CINEMATICA DEL CUERPO RIGIDO

1.

A) No es rígido B) No es rígido $\vec{V_Q} - \vec{V_P} \neq 0$

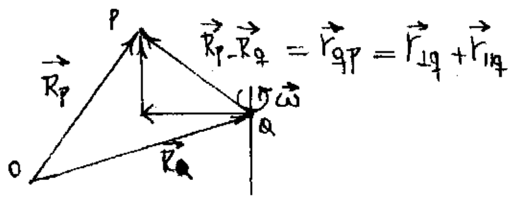
C) No es rígido D) Es rígido pues $V_P - V_Q = 0$

E) Es rígido F) No es rígido

2. i) Debe tener dirección coincidente con $\vec{P} - \vec{Q}$ Necesito $\vec{V_P} - \vec{V_Q} = \vec{0}$

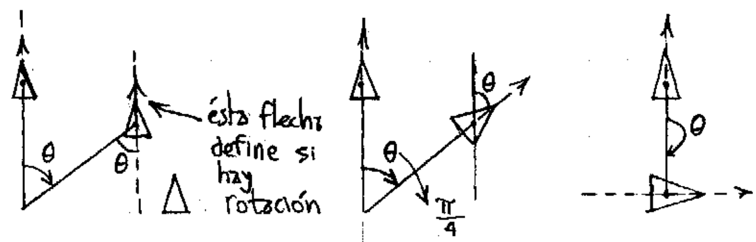
ii) $\vec{V_P} - \vec{V_Q} = \vec{\Omega} \times \vec{r_{QP}}$ forzamos $\vec{\Omega} \times \vec{r_{QP}} = 0 \iff \vec{\Omega} \parallel \vec{r_{PQ}}$

$\vec{V_P} = \vec{V_Q} + \vec{\Omega} \times \vec{r_{QP}}$ $\boxed{\text{satisface} \iff \vec{\Omega} \parallel \vec{r_{PQ}}}$



$\vec{R_P} - \vec{R_Q} = \vec{r_{QP}} = \vec{r_{\perp q}} + \vec{r_{\parallel q}}$

3.



del C.R. → $\omega = \boxed{0}$

$8t = T$

$t = \frac{1}{8} \cdot T$ $\dot{\theta} = \boxed{\frac{2\pi}{T}}$

del C.M. ángulo recorrido en un dado tiempo

$\omega = \frac{\pi}{4} \cdot \frac{8}{T} = \boxed{\frac{2\pi}{T}}$

$\dot{\theta} = \frac{2\pi/8}{T/8} = \boxed{\frac{2\pi}{T}}$

$\omega = \frac{\pi/2}{T/2} = \boxed{\frac{\pi}{T}}$

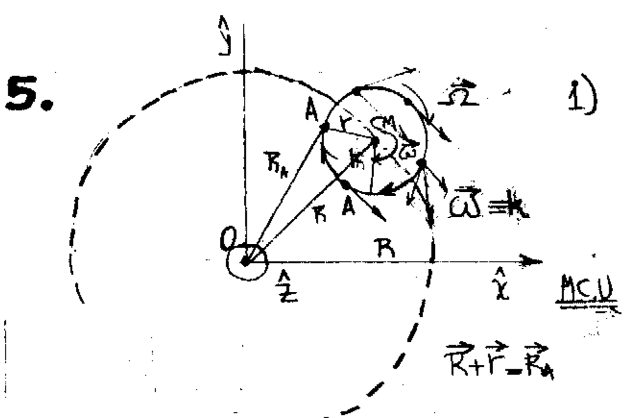
$\dot{\theta} = \frac{\pi}{T/2} = \boxed{\frac{2\pi}{T}}$

Solo coinciden ω [rotación del C.R] con la $\dot{\theta}$ [traslación circular] si se completa un giro cada una vuelta

4.

$$\frac{d\theta}{dt} = \Omega \qquad \int d\theta = \int \Omega \, dt$$
$$\theta = \Omega t + C$$

5.



$\vec{R} + \vec{r} = \vec{R_A}$

1) traslación rotación

$\vec{V_A} = \vec{V_{CM}} + \vec{\Omega} \times (\vec{r})$

$\vec{V_A} = \vec{\omega} \times \vec{R} + \vec{\Omega} \times \vec{r}$

$\omega . R \, \hat{\theta} + \Omega \, r \, \hat{\theta}$

$\boxed{\vec{V_A} = \left(\omega . R + \frac{2\pi}{\tau} r\right) \hat{\theta}}$

$r . \dot{\theta} \, \hat{\theta} = \left(\omega . R + \frac{2\pi r}{\tau}\right) \hat{\theta}$

$\boxed{\vec{\dot{\theta}} = \left(\omega . R / r + 2\pi / \tau\right) \hat{z}}$

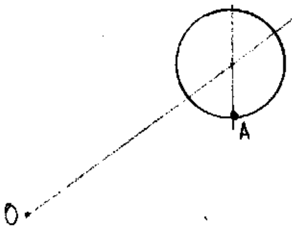
ω es k porque si hay $\hat{r} \, \hat{\theta} \, \hat{z}$

τ significa MCU

$\omega = \frac{2\pi}{\tau}$

$\omega . r = V_A$

ii)



$$\frac{2\pi}{t'} = \omega\frac{R}{r} + \frac{2\pi}{\tau}$$

$$\cancel{\frac{2\pi}{t'}} = \cancel{\frac{2\pi}{T}}\frac{R}{r} + \cancel{\frac{2\pi}{\tau}}$$

$$\frac{1}{t'} = \frac{R.\tau + T.r}{T.r.\tau}$$

$$\boxed{t' = \frac{T.r.\tau}{R.\tau + T.r}}$$

iii)

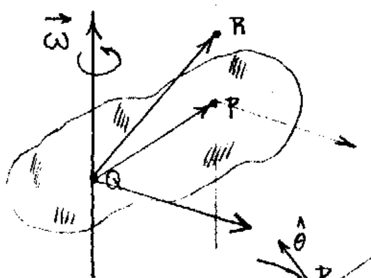
$$\Omega = \frac{2\pi \,.\, 23548}{3{,}15576.10^7} + \frac{2\pi}{86400} = \boxed{0{,}0047611}$$

6. i)

$$\vec{V}_P = \vec{V}_Q + \vec{\Omega}\times\vec{r}_{QP} \qquad \text{si } P \in \text{eje instantáneo de rotación} \Rightarrow$$

$$0 = \vec{V}_Q + \vec{\Omega}\times\vec{r}_{QP}$$

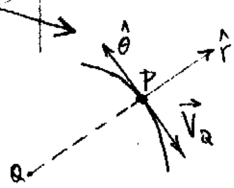
Eje ms $\times$ $\Omega = 0$



$$\vec{V}_Q = -\vec{\Omega}\times\vec{r}_{QP}$$

$$\vec{V}_Q = -\Omega\hat{z}\times r_{QP}\hat{r}$$

$$\vec{V}_Q = -\Omega\, r_{QP}\hat{\theta}$$

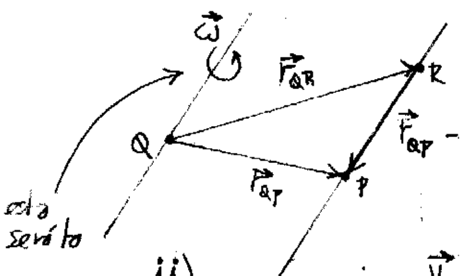


$$\vec{V}_P = \vec{V}_Q + \vec{\Omega}\times\vec{r}_{QP}$$

$$-\ \vec{V}_R = \vec{V}_Q + \vec{\Omega}\times\vec{r}_{QR}$$

$$0 = 0 + \vec{\Omega}\times\vec{r}_{QP} - \vec{\Omega}\times\vec{r}_{QR}$$

$$0 = \vec{\Omega}\times(\vec{r}_{QP} - \vec{r}_{QR}) \Rightarrow \boxed{\vec{\Omega} \text{ y } \vec{r}' \text{ son } \parallel}$$



$\vec{r}_{QP} - \vec{r}_{QR} = \vec{r}'$ → recta del eje instantáneo de rotación

ii) si $\vec{V}_P \cdot \vec{\Omega} \neq 0$ para algún punto $\vec{r}_P$

suponemos hay eje instantáneo de rotación $\Rightarrow$ Sea P cualquier punto y Q eje de rotación

$$\vec{V}_P = \vec{V}_Q + \vec{\Omega}\times\vec{r}_{QP}$$

$$\vec{V}_P\cdot\vec{\Omega} = (\vec{V}_Q + \vec{\Omega}\times\vec{r}_{QP})\cdot\vec{\Omega}$$

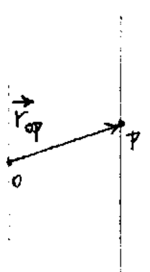
$$\vec{V}_P\cdot\vec{\Omega} = \vec{V}_Q\cdot\vec{\Omega} + \underbrace{(\underbrace{\vec{\Omega}\times\vec{r}_{QP}}_{\perp\,\vec{\Omega}})\cdot\vec{\Omega}}_{=0}$$

$$\boxed{\vec{V}_P = \vec{V}_Q}$$

$\Rightarrow$ si $\vec{V}_P = \vec{V}_Q$ no hay rotación en el cuerpo rígido

7.

$O \in$ eje instantáneo de rotación $\Rightarrow$

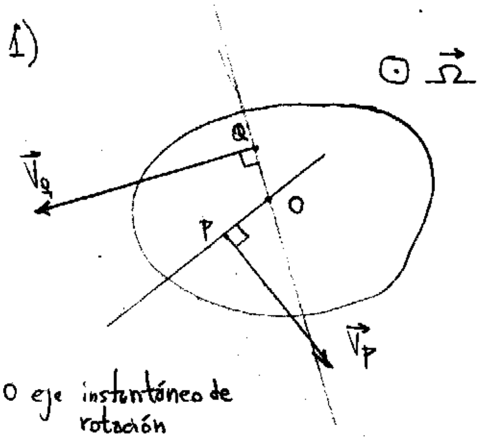


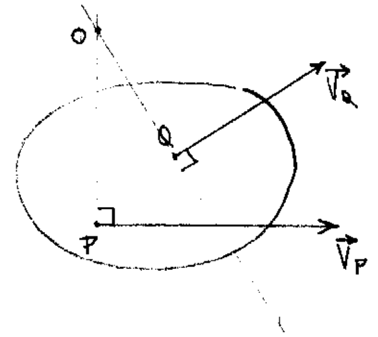
$$\vec{V}_P = \vec{V}_O + \vec{\omega}\times\vec{r}_{op}$$

$$\vec{V}_P = \vec{\omega}\times\vec{r}_{op} \Rightarrow \boxed{\begin{matrix}\vec{V}_P \perp \vec{\omega} \\ \vec{V}_P \perp \vec{r}_{op}\end{matrix}}$$

por propiedades del producto vectorial

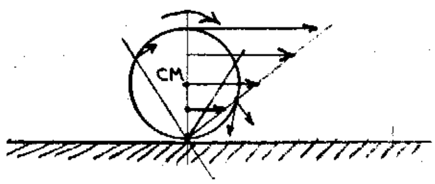
8. i) $\odot\ \vec{\Omega}$





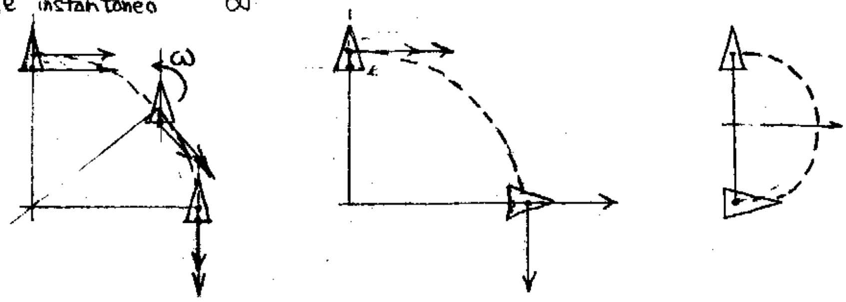
O eje instantáneo de rotación

ii)



iii)

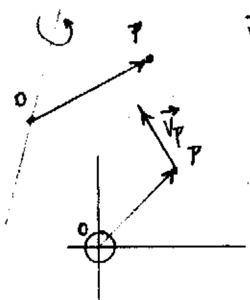
eje instantáneo ∞



9.

$$\vec{\omega} = (0,0,\omega)$$
$$\vec{V}_p = (V_x, V_y, 0) \quad \Rightarrow \quad \vec{r}_p = (r_x, r_y, \underbrace{r_z}_{r_z \equiv k \text{ en el tiempo}})$$

i)



$$\vec{V}_P = \underbrace{\vec{V}_o}_{\text{candidato a eje instantáneo}} + \vec{\omega} \times \vec{r}_{op} \quad \Rightarrow \quad V_x \hat{x} + V_y \hat{y} = 0 + (0,0,\omega) \times \vec{r}_{op}$$

$$V_x \hat{x} + V_y \hat{y} = -r_y \omega \hat{x} + r_x \omega \hat{y} \qquad \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & \omega \\ r_x & r_y & r_z \end{vmatrix}$$

si, $\boxed{\exists \text{ eje}}$ de rotación instantánea

$$r_x \omega \hat{y} - r_y \omega \hat{x}$$

ii) $\vec{V}_p = (V_x, V_y, \underset{\neq 0}{V_z})$ $\qquad \vec{V}_p = \vec{V}_o + \vec{\omega} \times \vec{r}_{op}$

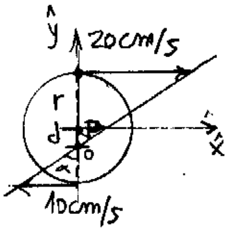
$$V_z \hat{z} + V_x \hat{x} + V_y \hat{y} = -r_y \omega \hat{x} + r_x \omega \hat{y} \quad \therefore \quad V_z \hat{z} = 0$$

lo que contradice hipótesis

no, $\boxed{\nexists \text{ eje}}$ instantáneo rotación

iii)

10.



$r = 10$ cm

i) $tg\,\alpha = \frac{20}{r+d} = \frac{10}{r-d} \Rightarrow 2 = \frac{r+d}{r-d}\cdot\frac{r+d}{r+d} = \frac{(r+d)^2}{r^2-d^2}$

$\boxed{\vec{o} = -\frac{r}{3}\hat{y}} = -\frac{10}{3}\hat{y}$

$2r - 2d = r + d$

$r = 3d$

$d = \frac{r}{3}$

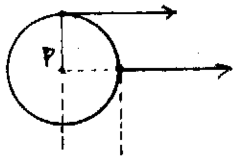
ii) $20\hat{x} = \vec{\Omega} \times \frac{4}{3} r \hat{y}$

$20 = \Omega \frac{4}{3} r$

$\boxed{\vec{\Omega} = 1{,}5\hat{z}}$

iii) $V_x\hat{x} = \Omega \frac{r}{3}\hat{x}$

$\boxed{V_{xp} = 5\frac{cm}{s}}$

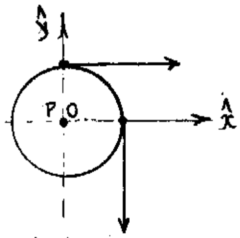


o en el ∞

i) o en el ∞

el cuerpo no rota

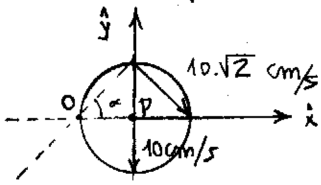
ii) $\boxed{\vec{\Omega} = 0}$ iii) $\boxed{V_{xp} = 20 \text{ cm/s}}$



i) $o = \vec{O} = p$ ii) $20\hat{x} = \Omega r \hat{x}$

$\boxed{\vec{\Omega} = 2\hat{z}}$

iii) $\boxed{\vec{V_p} = \vec{0}}$



ii) $\vec{V_o} = 0 = \vec{V_p} + \vec{\Omega} \times \vec{r_{op}}$

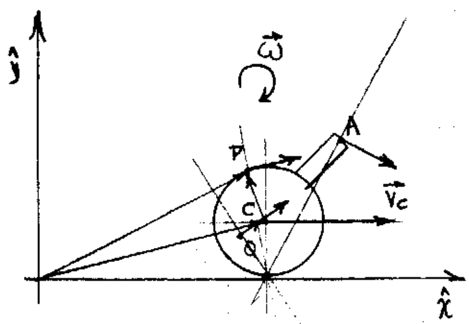
$0 = -10\hat{y} + \Omega\hat{z} \times 10\hat{x}$

$10\hat{y} = \Omega . 10\hat{y} \rightarrow \boxed{\vec{\Omega} = 1\hat{z}}$

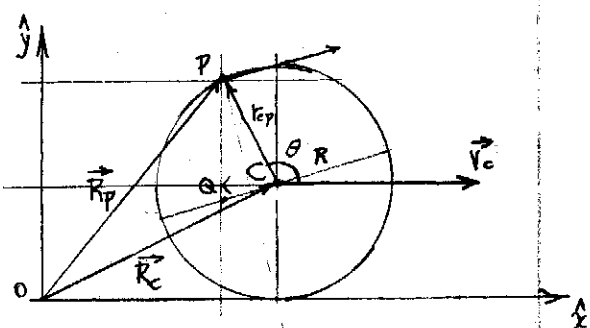
i) $\boxed{-10\hat{x} = \vec{o}}$

iii) $\boxed{\vec{V_p} = -10\hat{y}}$ $\boxed{V_{yp} = -10 \text{ cm/s}}$

11.



$R = 10$cm



$\vec{R_p} - \vec{R_c} = \vec{r_{cp}}$

$\frac{d}{dt}\vec{R_p} - \frac{d}{dt}\vec{R_c} = \frac{d}{dt}(\vec{r_{cp}})$

i) $\vec{V_p} = \vec{V_c} + \vec{\Omega} \times \vec{r_{cp}}$

$\vec{V_p} = V_c\hat{x} + \Omega\hat{z} \times (r_{cpx}\hat{x} + r_{cpy}\hat{y})$

$\vec{V_p} = V_c\hat{x} + \Omega r_{cpx}\hat{y} + \Omega . r_{cpy}\hat{x}$

$\vec{V_p} = V_c\hat{x} + \Omega . r_{cp} . \text{sen}\,\theta\,\hat{y} + \Omega . r_{cp} . \cos\theta\,\hat{x}$