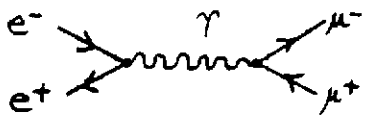
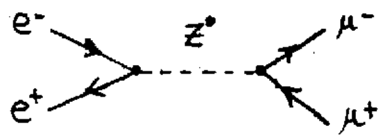
3.

$e^- e^+ \rightarrow \mu^- \mu^+$

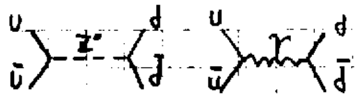
① 

② 

A bajas energías el ② está suprimido porque en el # que da la $\mathcal{A}$ figura la Masa del Z^0 que es no nula lo cual hace que $\mathcal{A}_2 < \mathcal{A}_1$ pues dicha masa aparece en el denominador.

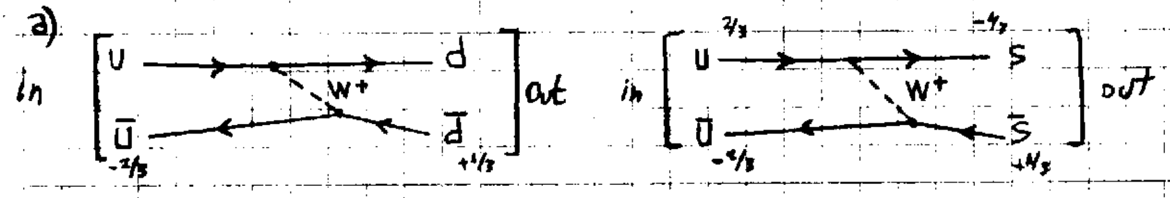
$$\left(\frac{e^2}{q^2}\right) J^{\mu}_{e^-e^+} J_{\mu}^{\mu^-\mu^+} \qquad \left(\frac{g^2}{M_Z^2 - q^2}\right) J^{\mu}_{e^-e^+} J_{\mu}^{\mu^-\mu^+}$$

4.



← también pueden darse así

a)



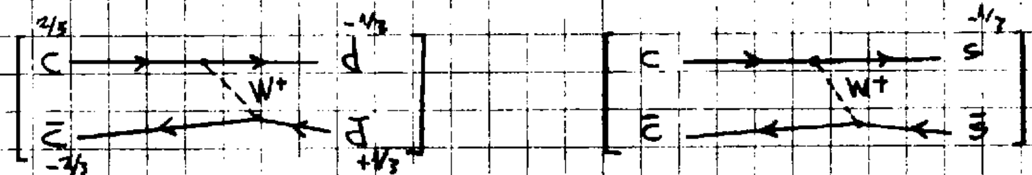
Las interacciones débiles acoplan quarks de la misma generación, similar al caso de los leptones, con cierta salvedad:

$$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix}$$ donde d' es mayoritariamente d y s' es mayoritariamente s

, en realidad: $$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \text{sen}\,\theta_c \\ -\text{sen}\,\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Entonces el proceso $u\bar{u} \to d\bar{d}$ es mucho más probable que $u\bar{u} \to s\bar{s}$ porque $\bar{u}$ se acopla principalmente con $\bar{d}$ que con $\bar{s}$

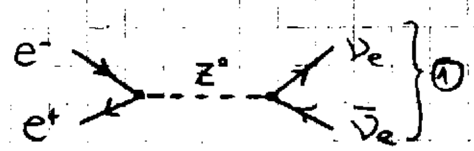
$\mathcal{A}_1 = \#.\cos^2\theta_c$ $\qquad \mathcal{A}_2 = \#.\,\text{sen}^2\theta_c \qquad \mathcal{A}_1/\mathcal{A}_2 = \cos^2\theta_c/\text{sen}^2\theta_c \sim 19$

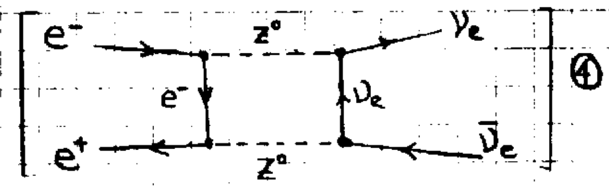


En este caso el suprimido es el primero porque $\bar{c}$ se acopla preferentemente a $\bar{s}$

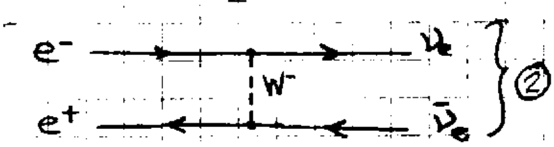
$\mathcal{A}_1 = \#.\,\text{sen}^2\theta_c \qquad \mathcal{A}_2 = \#.\cos^2\theta_c \qquad \dfrac{\mathcal{A}_1}{\mathcal{A}_2} = \dfrac{\text{sen}^2\theta_c}{\cos^2\theta_c} \sim 0{,}05$

b)





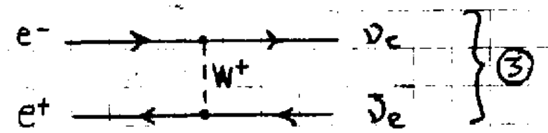
- Conserva carga
- conserva # leptónico $L_e = 0$
  $L_e = 0$

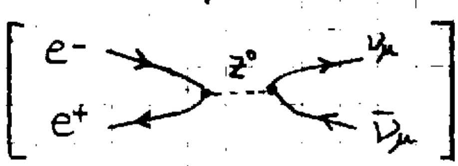


* Como hay neutrinos emergiendo es una reacción WEAK

* AL orden más bajo tengo 1, 2, 3

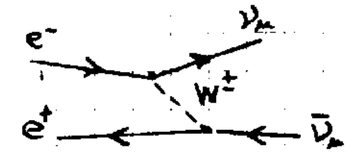
* ④ es de un orden superior





- Conserva la carga
- La interacción débil solo acopla leptones dentro de la misma familia

⇒ No existen →



5.

$$\mathcal{L} = i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - m\bar{\Psi}\Psi \qquad \mathcal{L} \text{ libre de Dirac}$$

a)

$$\Psi' = e^{i\gamma^5}\Psi \quad \rightarrow \quad \Psi'^{\dagger} = \left(e^{i\gamma^5}\Psi\right)^\dagger$$

Esto ya se hizo en Práctica 6 Problema 2 usando $\gamma^{5\dagger} = \gamma^5$

$$(\Psi')^\dagger = \left(\sum_{k=0}^{\infty}\frac{(i\gamma^5)^k}{k!}\Psi\right)^\dagger = \Psi^\dagger\sum_{k=0}^{\infty}\frac{(i\gamma^5)^{k\dagger}}{k!}$$

, pero como $(\gamma^5)^k = \gamma^5$ (con k impar) y $\mathbb{1}$ (con k par) $\rightarrow$ $(\gamma^5)^{k\dagger} = \begin{cases}\gamma^{5\dagger} = \gamma^5 \\ \mathbb{1}^\dagger = \mathbb{1}\end{cases}$

no jode el $\dagger$ $\Rightarrow$

$$(\Psi')^\dagger = \Psi^\dagger e^{-i\gamma^5} \rightarrow$$

$$\Psi'^\dagger\gamma^0 = \bar{\Psi}' = \Psi^\dagger e^{-i\gamma^5}\gamma^0$$

y basados en $\{\gamma^5, \gamma^\mu\} = 0$ es: $e^{-i\gamma^5}\gamma^0 = \gamma^0 e^{i\gamma^5} \Rightarrow$

$$\bar{\Psi}' = \bar{\Psi}e^{i\gamma^5}$$

$$\mathcal{L}' = i\bar{\Psi}'\gamma^\mu\partial_\mu\Psi' - m\bar{\Psi}'\Psi'$$

$$= i\bar{\Psi}e^{i\gamma^5}\gamma^\mu\partial_\mu e^{i\gamma^5}\Psi - m\bar{\Psi}e^{i\gamma^5}e^{i\gamma^5}\Psi$$

$$\mathcal{L}' = i\bar{\Psi}\gamma^\mu e^{-i\gamma^5}\partial_\mu e^{i\gamma^5}\Psi - m\bar{\Psi}e^{2i\gamma^5}\Psi$$

, usando que $e^{i\gamma^5}\gamma^\mu = \gamma^\mu e^{-i\gamma^5}$ (otra vez desde $\{\gamma^5, \gamma^\mu\} = 0$) es

Como ∂_μ no afecta a $e^{-i\gamma^5}$ $\Rightarrow$

$$\boxed{\mathcal{L}' = i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - m\, e^{2i\gamma^5}\bar{\Psi}\Psi}$$

Para la invariancia necesito $m = 0$

b) El teorema de Noether establece que si el $\mathcal{L}$ es invariante ante una transformación infinitesimal $\Rightarrow$ hay una cantidad conservada

$$\delta\mathcal{L} = 0 \rightarrow$$

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\bar{\Psi}}\cdot\delta\bar{\Psi} + \frac{\partial\mathcal{L}}{\partial\Psi}\cdot\delta\Psi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\Psi})}\delta\partial_\mu\bar{\Psi} + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\Psi)}\cdot\delta(\partial_\mu\Psi)$$

con
$\Psi \rightarrow (1 + i\alpha)\Psi \rightarrow$ $\quad \Psi' = \Psi + i\alpha\Psi \quad \Rightarrow \delta\Psi = i\alpha\Psi$
$\bar{\Psi} \rightarrow (1 - i\alpha)\bar{\Psi} \rightarrow$ $\quad \delta\bar{\Psi} = -i\alpha\bar{\Psi}$
$\partial_\mu\Psi \rightarrow (1 + i\alpha)\partial_\mu\Psi \rightarrow$ $\quad \delta(\partial_\mu\Psi) = i\alpha\partial_\mu\Psi$
$\partial_\mu\bar{\Psi} \rightarrow (1 - i\alpha)\partial_\mu\bar{\Psi} \rightarrow$ $\quad \delta(\partial_\mu\bar{\Psi}) = -i\alpha\partial_\mu\bar{\Psi}$

$$\delta\mathcal{L} = (i\gamma^\mu\partial_\mu\Psi - m\Psi)(i\alpha\bar{\Psi}) + (-m\bar{\Psi})(i\alpha\Psi) + 0\,(-i\alpha\partial_\mu\bar{\Psi}) + (i\bar{\Psi}\gamma^\mu)(i\alpha\partial_\mu\Psi)$$

$$\delta\mathcal{L} = \gamma^\mu\partial_\mu\Psi\alpha\bar{\Psi} + im\Psi\bar{\Psi}\alpha - im\bar{\Psi}\alpha\Psi - \bar{\Psi}\gamma^\mu\partial_\mu\Psi\alpha = 0$$

$\Rightarrow$ hay una cantidad conservada en esta transformación

- $\partial_\alpha\left(\dfrac{\partial\mathcal{L}}{\partial(\partial_\alpha\bar{\Psi})}\right) - \dfrac{\partial\mathcal{L}}{\partial\bar{\Psi}} = 0 \longrightarrow \quad -\dfrac{\partial\mathcal{L}}{\partial\bar{\Psi}} = 0 = +m\Psi - i\gamma^\mu\partial_\mu\Psi$

  $0 = i\gamma^\mu\partial_\mu\Psi - m\Psi$ → ecuación de Dirac

- $\partial_\alpha\left(\dfrac{\partial\mathcal{L}}{\partial(\partial_\alpha\Psi)}\right) - \dfrac{\partial\mathcal{L}}{\partial\Psi} = 0 \longrightarrow$ De acá también sale la ecuación de Dirac (conjugada)

* Cálculo de las cantidades conservadas

$$\delta\mathcal{L} = \sum_i\left(\frac{\partial\mathcal{L}}{\partial\phi_i}\delta\phi_i + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\delta(\partial_\mu\phi_i)\right)$$

pero: $\partial_\mu\left[\dfrac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\delta\phi_i\right] = \partial_\mu\left(\dfrac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\right)\delta\phi_i + \dfrac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\delta(\partial_\mu\phi_i)$

$$\delta\mathcal{L} = \sum_i \frac{\partial\mathcal{L}}{\partial\phi_i}\delta\phi_i + \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\delta\phi_i\right) - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\right)\delta\phi_i$$

$$\delta\mathcal{L} = \sum_i \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\delta\phi_i\right) - \underbrace{\left[\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\right)\delta\phi_i - \frac{\partial\mathcal{L}}{\partial\phi_i}\delta\phi_i\right]}_{=0 \text{ por Euler-Lagrange}}$$

$$\delta\mathcal{L} = 0 = \sum_i \partial_\mu\Big(\underbrace{\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\delta\phi_i}_{\text{cantidad conservada}}\Big)$$

$$\frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\Psi})}\delta\bar{\Psi} = 0 \;; \qquad \frac{\partial\mathcal{L}}{\partial(\partial_\mu\Psi)}\delta\Psi = i\bar{\Psi}\gamma^\mu(i\alpha\Psi) = -\alpha\bar{\Psi}\gamma^\mu\Psi$$

$$J^\mu \propto -\alpha\bar{\Psi}\gamma^\mu\Psi$$

Esto significa que la cantidad conservada es $\propto$ a la corriente a menos de una constante.

c)

$$\mathcal{L} = i\bar{\Psi}_R\gamma^\mu\partial_\mu\Psi_L = i\,\frac{1}{2}\bar{\Psi}(1-\gamma^5)\gamma^\mu\partial_\mu\,\frac{1}{2}(1-\gamma^5)\Psi$$

$\Psi_L = \frac{1}{2}(1+\gamma^5)\Psi$

$\Psi_R = \frac{1}{2}(1+\gamma^5)\Psi \quad\rightarrow\quad \Psi_R^\dagger = \frac{1}{2}\Psi^\dagger(1+\gamma^5)^\dagger = \frac{1}{2}\Psi^\dagger(1+\gamma^5)$

$\Psi_R^\dagger\gamma^0 = \frac{1}{2}(\Psi^\dagger\gamma^0 - \Psi^\dagger\gamma^0\gamma^5) = \frac{1}{2}\bar{\Psi}(1-\gamma^5)$

$$\mathcal{L} = \frac{i}{4}\bar{\Psi}(1-\gamma^5)\gamma^\mu\partial_\mu(1-\gamma^5)\Psi$$

$$\mathcal{L} = \frac{i}{4}\left(\bar{\Psi}\gamma^\mu\partial_\mu\Psi - \bar{\Psi}\gamma^5\gamma^\mu\partial_\mu\Psi - \bar{\Psi}\gamma^\mu\partial_\mu\gamma^5\Psi + \bar{\Psi}\gamma^5\gamma^\mu\partial_\mu\gamma^5\Psi\right)$$

$$= \frac{i}{4}\left(\cancel{\bar{\Psi}\gamma^\mu\partial_\mu\Psi} - \bar{\Psi}\gamma^5\gamma^\mu\partial_\mu\Psi - \bar{\Psi}\gamma^\mu\partial_\mu\gamma^5\Psi - \cancel{\bar{\Psi}\gamma^\mu\partial_\mu\Psi}\right)$$

$$= \frac{\partial\mathcal{L}}{\partial(\partial_\alpha\Psi)}\delta\Psi + \frac{\partial\mathcal{L}}{\partial(\partial_\alpha\bar{\Psi})}\delta\bar{\Psi} = \left(-\frac{i}{4}\bar{\Psi}\gamma^5\gamma^\mu\delta^\alpha_\mu\right)(i\alpha\Psi) + 0.(-i\alpha\bar{\Psi})$$

, donde considero $\Psi \to \Psi + i\alpha\Psi$, es decir $\Psi' = e^{i\alpha}\Psi$ a orden 1º

$$\Rightarrow \boxed{\alpha\frac{1}{4}\bar{\Psi}\gamma^5\gamma^\nu\Psi = J^\nu}$$

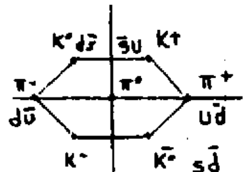
comente que se conserva en este nuevo lagrangiano

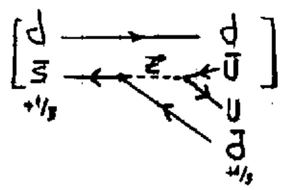
6. a) La idea es que en el $\mathcal{L}_W$ hay término de interacción que permitiría armar un proceso que no conserva # leptónico a través del vértice que define dicho término.

¿?

b) $K^0 \to \pi^+\pi^-$

$(d\bar{s}) \to (u\bar{d})(d\bar{u})$



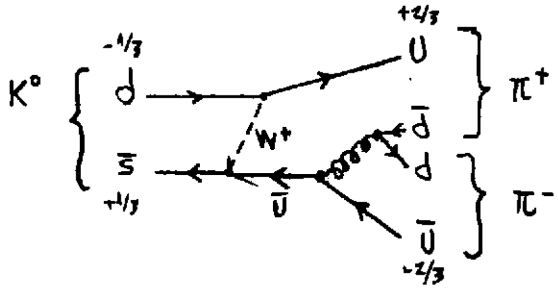


Estaríamos tentados a este diagrama, que está OK por conservación de carga pero que no está permitido porque $\bar{s} \to \bar{d}$ no pueden acoplarse débilmente:

$$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix} \begin{pmatrix} t \\ b' \end{pmatrix}$$

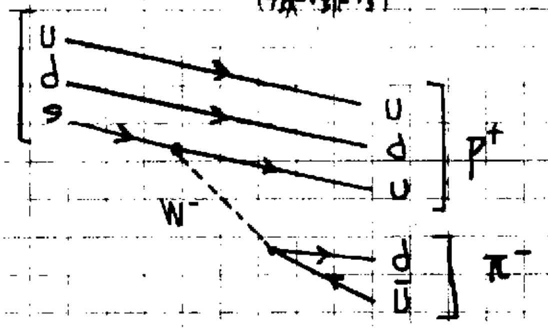
sólo hay acoples entre:

$u \to d, s, b$
$c \to d, s, b$
$t \to d, s, b$



c) $uds \to p^+ + \pi^{(-)}$ será π^- por la conservación de la carga

$(2/3)(-1/3)(-1/3)$

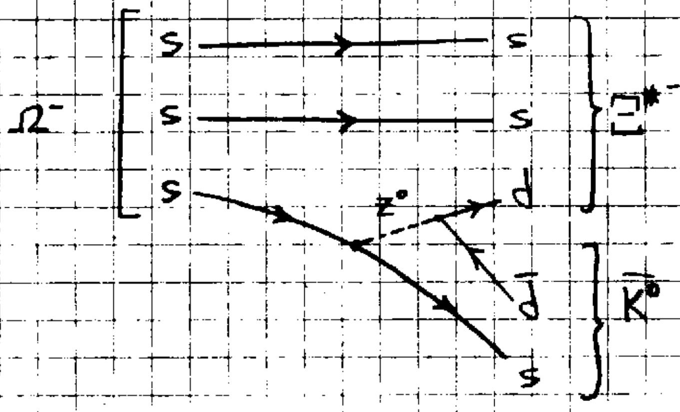


$\pi^-\ d\bar{u}$ $\quad \pi^0$ $\quad \pi^+\ u\bar{d}$

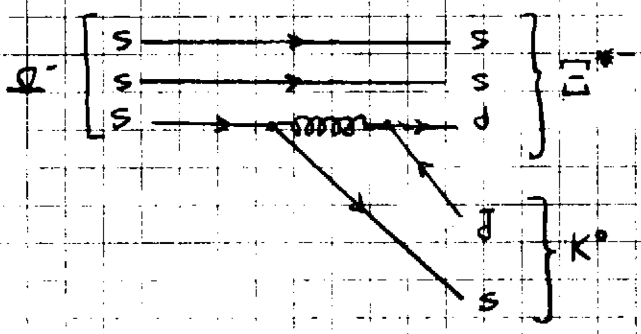
d) barión de $S=-3$ en un barión y un mesón neutro

$$\underset{Q=-1}{qqq} \to \underset{Q=-1}{qqq} + \underset{Q=0}{q\bar{q}}$$

→ la carga del barión será -1

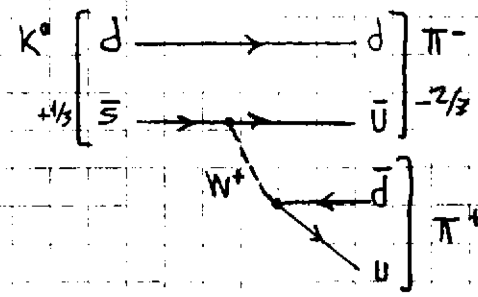


◄ Este es de lo más sencillo



◄ Proceso similar, pero fuerte

* observación:
Para el punto b) hay otro proceso posible: $\underset{d\bar{s}}{K^0} \to \underset{u\bar{d}}{\pi^+} + \underset{d\bar{u}}{\pi^-}$



Este proceso no incluye "contaminación" de decaimiento fuerte.
Además es de un orden menor que el planteado porque sólo incluye dos vértices.