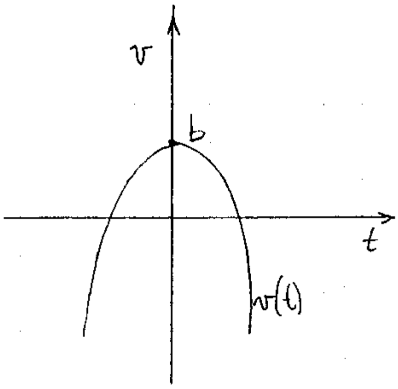
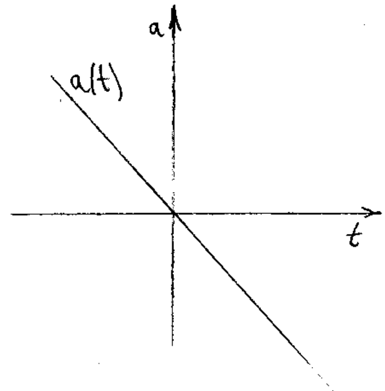
CINEMÁTICA

1. $x = -k\,t^3 + b.t \qquad k, b \geq 0$

a) $v = \dfrac{dx}{dt} \Rightarrow \boxed{v = -k.3.t^2 + b}$

$a = \dfrac{d^2x}{dt^2} \Rightarrow \boxed{a = -k.6.t}$





b) $0 = -3k.t^2 + b$

si $k \neq 0 \qquad t^2 = \dfrac{b}{3k}$

$t = \sqrt{\dfrac{b}{3k}} \qquad x = -k.\dfrac{b^{3/2}}{3^{3/2}k^{3/2}} + b.\sqrt{\dfrac{b}{3k}}$

c) $t < 0$: $t > 0$

mov. acelerado $(a > 0)$ — mov. desacelerado $(a < 0)$

2. $x = \sqrt{x_0^2 + 2kt} \qquad x_0, k > 0$

$[L]^2 + 2\dfrac{[L]}{[t]}$

$[k] = \dfrac{[L^2]}{[T]}$

a) $v = \dfrac{dx}{dt} = \dfrac{1}{\cancel{2}\sqrt{x_0^2 + 2k.t}}.\cancel{2}k = \boxed{\dfrac{k}{\sqrt{x_0^2 + 2kt}}}$

$a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = \dfrac{0.\sqrt{\quad} - k.\dfrac{1.\cancel{2}.k}{\cancel{2}\sqrt{x_0^2+2kt}}}{x_0^2 + 2kt} = -\dfrac{k.k}{\sqrt{x_0^2+2kt}.(x_0^2+2kt)}$

$\dfrac{L^4}{T^2} \quad \dfrac{1}{L^3}$

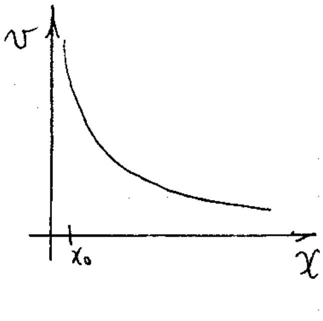
$\boxed{a = \dfrac{-k^2}{(x_0^2 + 2kt)^{3/2}}}$

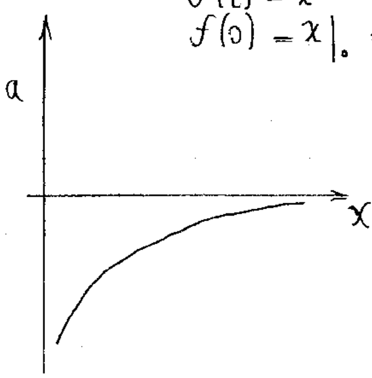
b) $v = \dfrac{k}{\sqrt{x_0^2 + 2kt}} = \dfrac{k}{x}$

$a = \dfrac{-k^2}{(\sqrt{x_0^2 + 2kt})^3} = -\dfrac{k^2}{x^3}$

$f(t) = x$

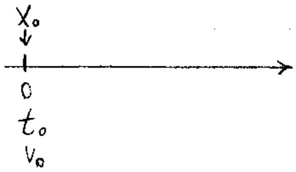
$f(0) = x|_0 = \sqrt{x_0^2 + 0} = |x_0| = x$





3.

$t=0$
$x_0 = 0$
Vento $V_0 \neq 0$



a) $a = kt^2 \quad , \quad k>0$

$$\frac{dv}{dt} = kt^2 \qquad dv = kt^2 . dt \quad \Rightarrow \quad \int dv = k \int t^2 . dt$$

$$V + C' = k . \frac{t^3}{3} + C''$$

$t=0 \qquad V_0 = \emptyset + C \quad \Rightarrow \quad C = V_0$ por condición inicial

$$V = k . t^3 . \frac{1}{3} + V_0$$

$$\int dx = \frac{k}{3} \int t^3 . dt + \int V_0 dt \Leftarrow \quad \frac{dx}{dt} = k . \frac{1}{3} . t^3 + V_0 \quad \text{OK UNID}$$

$$x + C' = \frac{k}{3} . \frac{t^4}{4} + V_0 + C''$$

$$x = \frac{k}{12} t^4 + V_0 + C$$

$t=0 \qquad x_0 = \emptyset + C = \emptyset \Rightarrow C=0 \quad \Rightarrow \quad x = \frac{k}{12} . t^4 + V_0$

$$V = \frac{k}{3} . t^3 \quad \Rightarrow \quad t = \sqrt[3]{\frac{3.v}{k}} \quad \text{OK}$$

$$f(t) = \boxed{x = \frac{k}{12} . t^4 + V_0} \quad \text{OK UNIDADES}$$

$$f(v) = \boxed{x = \frac{k}{12} \left(\frac{3.v}{k} \right)^{4/3}}$$

Análisis unidades

$$\frac{[L]}{[T]^2} = \frac{[L]}{[t]^4} [T]^2 \quad \Rightarrow \quad [k] = \frac{[l]}{[t]^4}$$

$$\frac{[l]}{[t]} . \frac{[t]}{[l]} \qquad \frac{[l]}{[t]^4} . [t]^{3 . \frac{4}{3}}$$

b) $a = -k . v^2 \quad , \quad k > 0$

Análisis unidades

$$\frac{[l]}{[t]^2} = \frac{1}{[l]} . \frac{[l]^2}{[t]^2} \quad \Rightarrow \quad \frac{1}{[l]} = [k]$$

$$\frac{dv}{dt} = -k . v^2 = -k . \frac{dx}{dt} . V$$

$$\frac{dv}{dt} = -k V . \frac{dx}{dt}$$

$$dv = -kV . dx$$

$$\int \frac{1}{V} . dv = -k \int dx$$

$$\text{LN } V + C_1 = -kx - kC_2$$

$$\text{LN } V = -kx \underline{-kC_2 - C_1}$$

$$\text{LN } V = -kx + C_3 \qquad V_0 = e^{C_3} \Rightarrow C_3 = \text{LN } V_0$$

$$V = e^{-kx + C_3}$$

$$\boxed{v = e^{-kx + \text{LN } V_0}}$$

$$v = e^{-kx + \text{LN} V_0} = e^{-kx} . e^{\text{LN} V_0} \quad \Rightarrow \quad v = e^{-kx} . v_0 \qquad \text{si } v_0 \neq 0$$

$$\frac{v}{v_0} = e^{-kx}$$

$$\boxed{-\frac{1}{k} . \text{LN} \left(\frac{v}{v_0} \right) = x} \qquad x(v)$$

$$\frac{dx}{dt} = e^{-kx + \ln V_0} = e^{-kx} . e^{\ln V_0} = e^{-kx} . V_0$$

$$\frac{1}{V_0} dx = e^{-kx} . dt$$

$$\int \frac{1}{e^{-kx}} . dx = \int V_0 . dt$$

$$\frac{1}{k} . e^{kx} + C_1 = V_0 . t + C_2$$

$$e^{kx} = k . V_0 . t + C_3$$

CA

$$\int e^{kx} . dx \qquad kx = U \quad k dx = du \quad dx = \frac{1}{k} . du$$

$$\int \frac{1}{k} . e^{u} . du$$

$$\frac{1}{k} . e^{kx} + C$$

si $\begin{cases} x = 0 \\ t = 0 \end{cases} \Rightarrow 1 = C_3 \quad \therefore \quad e^{kx} = k . v_0 . t + 1$

$$kx = \ln (k . v_0 . t + 1)$$

$$x(t) \qquad \boxed{x = \frac{1}{k} . \ln (k . v_0 . t + 1)}$$

c) $a = k . v . x \qquad k > 0$

$$\frac{dv}{dt} = k . \frac{dx}{dt} . x$$

Unidades

$$\frac{[l]}{[t]^2} = [t]^{-1} \frac{[l]}{[l]} \frac{[l]}{[t]} . [l]$$

$$\Downarrow [k] = \frac{1}{[l].[t]}$$

$$dv = k . x . dx$$

$$\int dv = k \int x . dx$$

$$v + C_1 = k \frac{x^2}{2} + k . C_2$$

$$V = k . \frac{x^2}{2} + [k . C_2 - C_1]$$

$$V = k . \frac{x^2}{2} + C_3 \Rightarrow$$

en $t = 0$ es $x = 0$ y $V_0 = C_3$

$$v(x) = k . \frac{1}{2} . x^2 + v_0$$

$$x^2 = \frac{(v - v_0) . 2}{k}$$

$$\boxed{x = \sqrt{\frac{2 (v - v_0)}{k}}} \qquad x = f(v)$$

$$\frac{dx}{dt} = \frac{k}{2} . x^2 + v_0 \Rightarrow \frac{dx}{dt} = \frac{k x^2 + 2 v_0}{2}$$

si $v_0 \neq 0$

$$\frac{2}{k x^2 + v_0} . dx = dt$$

$$\frac{1}{\frac{k x^2 + 2 v_0}{2}} dx = dt$$

$$\int \frac{1}{\frac{k x^2}{2} + v_0} dx = \int dt$$

$$\frac{\sqrt{k}\, x}{\sqrt{2} \sqrt{v_0}} = U$$

$$\sqrt{\frac{k}{2 v_0}} dx = du$$

$$dx = \sqrt{\frac{2 v_0}{k}} du$$

$$\int \frac{1}{v_0} . \frac{1}{\frac{k x^2}{v . 2} + 1} dx = t + C_1$$

$$\frac{1}{v_0} . \frac{\sqrt{2 v_0}}{\sqrt{k}} \int \frac{1}{u^2 + 1} du = t + C_1$$

$$\frac{\sqrt{2}}{\sqrt{v_0} \sqrt{k}} . \text{ARCotg} \sqrt{\frac{k}{2 v_0}}\, x + C_2 = t + C_1$$

$$\sqrt{\frac{2}{k.v_0}}.\text{ARCotg}\left(\sqrt{\frac{k}{2v_0}}.x\right)=t+C_3 \qquad \text{si } t=0 \qquad 0=C_3$$

$$\Rightarrow$$

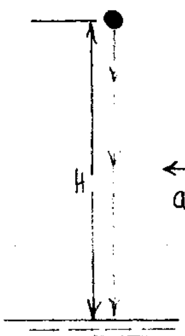
$$\sqrt{\frac{2}{kv_0}}.\text{ARCotg}\left(\sqrt{\frac{k}{2v_0}}.x\right)=t$$

$$x = tg\left(\sqrt{\frac{kv_0}{2}}\,t\right).\sqrt{\frac{2v_0}{k}}$$

$$\boxed{x(t) = tg\left(\sqrt{\frac{kv_0}{2}}\,t\right).\sqrt{\frac{2v_0}{k}}} \qquad tg\left(\sqrt{\frac{1}{[l].[t]}\frac{[l]}{[t]}}\,[t]\right) \quad \text{ok}$$

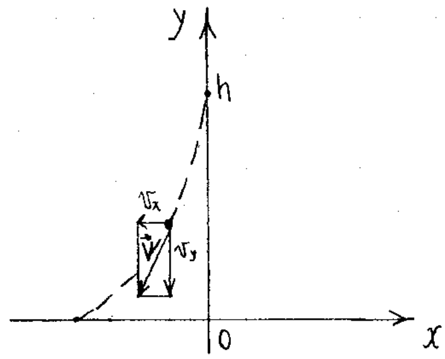
$$\sqrt{\frac{[l]}{[t]}.[l].[t]}\;[l] \text{ ok}$$

4. $t=0 \qquad \vec{V}_0=\vec{0}$



$g = a_y$

$a_x=-k.t^2 \quad , k>0$



a) Es un movimiento bidimensional

$$a_y=-g \qquad -g=\frac{dv_y}{dt} \qquad \Rightarrow \qquad \int dv_y=\int -g\,dt$$

$$a_x=-k.t^2 \qquad -k.t^2=\frac{dv_x}{dt}$$

$$v_y+C_1=-gt+C_2$$

$$v_y=-gt+\underbrace{C_2-C_1}$$

$$v_y=-gt+C_3$$

$$\boxed{v_y=-gt+v_{y0}}$$

$$\vec{v}=v_x.\hat{\imath}+v_y.\hat{\jmath} \qquad \vec{V}_0=v_{x0}\hat{\imath}+v_{y0}\hat{\jmath}=\vec{0}$$

$$\frac{d\vec{r}}{dt}=\frac{dx}{dt}.\hat{\imath}+\frac{dy}{dt}.\hat{\jmath}$$

unidades $\frac{[l]}{[t]^2}=\frac{[l].[t]^2}{[t]^4}$

$$[k]=\frac{[l]}{[t]^4}$$

$$\int dv_x=\int -k.t^2\,dt$$

$$v_x+C_1=-k.\frac{t^3}{3}+C_2$$

$$v_x=-k.\frac{t^3}{3}+C_3$$

$$\boxed{v_x=-k.\frac{t^3}{3}+v_{x0}}$$

$$v_x=\frac{dx}{dt}=-\frac{k.t^3}{3}$$

$$\int dx=\int -\frac{1}{3}.k.\,t^3\,dt$$

$$x=-\frac{1}{3}.k.\frac{t^4}{4}+C_3$$

$$\boxed{x(t)=-\frac{k}{12}.t^4}$$

$$\Downarrow$$

$$-\frac{12x}{k}=t^4$$

$$\sqrt{\frac{-12.x}{k}}=t^2$$

$$v_y=\frac{dy}{dt}=-gt$$

$$\int dy=\int -gt\,dt$$

$$y=-g\frac{t^2}{2}+C_3$$

$$\boxed{y(t)=-\frac{g}{2}.t^2+H}$$

$$\Downarrow$$

trayectoria $\boxed{y(x)=-g.\frac{1}{2}\sqrt{\frac{-12.x}{k}}+H}$

b)

$$0 = -g\cdot\frac{1}{2}\cdot\sqrt{\frac{-12\,x}{k}} + H$$

$$\frac{k}{-12}\left(-\frac{H.2}{-g}\right)^2 = x$$

$$\boxed{x = -\frac{k}{3}\cdot\frac{H^2}{(-g)^2}}$$ tocará el suelo aquí

unidades

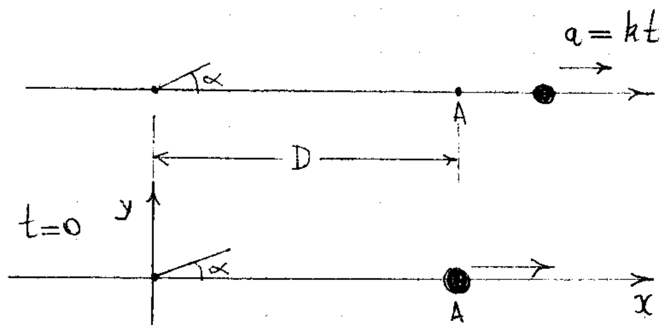
$$[x] = \frac{[l]}{\cancel{[t]^4}}\cdot\cancel{[l]^2}\cdot\frac{\cancel{[t]^4}}{\cancel{[l]^2}}$$ ok

Si $a_x = 0 \Rightarrow v_x = 0 \Rightarrow x(t) = 0$ es una caída libre

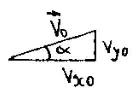
$v_y = -g t$ Toca el suelo en $x = y = 0$

5. En la carpeta

6.

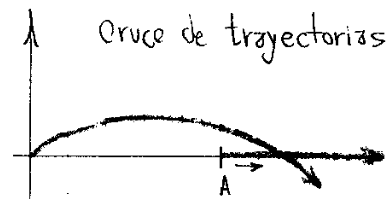


a)



bala

$$\begin{cases} V_{x0} = V_0 . \cos\alpha \\ V_{y0} = V_0 . \operatorname{sen}\alpha \end{cases}$$



$$\begin{cases} x(t) = V_0 . \cos\alpha . t \\ y(t) = V_0 . \operatorname{sen}\alpha . t - \frac{1}{2}\cdot g\, t^2 \end{cases}$$

$$t = \frac{x}{V_0 . \cos\alpha}$$

$$y(x) = (\operatorname{tg}\alpha).x - \frac{1}{2}\, g\cdot\left(\frac{x}{V_0.\cos\alpha}\right)^2$$

$$y(A) > 0$$

$$(\operatorname{tg}\alpha).A - \frac{1}{2}\, g\left(\frac{A}{V_0 . \cos\alpha}\right)^2 > 0$$

$$\frac{\cos\alpha}{A}\sqrt{\frac{2(\operatorname{tg}\alpha)A}{g}} > \frac{1}{V_0}$$

$$\boxed{\frac{1}{\cos\alpha}\sqrt{\frac{A\,g}{2\operatorname{tg}\alpha}} < V_0}$$

b)

bala

$$x(t) = V_0 \cos\alpha \, t$$
$$y(t) = V_0 \, \mathrm{sen}\,\alpha \, t - \frac{1}{2} g t^2$$

Pelotita

$$x(t) = \frac{k}{6} t^3 + A$$
$$y(t) = 0$$

$$a_x = k\,t$$
$$\int dv = k \int t \, dt$$
$$V_x = \frac{t^2}{2} k + \underbrace{C_3}_{V_0=0}$$
$$V_x = k \frac{t^2}{2}$$
$$\int dx = \frac{k}{2} \int t^2 dt$$
$$x = \frac{k}{2.3} t^3 + \underbrace{x_0}_{A}$$

condiciones de encuentro
$$\begin{cases} 0 = V_0 \, \mathrm{sen}\,\alpha (t_\varepsilon - t_0) - \frac{1}{2} g (t_\varepsilon - t_0)^2 \\ V_0 \cos\alpha (t_\varepsilon - t_0) = \frac{k}{6} (t_\varepsilon^3) + A \end{cases}$$

$$0 = (t_\varepsilon - t_0)\left[V_0 \, \mathrm{sen}\,\alpha - \frac{g}{2}(t_\varepsilon - t_0) \right]$$

$$\begin{cases} t_\varepsilon = t_0 \\ t_\varepsilon - t_0 = \frac{2 V_0 \, \mathrm{sen}\,\alpha}{g} \end{cases}$$

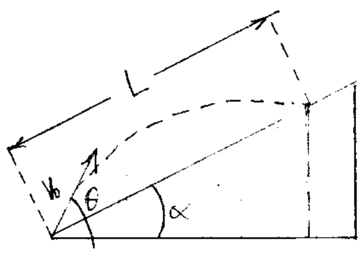
$$V_0 \cos\alpha \, \frac{V_0 \, \mathrm{sen}\,\alpha \, . 2}{g} = \frac{k}{6}(t_\varepsilon)^3 + A$$

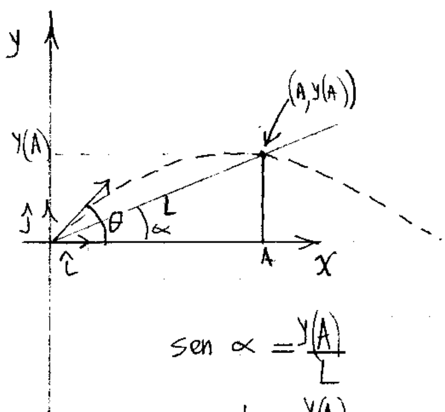
$$\sqrt[3]{\frac{6}{k}\left(\frac{V_0^2 . \cos\alpha . \mathrm{sen}\,\alpha . 2}{g} - A\right)} = t_\varepsilon$$

Instante en el que debe disparar para pegarle

$$\boxed{\sqrt[3]{\frac{6}{k}\left(\frac{V_0^2 . \cos\alpha . \mathrm{sen}\,\alpha . 2}{g} - A\right)} - \frac{2 V_0 \, \mathrm{sen}\,\alpha}{g} = t_0}$$

7.





a)

Pelota

$$a_x = 0$$
$$a_y = -g$$

$$V_{x0} = V_0 . \cos\theta$$
$$V_{y0} = V_0 . \mathrm{sen}\,\theta$$

$$\mathrm{sen}\,\alpha = \frac{y(A)}{L} \qquad \cos\alpha = \frac{A}{L}$$
$$\mathrm{sen}\,\alpha . L = y(A) \qquad \cos\alpha . L = A$$

$$\begin{cases} x(t) = V_0 \cos\theta . t \\ y(t) = V_0 . \mathrm{sen}\,\theta . t - \frac{1}{2} g . t^2 \end{cases}$$

$$t = \frac{x}{V_0 \cos\theta}$$

$$y(x) = \frac{V_0 \, \mathrm{sen}\,\theta}{V_0 . \cos\theta} . x - \frac{1}{2} . g . \frac{x^2}{V_0^2 . \cos^2\theta}$$

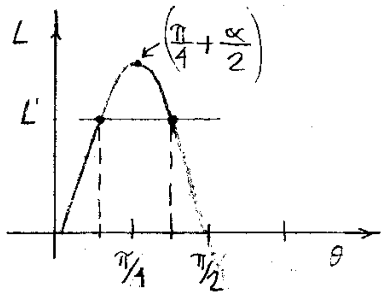
si $x = A = \cos\alpha . L \Rightarrow$
$$y(A) = \frac{\cancel{V_0} \, \mathrm{sen}\,\theta . \cos\alpha \, L}{\cancel{V_0} \cos\theta} - \frac{1}{2} . g \, \frac{\cos^2\alpha \, . \, L^2}{V_0^2 . \cos^2\theta}$$
$$0 = \mathrm{sen}\,\alpha . L - \frac{\mathrm{sen}\,\theta . \cos\alpha . L}{\cos\theta} + \frac{g}{2} . \frac{\cos^2\alpha \, L^2}{V_0^2 \cos^2\theta}$$
$$0 = L\left(\mathrm{sen}\,\alpha - \mathrm{tg}\,\theta . \cos\alpha + \frac{g}{2} \frac{1}{V_0^2} \left(\frac{\cos\alpha}{\cos^2\theta}\right)^2 L \right)$$

$$\begin{cases} L=0 \\ \dfrac{(\operatorname{tg}\theta.\cos\alpha-\operatorname{sen}\alpha)\,2V_0^2.\cos^2\theta}{g\,\cos^2\alpha}=L \end{cases}$$

$$\frac{\operatorname{sen}\theta}{\cos\theta}.\cos\alpha-\operatorname{sen}\alpha=\frac{\operatorname{sen}\theta\cos\alpha-\operatorname{sen}\alpha.\cos\theta}{\cos\theta}=\frac{\operatorname{sen}(\theta-\alpha)}{\cos\theta}\Rightarrow$$

$$\boxed{L=\frac{\operatorname{sen}(\theta-\alpha).2V_0^2\cos\theta}{g.\cos^2\alpha}}$$

b)



c)

$$L=\underbrace{\left(\frac{2V_0^2}{g.\cos^2\alpha}\right)}^{M}.\operatorname{sen}(\theta-\alpha).\cos\theta$$

$$L'=(M).\left[-\operatorname{sen}(\theta-\alpha).\operatorname{sen}\theta+\cos(\theta-\alpha).\cos\theta\right]$$

$$L'=M.\left[\cos(\theta-\alpha+\theta)\right]$$

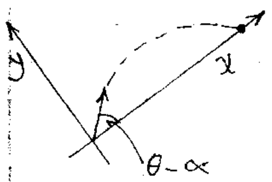
$$0=\cos(2\theta-\alpha)$$

$$\frac{\pi}{2}=2\theta-\alpha$$

$$\frac{\pi}{2}+\alpha=2\theta$$

$$\boxed{\frac{\pi}{4}+\frac{\alpha}{2}=\theta}$$

θ de alcance L máximo

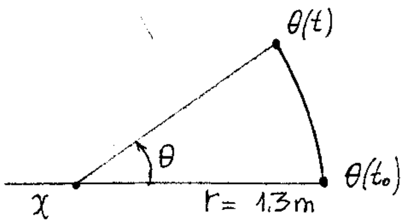


$$L=M.\operatorname{sen}(\theta-\alpha).\cos\theta$$

si $\theta=\pi/2$ $\quad L=0$

8.

en t_0 $\quad \theta_0=0$, $\omega_0=0$



$$\gamma=120\frac{1}{seg^4}t^2-48\frac{1}{seg^3}t+16\frac{1}{seg^2}$$

γ aceleración angular en $\frac{1}{seg^2}$

b) $\quad \gamma=\dfrac{d\omega}{dt}$

$$120t^2-48t+16=\frac{d\omega}{dt}$$

$$\int 120t^2dt-\int 48t\,dt+\int 16\,dt=\int d\omega$$

$$120\frac{t^3}{3}+C_3-48\frac{t^2}{2}+C_4\;16t+C_1=\omega+C_2$$

$$\boxed{40t^3-24t^2+16t+\omega_0=\omega(t)}$$

a) $\quad \omega=\dfrac{d\theta}{dt}$

$$40t^3-24t^2+16t+\omega_0=\frac{d\theta}{dt}$$

$$\int 40t^3dt-24\int t^2dt+16\int t\,dt=\int d\theta$$

$$+C_1+40\frac{t^4}{4}-24\frac{t^3}{3}+16\frac{t^2}{2}=\theta+C_2$$

$$\boxed{\theta(t)=\theta_0+8t^2-8t^3+10t^4}$$

d) $|\vec{V}| = |\vec{\omega}|.r \quad \frac{1}{s}.m$

$\vec{V} = 0.\hat{r} + r.\dot{\theta}.\hat{\theta}$

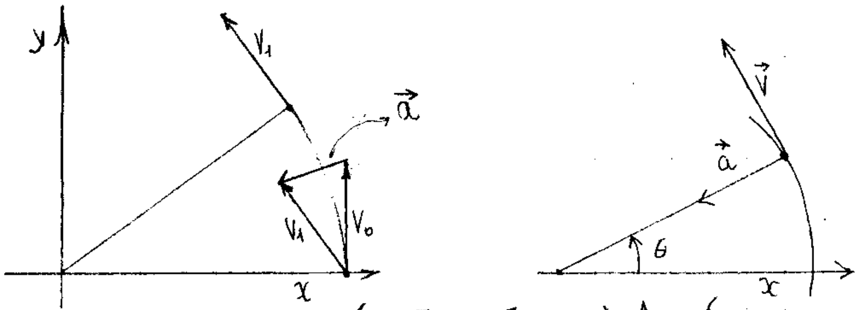
$\vec{V} = (1.3).256.\hat{\theta}$

$|\vec{V}| = |40(2)^3 - 24(2)^2 + 16(2) + 0|.\ 1.3$

$\vec{V} = 332{,}8.\hat{\theta}$

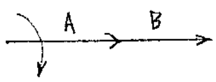
$|\vec{V}| = (320 - 96 + 32).1.3 = \boxed{332{,}8 \ \frac{m}{s}}$

c)

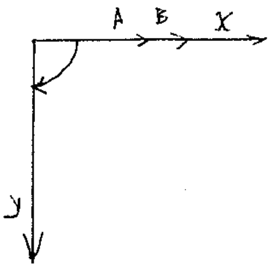


$\vec{a} = -1.3.(40t^3 - 24t^2 + 16t).\hat{r} + (120t^2 - 48t + 16).1.3.\hat{\theta}$

9.



Aguja A	Aguja B
$\omega_0 = k$	$\gamma_0 = k$
$\theta_{A0} = 0$	$\omega_{B0} = 2\omega_0$
	$\theta_{B0} = 0$



$\frac{d\theta}{dt} = \omega_0$

$\int d\theta = \omega_0 \int dt$

$\theta + C_1 = \omega_0.t + C_2$

$\theta(t) = \omega_0.t + \theta_0$

$\theta(t) = \omega_{0_A}.t$

$\frac{d\omega}{dt} = \gamma$

$\int d\omega = \gamma \int dt$

$\omega + C_1 = \gamma t + C_2$

$\omega = \gamma t + \omega_0$

$\frac{d\theta}{dt} = \gamma t + \omega_0$

$\int d\theta = \gamma \int t\,dt + \omega_0 \int dt$

$\theta + C_1 = \gamma \frac{t^2}{2} + \omega_0 t + C_2$

$\theta(t) = \frac{\gamma}{2}.t^2 + \omega_{0_B} t + \theta_0$

$\boxed{\theta(t) = \frac{\gamma}{2}.t^2 + \omega_{0_B} t}$

a) Coinciden si sus posiciones angulares son la misma:

$\omega_0.t = \frac{\gamma}{2}t^2 + 2\omega_0 t$

$0 = \left(\frac{\gamma}{2}t + \omega_0\right).t$

$\boxed{\begin{cases} t = 0 \\ t = \frac{-\omega_0.2}{\gamma} \end{cases}}$

b) Aguja A

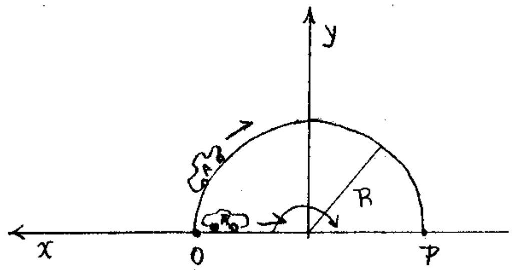
$\theta(t) = -\omega_{0_A}.t \Rightarrow$

$-\omega_0 t = \frac{\gamma}{2}t^2 + 2\omega_0 t$

$0 = \frac{\gamma}{2}t^2 + 3\omega_0 t = t\left(\frac{\gamma}{2}t + 3\omega_0\right)$

$\boxed{\begin{cases} t = 0 \\ t = \frac{-3\omega_0.2}{\gamma} \end{cases}}$

10.



$R = 90\,m$

AZUL $\Gamma_a = \alpha = k.t \quad \left(k = \frac{\pi}{6} \cdot \frac{1}{seg^3} \right.$

ROJO $a_r = -a_0 . \hat{x} \quad$ (constante)

AUTO AZUL

$$\alpha = \frac{\pi}{6}.t = \frac{d\omega}{dt}$$

$$d\omega = \frac{\pi}{6} t \, dt$$

$$\int_{\omega_0}^{\omega} d\omega = \frac{\pi}{6} \int_0^t t \, dt$$

$$\omega - \omega_0 = \frac{\pi}{6} \left(\frac{t^2}{2} - 0 \right)$$

$$\omega = \omega_0 + \frac{\pi t^2}{12}$$

$$\frac{d\theta}{dt} = \omega_0 + \frac{\pi t^2}{12}$$

$$d\theta = \omega_0 \, dt + \frac{\pi t^2}{12} dt$$

$$\int_{\theta_0}^{\theta} d\theta = \omega_0 \int_0^t dt + \frac{\pi}{12} \int_0^t t^2 \, dt$$

$$\theta - \theta_0 = \omega_0 t + \frac{\pi}{12} \frac{t^3}{3}$$

$$\theta = \theta_0 + \omega_0 t + \frac{\pi . t^3}{36}$$

a)

$$\pi = 0 + 0.t + \frac{\pi}{36} t^3$$

$$\sqrt[3]{36} = t$$

$$\boxed{3{,}302 \text{ seg} = t}$$

b) AUTO ROJO

$$a = -a_0$$

$$dv = -a_0 \, dt$$

$$\int_{v_0}^{v} dv = \int_{t_0}^{t} -a_0 \, dt$$

$$V = V_0 - a_0 (t - t_0)$$

$$\int_{x_0}^{x} dx = \int_{t_0}^{t} V_0 \, dt - a_0 \int_{t_0}^{t} (t - t_0) \, dt$$

$$x - x_0 = V_0 (t - t_0) - \frac{a_0}{2} (t - t_0)^2$$

$$x = x_0 + V_0 (t - t_0) - \frac{a_0}{2} (t - t_0)^2$$

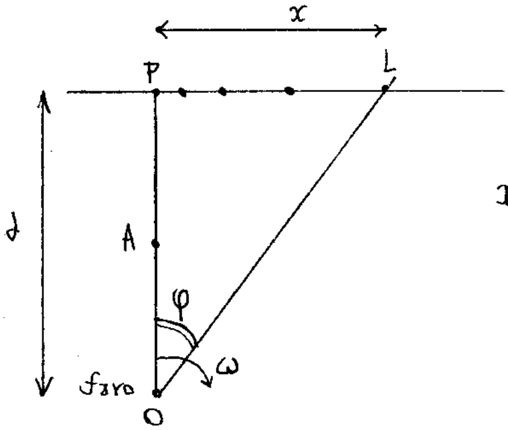
$$-90 = 90 + 0 \cdot (t - t_0) - \frac{a_0}{2} (t - 3)^2$$

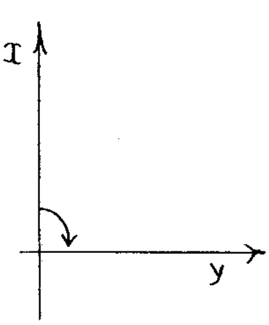
$$-180 = -\frac{a_0}{2} (t - 3)^2$$

$$\frac{-360}{(t-3)^2} = -a_0 \Rightarrow$$

$$\boxed{-3949 \frac{m}{seg^2} = -a_0}$$

11.





vel. angular constante

$$\omega_0 = k \Rightarrow$$

$$\frac{d\omega}{dt} = 0 \Rightarrow \alpha = 0$$

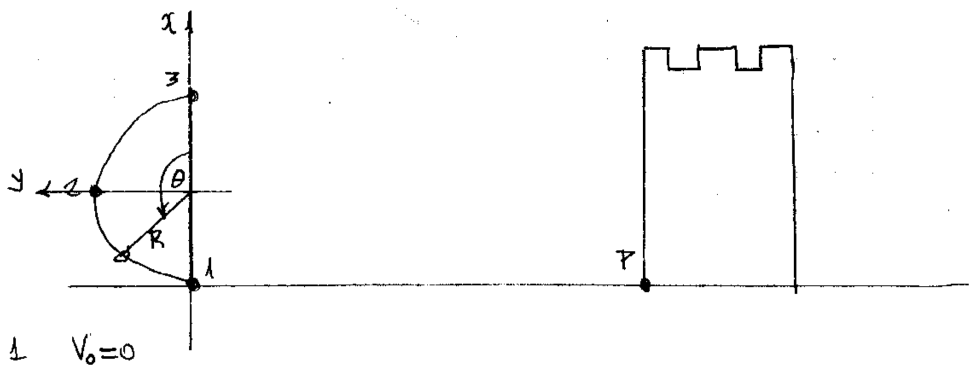
$$\omega = \frac{d\theta}{dt}$$

$$\int \omega . dt = \int d\theta$$

$$C_2 + \omega . t = \theta + C_1$$

$$\theta_0 + \omega . t = \theta(t)$$

12.



en 1 $\quad V_0 = 0$

$$\ddot{\theta} = \alpha = -\frac{(n+1)\cdot k}{\pi^{n+1}} \cdot \theta^n \qquad -\frac{5k}{\pi^5}\cdot\theta^4$$

Análisis dimensional

a) $\quad \ddot{\theta} = \dfrac{d^2\theta}{dt^2} = \dfrac{d\omega}{dt} \qquad V = R\cdot\omega$

un modo: $\qquad \dfrac{1}{[t]^2} = [k]$

$$\frac{d\omega}{d\theta}\cdot\frac{d\theta}{dt} = -\frac{5k}{\pi^5}\cdot\theta^4$$

$$\int \omega \cdot d\omega = -\frac{5k}{\pi^5}\int \theta^4\, d\theta$$

$$\frac{\omega^2}{2} + C_1 = -\frac{\cancel{5}\cdot k}{\pi^5}\frac{\theta^5}{\cancel{5}} + C_2$$

en (1) es $\theta = 180° = \pi$

$$0 = \frac{\omega_0^2}{2} = -\frac{k\,\cancel{\pi^5}}{\cancel{\pi^5}} + C_3$$

$$C_3 = +k$$

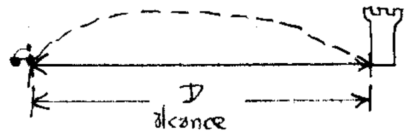
$$\omega^2 = -\frac{2k}{\pi^5}\,\theta^5 + 2k$$

$$\omega = \pm\sqrt[2]{-\frac{2k\cdot\theta^5}{\pi^5} + 2k} \Rightarrow \boxed{V = R\cdot \pm\sqrt[2]{-\frac{2k\cdot\theta^5}{\pi^5} + 2k}}$$

en (2) $\theta = 90° = \pi/2 \quad \Rightarrow V = R\cdot\pm\sqrt[2]{-\dfrac{2k\,\cancel{\pi^5}}{32\cdot\cancel{\pi^5}} + 2k} = \boxed{V(t=2) = -R\cdot\sqrt{-\dfrac{k}{16} + 2k}}$

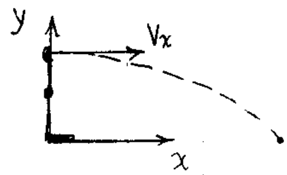
b)

$$\vec{V}_{(\text{posición } 3)} = -R.\sqrt{2K}\,.\hat{\theta}$$



Se interpreta que la altura de la catapulta interesa ⇒

$$\vec{V} = -R.\sqrt{2K}\,\hat{\theta} \quad \equiv \quad V_{x0} = R\sqrt{2K}$$



$$V_{y0} = 0$$
$$V_{x0} = R\sqrt{2K}$$

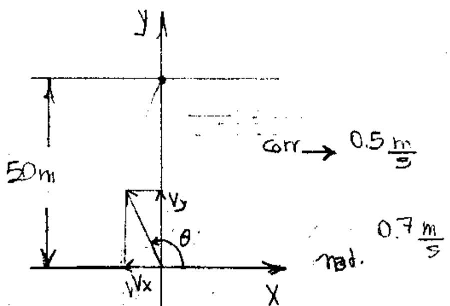
$$x = x_0 + V_{y0}.t = 0 + R\sqrt{2K}.t$$
$$y = y_0 + V_{x0}.t - g.t^2 = 2R + 0.t - g.t^2$$

$$\frac{x}{R\sqrt{2K}} = t \qquad y_{(x)} = 2R - g\frac{x^2}{R^2.2K}$$

donde $y(x)$ corta al eje x

$$0 = 2R - g.\frac{x^2}{R^2.2K}$$
$$x^2 = \frac{2R.R^2.2k}{g}$$
$$\boxed{x = \sqrt{\frac{4R^3K}{g}} = D}$$

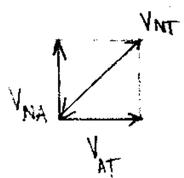
13.



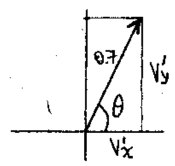
$$V = k$$

	s'	s'' (agua con respecto a la tierra)	
$V_x =$	$V'_x.\hat{i}$	$+ V''_x.\hat{i}$	$x = x' + x''$
$V_y =$	$V'_y\,\hat{j}$	$+ \underbrace{V''_y.\hat{j}}_{0}$	$y = y' + \underbrace{y''}_{0}$

$$\begin{cases} V_x = 0{,}7.\cos\theta.\hat{i} + 0.5.\hat{i} \\ V_y = 0{,}7.\text{sen}\,\theta.\hat{j} + 0.\hat{j} \end{cases}$$

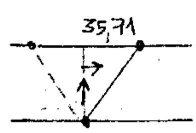


$$\vec{V}_{NA} + \vec{V}_{AT} - \vec{V}_{TN} = 0$$



$$V'_x = 0.7.\cos\theta$$
$$V'_y = 0.7.\text{sen}\,\theta$$

Nado en línea recta



$$x = x_0 + V_x(t) = 0 + (0.7\cos\theta + 0.5)t$$
$$y = y_0 + V_y(t) = 0 + 0.7.\text{sen}\theta.t$$

a) Punto opuesto orilla:

$$0 = (0{,}7\cos\theta + 0{,}5).t$$
$$50 = 0{,}7\,\text{sen}\,\theta.\,t$$

$$-\frac{0{,}5}{0{,}7} = \cos\theta \Rightarrow \theta = 135{,}5847 = \boxed{135^\circ 35' 4''}$$

$$t = \frac{50}{0.7.\,\text{sen}(135^\circ 35' 4'')} = \boxed{102{,}06 \text{ seg}}$$

b)

$$\frac{50}{0{,}7} = \operatorname{sen}\theta \cdot t \quad \overset{\text{si } \theta \neq 0}{\Rightarrow} \quad \frac{50}{0.7}\cdot\frac{1}{\operatorname{sen}\theta} = t$$

$$-\frac{50}{0.7}\cdot\frac{\cos\theta}{\operatorname{sen}^2\theta} = t'$$

$$\cos\theta = 0$$

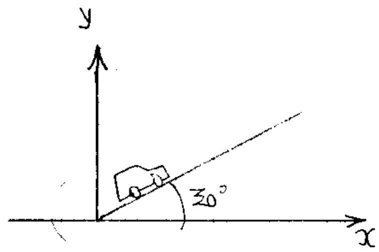
$$\boxed{\theta = 90^\circ}$$

$$t = \frac{50}{0.7}\cdot 1 = \boxed{71{,}42 \text{ seg}}$$

$$X = 0.5\,(71{,}42\,s) = \boxed{35{,}714 \text{ m}}$$

$$y = \boxed{50 \text{ m}}$$

14.



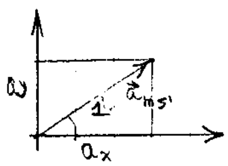
$S' \qquad S''$

$$\vec{a} = \vec{a}_{mS'} + \vec{a}_{S'S''}$$

$$\vec{a} = (0.866,\ 0.5) + (0{,}5,\ 0)$$

$$\vec{a} = 1{,}3660\,\hat{i} + 0{,}5\,\hat{j}$$

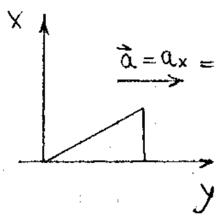
$\underline{\underline{S'}}$



$a_x = \cos 30^\circ \cdot 1$
$a_x = 0.866$

$a_y = \operatorname{sen} 30^\circ \cdot 1$
$a_y = 0.5$

$\underline{\underline{S''}}$

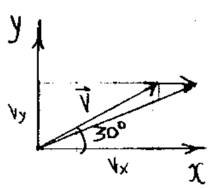


a) $\boxed{\vec{a} = 1{,}3660\,\hat{i} + 0{,}5\,\hat{j}}$, $|\vec{a}| = 1{,}454$

b) $V_x = V'_x + V''_x$
$V_y = V'_y + V''_y$

$\underline{\underline{S'}}$

$$\vec{V} = \vec{V}_0 + \vec{a}\cdot t$$



$V_x = V_{0x} + a_x \cdot t$
$V_y = V_{0y} + a_y \cdot t$

$V_x = 0 + (0{,}866)\cdot 1$
$V_y = 0 + (0.5)\cdot 1$

en $t=1$ $\quad |\vec{V}| = V = \boxed{1 \ \frac{m}{s}}$ respecto de la rampa

respecto de la rampa

$$\vec{V} = \vec{V}_0 + \vec{a}\cdot t$$

$|\vec{V}| = |\vec{a}|\cdot t$ en $t=1 \Rightarrow$

$|\vec{V}| = |\vec{a}| = \boxed{1{,}454 \ \frac{m}{s}}$ respecto de la tierra