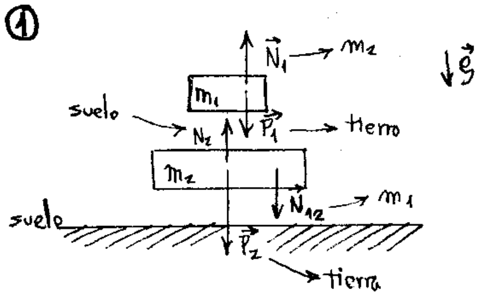
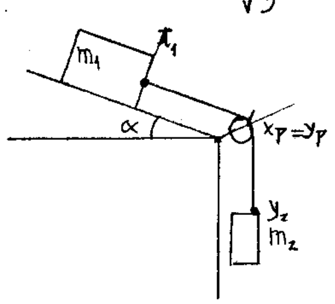
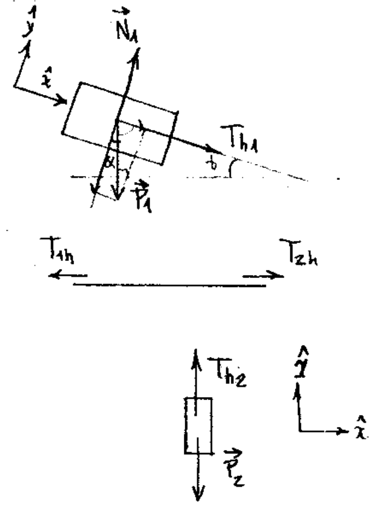
Dinámica - Interacciones

①



② a)





b)

m_1

$\hat{x})\quad m_1 \cdot \operatorname{sen}\alpha \cdot g + T_{h1} = m_1 \cdot a_{1x}$

$\hat{y})\quad -m_1 \cdot \cos\alpha \cdot g + N_1 = 0$

m_2

$\hat{y})\quad -m_2 \cdot g + T_{h2} = m_2 \cdot a_{2y}$

hilo

$\hat{x})\quad T_{2h} - T_{1h} = \underbrace{m_h}_{\to 0} \cdot a_h = 0 \Rightarrow T_{2h} = T_{1h} \Rightarrow T_{h2} = T_{h1}$ por pares acción-reacción

$\therefore\ T_{h2} = T_{h1} = T$ y

$$L = x_p - x_1 + y_p - y_2$$
$$0 = -\dot{x}_1 - \dot{y}_2$$
$$0 = -\ddot{x}_1 - \ddot{y}_2 \Rightarrow \ddot{x}_1 = -\ddot{y}_2$$
$$a_{1x} = -a_{2y}$$

$\Rightarrow$

$$m_1 \cdot \operatorname{sen}\alpha \cdot g + T = m_1 \cdot a_{1x}$$
$$-m_2 \cdot g + T = m_2 \cdot (-a_{1x})$$
$$m_1 \operatorname{sen}\alpha \cdot g + m_2 g = (m_1 + m_2) \cdot a_{1x}$$
$$\boxed{\frac{(m_1 \operatorname{sen}\alpha + m_2)}{m_1 + m_2} \cdot g = a}$$

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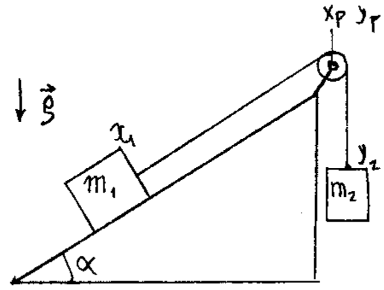
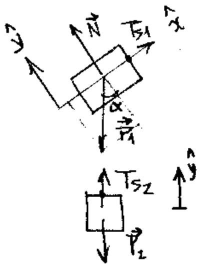
③



$\frac{m_1}{m_2} = 9$

Conf. 1



m_1

$\hat{x})\quad T_{s1} - m_1 \cdot g \cdot \operatorname{sen}\alpha = m_1 \cdot a_{1x}$

$\hat{y})\quad N_1 - m_1 \cdot g \cdot \cos\alpha = 0$

m_2

$\hat{x})\quad 0 = 0$

$\hat{y})\quad T_{s2} - m_2 \cdot g = m_2 \cdot a_{2y} = -m_2 \cdot a_{1x}$

soga tiene $m_s \to 0 \Rightarrow T_{s2} = T_{s1}$

$$-m_1 \cdot g \cdot \operatorname{sen}\alpha + m_2 \cdot g = (m_1 + m_2) \cdot a$$
$$\frac{(-m_1 \operatorname{sen}\alpha + m_2)g}{m_1 + m_2} = a$$

$$L = x_p - x_1 + y_p - y_2$$
$$0 = -\ddot{x}_1 - \ddot{y}_2$$
$$\ddot{x}_1 = -\ddot{y}_2 \Rightarrow a_{1x} = -a_{2y}$$

conf. 2

m_2

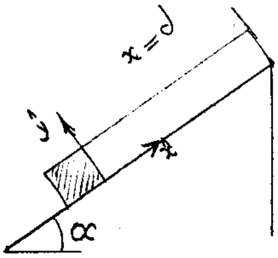
$\hat{x})\quad T_{s2} - m_2 g \operatorname{sen}\alpha = m_2 \cdot a_{2x}$

$\hat{y})\quad N_2 - m_2 g \cos\alpha = 0$

m_1

$\hat{y})\quad T_{s1} - m_1 \cdot g = -m_1 \cdot a_{2x}$

$$-m_2 \cdot g \operatorname{sen}\alpha + m_1 g = (m_2 + m_1) \cdot a' \Rightarrow a' = g \frac{(-m_2 \operatorname{sen}\alpha + m_1)}{(m_1 + m_2)}$$



$$x = x_0 + v_0 \cdot t + \frac{1}{2} \cdot a \cdot t^2$$

$$\left.\begin{array}{l} d = \frac{1}{2} \cdot a \cdot T^2 \\ d = \frac{1}{2} \cdot a' \cdot \left(\frac{T}{4}\right)^2 \end{array}\right\} \Rightarrow \quad \frac{1}{\cancel{2}} a \cancel{T^2} = \frac{1}{\cancel{2}} a' \frac{\cancel{T^2}}{16}$$

$$16 a = a'$$

$$\frac{(-m_1 \cdot \operatorname{sen}\alpha + m_2)}{\cancel{(m_1+m_2)}} \cdot \cancel{g} \cdot 16 = \frac{(-m_2 \operatorname{sen}\alpha + m_1)}{\cancel{(m_1+m_2)}} \cancel{g}$$

$$\cancel{m_2}\left(-\frac{m_1}{m_2} \operatorname{sen}\alpha + 1\right) \cdot 16 = \cancel{m_2}\left(-\operatorname{sen}\alpha + \frac{m_1}{m_2}\right)$$

$$(-9 \operatorname{sen}\alpha + 1) \cdot 16 = -\operatorname{sen}\alpha + 9$$

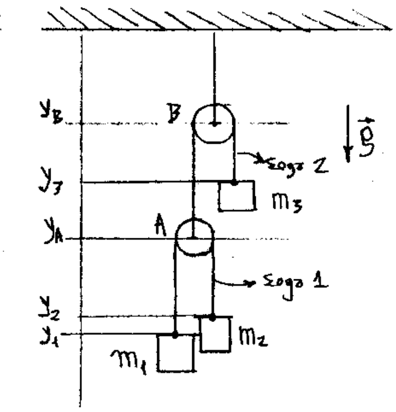
$$7 = 143 \operatorname{sen}\alpha$$

$$\text{arco sen}\left(\frac{7}{143}\right) = \alpha$$

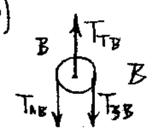
$$\boxed{\alpha = 2^\circ\ 48'\ 21''}$$

Dinámica

1

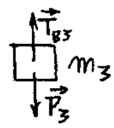


a)

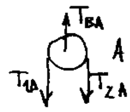


$\underset{\approx}{B}$ $\hat{y})$ $T_{TB} - T_{AB} - T_{3B} = \underbrace{m_B}_{\to 0} \cdot \underbrace{a_B}_{\to 0}$

$$T_{TB} = T_{AB} + T_{3B}$$



$\underset{\approx}{m_3}$ $\hat{y})$ $T_{B3} - m_3 \cdot g = m_3 \cdot a_3$

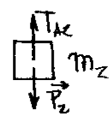


$\underset{\approx}{A}$ $\hat{y})$ $T_{BA} - T_{1A} - T_{2A} = \underbrace{m_A}_{\to 0} \cdot a_A$

$$T_{BA} = T_{1A} + T_{2A}$$

$$L_1 = y_A - y_1 + \pi \cdot R_A + y_A - y_2$$

$$L_2 = y_B - y_A + \pi \cdot R_B + y_B - y_3$$



$\underset{\approx}{m_2}$ $\hat{y})$ $T_{A2} - m_2 \cdot g = m_2 \cdot a_2$

$\underset{\approx}{L_1}$

$\underset{\approx}{m_1}$ $\hat{y})$ $T_{A1} - m_1 \cdot g = m_1 \cdot a_1$

$\underset{\approx}{L_1}$: $-T_{1A} + T_{2A} = \underbrace{m_{L_1}}_{\to 0} \cdot a_{L_1} = 0$

$T_{2A} = T_{1A}$ T_{AB} T_{3B} $\left.\begin{array}{l} T_{2A} = -T_{A2} \\ T_{1A} = -T_{A1} \end{array}\right\} \Rightarrow T_{A2} = T_{A1}$ $\left.\begin{array}{l} T_{AB} = -T_{BA} \\ T_{3B} = -T_{B3} \end{array}\right\} \Rightarrow T_{BA} = T_{B3}$

$\underset{\approx}{L_2}$: $-T_{AB} + T_{3B} = \underbrace{m_{L_2}}_{\to 0} \cdot a_{L_2} = 0$

$$T_{B3} = T_{1A} + T_{2A}$$

$T_{3B} = T_{AB}$ $\Rightarrow T_{3B} = T_{AB} \quad \therefore \quad T_{TB} = 2T_{AB} = 2T_{3B}$

b) $0 = \ddot{y}_A - \ddot{y}_1 + \ddot{y}_A - \ddot{y}_2$ $\qquad 2\ddot{y}_A = \ddot{y}_1 + \ddot{y}_2$

$0 = 0 - \ddot{y}_A + 0 - \ddot{y}_3 \Rightarrow \ddot{y}_A = -\ddot{y}_3$ $\Bigg\} \Rightarrow \quad \ddot{y}_3 = -\frac{(\ddot{y}_1 + \ddot{y}_2)}{2}$

$$-2\ddot{y}_3 - \ddot{y}_2 = \ddot{y}_1$$

$\begin{array}{l} a_3 = \ddot{y}_3 \\ a_2 = \ddot{y}_2 \\ a_1 = \ddot{y}_1 \end{array} \Rightarrow$

$$T_{B3} - m_3 \cdot g = -m_3 \cdot \ddot{y}_A$$

$$-m_2 g + m_1 g = m_2 a_2 - m_1 \cdot a_1$$

$$0 = \ddot{y}_A - \ddot{y}_A - \ddot{y}_2 - \ddot{y}_3$$

$$g(m_1 - m_2) = m_2(-2\ddot{y}_3 - \ddot{y}_1) - m_1(-2\ddot{y}_3 - \ddot{y}_2)$$

$$-m_2 2\ddot{y}_3 - m_2 \ddot{y}_1 + m_1 \cdot 2\ddot{y}_3 + m_1 \cdot \ddot{y}_2$$

$$g(m_1 - m_2) = 2\ddot{y}_3(m_1 - m_2) - m_2 \cdot \ddot{y}_1 + m_1 \cdot \ddot{y}_2$$

$$\underbrace{T_{1A}+T_{2A}}^{2T_{A2}} - m_3 \cdot g = m_3 \cdot \ddot{y}_3 \Rightarrow -m_3 \cdot \left(\frac{\ddot{y}_2+\ddot{y}_1}{2}\right) = -m_3 \cdot g + 2T_{1A}$$

$$-T_{2A} - m_2 g = m_2 \cdot \ddot{y}_2 \Rightarrow -T_{1A} - m_2 \cdot g = m_2 \cdot \ddot{y}_2$$

$$-T_{1A} - m_1 g = m_1 \cdot \ddot{y}_1 \Rightarrow -T_{1A} - m_1 \cdot g = m_1 \cdot \ddot{y}_1$$

$$-m_2 \cdot g + m_1 \cdot g = m_2 \cdot \ddot{y}_2 - m_1 \cdot \ddot{y}_1$$

$$m_1 \cdot \ddot{y}_1 + m_1 \cdot g = m_2 \cdot \ddot{y}_2 + m_2 \cdot g$$

$$\boxed{m_1 \cdot (\ddot{y}_1 + g) = m_2 (\ddot{y}_2 + g)}$$

$$\frac{m_1}{m_2}(\ddot{y}_1 + g) - g = \ddot{y}_2$$

$$T_{1A} = -m_2 (g + \ddot{y}_2)$$

$$\frac{m_3}{2}\ddot{y}_2 - \frac{m_3}{2} \ldots$$

$$-(m_3 + m_2 + m_1) \cdot g = m_2 \ddot{y}_2 + m_1 \ddot{y}_1 - \ldots$$

$$-m_1 g - m_1 \ddot{y}_1 =$$

$$-m_1 (g + \ddot{y}_1) - m_2 (g + \ddot{y}_2) = m_3 g - \frac{m_3}{2}(\ddot{y}_2 + \ddot{y}_1)$$

$$-2m_2 (\ddot{y}_2 + g) = m_3 \left(g - \frac{\ddot{y}_2}{2} - \frac{\ddot{y}_1}{2}\right)$$

$$-2\cancel{m_2}\left(\frac{m_1}{\cancel{m_2}}(\ddot{y}_1+g) \cancel{-g} \cancel{+g}\right) = m_3 \left(g - \frac{m_1}{m_2 \cdot 2}(\ddot{y}_1 + g) + \frac{g}{2} - \frac{\ddot{y}_1}{2}\right)$$

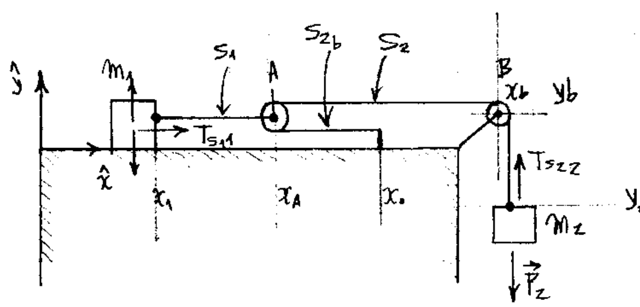
$$-\frac{2 \cdot m_1}{m_3}[\ddot{y}_1 + g] = \frac{3}{2} \cdot g - \frac{\ddot{y}_1}{2} - \frac{m_1}{2m_2}(\ddot{y}_1 + g)$$

$$-\frac{2m_1}{m_3} \cdot g + \frac{m_1}{2m_2} \cdot g - \frac{3}{2} g = \frac{2m_1}{m_3} \cdot \ddot{y}_1 - \ddot{y}_1 - \frac{m_1}{2m_2}\ddot{y}_1$$

$$\left(-\frac{2m_1}{m_3} + \frac{m_1}{2m_2} - \frac{3}{2}\right) \cdot g = \ddot{y}_1 \left(\frac{2m_1}{m_3} - 1 - \frac{m_1}{2m_2}\right)$$

$$g\left(\frac{-4m_2 m_1 + m_1 m_3 - 3m_3 m_2}{2m_3 m_2}\right) = \ddot{y}_1 \left(\frac{4m_1 m_2 - 2m_3 m_2 \ldots}{2m_3 m_2}\right)$$

②



$\vec{g} \downarrow$

a) $\underline{m_1}$

x̂) $T_{S_1 1} = m_1 \cdot a_{1x}$

ŷ) $N_1 - m_1 \cdot g = 0$

$\underline{m_2}$

x̂) $0 = 0$

ŷ) $T_{S_2 2} - m_2 \cdot g = m_2 \cdot a_{2y}$

$$L_{S_1} = x_A - x_1$$
$$L_{S_2} = x_0 - x_A + x_b - x_A + \pi R + y_b - y_2 + \frac{\pi R}{2}$$

$$0 = \ddot{x}_A - \ddot{x}_1 \Rightarrow \ddot{x}_A = \ddot{x}_1$$
$$0 = -\ddot{x}_A - \ddot{x}_A - \ddot{y}_2 \Rightarrow 2\ddot{x}_A = 2\ddot{x}_1 = -\ddot{y}_2$$

b)

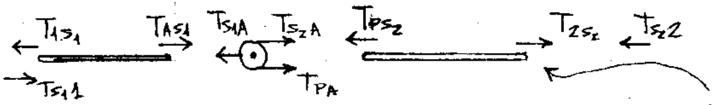
$d_1 = \frac{1}{2} \cdot \ddot{x}_1 \cdot t^2$ si x_1 inicial es 0

$d_2 = \frac{1}{2} \cdot 2\ddot{x}_1 \cdot t^2$ si y_2 inicial es 0

$$\frac{d_1}{d_2} = \frac{1}{\cancel{2}} \cdot \cancel{\ddot{x}_1} \cdot \cancel{t^2} \cdot \frac{\cancel{2}}{2\cancel{\ddot{x}_1} \cdot \cancel{t^2}} = \frac{1}{2}$$

$$\boxed{2 \cdot d_1 = d_2}$$

c)



$T_{1S_1} = T_{AS_1}$ $\qquad$ $T_{S_1A} = \underbrace{T_{S_2A}}_{} + \underbrace{T_{PA}}_{}$ $\qquad$ $T_{PS_2} = T_{2S_2} \Rightarrow T_{S_2P} = T_{S_22}$

$T_{1S_1} = -T_{S_11} = T_{AS_1}$ $\qquad$ $T_{2S_2} + T_{PS_2}$

$T_{S_1A} = 2T_{2S_2}$

$-T_{AS_1} = -2T_{S_22}$

$T_{S_11} = -T_{1S_1} = T_{AS_1} = 2T_{S_22} \Rightarrow T_{S_11} = 2T_{S_22}$ $\quad\therefore$

$T_{S_11} = 2T_{S_22}$

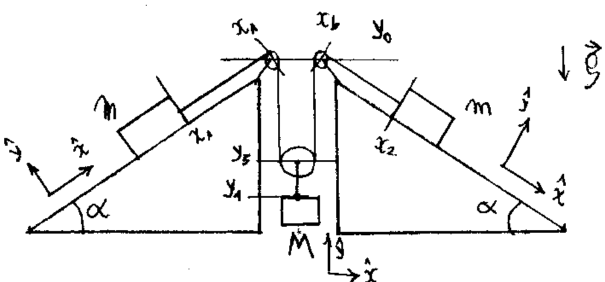
$$\begin{cases} T_{S_11} = m_1 . \ddot{x}_1 \\ \dfrac{T_{S_11}}{2} - m_2 . g = -m_2 . 2 . \ddot{x}_1 \end{cases} \Rightarrow \begin{array}{l} T_{S_11} = m_1 \ddot{x}_1 \\ T_{S_11} - 2m_2 g = -4m_2 \ddot{x}_1 \end{array}$$

$2m_2 g = m_1 \ddot{x}_1 + 4m_2 \ddot{x}_1$

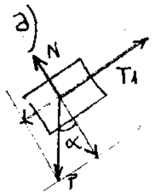
$T_{S_11} = \dfrac{m_1 2m_2}{(m_1 + 4m_2)}$ $\qquad$ $\boxed{\dfrac{2m_2}{(m_1+4m_2)} g = \ddot{x}_1}$

$\boxed{-\dfrac{4m_2}{(m_1+4m_2)} g = \ddot{y}_2}$

③

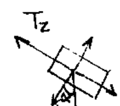


a)



$\hat{x})$ $T_{S1} - m.g.\text{sen}\,\alpha = m.a_1$

$\hat{y})$ $N_1 - m.g.\cos\alpha = 0$

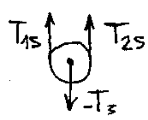


$\hat{x})$ $m.g.\text{sen}\,\alpha - T_{S2} = m.a_2$

$\hat{y})$ $N_2 - m.g.\cos\alpha = 0$

$\hat{x})$ $0 = 0$

$\hat{y})$ $T_3 - M.g = M.a_3$



$\Rightarrow$ $T_3 = T_{1S} + T_{2S}$

$T_3 = 2T_{1S} = 2T_{2S}$

Soga

$T_{1S} = T_{2S} \Rightarrow T_{S1} = T_{S2}$

$S = x_A - x_1 + y_0 - y_3 + y_0 - y_3 + \pi R$
$\quad x_2 - x_b$

b)

$-m.g.\text{sen}\,\alpha + m.g.\text{sen}\,\alpha = m.\ddot{x}_1 + m.\ddot{x}_2$ $\qquad$ $0 = -\ddot{x}_1 - 2\ddot{y}_3 + \ddot{x}_2$

$0 = m(\ddot{x}_1 + \ddot{x}_2)$ $\qquad$ $2\ddot{y}_3 = -\ddot{x}_1 + \ddot{x}_2$

$\ddot{x}_1 = -\ddot{x}_2$

$T_{S1} - mg\,\text{sen}\,\alpha - mg\,\text{sen}\,\alpha + T_{S1} = m(\ddot{x}_1 - \ddot{x}_2)$ $\qquad$ $2\ddot{y}_3 = \ddot{x}_2 + \ddot{x}_2 = 2\ddot{x}_2 \Rightarrow \ddot{y}_3 = \ddot{x}_2$

$\ddot{y}_3 = -\ddot{x}_1$

$T_3 - 2mg\,\text{sen}\,\alpha = -m(-\ddot{x}_1 + \ddot{x}_2)$

$-M(g + \ddot{y}_3) - 2mg\,\text{sen}\,\alpha = -m2\ddot{y}_3$

$-Mg - 2mg\,\text{sen}\,\alpha = -m2\ddot{y}_3 + M\ddot{y}_3$

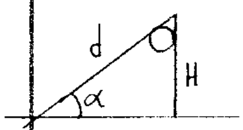
$(-M - 2m\,\text{sen}\,\alpha)\,g = \ddot{y}_3(-2m + M)$

$\boxed{\dfrac{-(2m\,\text{sen}\,\alpha + M)}{(-2m + M)} g = \ddot{y}_3}$

$\boxed{\ddot{x}_1 = \dfrac{(2m\,\text{sen}\,\alpha + M)}{(M - 2m)} . g}$

$\boxed{\ddot{x}_2 = \dfrac{-(2m\,\text{sen}\,\alpha + M)}{(M - 2m)} . g}$

c)



$H = \operatorname{sen}\alpha \cdot d$

$$H = \frac{1}{2} \cdot \ddot{y}_y \cdot T^2$$

$$H = -\frac{1}{2} \cdot \ddot{x}_1 \cdot T^2$$

$$H = \frac{1}{2} \cdot \ddot{x}_2 \cdot T^2$$

$$\frac{H \cdot 2}{T^2 g} = \frac{-2m \operatorname{sen}\alpha - M}{-2m + M} =$$

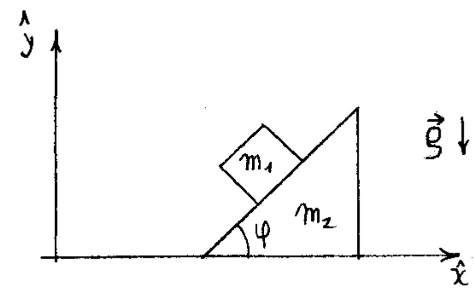
$$g \cdot \frac{-(2m \operatorname{sen}\alpha + M)}{(-2m + M)} > 0$$

$$\frac{2m \operatorname{sen}\alpha + M}{2m - M} > 0$$

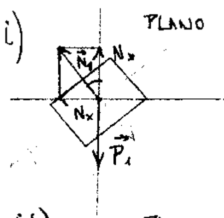
Sean:
$-2m + M < 0$
$-2m < -M$
$2m > M \Rightarrow \boxed{m > \frac{M}{2}}$

$$m > \frac{-M}{2 \operatorname{sen}\alpha}$$

④



$\vec{g} \downarrow$

i) 

PLANO FIJO:

ii) PLANO LIBRE:

$\underline{m_1}$: bloque

$\hat{x}$) $-N_1 \operatorname{sen}\varphi = m_1 \cdot a_{1x}$

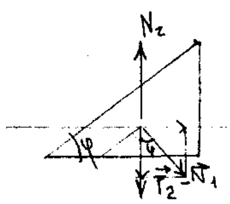
$\hat{y}$) $N_1 \cos\varphi - m_1 \cdot g = m_1 \cdot a_{1y}$

$\underline{m_2}$: plano

$\hat{x}$) $N_1 \cdot \operatorname{sen}\varphi = m_2 \cdot a_{2x}$

$\hat{y}$) $N_2 - m_2 g - N_1 \cdot \cos\varphi = m_2 \cdot \underbrace{a_{2y}}_{=0} = 0$

$N_2 = m_2 \cdot g + N_1 \cdot \cos\varphi$



$\underline{\text{vínculo}}$

$$\boxed{N_1 = \cos\varphi \cdot m_1 \cdot g}$$

$N_1 \cdot \operatorname{sen}\varphi$
N_1

sumando $\underline{m_1}\,\hat{x}$ y $\underline{m_2}\,\hat{x} \Rightarrow 0 = m_1 \cdot a_{1x} + m_2 \cdot a_{2x}$

$$m_1 \cdot a_{1x} = -m_2 \cdot a_{2x}$$

$-m_1 g + N_2 - m_2 g = m_1 a$
$(-m_1 - m_2) \cdot g + N_2 = m_1 a$

$$a_{1x} = -\frac{m_2}{m_1} \cdot a_{2x} = -\frac{m_2}{m_1} \cdot \frac{N_1 \cdot \operatorname{sen}\varphi}{m_2}$$

$$= -\frac{m_2 \cdot \operatorname{sen}\varphi \cdot \cos\varphi \cdot m_1 \cdot g}{m_1 \cdot m_2} = \frac{}{\frac{m_1 m_2 \cos\varphi}{1 - \operatorname{sen}^2\varphi}}$$

$\operatorname{sen}^2\varphi + \cos^2\varphi = 1$
$tg^2 + 1 = \sec^2$
$1 = \sec^2 - tg^2$

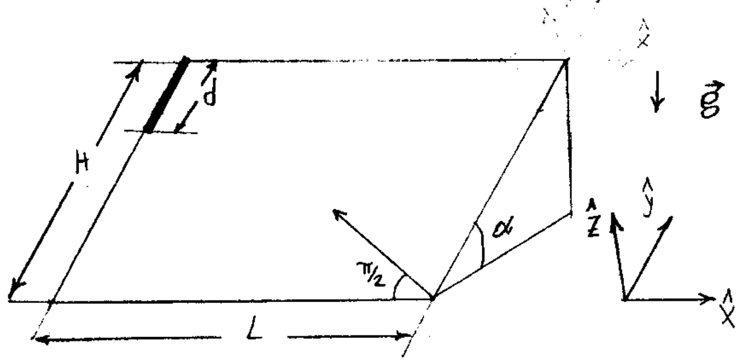
$1 - \operatorname{sen}^2 = \cos^2$
$\sec^2 - tg^2 = 1$

$$a_{1y} = -g + \cos^2\varphi \cdot g = g\left(-1 + \cos^2\varphi\right) = -g \cdot \operatorname{sen}^2\varphi$$

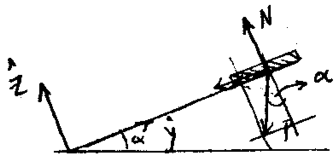
$$= \frac{-m_2 \cdot \operatorname{tang}\varphi \cdot \cos^2\varphi \cdot m_1 \cdot g}{m_1 \cdot m_2} =$$

5

⑤



a)

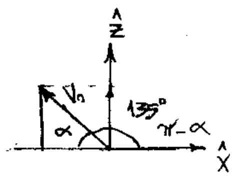


Varilla

$\hat{y})\quad -m_v \cdot g \cdot \text{sen}\,\alpha = m_v \cdot a_v$

$\hat{z})\quad N - m_v \cdot g \cdot \cos\alpha = 0$

Proyectil

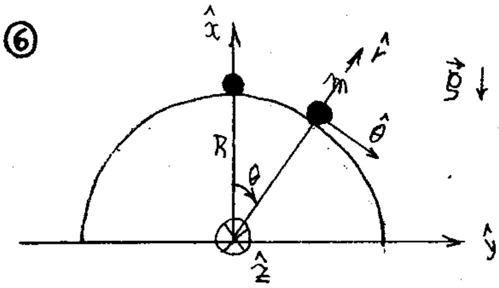


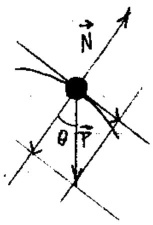
$V_0 \cdot \text{sen}\, 135° = V_{0z}$

$\hat{x})\quad x = V_0 \cdot \cos(\pi-\alpha) \cdot t$

$\hat{z})\quad z = z_0 + \underbrace{V_0 \cdot \text{sen}\,(\pi-\alpha)}_{V_{0\hat{z}}}\, t - \frac{1}{2} \cdot g\, t^2$

⑥





$R = k \Rightarrow \dot{r} = \ddot{r} = 0$

$\hat{r})\quad N - m \cdot g \cdot \cos\theta = -m \cdot R \cdot \dot{\theta}^2$

$\hat{\theta})\quad \cancel{m} \cdot g \cdot \text{sen}\,\theta = \cancel{m} \cdot R \cdot \ddot{\theta}$

$\frac{1}{R} g \cdot \text{sen}\,\theta = \ddot{\theta}$

Se separa si $N = 0$, es decir si:

$-\cancel{m} \cdot g \cdot \cos\theta_d = -\cancel{m} \cdot R \cdot \dot{\theta}^2 \qquad \cos\theta = \frac{R}{g} \cdot \dot{\theta}^2$

$$\ddot\theta = \frac{d\dot\theta}{d\theta}\cdot\frac{d\theta}{dt} \Rightarrow \int_{\dot\theta_0=0}^{\dot\theta(\theta_d)} \dot\theta \cdot d\dot\theta = \frac{g}{R}\int_{\theta_0=0}^{\theta_d} \operatorname{sen}\theta \cdot d\theta$$

$$\frac{\dot\theta_{\theta d}^2}{2} = +\frac{g}{R}\left[-\cos\theta_d + \cos 0\right]$$

$$\frac{\dot\theta_{\theta d}^2}{2} = -\frac{g}{R}\cdot\cos\theta_d + \frac{g}{R}$$

$$\cos\theta_d = \frac{R}{g}\dot\theta_{\theta_d}^{2}$$

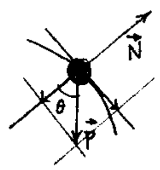
$$\dot\theta_{\theta d}^2 = g\frac{\cos\theta_d}{R}$$

$$\frac{1}{2}\cdot g\cdot\frac{\cos\theta_d}{R} = -\frac{g}{R}\cos\theta_d + \frac{g}{R}$$

$$\frac{3}{2}\cdot\frac{g}{R}\cos\theta_d = \frac{g}{R}$$

$$\cos\theta_d = \frac{2}{3} \Rightarrow \boxed{\theta_d = \arccos\left(\frac{2}{3}\right)}$$

b)



$$\dot\theta^2 = 2\cdot\frac{g}{R}\left(-\cos\theta + 1\right)$$

$$\dot\theta = +\sqrt{2\cdot\frac{1}{R}\cdot g}$$

$$2\cdot\frac{m}{m\,s^2}$$

angulares

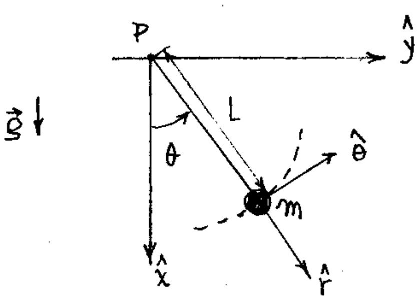
$$\boxed{\dot\theta_{\pi/2} = +\sqrt{\frac{2}{R}\cdot g}}$$

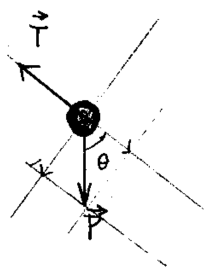
$$\boxed{\ddot\theta = g\cdot\frac{1}{R}}$$

tang. $\boxed{R\cdot\ddot\theta = g}$

$$\boxed{R\cdot\dot\theta = R\sqrt{\frac{2}{R}\cdot g} = \sqrt{R\cdot g\cdot 2}}$$

7





V en $\theta = 0 \Rightarrow V_0 = V_{\theta=0}$

$L = k \Rightarrow \dot L = \ddot L = 0$

a)

$\hat r)$ $m\cdot g\cdot\cos\theta - T = -m\cdot L\cdot\dot\theta^2$

$\hat\theta)$ $-m\cdot g\cdot\operatorname{sen}\theta = m\cdot L\cdot\ddot\theta$

$$\ddot\theta = -\frac{1}{L}\cdot g\cdot\operatorname{sen}\theta$$

$$\int_{\dot\theta_0=\dot\theta_0}^{\dot\theta}\dot\theta\cdot d\dot\theta = -\frac{1}{L}\cdot g\int_{\theta_0=0}^{\theta}\operatorname{sen}\theta\cdot d\theta$$

$$\frac{\dot\theta^2}{2} - \frac{\dot\theta_0^2}{2} = -\frac{1}{L}\cdot g\cdot\left(-\cos\theta + 1\right)$$

$$\frac{\dot\theta^2}{2} - \frac{\dot\theta_0^2}{2} = \frac{1}{L}\cdot g\cdot\left(\cos\theta - 1\right)$$

$V_t = L\cdot\dot\theta_0$
$V_0^2 = L^2\cdot\dot\theta_0^2$

$$-\frac{\dot\theta_0^2}{2} = \frac{1}{L}\cdot g\left(\cos\theta - 1\right)$$

$$-\frac{L\cdot\dot\theta_0^2}{2g} + 1 = \cos\theta = \left(1 - \frac{V_0^2}{2gL}\right) \Rightarrow \boxed{\theta = A\cos\left(1 - \frac{V_0^2}{2gL}\right)}$$

b)

$$m.g.\cos\theta - T = -m.\underbrace{L.\dot{\theta}^2}_{a.c.} \qquad \text{si } T=0 \Rightarrow$$

$$\frac{1}{L}g.\cos\theta = -\dot{\theta}^2 = -\left(\frac{2}{L}g\left[\cos\theta - 1\right] + \dot{\theta}_0^2\right)$$

$$\frac{1}{L}g\cos\theta = -\frac{2}{L}g\cos\theta + \frac{2g}{L} - \dot{\theta}_0^2$$

$$\frac{3}{L}g\cos\theta = \frac{2g}{L} - \dot{\theta}_0^2$$

$$\cos\theta = \frac{2}{3} - \frac{L\dot{\theta}_0^2}{3g}$$

$$\theta = A\cos\left(\frac{2}{3} - \frac{V_0^2}{3gL}\right)$$

$$-1 \leq \frac{2}{3} - \frac{V_0^2}{3gL} \leq 1$$

$$-\frac{5}{3} \leq -\frac{V_0^2}{3gL} \leq \frac{1}{3}$$

$$-5gL \leq -V_0^2 \leq gL$$

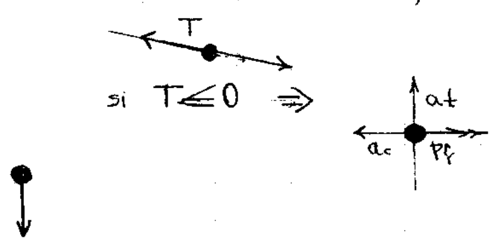
$$5gL \geq V_0^2 \geq -gL$$

$$\underbrace{5gL \geq V_0^2}_{\text{Para que } \exists \text{ ángulo}} \geq 0 > -gL$$

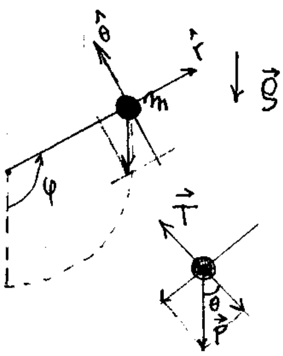
c) Se puede reemplazar en tanto y en cuanto la

$T > 0$ (en $\hat{r}$)

si $T \leq 0 \Rightarrow$



⑧



M.C.U $\qquad \omega_0 = k, \ R = k \Rightarrow \dot{\theta}_0 = k, \ \dot{r} = \ddot{r} = 0 \Rightarrow \ddot{\theta} = 0$

a)

$\hat{r}$) $m.g.\cos\theta - T = -m.R.\dot{\theta}_0^2$

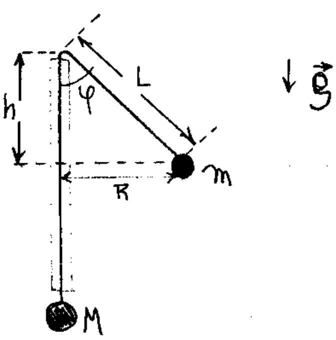
$\hat{\theta}$) $-m.g.\text{sen}\,\theta = m.R.\underbrace{\ddot{\theta}}_{0} \Rightarrow 0 = 0$

b)

$$T = m.g.\cos\theta + m.R.\dot{\theta}_0^2$$

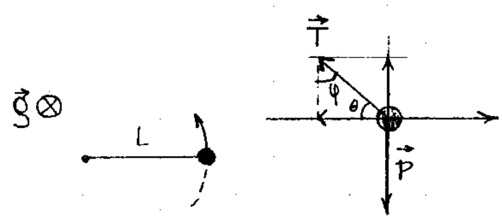
$$\boxed{T = m.\left[g.\cos\theta + \frac{V_0^2}{R}\right]}$$

⑨



$\text{sen}\,\varphi = \frac{R}{L}$

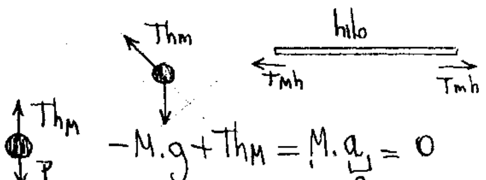
$\cos\varphi = \frac{h}{L}$



$\hat{r}$) $-T.\text{sen}\,\varphi = -m.R.\dot{\theta}^2$

$\hat{z}$) $T.\cos\varphi - m.g = m.\underbrace{a_z}_{0} \Rightarrow \quad T.\cos\varphi = m.g$

a)



$-M.g + Th_M = M.\underbrace{a}_{0} = 0$

$Th_M = M.g$

$Tmh = Tmh \Rightarrow ThM = Thm$

$$\Rightarrow \quad \text{sen}\,\varphi = \frac{m.R.\dot{\theta}^2}{M.g}$$

$$T.\cos\varphi = m.g$$

$$M.g.\cos\varphi = m.g$$

$$\boxed{\varphi = A\cos\left(\frac{m}{M}\right)}$$

b) L en función de τ, m, M y g

$$\tau = \frac{2\pi}{\dot\theta} \qquad \dot\theta^2 = \left(\frac{2\pi}{\tau}\right)^2 \qquad \cos\varphi = \frac{h}{L} = \frac{m}{M}$$

$$\dot\theta^2 = \frac{T \cdot \operatorname{sen}\varphi}{m \cdot R} \qquad\qquad \frac{M \cdot h}{m} = L$$

$$\dot\theta^2 = \frac{T \cdot \cancel{\operatorname{sen}\varphi}}{m \cdot \cancel{\operatorname{sen}\varphi} \cdot L}$$

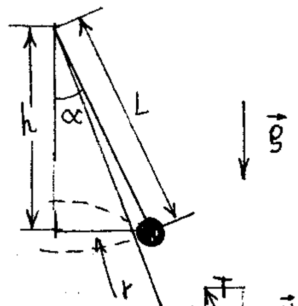
$$\dot\theta^2 = \frac{M \cdot g}{m \cdot L} \qquad\qquad [l] = \frac{\frac{1}{[t]^2}[m]\,[l]/[t]^2}{[m]}$$

$$\frac{4\pi^2}{\tau^2} = \frac{M \cdot g}{m \cdot L} \Rightarrow \boxed{L = \frac{\tau^2 \cdot M \cdot g}{4\pi^2 \cdot m}}$$

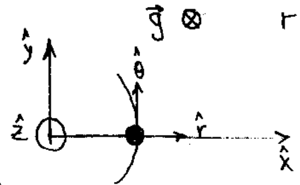
c) $$\tau = \frac{2\pi}{\dot\theta} = \frac{2\pi}{\sqrt{\frac{M \cdot g}{m \cdot L}}} = \frac{2\pi}{\sqrt{\frac{\cancel{L}}{h} \cdot \frac{g}{\cancel{L}}}} = \frac{2\pi}{\sqrt{\frac{g}{h}}} = \boxed{2\pi \cdot \sqrt{\frac{h}{g}}}$$

$$\frac{[l]}{\frac{[l]}{[t^2]}} = \frac{1}{\frac{1}{[t]^2}} = [t]^2$$

(10)



$$\omega_0 = \dot\theta$$



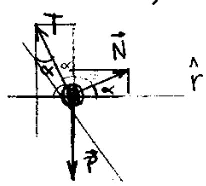
$$r = k \Rightarrow \dot r = \ddot r = 0$$

$$\operatorname{sen}\alpha = \frac{r}{L} \qquad \cos\alpha = \frac{h}{L}$$

$$\frac{r^2}{L^2} + \frac{h^2}{L^2} = 1$$

$$L^2 = h^2 + r^2$$
$$L^2 - r^2 = h^2$$
$$h \quad L^2 - r^2$$

a)



$\hat{r}$) $$-m \cdot r \cdot \dot\theta^2 = N \cdot \cos\alpha - T \cdot \operatorname{sen}\alpha$$

$\hat\theta$) $$m \cdot r \cdot \ddot\theta = 0 \Rightarrow 0 = 0$$

$\hat{z}$) $$T\cos\alpha + N \cdot \operatorname{sen}\alpha - m \cdot g = 0$$

$$T\cos\alpha + N\operatorname{sen}\alpha = m \cdot g$$

b)

$$\vec{a} = (-r \cdot \dot\theta^2) \cdot \hat{r} + \underbrace{(r \cdot \ddot\theta)}_{0} \hat\theta \Rightarrow \text{es MCU}$$

$$\vec{a} = (-r \cdot \omega_0^2) \cdot \hat{r}$$

c)

$$T \cdot \operatorname{sen}\alpha = N \cdot \cos\alpha + m \cdot r \cdot \omega_0^2$$

$$T = N \cdot \frac{\cos\alpha}{\operatorname{sen}\alpha} + \frac{m \cdot r \cdot \omega_0^2}{\operatorname{sen}\alpha}$$

$$T = \frac{m \cdot g}{\cos\alpha} - N \cdot \frac{\operatorname{sen}\alpha}{\cos\alpha}$$

$$N\frac{\cos\alpha}{\operatorname{sen}\alpha} + N\frac{\operatorname{sen}\alpha}{\cos\alpha} = -\frac{m \cdot r \cdot \omega_0^2}{\operatorname{sen}\alpha} + \frac{m g}{\cos\alpha}$$

$$N\left(\frac{\cos\alpha}{\operatorname{sen}\alpha} + \frac{\operatorname{sen}\alpha}{\cos\alpha}\right) = -m\left(\frac{r \cdot \omega_0^2}{\operatorname{sen}\alpha} - \frac{g}{\cos\alpha}\right)$$

$$N\left(\frac{h}{\cancel{L}} \cdot \frac{\cancel{L}}{r} + \frac{r}{\cancel{L}} \cdot \frac{\cancel{L}}{h}\right) = N\left(\frac{h}{r} + \frac{r}{h}\right) = -m\left(L \cdot \omega_0^2 - \frac{g}{\cos\alpha}\right)$$

$$\frac{h^2 + r^2}{r \cdot h}$$

$$N = -\frac{m \cdot r \cdot h}{(h^2 + r^2)}\left(L \cdot \omega_0^2 - \frac{g}{\cos\alpha}\right)$$

$$N = -\frac{m \cdot r \cdot h}{L^2}$$

$$\Leftarrow N = -\frac{m \cdot r \cdot \cos\alpha}{L}\left(L\omega_0^2 - \frac{g}{\cos\alpha}\right)$$

$$N = -\frac{m \cdot r \cdot \cancel{\cos\alpha} \cdot g \cdot \cancel{L}}{\cancel{L}\,\cancel{\cos\alpha}}\left(\frac{\cos\alpha \cdot \omega_0^2}{g} - \frac{1}{L}\right)$$

$$\boxed{N = m \cdot g \cdot r\left(\frac{1}{L} - \frac{\omega_0^2}{g} \cdot \cos\alpha\right)}$$

$$T = \frac{m.g}{\cos\alpha} - \frac{\operatorname{sen}\alpha}{\cos\alpha} \cdot \frac{m.g.r}{L} + \frac{\operatorname{sen}\alpha}{\cancel{\cos\alpha}} \frac{\omega_0^2}{g} \cdot \cancel{\cos\alpha}$$

$$\frac{m.g}{\cos\alpha} - \frac{\operatorname{sen}^2\alpha . m g}{\cos\alpha} + g \cdot \omega_0^2 . \operatorname{sen}\alpha$$

$$m.g.\cos\alpha \left(\frac{1}{\cos^2\alpha} - \frac{\operatorname{sen}^2\alpha}{\cos^2\alpha} + \frac{\omega_0^2 . \operatorname{sen}\alpha}{m.g.\cos\alpha} \right)$$

$$T = m.g.\cos\alpha \left(\frac{1 - \operatorname{sen}^2\alpha}{\cos^2\alpha} + \frac{\omega_0^2 . \operatorname{sen}\alpha}{m.g.\cos\alpha} \right)$$

$$\boxed{T = m.g.\cos\alpha \left(1 + \frac{\omega_0^2}{m.g} . tg\,\alpha \right)}$$

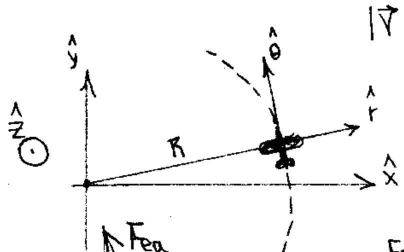
N se anula si

$$\frac{1}{L} - \frac{\omega_0^2}{g} \cos\alpha = 0$$

$$\frac{1}{L} = \frac{\omega_0^2}{g} . \cos\alpha$$

$$\frac{1}{\cos\alpha} . g . \frac{1}{L} = \omega_0^2 \qquad \text{si} \qquad \boxed{\omega_0 = \sqrt{\frac{g}{\cos\alpha . L}}}$$

⑪



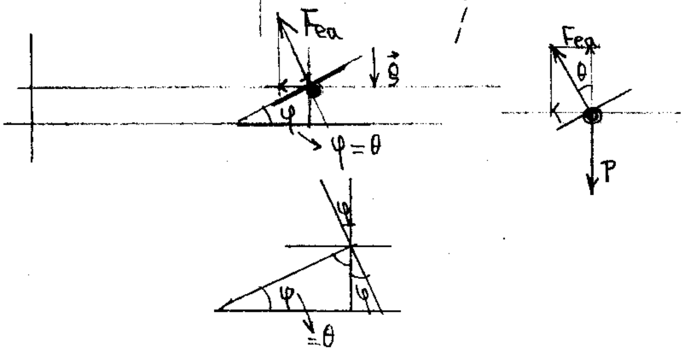
$$|\vec{V}| = k \quad \Rightarrow \quad \left.\begin{array}{l} V = R.\omega = R.\dot{\theta} = k \\ R = k \Rightarrow \dot{r} = \ddot{r} = 0 \\ \ddot{\theta} = 0 \end{array}\right\} \Rightarrow$$

a)

$$\hat{r}) \; -m.R.\dot{\theta}^2 = -F_{ea}.\operatorname{sen}\theta$$

$$\hat{z}) \; F_{ea}.\cos\theta - m.g = 0$$

$$F_{ea}.\cos\theta = m.g$$



$$-m.R.\dot{\theta}^2 = -m.g.\frac{\operatorname{sen}\theta}{\cos\theta}$$

$$-m.\underbrace{R.\dot{\theta}^2} = -m.g.tg\,\theta$$

$$\boxed{\frac{V^2}{R} = g.tg\,\theta}$$

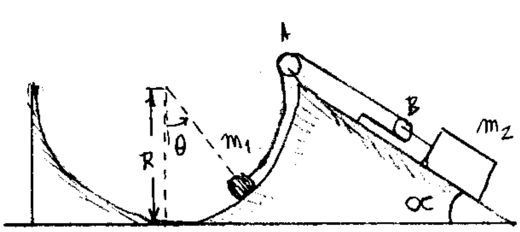
b)

$$\frac{3600 \; m^2/s^2}{1000 \; m} = 9.8 \frac{m}{s^2} . tg\,\theta$$

$$\operatorname{atan}(0.367) = \theta$$

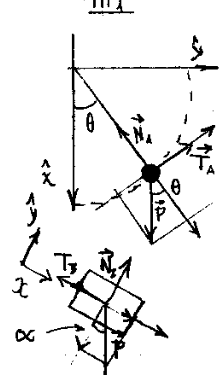
$$\boxed{\theta = 20^\circ \, 10' \, 14''}$$

⑫



a)

m_1



$$\hat{r}) \; m_1.g.\cos\theta - N_1 = -m_1 R.\dot{\theta}^2$$

$$\hat{\theta}) \; T_A - m_1 g.\operatorname{sen}\theta = m_1 R.\ddot{\theta}$$

m_2

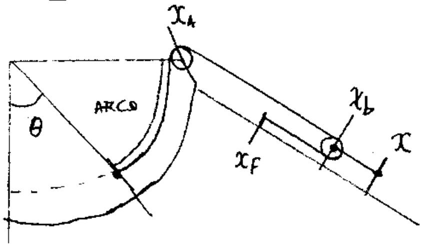
$$\hat{x}) \; -T_B + m_2.g.\operatorname{sen}\alpha = m_2.\ddot{x}$$

$$\hat{y}) \; N_2 - m_2.g.\cos\alpha = 0$$

vínculos

$\frac{s}{R} = \text{radian}$

L



ARCO

$$\left(\frac{\pi}{2} - \theta\right) . R$$

$$L_1 = x_b - x_A + x_b - x_F + \frac{\pi r}{2} + R\frac{\pi}{2} - \theta R$$

$$0 = \ddot{x}_b + \ddot{x}_b - \ddot{\theta} R$$

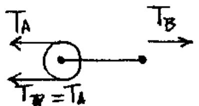
$$\ddot{\theta} R = 2\ddot{x}_b$$

$$L_2 = x - x_b$$

$$0 = \ddot{x} - \ddot{x}_b$$

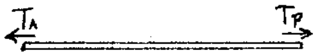
$$\boxed{\ddot{x}_b = \ddot{x}}$$

$$\boxed{\ddot{x} = \frac{R}{2}\ddot{\theta}}$$



Polea

$$-T_A - T_A + T_B = \underbrace{m}_{\to 0} \ddot{x}_b = 0 \Rightarrow \boxed{T_B = 2T_A}$$



$$T_A = T_P \Rightarrow$$

b)

como en reposo

c)

$$\begin{cases} -2T_A + m_2 g \,\text{sen}\,\alpha = m_2 \frac{R}{2}\ddot{\theta} \\ T_A - m_1 . g . \text{sen}\,\theta = m_1 . R . \ddot{\theta} \end{cases}$$

$$-2T_A + m_2 . g . \text{sen}\,\alpha = m_2 . R . \ddot{\theta} . \frac{1}{2}$$

$$2T_A - 2m_1 g \,\text{sen}\theta = 2m_1 R \ddot{\theta}$$

$$m_2 . g \,\text{sen}\,\alpha - 2m_1 g \,\text{sen}\,\theta = \frac{m_2 . R . \ddot{\theta}}{2} + 2m_1 R \ddot{\theta}$$

$$= \ddot{\theta} \left(\frac{m_2 R}{2} + 2m_1 R\right)$$

$$2m_2 g \,\text{sen}\alpha - 4m_1 g \,\text{sen}\,\theta = \ddot{\theta} . \left(\frac{m_2 R + 4m_1 R}{2}\right)$$

$$\frac{2m_2 g \,\text{sen}\,\alpha - 4m_1 g \,\text{sen}\theta}{m_2 R + 4m_1 R} = \ddot{\theta}$$

$$T_A = m_1 . g . \text{sen}\,\theta$$

$$T_A = m_2 g \frac{\text{sen}\,\alpha}{2}$$

$$m_1 . g . \text{sen}\theta = m_2 . g . \text{sen}\,\alpha . \frac{1}{2}$$

$$\boxed{\theta = \text{Asen}\left(\frac{m_2 \,\text{sen}\,\alpha}{2m_1}\right)}$$

$$\left(\frac{2g}{R}\right) \frac{m_2 \,\text{sen}\,\alpha - 2m_1 \,\text{sen}\,\theta}{(m_2 + 4m_1)} = \ddot{\theta}$$

$$\underbrace{\left(\frac{2g}{R}\right) \frac{m_2 . \text{sen}\,\alpha}{(m_2 + 4m_1)}}_{M_1} \int_{\theta_0}^{\theta} d\theta - \underbrace{\left(\frac{2g}{R}\right) \frac{2m_1}{(m_2 + 4m_1)}}_{M_2} \int_{\theta_0}^{\theta} \text{sen}\,\theta \, d\theta = \int \dot{\theta} . d\dot{\theta}$$

$$M_1 (\theta - \theta_0) - M_2 (-\cos\theta + \cos\theta_0) = \frac{\dot{\theta}^2 - \dot{\theta}_0^2}{2}$$

$$2 . M_1 . \theta + 2 . M_2 (\cos\theta - 1) = \dot{\theta}^2$$

$$\boxed{\frac{4}{R} g . \frac{m_2 . \text{sen}\,\alpha}{(m_2 + 4m_1)} . \theta + \frac{8}{R} g . \frac{m_1}{(m_2 + 4m_1)} . (\cos\theta - 1) = \dot{\theta}^2}$$