} \in$ Eje instantáneo rotación $\Rightarrow$ $\vec{V}_P = 0 = \vec{V}_{CM} + \vec{\omega} \times \vec{r}_{CMP}$



$\vec{V}_{CM} = \vec{\omega} \times \vec{r}_{CMP}$

$\vec{V}_A = \vec{V}_B + \vec{\omega} \times \vec{r}_{BA}$

$V_A = \left(-V_A \operatorname{sen} \frac{\pi}{4}\,\hat{x} - V_A \cos \frac{\pi}{4}\,\hat{y}\right)$

$V_A = \left(-V_A \frac{\sqrt{2}}{2}\,\hat{x} - V_A \frac{\sqrt{2}}{2}\,\hat{y}\right)$

$\|\vec{V}_A\| = \sqrt{\frac{V_A^2}{2} + \frac{V_A^2}{2}} = V_A$ $\qquad V_B = \left(V_B \sqrt{2}/2\,\hat{x} - V_B \sqrt{2}/2\,\hat{y}\right)$

$V_A = \left(V_B \phi\,\hat{x} - V_B \phi\,\hat{y}\right) + \omega\,\hat{z} \times (-\sqrt{2}\,R)\,\hat{y}$

$-V_A \phi\,\hat{x} - V_A \phi\,\hat{y} = \left(V_B \phi\,\hat{x} - V_B \phi\,\hat{y}\right) + \omega \sqrt{2}\,R\,\hat{x}$

$\hat{y})\quad -V_A \sqrt{2}/2 = -V_B \sqrt{2}/2 \Rightarrow V_A = V_B \quad \therefore$

$\hat{x})\quad -V_A \sqrt{2}/2 = V_B \sqrt{2}/2 + \omega \sqrt{2}\,R \Rightarrow -\frac{2}{R}\frac{V_A}{\sqrt{2}}\frac{\sqrt{2}}{2} = \omega \Rightarrow \omega = -\frac{V_A}{R}$

$\vec{V}_P = \vec{V}_A + \left(\frac{-V_A}{R}\right)\hat{z} \times \vec{r}_{AP} = 0$

b)

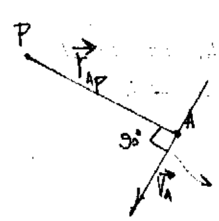
La E se conserva pues

$W_T = \underset{F_C}{W_P} \Rightarrow \Delta E = 0$

$\vec{V}_A = -\left(-\frac{V_A}{R} \times \vec{r}_{AP}\right) =$

$\perp\ \frac{V_A}{R}\ \ \operatorname{sen} \perp$

$\perp\ \vec{r}_{AP}$



$E_i = E_f \Rightarrow$

$E_i = \frac{1}{2} M \cdot 0^2 + \frac{1}{2} \frac{(\sqrt{2}\,R)^2}{12} M \omega^2$

$-M g \frac{R}{2} = \frac{M}{2} V_{CM}^2 + \frac{MR^2}{12} \omega^2 - M g \frac{R}{\sqrt{2}}$

$V_A = -\left(-\frac{V_A}{R} \cdot r_{AP}\right)\hat{\theta}$

$\boxed{R = \vec{r}_{AP}}$

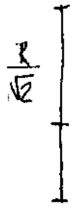
$-M g \frac{R}{\sqrt{2}} = \frac{M}{2}\left(\frac{R^2}{2}\omega^2\right) + \frac{MR^2}{12}\omega^2$

$R\,Mg\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right) = \frac{1}{3} R\,\omega^2$

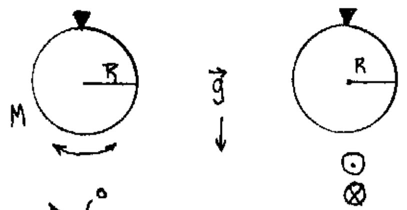
$$g\left(\frac{1}{\sqrt{2}}-\frac{1}{2}\right)=\frac{1}{3}R\,\omega^2 \Rightarrow \frac{3g}{2R}(\sqrt{2}-1)=\omega^2$$

$$V_{CM}=\sqrt{\frac{3g(\sqrt{2}-1)}{2R}}\cdot\frac{R}{\sqrt{2}} = \boxed{\sqrt{\frac{3gR(\sqrt{2}-1)}{4}} = V_{CM}}$$

$$\frac{2-\sqrt{2}}{\sqrt{2}.2}$$
$$\frac{1-\sqrt{2}/2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$
$$\frac{\sqrt{2}-1}{2}$$

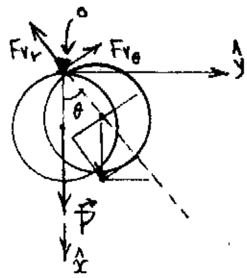


7.



$$T=\frac{2\pi}{\omega}$$

a)



■ *momento lineal*

$$\sum\vec{F}e=\vec{F}_v+\vec{P}=\frac{d\vec{P}}{dt}=M.A_{cm}$$

$\hat{r})\quad M.(-R.\omega^2)= M.g.\cos\theta - Fv_{\hat{r}}$ [1]

$\hat{\theta})\quad M.R.\dot{\omega}=-M.g.\operatorname{sen}\theta+Fv_{\hat{\theta}}$ [2]

■ momento angular

$$\sum\tau_o^{ext}=0\times\vec{F}v_{\hat{r}}+0\times\vec{F}v_{\hat{\theta}}+R\hat{r}\times M.g\left[\cos\theta\,\hat{r}-\operatorname{sen}\theta\,\hat{\theta}\right]$$
$$\sum\vec{\tau}_o^{ext}=(-R.M.g.\operatorname{sen}\theta)\hat{z}=\frac{d\vec{L}_o}{dt}$$

$$\vec{R}_{cm}\times M.\vec{A}_{cm}+I_{cm}.\vec{\dot{\omega}}=(-R.M.g.\operatorname{sen}\theta)\hat{z}$$
$$R\hat{r}\times M.\vec{A}_{cm}+M.R^2.\vec{\dot{\omega}}=$$
$$(R.M.R.\dot{\omega}+M.R^2.\dot{\omega})\hat{z}=(-MgR\operatorname{sen}\theta)\hat{z}$$

$\hat{z})$
$$2MR^2\dot{\omega}=-MgR\operatorname{sen}\theta$$
$$\dot{\omega}=-\frac{g.\operatorname{sen}\theta}{2R}\qquad [3]$$

■ Ecuaciones de movimiento

$$\dot{\omega}=\frac{d\omega}{dt}=\frac{d\omega}{d\theta}\frac{d\theta}{dt}=\frac{d\omega}{d\theta}.\omega=\omega.\frac{d\omega}{d\theta}\Rightarrow$$
$$\ddot{\theta}+\frac{g}{2R}\theta=0$$

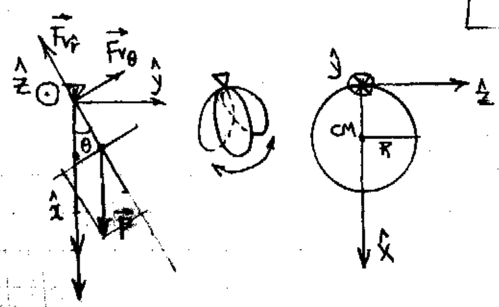
pequeñas oscilaciones $\quad \operatorname{sen}\theta\cong\theta \Rightarrow \quad \omega^2\cong\frac{g}{2R}$

$\Rightarrow$ Luego

$$\boxed{T_1=2\pi\sqrt{\frac{2R}{g}}}\quad\leftarrow\quad \omega=\sqrt{\frac{g}{2R}}$$

$\frac{m}{s^2}$ / m

b)



■ momento lineal

$$\sum\vec{F}e=\frac{d\vec{P}}{dt}=\vec{F}v_r+\vec{F}v_\theta+\vec{P}$$

$\hat{r})\quad M.(-R.\omega^2)=M.g.\cos\theta - Fv_{\hat{r}}$

$\hat{\theta})\quad M.R.\dot{\omega}=-M.g.\operatorname{sen}\theta+Fv_{\hat{\theta}}$

■ Momento Angular

$$\sum \vec{\tau}_0^{ext} = R\hat{r} \times M.g\,[\cos\theta\,\hat{r} + (-\operatorname{sen}\theta)\hat{\theta}] = (-M.g.R.\operatorname{sen}\theta)\hat{z}$$

$$\vec{R}_{cm} \times M.\vec{A}_{cm} + I_{cm}.\dot{\vec{\omega}} = -M.g\,R\,\operatorname{sen}\theta$$

$$\hat{z})\qquad R^2.M.\dot{\omega} + \frac{M}{2}R^2.\dot{\omega} = -M.g\,R\,\operatorname{sen}\theta$$

$$\frac{3}{2}MR^2\dot{\omega} = -M\,g\,R\,\operatorname{sen}\theta$$

$$\dot{\omega} = -\frac{2g}{3R}\operatorname{sen}\theta$$

$$\dot{\omega} + \frac{2}{3}g.\frac{\operatorname{sen}\theta}{R} = 0$$

aprox. ángulos pequeños $\Rightarrow$ $\dot{\omega} + \underbrace{\frac{2g}{3R}}_{\omega^2}\theta = 0$

$$\boxed{T_2 = 2\pi\sqrt{\frac{3R}{2g}}}$$

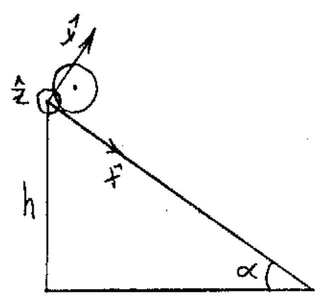
$$T_1 > T_2$$

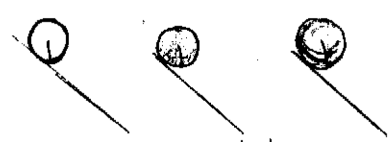
$$\cancel{2\pi}\sqrt{\frac{2R}{\cancel{g}}} > \cancel{2\pi}\sqrt{\frac{3\cancel{R}}{2\cancel{g}}}$$

$$2 > \frac{3}{2}$$

$$4 > 3$$

9.

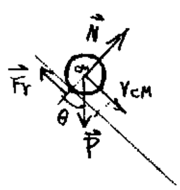




aro	cilindro	esfera
mr^2	$\frac{1}{2}mr^2$	$\frac{2}{5}mr^2$

suponemos r, m cualesquiera

bajan rodando → hay fricción, no deslizamiento



* momento lineal:

$$\sum F_{ext} =$$

$$\vec{P} + \vec{N} + \vec{Fr}$$

$\frac{d\vec{P}}{dt} = M.\vec{A}_{cm} =$	$\hat{x})$ $mg.\operatorname{sen}\theta - F_{roz}$	$mg\operatorname{sen}\theta - Fr$	$mg\operatorname{sen}\theta - Fr$
$0 =$	$\hat{y})$ $N - m.g.\cos\theta$	$N - mg\cos\theta$	$N - mg\cos\theta$

$\vec{P} \neq k$

vínculos

en O hay eje instantáneo de rot

$$V_0 = 0 = \vec{V}_{cm} = -\vec{\omega} \times \vec{r}$$

$$\vec{V}_{CM} = -\omega\hat{z} \times -r\hat{y}$$

$$\vec{V}_{CM} = -\omega r\,\hat{x}$$

$$\boxed{\vec{A}_{CM} = -\dot{\omega}.r\,\hat{x}}$$

* momento angular

$$\sum \vec{\tau}_{cm}^{ext} = 0 \times \vec{N} + 0 \times \vec{P} + \vec{r} \times \vec{Fr} = (-r)\hat{y} \times (-Fr)\hat{x} = -rFr\hat{z}$$

$$\frac{d\vec{L}_0}{dt} = \underbrace{\vec{R}_{cm} \times M.\vec{A}_{cm}}_{0} + I_{cm}.\frac{d\vec{\omega}}{dt} = -r.Fr\,\hat{z}$$

del cuadro de abajo vemos que las aceleraciones son en orden decreciente:

$$\boxed{a_{cm\,esfera} > a_{cm\,cilindro} > a_{cm\,aro}}$$

lo cual prueba el orden de llegada

aro

$$mr^2.\dot{\omega} = -r.F_{roz}$$

$$m\,r.\dot{\omega} = -\mu_d.N$$

$$r.\dot{\omega} = -\frac{\mu_d N}{m}$$

$$r.\dot{\omega} = -\mu_d.g.\cos\theta$$

cilindro

$$\frac{1}{2}mr^2\dot{\omega} = -r.F_{roz}$$

$$r.\dot{\omega} = \frac{\mu_d.N.2}{m}$$

$$r.\dot{\omega} = -\mu_d.g.\cos\theta\,2$$

esfera

$$\frac{2}{5}mr^2\dot{\omega} = -rF_{roz}$$

$$r\dot{\omega} = \frac{\mu_d N.5}{2m}$$

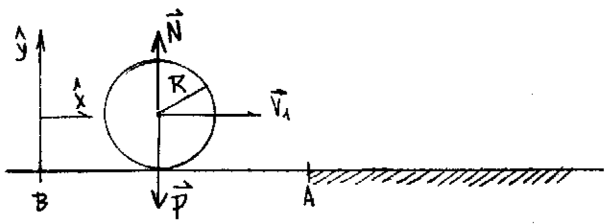
$$r.\dot{\omega} = \frac{5}{2}.g.\cos\theta$$

9.

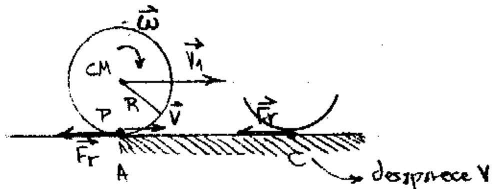
$$I_{CM} = \frac{1}{2} M R^2$$

$\hat{x})\quad 0 = 0$

$\hat{y})\quad 0 = N - M.g \Rightarrow N = Mg$



a)



si hay rodadura: $\Rightarrow$

$$\vec{V_P} = 0 = \vec{V_{CM}} + \vec{\omega} \times \vec{R}$$

al entrar en A

* momento lineal $\quad \Sigma \vec{F}^{ext} = \vec{F_r} + \underbrace{\vec{N} + \vec{P}}_{=0} \Rightarrow (-\mu_d . N)\hat{x} = \frac{d\vec{P}}{dt}$

$$M.A_{cm} = -\mu_d . N = \mu_d . M.g$$

$$A_{cm} = -\mu_d . g \quad [1] \rightarrow \text{aceleración de traslación}$$

$\hat{x})\quad M.a_{cm} = -\mu_d.M.g$

$\hat{y})\quad 0 = N - M.g$

* momento angular $\quad \Sigma \vec{\tau}^{ext}_{CM} = \frac{d\vec{L_0}}{dt} = \vec{R} \times \vec{F_r} = -R\hat{y} \times -Fr\hat{x} = -RFr\hat{z}$

$$I_{CM}.\dot{\omega} = \frac{d\vec{L_0}}{dt} = (-R.\mu_d.M.g)\hat{z}$$

$$\hat{z})\quad \frac{1}{2}MR^2\dot{\omega} = -R\mu_d M.g$$

$$R\dot{\omega} = -2\mu_d g \quad [2]$$

$$\dot{\omega} = \frac{-2\mu_d g}{R} \rightarrow \text{aceleración de rotación}$$

hagamos $A = x_i = 0$ $\quad$ Para

$$x_f = x_i + V_1.t + \frac{1}{2}(-\mu_d.g).t^2$$

(P no desliza más)

$$C = \underbrace{V_1}_{(V_{CM_1} + 0 \times R)}.t - \frac{\mu_d g}{2}t^2$$

$V_{CM} \times V_{CM}$

$0 = V_1 - ($

$\underline{V_{CM}}\ \hat{x}$ $\quad$ $\underline{\text{Velocidad angular}}$

$$V_{CM_f} = \underset{V_{CM_i}}{V_1} - \mu_d g t \qquad \omega_f = \overset{\omega_i}{0} - 2\frac{\mu_d g}{R}t$$

$$V_{P en C} = (V_1 - \mu_d g t)\hat{x} + \left(-2\frac{\mu_d g t}{R}\right)\hat{z} \times -R\hat{y}$$

$$V_1\hat{x} - \mu_d g t\hat{x} - 2\mu_d \frac{g}{\cancel{R}}\cancel{R}t\hat{x}$$

$$0 = V_1\hat{x} - 3\mu_d g t\hat{x}$$

$$\hat{x})\qquad t = \frac{V_1}{3\mu_d g}$$

$$\omega_f = -\frac{2\cancel{\mu_d g}.V_1}{R\ 3\cancel{\mu_d g}}$$

$$\omega_f = -\frac{2}{3}\frac{V_1}{R}$$

$$C = \frac{V_1.V_1}{3\mu_d g} - \frac{\cancel{\mu_d g}}{2}\frac{V_1^2}{9\mu_d^2 g^2} = \boxed{\frac{5}{18}\frac{V_1^2}{\mu_d.g} = C}$$

empieza la rodadura aquí

$$V_{CM_f} = V_1 - \frac{\cancel{\mu_d g}.V_1}{3\cancel{\mu_d g}} = \boxed{\frac{2}{3}V_1}$$

b)

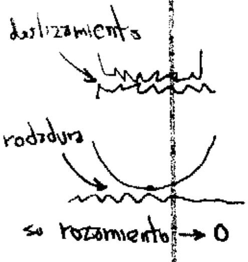
$$\Sigma \vec{F}_{ext} = \vec{0} \Rightarrow$$

El rozamiento solo se opone al deslizamiento no a la rodadura

$$M.\vec{A}_{cm} = 0 \Rightarrow \vec{P} \equiv k$$

$\downarrow$ supongamos $\vec{A}_{cm} \neq 0$

$$M\vec{A}_{cm} = -F_{roz}$$



$$\frac{d\vec{L}_o}{dt} = (-R.F_r)\hat{z} = \frac{1}{2}MR^2 .\dot{\omega}$$

$$-R.\underbrace{\mu_d.Mg}_{} = \frac{1}{2}MR^2\dot{\omega}$$

$$-F_{roz} = \frac{1}{2}MR\dot{\omega}$$

$\hat{x}$) $\underbrace{-M.A_{cm} = M\cancel{R}\dot{\omega} = -\frac{1}{2}MR\dot{\omega}}$ $\rightarrow \Leftrightarrow \dot{\omega} = 0$ lo cual no es cierto luego $\boxed{\vec{A}_{cm} = 0}$ $\boxed{F_{roz} = 0}$

A partir de C hay rodadura

$$\vec{V}_P = \vec{V}_{CM} + \vec{\omega} \times \vec{R}$$

$$0 = \vec{V}_{CM} + -\frac{2V_1}{3R}\hat{z} \times -R\hat{y}$$

$$\vec{V}_{CM} = \frac{2}{3}V_1\hat{x}$$

$$\vec{A}_{cm} = \frac{d}{dt}\left(\frac{2}{3}V_1\right)\hat{x} = 0$$

$$\vec{A}_{CM} = 0$$

c)

$$W_{NC} = \Delta E_M$$

$$W_{Fr} = \Delta E_M = E_{k(C)} - E_{k(A)}$$

$$\Delta E_{M\,A\to C} = \frac{1}{2}MV_{cm_f}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_f^2 - \left(\frac{1}{2}MV_{cm_i}^2 + \frac{1}{4}MR^2\underbrace{\omega_i^2}_{=0}\right)$$

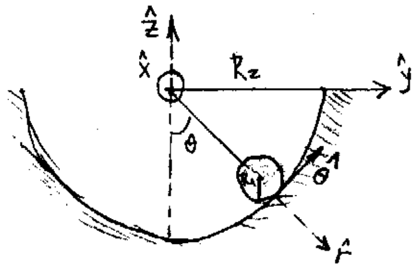
porque hasta A no rodaba

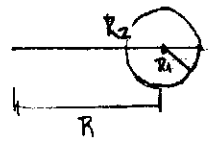
$$= \frac{1}{\cancel{2}}M\frac{\cancel{4}}{9}V_1^2 + \frac{1}{4}M\cancel{R^2}.\frac{4}{9}\frac{V_1^2}{\cancel{R^2}} - \frac{1}{2}MV_1^2$$

$$\boxed{\Delta E_{M\,A\to C} = -\frac{1}{6}MV_1^2}$$

Esta pérdida se debe a la acción de la Froz que no es conservativa.

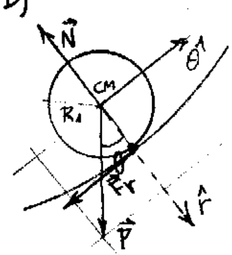
11.





defino:
$R_2 - R_1 = R$

a) y b)



$$* \quad \Sigma \vec{F}_e = \vec{P} + \vec{N} + \vec{F}_r = \frac{d\vec{P}}{dt}$$

$$\hat{r})\ -M.R.\dot{\theta}^2 = -N + M.g.\cos\theta$$

$$\hat{\theta})\ M.\underline{R.\ddot{\theta}} = -F_r - M.g.\operatorname{sen}\theta$$

según coordenadas polares

$$\boxed{\begin{aligned}\vec{V}_{CM} &= R.\dot{\theta}\,\hat{\theta}\\ \vec{A}_{CM} &= -R\dot{\theta}^2\hat{r} + R.\ddot{\theta}\,\hat{\theta}\end{aligned}}$$

NB

$\dot{\theta}, \ddot{\theta}$	refieren traslación del CR
$\omega, \dot{\omega}$ $\Omega, \dot{\Omega}$	refieren rotación del CR

Vínculos
rodadura

$$0 = \vec{V}_{CM} + \vec{\omega} \times \vec{R}_1$$

$$\vec{V}_{CM} = -\omega\hat{z} \times R_1\hat{r}$$

$$\vec{V}_{CM} = -\omega R_1\hat{\theta}$$ → cambia con el tiempo

$$\vec{A}_{CM} = -R_1.\dot{\omega}\,\hat{\theta} + R_1\omega\dot{\theta}\,\hat{r}$$

$$\frac{d\vec{V}_{CM}}{dt} = -R_1\dot{\omega}\hat{\theta} + \omega R_1\dot{\theta}\hat{r}$$

$$\vec{\dot{\omega}} = -\dot{\omega}\hat{\theta} + \omega\dot{\theta}\hat{r} + \frac{R\dot{\theta}^2}{R_1}\hat{r}$$

$$\boxed{\begin{aligned}\omega &= -\frac{V_{CM}}{R_1} = -\frac{R.\dot{\theta}}{R_1}\,\hat{\theta}\\ \dot{\omega} &= -\frac{A_{cm}}{R_1} + \frac{\cancel{R_1\omega\dot{\theta}}}{\cancel{R_1}}\hat{r} = +\frac{R\dot{\theta}^2}{R_1}\hat{r} - \frac{R\ddot{\theta}\,\hat{\theta}}{R_1} - \frac{R\dot{\theta}^2}{R_1}\hat{r}\end{aligned}}$$

$$\Sigma \vec{\tau}_{CM}^{ext} = \frac{d\vec{L}_{CM}}{dt} = R_1\hat{r} \times -F_r\hat{\theta} = (-R_1.F_r)\hat{z}$$

$$\vec{L}_{CM} = I_{CM}.\vec{\omega} \Rightarrow \frac{d\vec{L}_{CM}}{dt} = \frac{1}{2}MR_1^2\dot{\omega}\,\hat{z}$$

$$\hat{z})\ \frac{1}{2}MR_1^{\cancel{2}}\dot{\omega} = -\cancel{R_1}.F_r$$

$$R.\ddot{\theta}.M = \frac{1}{2}MR_1\dot{\omega} - Mg\operatorname{sen}\theta$$

$$\ddot{\theta} = \frac{R_1\dot{\omega}}{2R} - \frac{g}{R}\operatorname{sen}\theta$$

c)

$\hat{r})$ $-MR\dot{\theta}^2 = -N + Mg\cos\theta$ pero $\dot{\omega} = -\frac{R.\ddot{\theta}}{R_1}\hat{\theta}$

$\hat{\theta})$ $MR\ddot{\theta} = -Fr - Mg\,\text{sen}\,\theta$

$\hat{z})$ $\frac{1}{2}MR_1\dot{\omega} = -Fr$

$$z)\quad \cancel{M}R\ddot{\theta} - \frac{1}{2}\cancel{M}\cancel{R_1}\left[-\frac{R.\ddot{\theta}}{\cancel{R_1}}\right] = -\cancel{M}.g\,\text{sen}\theta$$

$$R\ddot{\theta} + \frac{1}{2}R\ddot{\theta} = -g.\text{sen}\theta$$

$$\frac{3}{2}R\ddot{\theta} + g\,\text{sen}\theta = 0$$

ecuación diferencial de movimiento oscilatorio $\blacktriangleright$ $\ddot{\theta} + \frac{2g}{3R}\text{sen}\theta = 0$

d) Ángulos pequeños $\text{sen}\,\theta \cong \theta \Rightarrow$

$$\ddot{\theta} + \frac{2g}{3R}\theta = 0$$

$$\omega^2 = \frac{2g}{3R}$$

$0 = A.\cos\varphi \Leftrightarrow \varphi = \pi/2$ $\theta(t) = A.\cos\left(\sqrt{\frac{2g}{3R}} + \varphi\right)$

$\omega_0 = -A.\sqrt{\frac{2g}{3R}}$

$\dot{\theta}(t) = -A.\sqrt{\frac{2g}{3R}}.\text{sen}\left(\sqrt{\frac{2g}{3R}} + \varphi\right)$

$\sqrt{\frac{3R}{2g}}.(-\omega_0) = A$

$$\boxed{\theta(t) = -\omega_0.\sqrt{\frac{R3}{2g}}.\cos\left(\sqrt{\frac{2g}{3R}} + \pi/2\right)}$$

12.

Problema 11

a) $\vec{L}_A = I_A\vec{\Omega}$

b) $\vec{L}_0 = I_0\vec{\Omega}$

c) $\vec{L}_{CM} = I_{cm}\vec{\Omega}$

a) $\vec{L}_A = \left(I_{cm} + M[\overset{d(A,CM)}{d(t)}]^2\right).\vec{\Omega}$

$\vec{L}_{cm} + I_{cm}.\vec{\Omega} = I_{cm}.\vec{\Omega} + M[d(t)]^2.\vec{\Omega}$

$\vec{L}_{CM} = M[d(t)]^2.\vec{\Omega}$

$\vec{R}_{cm} \times M.\vec{V}_{cm} = M[d(t)]^2.\vec{\Omega}$