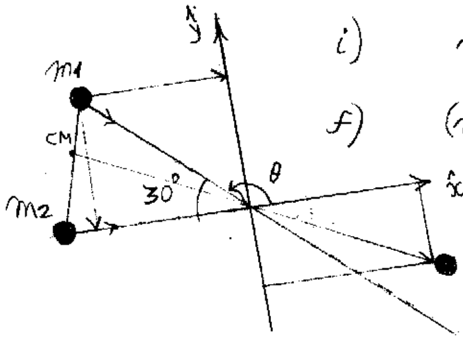
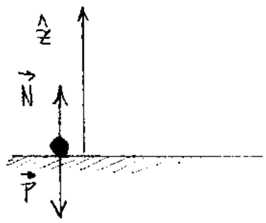
CANTIDAD DE MOVIMIENTO

1.



i) $m_1 . \vec{v}_1 + m_2 . \vec{v}_2 = P$

f) $(m_1 + m_2) . \vec{v} = \vec{P}$



$\underline{m_1}$ $\hat{z})$ $N_1 = m_1 g$

$\underline{m_2}$ $\hat{z})$ $N_2 = m_2 g$

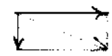
$$\sum F_{ext} = N_1 - m_1 g + N_2 - m_2 . g = 0 \quad (\text{por Newton})$$

$$\Rightarrow \quad \vec{P} = k \qquad \frac{d\vec{P}}{dt} = 0$$ El ímpetu se conserva

$\cos 30° = \frac{V_{1x}}{V_1}$

inicial

$\hat{x})$ $m_1 \overbrace{v_1 . \cos 30°}^{V_{1x}} + m_2 . \overbrace{v_2}^{V_{2x}} = 5212{,}43 \ kg \frac{m}{s}$



$\hat{y})$ $m_1 \overbrace{v_1 . \operatorname{sen} 30°}^{V_{1y}} + 0 = 700 \ kg . \frac{m}{s}$

final

$$(m_1 + m_2) . \vec{v} = 5259{,}223 \ \vec{kg \frac{m}{s}}$$

$$\vec{v} = 30{,}936 \ \vec{\frac{m}{s}}$$

$\vec{P} = M . \vec{V}_{CM} \equiv k$

$\Rightarrow \quad \vec{V}_{CM} \equiv k$

$\hat{x})$ $M . v . \cos \theta = P_x$

$\vec{v} = \vec{V}_{CM}$

$\hat{y})$ $M . v . \operatorname{sen} \theta = P_y$

$\hat{x})$ $V_{CM} = \frac{m_1 . v_1 . \cos 30° + m_2 . v_2}{m_1 + m_2} = 30{,}66 \ \frac{m}{s}$

$\hat{y})$ $V_{CM} = \frac{m_1 . v_1 . \operatorname{sen} 30°}{m_1 + m_2} = 4{,}11 \ \frac{m}{s}$

$v . \cos \theta = 30{,}66$

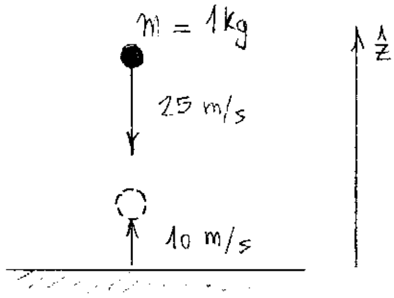
$v . \operatorname{sen} \theta = 4{,}11$

$\theta = 7° 40' 3{,}15''$

$$\boxed{\vec{V}_f = 30{,}66 \ \hat{x} + 4{,}11 \ \hat{y}}$$

$$\boxed{|\vec{V}_f| = 30{,}936}$$

2.



a) $\sum \vec{F}_{ext} = -m . g . \hat{z}$

$\hat{z}$ $\quad -m . g = M . \vec{a}_{CM}$

$$\frac{d\vec{P}}{dt} = M . \vec{a}_{CM} = -m . g . \hat{z}$$

$$\frac{d\vec{P}}{dt} = -9.8 \left[kg . \frac{m}{s} \right] \hat{z}$$

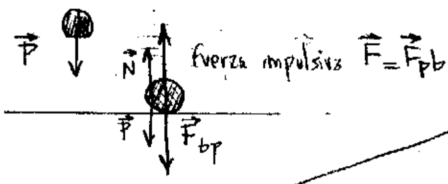
$\vec{p}_i = m.\vec{v}_i$ $\quad \hat{z})\quad p_i = 1kg.-25\,\frac{m}{s} = -25\,\frac{kg\,m}{s}$

$\vec{p}_F = m.\vec{v}_F$ $\quad \hat{z})\quad p_F = 1kg.\,10\,\frac{m}{s} = 10\ kg.\frac{m}{s}$

$$\vec{p}_F - \vec{p}_i = 10\hat{z} - (-25)\hat{z} = 35\,\hat{z} \Rightarrow \boxed{\Delta p = 35\ kg.\frac{m}{s}\,\hat{z}}$$

b) $\quad t_F - t_i = 0{,}02$ seg

durante la colisión $\sum \vec{F}_{ext} = 0$



$$\frac{d\vec{p}}{dt} = \vec{F}_{imp}$$

$$d\vec{p} = \vec{F}_{imp}.dt$$

$$\int_{p_i}^{p_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}_{imp}.dt = \vec{p}_f - \vec{p}_i$$

Fimp es externa porque el sistema es solo la bola

NB durante la colisión actúan F_{pb}. (fuerza impulsiva) sobre la bola porque $\vec{P}$ y $\vec{N}$ se cancelan

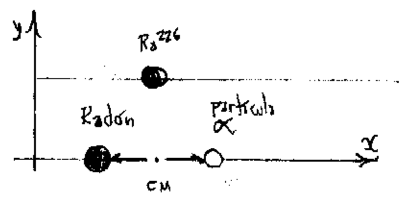
$\vec{p}_f - \vec{p}_i \Rightarrow \quad \hat{z})\quad 35 = \int_0^{0{,}02} \underbrace{F_{pb}}\,.dt \longrightarrow$ No depende del tiempo

$$35 = F_{pb}\,.\,0{,}02$$

$$\boxed{1750 = F_{pb}}$$

$$\boxed{\vec{F}_{pb} = 1750\ N\ \hat{z}}$$

3.



La emisión es en línea recta

$$\sum \vec{F}_{ext} = 0 \Rightarrow \frac{d\vec{P}}{dt} = 0 \Rightarrow \vec{P} = k$$

$$\vec{P} = M.\vec{V}_{CM}$$

inicial

$\hat{x})\quad 3{,}8.10^{-22}\,g.\ V_i = P_i$

final

$\hat{x})\quad 3{,}7.10^{-22}\,g.\ V_{F_1} + 6{,}7.10^{-24}\,g.\ \underbrace{V_{F_2}}_{0{,}05\,c} = P_f$

Como $P \equiv k \Rightarrow$

$$3{,}8.10^{-25}\,kg.\ \underbrace{V_i}_{0} = 3{,}7.10^{-25}\,kg.V_F + 6{,}7.10^{-27}\,kg.\,1{,}5.10^{7}\,\frac{m}{s}$$

$$3{,}7.10^{-25}\,kg.V_F = 10{,}05.10^{-20}\,kg.\frac{m}{s}$$

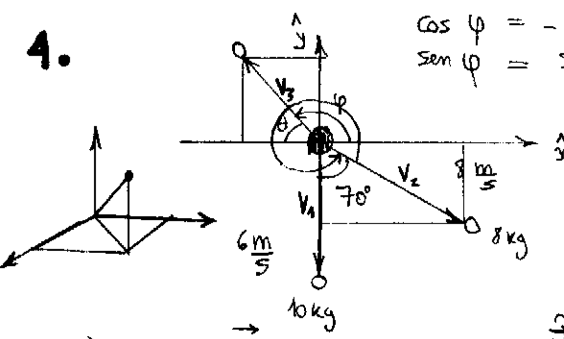
$$\boxed{V_F = 271\,621{,}62\,\frac{m}{s}}$$

15.000 km/s

15.000.000 m/s

$1{,}5.10^{7}\,\frac{m}{s}$

4.



$\cos\varphi = -\cos\theta$

$\text{sen}\,\varphi = \text{sen}\,\theta$

$30\,kg = 10\,kg + 8\,kg + \underbrace{12\,kg}_{m}$ tercer trozo

tomo como plano xy el determinado por $\vec{V}_1$ y $\vec{V}_2$

$$\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} = M.\vec{A}_{CM} = 0 \Rightarrow$$

$$\vec{P} = M.\vec{V}_{CM} \equiv k$$

a) $\quad \vec{P}_i = 0 = 30\,kg.\,\underbrace{\vec{V}_{CM}}_{=0} = 0$

$\quad \vec{P}_f = 0 = 10\,kg.\ \vec{V}_1 + 8\,kg.\ \vec{V}_2 + 12\,kg\ \vec{V}_3$

$\hat{x})\quad 0 = 10\,kg.\ 0 + 8\,kg\ \cos 340°.\,8\,\frac{m}{s} + 12\,kg.|\vec{V}_3|.\cos\varphi$

$\hat{y})\quad 0 = 10\,kg.\ V_1.\overset{\text{sen}\,270°}{} + 8\,kg.\ \text{sen}\,340°.\,8\,\frac{m}{s} + 12\,kg\ |\vec{V}_3|.\ \text{sen}\,\varphi$

$\hat{z})\quad 0 = 0 + 0 + \underbrace{0}$ → no puede tener componente en $\hat{z}$

$\therefore \quad \vec{V}_3 \notin$ plano zy ni a plano zx.

b) $\hat{x}$) $\quad 0 = 60{,}140 \ \mathrm{kg}\frac{m}{s} + 12 \cdot V_3 \cdot \cos\varphi$

$\hat{y}$) $\quad 0 = -60 \ \mathrm{kg}\frac{m}{s} + 21{,}88 \frac{m}{s} + 12 \cdot V_3 \cdot \operatorname{sen}\varphi$

$\hat{x}$) $\quad -\frac{60{,}14}{12} = V_3 \cdot \cos\varphi = -5{,}011$

$\hat{y}$) $\quad \frac{\cancel{81{,}87}}{12} = V_3 \cdot \operatorname{sen}\varphi = 6{,}823$

no es un buen método

$\frac{60{,}140}{12 \cdot V_3} =$

$V_3 = \frac{-5{,}011}{\cos\varphi} \Rightarrow \quad -\frac{5{,}011}{\cos\varphi} \cdot \operatorname{sen}\varphi = 6{,}823$

$\operatorname{tg}\varphi = -1{,}361$

$-\cancel{53}°41'35''$

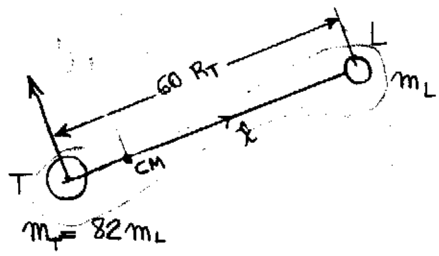
$\boxed{126°18}$

$$\boxed{\vec{V}_3 = -5{,}011\ \hat{x} + 6{,}823\ \hat{y}}$$

$$|\vec{V}_3| = 8{,}465$$

5. $\nexists$ este ejercicio

6.



sistema Tierra-Luna aislado, no considero $\vec{F}_{ext.}$; Luego:

$$0 = \frac{d\vec{P}}{dt} \quad \therefore \quad \vec{P} \equiv k$$

$\hat{x}$)

$M \cdot r_{cm} = m_T \cdot r_t + m_L \cdot r_L$

$83 \cdot 0 = 82 \cdot \quad + 1 \cdot$

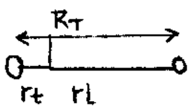
$83 \cdot r_{cm} = 82 \cdot \underbrace{r_t}_{0} + 1 \cdot r_L$

$r_{cm} = \frac{82}{83} \cdot 0 + \frac{60}{83} R_T$

$r_{cm} = \frac{60}{83} R_T$

$$\boxed{r_{cm} = 0.7229\ R_T}$$

NB
La ubicación del CM depende del sistema de referencia elegido



OTRA FORMA:

$-r_t + r_L = 60 R_T$

$0 = 82 r_t + 1 r_L$

$0 = 82 r_t + 1(60 R_T - r_t)$

$0 = 81 r_t + 60 R_T$

$-r_L = 82 r_t$

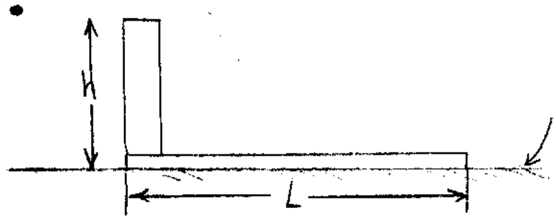
$-60 R_T - r_t = 82 r_t$

$-60 R_T = 83 r_t$

$\boxed{-0.723 R_T = r_t}$

$\boxed{r_L = 59{,}277\ R_T}$

7.

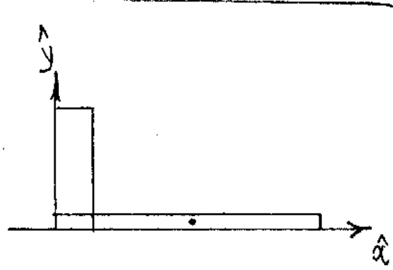


$\nexists$ roz. tabla-suelo

$\sum \vec{F}_{ext} = 0 \Rightarrow \frac{d\vec{P}}{dt} = 0 \Rightarrow \vec{P} \equiv k$ (se conserva el momento lineal)

sistema (hombre-tabla)

a)



$x_{cm} = \frac{M \cdot \overset{0}{x_h} + m \cdot x_t}{M+m} = \frac{m \cdot (L/2)}{M+m}$

$y_{cm} = \frac{M \cdot y_h + m \cdot \overset{0}{y_t}}{M+m} = \frac{M \cdot (H/2)}{M+m}$

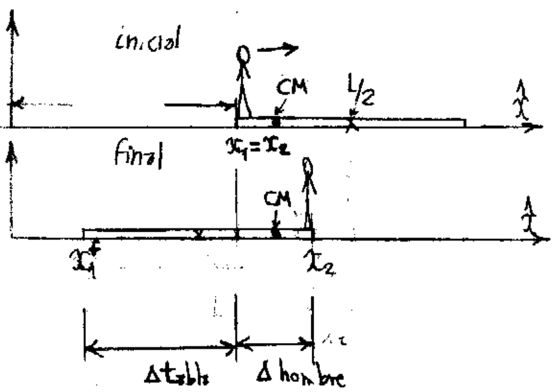
$$\boxed{\vec{r}_{cm} = \frac{m}{M+m}\left(\frac{L}{2}\right)\hat{x} + \frac{M}{M+m}\cdot\left(\frac{H}{2}\right)\cdot\hat{y}}$$

b) En el instante inicial hay reposo $\Rightarrow \vec{P} = M_t \cdot \vec{V}_{CM} = 0 \Rightarrow \boxed{\vec{V}_{CM} = 0}$ $\forall$ instante por conservación momento lineal, luego:

$$m \cdot \vec{V}_t + M \cdot \vec{V}_h = 0$$

$$\hat{x}) \quad m \cdot v_t + M \cdot v_h = 0 \quad \Rightarrow$$

c) si $\vec{V}_{CM} = 0 \Rightarrow \vec{R}_{CM} \equiv k$ (el CM permanece estable)



$$x_{CM} = \frac{M \cdot \overbrace{x_1^i}^{= x_2^i} + m \cdot (x_1^i + L/2)}{M+m} =$$

$$x_{CM} = \frac{M \cdot x_2^f + m \cdot (x_1^f + L/2)}{M+m} =$$

$$M x_1^i + m x_1^i + m \cancel{\frac{L}{2}} = M x_2^f + m x_1^f + m \cancel{\frac{L}{2}}$$

$$M \cdot x_2^i + m \cdot x_2^i = M x_2^f + m (x_2^f - L)$$

$$(M+m) x_2^i = M x_2^f + m x_2^f - m L$$

$$= (M+m) x_2^f - m \cdot L$$

$$m \cdot L = (M+m)(x_2^f - x_2^i)$$

$$\boxed{\frac{m}{M+m} \cdot L = x_2^f - x_2^i}$$ desplazamiento hombre

$$M \cdot x_1^i + m \cdot x_1^i + m \cancel{\frac{L}{2}} = M \cdot (x_1^f + L) + m \cdot \left(x_1^f + \cancel{\frac{L}{2}}\right)$$

$$(M+m) \cdot x_1^i = (M+m) \cdot x_1^f + M \cdot L$$

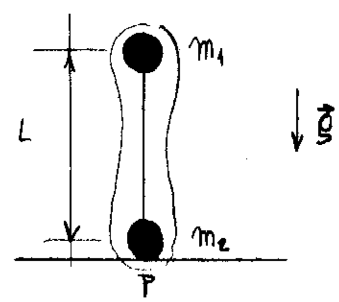
$$(M+m) \cdot (x_1^i - x_1^f) = M \cdot L$$

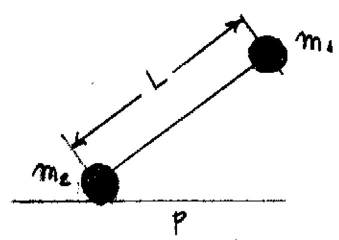
$$\boxed{(x_1^i - x_1^f) = \frac{M \cdot L}{(M+m)}}$$ desplazamiento tabla

$$L - \frac{M \cdot L}{(M+m)} = x_1^f + L - x_1^i$$

$$\boxed{L \left(1 - \frac{M}{M+m}\right) = x_1^f + L - x_1^i}$$ desplazamiento hombre

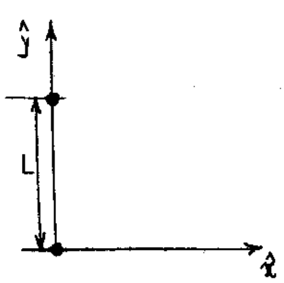
8.





sistema: barra, m_1, m_2

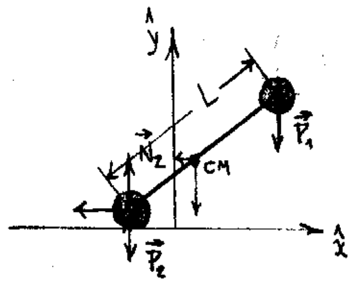
a) inicialmente



$$y_{CM} = \frac{m_1 \cdot L + m_2 \cdot 0}{m_1 + m_2} = \boxed{\frac{m_1}{m_1 + m_2} \cdot L}$$

$$x_{CM} = 0$$

b)

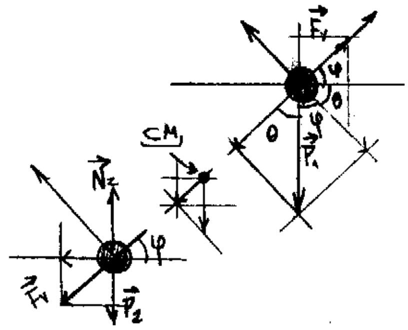


$$\sum \vec{F}_{ext} = \underbrace{\vec{N}_2 + \vec{P}_2}_{\text{por Newton}} + \vec{P}_1 \neq 0 \Rightarrow \frac{d\vec{P}}{dt} = M.\vec{A}_{CM}$$

$$\frac{d\vec{P}}{dt} = M.\vec{A}_{CM} \neq 0$$

m_2 $\hat{y})$ $N_2 - P_2 = N_2 - m_2.g = m.\overset{0}{\widehat{a_{2\hat{y}}}} = 0$

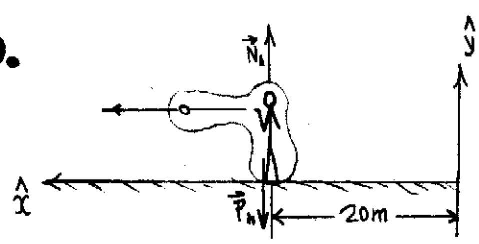
$$N_2 = m_2 . g$$



$$\frac{\pi}{2} = \varphi + \theta$$

$$M.\vec{A}_{CM} = m_1.\vec{a}_1 + m_2.\vec{a}_2$$

9.



$\vec{P}_h = 100 \text{ kgf} = m_h . g = 980 \text{ N}$
$m_h = 100 \text{ kg}$

$m_p = 1 \text{ kg}$

$\sum \vec{F}_{ext} = 0$ (por Newton)

$\frac{d\vec{P}}{dt} = 0 \Rightarrow \vec{P} = k$ inicialmente el sistema está en reposo $\vec{P} = 0$
Luego de lanzar el proyectil $\vec{P} = 0$ por conservación momento lineal

$\hat{x})$

$$M.V_{CM} = m_h.V_h + m_p.V_p$$
$$0 = 100 \text{ kg}.V_h + 1 \text{ kg} (-10 \text{ m/s})$$
$$0 = 100 \text{ kg}.V_h - 10 \text{ kg} \frac{m}{s}$$

$$\frac{10 \text{ kg} \frac{m}{s}}{100 \text{ kg}} = V_h$$
$$0.1 \frac{m}{s} = V_h$$

Velocidad constante

$$x = x_0 + V_0.t + \frac{1}{2} a t^2$$

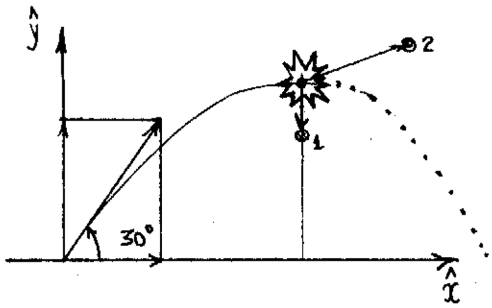
$$0 = -20\text{m} + 0,1 \frac{m}{s}.t$$

$$+\frac{20\text{m}}{0,1 \frac{m}{s}} = t$$

$$200 \text{ s} = t$$

$$\boxed{3,33 \text{ min} = t}$$

10.



$$\sum \vec{F}_{ext} = \vec{P}_{eso}$$

En todo momento la única Fexterna es el peso; Luego:

$$\frac{d\vec{P}}{dt} = -m \cdot g \cdot \hat{y} = M \cdot \vec{a}_{CM}$$

$$\vec{a}_{CM} = -g\,\hat{y}$$

$\hat{y})$ $\quad m \cdot a_{CM} = -\frac{m}{2} \cdot g + \frac{m}{2} \cdot a_2 = -m \cdot g$

$$\frac{m}{2}\left(-g + a_2\right) = -m \cdot g$$

$$-g + a_2 = -2g$$

$$\longleftarrow \quad a_2 = -g$$

$\hat{x}$

$V_{0x} = 86{,}60 \ \frac{m}{s}$

$x = 86{,}60 . t$

$V_x = 86{,}60$

$\boxed{x(5s) = 433 \text{ m}}$

$\hat{y}$

$V_{0y} = 50 \ \frac{m}{s}$

$y = 50.t - \frac{1}{2} . 10 . t^2$

$y_m = 50t - 10.t$

$y_{max} = 40t$

$V_y = V_{0y} - g.t$

$0 = 50 - 10t$

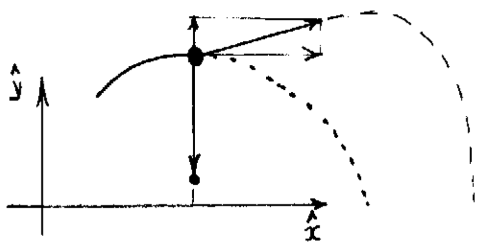
$10t = 50$

$t = 5 \text{ seg}$

$\boxed{y_{max} = 200 \text{ m}}$

antes: $\vec{P} = M . \vec{V}_{CM}$

después: $\vec{P} = \frac{m}{2} . \vec{V}_1 + \frac{m}{2} . \vec{V}_2$



• antes:

x: $P_x = 40 kg . 86{,}60 \frac{m}{s}$ $\qquad$ y: $P_y = 40 kg . 0$

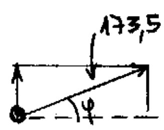
$P_x = 3464 \ kg \frac{m}{s}$ $\qquad$ $P_y = 0$

• después:

x: $P_x = 20kg . 0 + 20kg . V_{2x}$ $\qquad$ y: $P_y = 20kg . -10 \frac{m}{s} + 20kg . V_{2y}$

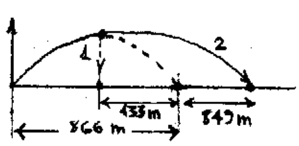
$3464 \ kg \frac{m}{s} = 20kg . V_{2x}$ $\qquad$ $200 \ kg\frac{m}{s} = 20kg . V_{2y}$

$173{,}2 \ \frac{m}{s} = V_{2x}$ $\qquad$ $10 \ \frac{m}{s} = V_{2y}$

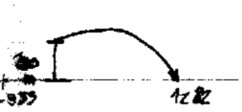


$\varphi = 3°18'15''$

Al caer al piso (un instante anterior) $\quad$ CM



NUEVO TIRO



$V_{0x} = 173{,}2 \ m/s$

$V_{0y} = 10 \ m/s$

$x = 173{,}2 . t$

$y = 200 + 10.t - 5t^2$

$y(x) = 200 + \frac{10.x}{173{,}2} - 5 . \frac{x^2}{173{,}2^2}$

$= 200 + 0{,}0577 - 0{,}0001666 \, x^2$

$x_1 = -939$

$x_2 = 1282 \quad \Rightarrow \quad 1282 - 433 = 849 \text{ m}$

$y = 50 \frac{x}{86{,}60} - \frac{1}{2} 10 . \frac{x^2}{86{,}60^2}$

$y = 0{,}577 \, x - 0{,}0006667 \, x^2$

$0 = (0{,}577 - 0{,}0006667 \, x) . x$

$x_1 = 0$

$x_2 = 866$

t_{final}:

$866 \text{ m} = 86{,}60 \frac{m}{s} . t$

$10 \text{ s} = t$

Llegada al piso:

CM

$\hat{x})$ $\quad P_x = 40kg . 86{,}60 \frac{m}{s}$

$P_x = 3464 \ kg\frac{m}{s}$

$\hat{y})$ $\quad P_y = 40kg . \left(50\frac{m}{s} - 10\frac{m}{s^2}.t\right)$

$P_y = -40kg . 50\frac{m}{s} = -2000 \ kg\frac{m}{s}$

Fragmentos

$\hat{x})$ $\quad$ llegada

$3464 = 20.0 + 20 . V_{2x}$

$173{,}2 = V_{2x}$

$\hat{y})$ $\quad$ llegada

$-2000 = 20 . V_{1y} + 20 . V_{2y}$

$-200 = V_{1y} + V_{2y}$

$\boxed{\text{Los fragmentos caen a } 433 \text{ m y a } 1715 \text{ m}}$