Practica 3: Isospin y Sabor

detalle

d = deuterio (1 p + 1 n)

$$|0,0\rangle = \frac{|+-\rangle - |-+\rangle}{\sqrt{2}}$$

1.

① $p + p \longrightarrow d + \pi^+$

		p	p	d	π^+	
(isospin)	I	$1/2$	$1/2$	0	1	1
(isospin en $\hat{3}$)	I_3	$+1/2$	$+1/2$	0	$+1$	1

$$|\overset{I_3}{1/2}\ \overset{I_3}{1/2}\rangle = |\overset{I}{1}, \overset{I_3}{1}\rangle \qquad |0,0\rangle \otimes |1,1\rangle = |\overset{I}{1}, \overset{I_3^{\text{total}}}{1}\rangle$$

$$\sigma_1 = |\langle p|\langle p| e^{-\frac{i}{\hbar}\int H dt} |d\rangle|\pi^+\rangle|^2 \propto \left(\langle p|\otimes\langle p|\right)_I \left(|d\rangle\otimes|\pi^+\rangle\right)_I = \left(\langle 1,1|\right)\left(|1,1\rangle\right)_I \Rightarrow$$

$$\sigma_1 \propto |\langle 1,1|1,1\rangle|^2 = 1$$

② $p + n \longrightarrow d + \pi^0$

	p	n	d	π^0	
I	$1/2$	$1/2$	0	1	1
I_3	$+1/2$	$-1/2$	0	0	0

una papa: suma momentos $J_1 = 0$ y $J_2 = 1$ → $J = 1$ $M = 0$

$$|\overset{I_3}{1/2}, \overset{I_3}{-1/2}\rangle = \frac{1}{\sqrt{2}}\left(|1,0\rangle - |0,0\rangle\right) \qquad |0,0\rangle\otimes|1,0\rangle = |1,0\rangle$$

$$\sigma_2 \propto \left|\left(-\frac{\langle 0,0| + \langle 1,0|}{\sqrt{2}}\right)|1,0\rangle\right|^2 = \frac{|\langle 1,0|1,0\rangle|^2}{2} = \frac{1}{2}$$

③ $n + n \longrightarrow d + \pi^-$

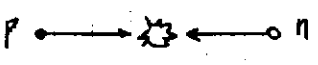
	n	n	d	π^-	
I	$1/2$	$1/2$	0	1	1
I_3	$-1/2$	$-1/2$	0	-1	-1

$$|\overset{I_3}{-1/2}, \overset{I_3}{-1/2}\rangle = |1,-1\rangle \qquad |0,0\rangle\otimes|1,-1\rangle = |1,-1\rangle$$

$$\sigma_3 \propto |\langle 1,-1|1,-1\rangle|^2 = 1$$

Con lo cual: $\sigma_1 = \sigma_3 = 1$ pero $\sigma_2 = \frac{1}{2}$ → $\boxed{\sigma_1 = \sigma_3 = 2\sigma_2}$

Nota:



El $|\ \rangle_{\text{inicial}}$ puede verse como $|p+n\rangle_I$ pero no debe confundirse con un estado ligado de p y n (como el deuterio). En este último caso:

$$|p+n\rangle_I = |p\rangle_I|n\rangle_I = |1/2, +1/2\rangle \otimes |1/2, -1/2\rangle$$
$$= |1/2, 1/2; +1/2, -1/2\rangle = |+1/2, -1/2\rangle$$
$$= \frac{1}{\sqrt{2}}\left(|1,0\rangle + |0,0\rangle\right)$$

$$|d\rangle \equiv |p.n\rangle = |p\rangle\otimes|n\rangle \underset{\text{singlete}}{=} |0,0\rangle$$

pero esto surge de consideraciones experimentales (pág 118, Griffiths, Introduction to Elementary Particles).

2.

a) $\Sigma^{*0} \longrightarrow \Sigma^- + \pi^+$

b) $\Sigma^{*0} \longrightarrow \Sigma^0 + \pi^0$ $\qquad \langle final | e^{\frac{i}{\hbar}\int H.dt} | inicial \rangle \equiv \mathcal{A}$

c) $\Sigma^{*0} \longrightarrow \Sigma^+ + \pi^-$ $\qquad$ Desconocemos como es H de este proceso (interacción fuerte) pero sabemos que: $[H,I]=0$, $[H,I_z]=0$

$$|\Sigma^{*0}\rangle = |\psi_{spa}^{\Sigma^{*0}}\rangle \otimes |\psi_{spin}^{\Sigma^{*0}}\rangle \otimes |1,0\rangle$$

$$|\Sigma^- + \pi^+\rangle = |\psi_{spa}^{\Sigma^-\pi^+}\rangle \otimes |\psi_{spin}^{\Sigma^-\pi^+}\rangle \otimes \left(\underset{\Sigma^-}{|1,-1\rangle} \otimes \underset{\pi^+}{|1,+1\rangle}\right)_I$$

$$|\Sigma^0 + \pi^0\rangle = |\quad\rangle \otimes |\quad\rangle \otimes \left(|1,0\rangle \otimes |1,0\rangle\right)_I$$

$$|\Sigma^+ + \pi^-\rangle = |\quad\rangle \otimes |\quad\rangle \otimes \left(|1,+1\rangle \otimes |1,-1\rangle\right)_I$$

a) $\langle \Sigma^- + \pi^+ | e^{\frac{i}{\hbar}\int H.dt} | \Sigma^{*0}\rangle = \#_a \langle 1,0 | \underbrace{\left(|1,-1\rangle \otimes |1,+1\rangle\right)}_{\text{hay que sumar isospines} \rightarrow \text{paso a la base acoplada } |I, I_3^{tot}\rangle}$

$$= \#_a \langle 1,0 | \left(\frac{1}{\sqrt{6}}|2,0\rangle - \frac{1}{\sqrt{2}}|1,0\rangle + \frac{1}{\sqrt{3}}|0,0\rangle \right) = \#_a \cdot \frac{-1}{\sqrt{2}}$$

en el canal de $I=1$

b) $\langle \Sigma^0 + \pi^0 | e^{\frac{i}{\hbar}\int H.dt} | \Sigma^{*0}\rangle = \#_b \langle 1,0 | \left(|1,0\rangle \otimes |1,0\rangle\right)$

$$= \#_b \langle 1,0 | \left(\sqrt{\frac{2}{3}}|2,0\rangle - \sqrt{\frac{1}{3}}|0,0\rangle \right)$$

$$= \#_b \; 0 = 0 \Rightarrow$$ No se esperará decaimiento aquí

c) $\langle \Sigma^+ + \pi^- | e^{\frac{i}{\hbar}\int H.dt} | \Sigma^{*0}\rangle = \#_c \langle 1,0 | \left(|1,+1\rangle \otimes |1,-1\rangle\right)$

$$= \#_c \langle 1,0 | \left(\frac{1}{\sqrt{6}}|2,0\rangle + \frac{1}{\sqrt{2}}|1,0\rangle + \frac{1}{\sqrt{3}}|0,0\rangle \right)$$

$$= \#_c \frac{1}{\sqrt{2}}$$ en el canal de $I=1$

En el canal b) se espera 0% de decaimientos debido a la ortogonalidad de los isospines. En a) y c) se esperan 50% y 50%, las secciones eficaces $\sigma_a = \sigma_c$ serán iguales. → $\begin{cases} \sigma_a = \left|\#_a \cdot \frac{-1}{\sqrt{2}}\right|^2 = \#_a^2/2 \\ \sigma_c = \#_c^2/2 \end{cases}$

3.

① $\underset{I\;(1,\,+1/2)}{\pi^- + p} \rightarrow \underset{(1/2\,,\,1)}{K^0 + \Sigma^0}$ $\qquad I_z\,(-1 + 1/2) \dashrightarrow (0 - 1/2)$

$$|\pi^-\rangle \otimes |p\rangle = \sqrt{\frac{1}{3}}|3/2,-1/2\rangle - \sqrt{\frac{2}{3}}|1/2,-1/2\rangle \qquad |\Sigma^0\rangle \otimes |K^0\rangle = \sqrt{2/3}\,|3/2,-1/2\rangle + \sqrt{1/3}\,|1/2,-1/2\rangle$$

$$\sigma_1 \propto \left| \underset{I}{\langle} K^0 | \langle \Sigma^0 | U_T | \pi^-\rangle | p \underset{I}{\rangle} \right|^2 = \left| \left(\langle f, I=3/2| + \langle f, I=1/2| \right) U_T \left(|i, I=3/2\rangle + |i, I=1/2\rangle \right) \right|^2$$

$$= \left| \sqrt{\frac{2}{3}\cdot\frac{1}{3}} \right|^2 + \left| -\sqrt{\frac{2}{3}\cdot\frac{1}{3}} \right|^2$$

$$= \frac{2}{9}\#_{3/2} + \frac{2}{9}\#_{1/2}$$

② $\dfrac{\pi^0 + p \longrightarrow K^+ + \Sigma^0}{I\,(1\ ;\ 1/2)\qquad(1/2\ ;\ 1)}$

$I_3\ (0\ \ +1/2) \dashrightarrow (+1/2\ \ 0)$

$|K^+ + \Sigma^0\rangle = |1/2, +1/2\rangle + |1, 0\rangle$

$${}_I\Big(\langle K^+| \otimes \langle \Sigma^0|\Big)\Big(|\pi^0\rangle \otimes |p\rangle\Big)_I$$

$$\left|\left(\sqrt{\frac{2}{3}}\langle 3/2, +1/2| - \sqrt{\frac{1}{3}}\langle 1/2, +1/2|\right)\left(\sqrt{\frac{2}{3}}\,|3/2, +1/2\rangle - \sqrt{\frac{1}{3}}\,|1/2, +1/2\rangle\right)\right|$$

$$= \frac{4}{9}\#_{3/2} + \frac{1}{9}\#_{1/2}$$

③ $\dfrac{\pi^+ + p \longrightarrow K^+ + \Sigma^+}{I\ \ (1\ +1/2)\qquad(1/2\ +1)}$

$I_3\ (+1\ \ +1/2) \dashrightarrow (+1/2\ \ +1)$

$${}_I\Big(\langle \Sigma^+| \otimes \langle K^+|\Big)\Big(|\pi^+\rangle \otimes |p\rangle\Big)_I$$

$$\sigma_3 \propto \Big|\Big(\langle 3/2, 3/2|\Big)\Big(|3/2, 3/2\rangle\Big|^2 \rightarrow \sigma_3 = \#_{3/2}\, 1$$

* Para isospin $1/2$ tendremos:

$\sigma_1 \propto 2/9 \qquad \sigma_2 \propto 1/9 \qquad \sigma_3 = 0$

$$\boxed{\frac{\sigma_1}{\sigma_2} = 2\ ;\ \sigma_3 = 0}$$

* Para isospin $3/2$ tendremos:

$\sigma_1 \propto 2/9 \qquad \sigma_2 \propto 4/9 \qquad \sigma_3 \propto 1$

$$\boxed{\begin{array}{c}\dfrac{\sigma_1}{\sigma_2} = \dfrac{1}{2}\\[2mm] \dfrac{\sigma_1}{\sigma_3} = \dfrac{1}{9}\end{array}}$$

*<u>Nota</u>:

La interacción fuerte tiene un hamiltoniano H que conmuta con isospin I ⇒

$$[H, I] = 0 \quad\Rightarrow\quad [H, I_3] = 0$$

Sea $U_T = e^{-\frac{i}{\hbar}\int H.dt}$ el evolucionador temporal, entonces si tenemos:

$$\Big(\langle I''' I_3'''| \otimes \langle I^{IV}, I_3^{IV}|\Big)\, U_T\Big(|I', I_3'\rangle \otimes |I'', I_3''\rangle\Big) =$$

$$\Big(\langle I_1, \#| + \langle I_2, \#| + \dots\Big)\Big(|I_1, \#\rangle + |I_2, \#\rangle + \dots\Big)$$

Esto ya es en la base suma de $I' + I''$ (kets) y en la de $I''' + I^{IV}$ (bras).

Tenemos que sumar Isospines y puede darse el caso de obtener I^{total} de más de un valor; es decir: si suma, por ejemplo $I_1 = 1$ $I_2 = 1/2$ tengo en principio:

$$a|1/2, \#_a\rangle + b|3/2, \#_b\rangle$$

como estado general de I^{total}; en este caso cabe distinguir dos canales de decaimiento: $I = 1/2$ e $I = 3/2$

CA: $j_1 = 1 \atop j_2 = 1/2$ → $3 \otimes 2 = \sum_{1/2}^{3/2} 2 \oplus 4$

4.

Si sumamos 2 representaciones en SU(2) tendremos:

$$2 \otimes 2 = 3 \oplus 1$$

Si sumamos 3 representaciones en SU(2) → $2 \otimes 2 \otimes 2 = \underset{\text{simetrico}}{4} \oplus \underset{M_S}{2} \oplus \underset{M_A}{2}$

$[(2 \otimes 2) \otimes 2 = (3 \oplus 1) \otimes 2 = \underbrace{3 \otimes 2}_{CA} \oplus \underbrace{1 \otimes 2}_{=2} = ↗]$

- $2 \otimes 2 = \underset{\text{sim.}}{3} \oplus \underset{\text{antisim.}}{1}$

I_3: (dd) en -1, $\left(\frac{du+ud}{\sqrt{2}}\right)$ en 0, (uu) en 1 — 3 estados simet.

I_3: $\left(\frac{du-ud}{\sqrt{2}}\right)$ — 1 estado asim.

- $2 \otimes 2 \otimes 2 = 4 \oplus 2 \oplus 2 \oplus 1$

4 estados sim

I_3: (ddd) en $-3/2$; $-1/2$; $\frac{(duu+udu+uud)}{\sqrt{3}}$ en $1/2$; (uuu) en $3/2$

$$= \frac{ddu + dud + ddu + udd + dud + udd}{\sqrt{6}}$$

$$= \frac{1}{\sqrt{3}}(ddu + dud + udd)$$

2 estados Mixtos simetricos

I_3: $-1/2$: $\frac{1}{\sqrt{3}}\left[\frac{(ud+du)d}{\sqrt{2}} - ddu\sqrt{2}\right]$; $1/2$: $\frac{1}{\sqrt{3}}\left[\frac{(ud+du)u}{\sqrt{2}} - uud\sqrt{2}\right]$

$$\frac{1}{\sqrt{6}}(-udd - dud + 2ddu)$$

$$\frac{1}{\sqrt{6}}(udu + duu - 2uud) = \#$$

$$I\,\# = \frac{1}{\sqrt{6}}(ddu + udd + ddu + dud - 2dud - 2udd)$$

$$= \frac{1}{\sqrt{6}}(2ddu - dud - udd)$$

2 estados mixtos antisimétricos

I_3: $-1/2$: $\frac{1}{\sqrt{2}}(udd - dud)$; $+1/2$: $\frac{1}{\sqrt{2}}(udu - duu)$

Cada una de las combinaciones u,d será la $|\Psi\rangle$ [función de onda de isospin] del caso considerado.

* Para la carga se tiene:

$$Q = e\left(I_3 + \frac{B+S}{2}\right)$$

B → # bariónico

S → extrañeza

$2 \otimes 2$

$-1/6\,e$, $1/3\,e$, $5/6\,e$ } $Q = 1e$, $I = 1$

$1/3\,e$ } $Q = 1/3\,e$, $I = 0$

$2 \otimes 2 \otimes 2$

$Q = 2e$, $I = 3/2$ { $-1e$, 0, $1e$, $2e$

$Q = 1e$, $I = 1/2$ { 0, $1e$

$Q = 1e$, $I = 1/2$ { 0, $1e$

Como en SU(2) isospin es una copia de SU(2) de spin ⇒ isospin → spin

u → ↑

d → ↓

Si $|\Psi\rangle \otimes |\chi\rangle$ es simétrica ⇒ en $2 \otimes 2$

S: (3,3)

$\lvert 1\rangle \otimes$	$\lvert dd\rangle\lvert\uparrow\uparrow\rangle$	$\lvert dd\rangle\lvert\downarrow\downarrow\rangle$	$\lvert dd\rangle\left(\frac{\lvert\downarrow\uparrow\rangle + \lvert\uparrow\downarrow\rangle}{\sqrt{2}}\right)$
$\lvert -1\rangle \otimes$	$\lvert uu\rangle\lvert\uparrow\uparrow\rangle$	$\lvert uu\rangle\lvert\downarrow\downarrow\rangle$	$\lvert uu\rangle\left(\frac{\lvert\downarrow\uparrow\rangle + \lvert\uparrow\downarrow\rangle}{\sqrt{2}}\right)$
$\lvert 0\rangle \otimes$	$\left\lvert\frac{du+ud}{\sqrt{2}}\right\rangle\lvert\uparrow\uparrow\rangle$	$\left\lvert\frac{du+ud}{\sqrt{2}}\right\rangle\lvert\downarrow\downarrow\rangle$	$\left\lvert\frac{du+ud}{\sqrt{2}}\right\rangle\left(\frac{\lvert\downarrow\uparrow + \uparrow\downarrow\rangle}{\sqrt{2}}\right)$
	$\otimes\lvert 1\rangle$	$\otimes\lvert -1\rangle$	$\otimes\lvert 0\rangle$

5.

protón: es un estado $\overset{\text{Ligado}}{\downarrow}$ qqq (barión) → $\begin{cases} Q=1 \\ S=0 \\ I_3=+1/2 \\ \text{Spin}=1/2 \end{cases}$ → El protón es en realidad un nucleón con $I_3=+1/2$

es un fermión → es $|p\rangle$ antisimétrica respecto a los quarks

$$|p\rangle = |\psi\rangle_E \otimes \underbrace{|\phi\rangle_{Sabor} \otimes |\chi\rangle_{Spin}}_{\text{será simétrico}} \otimes |\psi\rangle_{Color} \longrightarrow \text{la antisimetría estará aquí}$$

Como tiene spin 1/2 → pertenece al octete. Para tener $\begin{cases} I_3=1/2 \\ Q=1 \end{cases}$ necesito uud

$$|\phi_p\rangle_{Ms} = \frac{1}{\sqrt{6}}\left(|udu\rangle + |duu\rangle - 2|uud\rangle\right) \qquad (I_3=+1/2)$$

$$|\phi_p\rangle_{Ma} = \frac{1}{\sqrt{2}}\left(|udu\rangle - |duu\rangle\right) \qquad (I_3=+1/2)$$

$$|\chi\rangle^{\uparrow}_{Ms} = \frac{1}{\sqrt{6}}\left(|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle - 2|\uparrow\uparrow\downarrow\rangle\right) \qquad (\text{spin } +1/2)$$

$$|\chi\rangle^{\uparrow}_{Ma} = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle\right) \qquad (\text{spin } +1/2)$$

$$|\psi_p\rangle_{\text{tot. sim}} = \left(|\phi_p\rangle_{MS} \otimes |\chi\rangle_{MS} + |\phi_p\rangle_{MA} \otimes |\chi\rangle_{MA}\right)\frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{6}}\left[|udu\rangle + |duu\rangle - 2|uud\rangle\right]\left[|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle - 2|\uparrow\uparrow\downarrow\rangle\right]\right.$$

$$\left. + \frac{1}{2}\left[|udu\rangle - |duu\rangle\right]\left[|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle\right]\right)$$

$$= \frac{1}{\sqrt{2}}\left[|udu\rangle\left(|\uparrow\downarrow\uparrow\rangle 2/3 - 1/3|\downarrow\uparrow\uparrow\rangle - 1/3|\uparrow\uparrow\downarrow\rangle\right) + |duu\rangle\left(-1/3|\uparrow\downarrow\uparrow\rangle\right.\right.$$

$$\left. + 2/3|\downarrow\uparrow\uparrow\rangle - 1/3|\uparrow\uparrow\downarrow\rangle\right) + |uud\rangle\left(-1/3|\uparrow\downarrow\uparrow\rangle - 1/3|\downarrow\uparrow\uparrow\rangle + 2/3|\uparrow\uparrow\downarrow\rangle\right)$$

$$\boxed{\begin{aligned} |\psi_{proton}\rangle_{sabor} \otimes |\psi_{prot}\rangle_{spin} = \frac{1}{3\sqrt{2}}\Big[& 2|udu\rangle|\uparrow\downarrow\uparrow\rangle - |udu\rangle|\downarrow\uparrow\uparrow\rangle - |udu\rangle|\uparrow\uparrow\downarrow\rangle \\ & - |duu\rangle|\uparrow\downarrow\uparrow\rangle + 2|duu\rangle|\downarrow\uparrow\uparrow\rangle - |duu\rangle|\uparrow\uparrow\downarrow\rangle \\ & - |uud\rangle|\uparrow\downarrow\uparrow\rangle - |uud\rangle|\downarrow\uparrow\uparrow\rangle + 2|uud\rangle|\uparrow\uparrow\downarrow\rangle \Big] \end{aligned}}$$

función de onda simétrica para el protón

$$|\psi_p\rangle_{\text{tot as}} = \frac{1}{\sqrt{2}}\left(|\phi_p\rangle_{MA} \otimes |\chi_{Ms}\rangle - |\phi_p\rangle_{Ms} \otimes |\chi_{Ma}\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left[\left(\frac{|udu\rangle - |duu\rangle}{\sqrt{2}}\right)\left(\frac{|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle - 2|\uparrow\uparrow\downarrow\rangle}{\sqrt{6}}\right)\right.$$
$$\left. - \left(\frac{|udu\rangle + |duu\rangle - 2|uud\rangle}{\sqrt{6}}\right)\left(\frac{|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle}{\sqrt{2}}\right)\right]$$

$$\begin{aligned} = \frac{1}{2\sqrt{6}}\Big[& |udu\rangle\left(2|\downarrow\uparrow\uparrow\rangle - 2|\uparrow\uparrow\downarrow\rangle\right) \\ & - |duu\rangle\left(2|\uparrow\downarrow\uparrow\rangle - 2|\uparrow\uparrow\downarrow\rangle\right) \\ & |uud\rangle\left(2|\uparrow\downarrow\uparrow\rangle - 2|\downarrow\uparrow\uparrow\rangle\right)\Big] \end{aligned}$$

$$\boxed{\begin{aligned} |\psi_p\rangle\otimes|\chi_p\rangle = \frac{1}{\sqrt{6}}\Big[& |udu\rangle|\downarrow\uparrow\uparrow\rangle - |udu\rangle|\uparrow\uparrow\downarrow\rangle - |duu\rangle|\uparrow\downarrow\uparrow\rangle \\ & - |duu\rangle|\uparrow\uparrow\downarrow\rangle + |uud\rangle|\uparrow\downarrow\uparrow\rangle - |uud\rangle|\downarrow\uparrow\uparrow\rangle\Big] \end{aligned}}$$

función de onda totalmente antisimétrica para el protón

* Se repite el cálculo para el neutrón

$|n\uparrow\rangle \rightarrow$ el neutrón es un estado (udd) $Q=0$ $I_z=-1/2$ $2/3-1/3-1/3$

$$|\psi_n\rangle_{M_S} = -\frac{1}{\sqrt{6}}\left(udd + dud - 2ddu\right)$$

$$|\psi_n\rangle_{M_a} = \frac{1}{\sqrt{2}}\left(udd - dud\right)$$

$$|\chi\rangle_{M_S}^{\uparrow} = \frac{1}{\sqrt{6}}\left(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow\right)$$

spin $\left(+\frac{1}{2}\right)$

$$|\chi\rangle_{M_a}^{\uparrow} = \frac{1}{\sqrt{2}}\left(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow\right)$$

$$\underset{\text{simétrica}}{|\psi_n\rangle\otimes|\chi_n\rangle} = \frac{1}{\sqrt{2}}\left[-\frac{1}{\sqrt{6}}\frac{1}{\sqrt{6}}\left(udd + dud - 2ddu\right)\left(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow\right)\right.$$
$$\left. + \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\left(udd - dud\right)\left(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow\right)\right]$$

$$\begin{aligned} = \frac{1}{\sqrt{2}}\Big[& udd\left(1/3\uparrow\downarrow\uparrow - 2/3\downarrow\uparrow\uparrow + 1/3\uparrow\uparrow\downarrow\right) + dud\left(-2/3\uparrow\downarrow\uparrow + 1/3\downarrow\uparrow\uparrow + 1/3\uparrow\uparrow\downarrow\right) \\ & ddu\left(1/3\uparrow\downarrow\uparrow + 1/3\downarrow\uparrow\uparrow - 2/3\uparrow\uparrow\downarrow\right)\Big] \end{aligned}$$

$$\boxed{\begin{aligned} \underset{\text{simétrica}}{|\psi_n\rangle\otimes|\chi_n\rangle} = \frac{1}{\sqrt{2.3}}\Big[& |udd\rangle|\uparrow\downarrow\uparrow\rangle - 2|udd\rangle|\downarrow\uparrow\uparrow\rangle + |udd\rangle|\uparrow\uparrow\downarrow\rangle \\ & -2|dud\rangle|\uparrow\downarrow\uparrow\rangle + |dud\rangle|\downarrow\uparrow\uparrow\rangle + |dud\rangle|\uparrow\uparrow\downarrow\rangle \\ & |ddu\rangle|\uparrow\downarrow\uparrow\rangle + |ddu\rangle|\downarrow\uparrow\uparrow\rangle - 2|ddu\rangle|\uparrow\uparrow\downarrow\rangle\Big] \end{aligned}}$$

$$\underset{\text{antisimétrica}}{|\psi_n\rangle \otimes |\chi_n\rangle} = \frac{1}{\sqrt{2}}\left[+\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6}}(udd - dud)(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow) - \frac{1}{\sqrt{2}}\frac{-1}{\sqrt{6}}(udd + dud - 2ddu)(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)\right]$$

$$= \frac{1}{2\sqrt{6}}\left[udd\,(2\uparrow\downarrow\uparrow - 2\uparrow\uparrow\downarrow) - dud\,(2\downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow) - ddu\,(2\uparrow\downarrow\uparrow - 2\downarrow\uparrow\uparrow)\right]$$

$$\boxed{\underset{\text{antisimétrica}}{|\psi_n\rangle \otimes |\chi_n\rangle} = \frac{1}{\sqrt{6}}\left[|udd\rangle|\uparrow\downarrow\uparrow\rangle - |udd\rangle|\uparrow\uparrow\downarrow\rangle - |dud\rangle|\downarrow\uparrow\uparrow\rangle + |dud\rangle|\uparrow\uparrow\downarrow\rangle - |ddu\rangle|\uparrow\downarrow\uparrow\rangle + |ddu\rangle|\downarrow\uparrow\uparrow\rangle \right]}$$

* Ahora hay que calcular los momentos magnéticos, definidos como:

$$\mu_A = \sum_{i=1}^{3} \langle A\uparrow | \mu_i (\sigma_3)_i | A\uparrow\rangle \longleftarrow$$

Por convención los momentos magnéticos se evalúan entre estados spin-up

, donde A es la partícula (barión) en cuestión.

$$\mu_i(\sigma_3)_i = Q_i \frac{e}{\cancel{2}m_i}\cancel{2}S_z \rightarrow \mu_i(\sigma_3)_i = \mu_i\,\sigma_z \quad \text{con } \sigma_z\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$\underset{\psi\ \text{simétrica}}{\mu_p} = \left(\frac{1}{3\sqrt{2}}\right)^2 \Big(\langle p\uparrow| \Big[2\left(\mu_u - \mu_d + \mu_u\right)^{\substack{|udu\rangle\\ \uparrow\downarrow\uparrow}} - \left(-\mu_u + \mu_d + \mu_u\right)^{\substack{|udu\rangle\\ \downarrow\uparrow\uparrow}} - \left(\mu_u + \mu_d - \mu_u\right)^{\substack{|udu\rangle\\ \uparrow\uparrow\downarrow}}$$
$$- \left(\mu_d - \mu_u + \mu_u\right)^{\substack{|duu\rangle\\ \uparrow\downarrow\uparrow}} + 2\left(-\mu_d + \mu_u + \mu_u\right)^{\substack{|duu\rangle\\ \downarrow\uparrow\uparrow}} - \left(\mu_d + \mu_u - \mu_u\right)^{\substack{|duu\rangle\\ \uparrow\uparrow\downarrow}}$$
$$- \left(\mu_u - \mu_u + \mu_d\right)^{\substack{|uud\rangle\\ \uparrow\downarrow\uparrow}} - \left(-\mu_u + \mu_u + \mu_d\right)^{\substack{|uud\rangle\\ \downarrow\uparrow\uparrow}} + 2\left(\mu_u + \mu_u - \mu_d\right)^{\substack{|uud\rangle\\ \uparrow\uparrow\downarrow}} \Big]$$

Recordar

$\sigma_3(\uparrow) = +\uparrow$

$\sigma_3(\downarrow) = -\downarrow$

$$= \frac{1}{9.2}\left[4(2\mu_u - \mu_d) + (\mu_d) + \mu_d + \mu_d + 4(2\mu_u - \mu_d) + \mu_d + \mu_d + \mu_d + 4(2\mu_u - \mu_d)\right]$$

$$\mu_p \doteq \frac{1}{18}\left[6\mu_d + 3.8\mu_u - 3.4\mu_d\right] = \frac{1}{18}\left(24\mu_u - 6\mu_d\right) = \boxed{\frac{1}{3}\left(4\mu_u - \mu_d\right)}$$

con $\underset{\text{simétrica}}{|\psi\rangle\otimes|\chi\rangle}$

$$\underset{\psi\ \text{antisimétrica}}{\mu_p} = \frac{1}{6}\Big(\langle p\uparrow| \Big[(-\mu_u + \mu_d + \mu_u)\,|udu\rangle|\downarrow\uparrow\uparrow\rangle - (\mu_u + \mu_d - \mu_u)\,|udu\rangle|\uparrow\uparrow\downarrow\rangle$$
$$- (\mu_d - \mu_u + \mu_u)\,|duu\rangle|\uparrow\downarrow\uparrow\rangle - (\mu_d + \mu_u - \mu_u)\,|duu\rangle|\uparrow\uparrow\downarrow\rangle$$
$$+ (\mu_u - \mu_u + \mu_d)\,|uud\rangle|\uparrow\downarrow\uparrow\rangle - (-\mu_u + \mu_u + \mu_d)\,|uud\rangle|\downarrow\uparrow\uparrow\rangle \Big]$$

$$\mu_p = \frac{1}{6}\left(\mu_d + \mu_d + \mu_d + \mu_d + \mu_d + \mu_d \right) = \boxed{\mu_d}$$

con $\underset{\text{antisimétrica}}{|\psi\rangle\otimes|\chi\rangle}$

$$\underset{\Psi_{\text{simétrica}}}{\mu_n} = \frac{1}{9.2}\Big[(\mu_u - \mu_d + \mu_d) + 4(-\mu_u + \mu_d + \mu_d) + (\mu_u + \mu_d - \mu_d)$$
$$+4(\mu_d - \mu_u + \mu_d) + (-\mu_d + \mu_u + \mu_d) + (\mu_d + \mu_u - \mu_d)$$
$$(\mu_d - \mu_d + \mu_u) + (-\mu_d + \mu_d + \mu_u) + 4(\mu_d + \mu_d - \mu_u)\Big]$$

$$= \frac{1}{18}\Big[\mu_u + 4(2\mu_d - \mu_u) + \mu_u + 4(2\mu_d - \mu_u) + \mu_u + \mu_u + \mu_u + \mu_u + 4(2\mu_d - \mu_u)\Big]$$

$$\mu_n = \frac{1}{18}\Big[6\mu_u + 24\mu_d - 12\mu_u\Big] = \boxed{\frac{1}{3}(4\mu_d - \mu_u)} \quad \text{con } |\psi\rangle\otimes|\chi\rangle_{\text{simétrica}}$$

$$\underset{\Psi_{\text{antisimétrica}}}{\mu_n} = \frac{1}{6}\Big[(\mu_u - \cancel{\mu_d} + \cancel{\mu_d}) + (\mu_u + \cancel{\mu_d} - \cancel{\mu_d}) + (-\cancel{\mu_d} + \mu_u + \cancel{\mu_d}) + (\cancel{\mu_d} + \mu_u - \cancel{\mu_d})$$
$$+ (\cancel{\mu_d} - \cancel{\mu_d} + \mu_u) + (-\cancel{\mu_d} + \cancel{\mu_d} + \mu_u)\Big]$$

$$= \frac{1}{6}(\mu_u + \mu_u + \mu_u + \mu_u + \mu_u + \mu_u) \Rightarrow \boxed{\mu_n = \mu_u} \quad \text{con } |\psi\rangle\otimes|\chi\rangle_{\text{antisimétrica}}$$

Ψ simétrica

$$\boxed{\frac{\mu_p}{\mu_n} = \frac{1/3(4\mu_u - \mu_d)}{1/3(4\mu_d - \mu_u)}}$$

Ψ antisimétrica

$$\boxed{\frac{\mu_p}{\mu_n} = \frac{\mu_d}{\mu_u}}$$

Pero si vemos ahora que $m_u \approx m_d \rightarrow$

$$\mu_d = \frac{Qe}{2m_d} \qquad \mu_u = \frac{Qe}{2m_u}$$

$$\mu_d = \frac{-1e}{3.2m_d} \qquad \mu_u = \frac{2\,e}{3\,2m_u} \rightarrow \qquad \frac{\mu_d}{\mu_u} = \frac{-\cancel{3}\,e\,.\cancel{2m}}{\cancel{3}.\cancel{2m}\,2\cancel{e}} \rightarrow \mu_d = -\frac{\mu_u}{2}$$

$$\Psi_{\text{sim.}} \quad \frac{\mu_p}{\mu_n} \approx \frac{4\mu_u + \mu_u/2}{4\left(-\frac{\mu_u}{2}\right) - \mu_u} = \frac{9/2\,\mu_u}{-3\,\mu_u} = -\frac{3}{2}$$

$$\Psi_{\text{antisim.}} \quad \frac{\mu_p}{\mu_n} \approx \frac{-\mu_u}{2\mu_u} = -\frac{1}{2} \longrightarrow$$

Pero experimentalmente obtenemos:

$$\frac{\mu_n}{\mu_p} \approx -0{,}6849.$$

con lo cual el $\frac{\mu_n}{\mu_p} = -\frac{2}{3} = 0{,}666$ que obtuvimos con las funciones $|\psi\rangle\otimes|\chi\rangle_{\text{simétricas}}$ es el correcto.

Asimismo con $|\psi\rangle\otimes|\chi\rangle_{\text{antisimétricas}}$ hemos llegado a que

$$\mu_n = \mu_u = \frac{\cancel{2}}{3}\frac{e}{\cancel{2}m_u} = \frac{e}{3.m_u} > 0 \quad \Rightarrow \mu_n > 0$$

pero experimentalmente se ve que $\mu_n < 0$ (−1,913 según Griffiths) con lo cual las $|\Psi\rangle = |\psi\rangle\otimes|\chi\rangle$ correctas para protón y neutrón (bariones) son las simétricas. La antisimetría necesaria la dará la ($|C\rangle_{\text{color}}$) función de onda de color.

7.

Λ^0 Son bariones pertenecientes al octete ⇒ spin 1/2

$$|\Psi_{\Lambda^0}\rangle_{MA} = \frac{1}{\sqrt{12}}\left[2(ud-du)s+(us-su)d+(sd-ds)u\right]$$

$$|\Psi_{\Lambda^0}\rangle_{AS} = \frac{1}{2}\left[(ds+sd)u-(us+su)d\right]$$

$$\frac{1}{\sqrt{2}}\left(|\Psi_{\Lambda^0}\rangle_{AS}\otimes|\chi_{MS}\uparrow\rangle + |\Psi_{\Lambda^0}\rangle_{MA}\otimes|\chi_{MA}\uparrow\rangle\right) =$$

$$= \Bigg(\frac{1}{2}\left[(dsu+sdu)-(usd+sud)\right]\left[\frac{1}{\sqrt{6}}\left(|\uparrow\downarrow\uparrow\rangle+|\downarrow\uparrow\uparrow\rangle-2|\uparrow\uparrow\downarrow\rangle\right)\right]$$

$$+\frac{1}{\sqrt{12}}\left[2uds-2dus+usd-sud+sdu-dsu\right]\left[\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\uparrow\rangle-|\downarrow\uparrow\uparrow\rangle\right)\right]\Bigg)\frac{1}{\sqrt{2}}$$

$\frac{1}{\sqrt{24}}=\frac{1}{2\sqrt{6}}$

$$=\frac{1}{\sqrt{2}}\frac{1}{2\sqrt{6}}\Big(dsu\left[2\downarrow\uparrow\uparrow-2\uparrow\uparrow\downarrow\right]+sdu\left[2\uparrow\downarrow\uparrow-2\uparrow\uparrow\downarrow\right]+usd\left[-2\downarrow\uparrow\uparrow+2\uparrow\uparrow\downarrow\right]$$
$$sud\left[-2\uparrow\downarrow\uparrow+2\uparrow\uparrow\downarrow\right]+uds\left[2\uparrow\downarrow\uparrow-2\downarrow\uparrow\uparrow\right]+dus\left[-2\uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow\right]\Big)$$

$$|\Psi\rangle\otimes|\chi\uparrow\rangle_{\text{simétrica}}=\frac{1}{\sqrt{12}}\Big(dsu\downarrow\uparrow\uparrow - dsu\uparrow\uparrow\downarrow + sdu\uparrow\downarrow\uparrow - sdu\uparrow\uparrow\downarrow$$
$$-usd\downarrow\uparrow\uparrow + usd\uparrow\uparrow\downarrow - sud\uparrow\downarrow\uparrow + sud\uparrow\uparrow\downarrow$$
$$-dus\uparrow\downarrow\uparrow + dus\downarrow\uparrow\uparrow + uds\uparrow\downarrow\uparrow - uds\downarrow\uparrow\uparrow\Big)$$

$$\mu_{\Lambda} = \sum_{i=1}^{3}\underset{\text{sim.}}{\left(\langle\chi\uparrow|\otimes\langle\Psi_{\Lambda}|\right)}\mu_i(\sigma_3)_i\left(|\Psi_{\Lambda}\rangle\otimes|\chi\uparrow\rangle\right)$$

$$\mu_{\Lambda^0} = \frac{1}{12}\Big[\left(\cancel{-\mu_d}+\mu_s+\cancel{\mu_u}\right)+\left(\cancel{\mu_d}+\mu_s-\cancel{\mu_u}\right)+\left(\mu_s\cancel{-\mu_d}\cancel{+\mu_u}\right)+\left(\mu_s\cancel{+\mu_d}\cancel{-\mu_u}\right)$$
$$\left(\cancel{-\mu_u}+\mu_s+\cancel{\mu_d}\right)+\left(\cancel{\mu_u}+\mu_s-\cancel{\mu_d}\right)+\left(\mu_s-\cancel{\mu_u}+\cancel{\mu_d}\right)+\left(\mu_s\cancel{+\mu_u}\cancel{-\mu_d}\right)$$
$$\left(\cancel{\mu_d}-\cancel{\mu_u}+\mu_s\right)+\left(\cancel{-\mu_d}+\cancel{\mu_u}+\mu_s\right)+\left(\cancel{\mu_u}-\cancel{\mu_d}+\mu_s\right)+\left(\cancel{-\mu_u}+\cancel{\mu_d}+\mu_s\right)\Big]$$

$$\mu_{\Lambda^0} = \frac{1}{12}\left[12\mu_s\right] \rightarrow \boxed{\mu_{\Lambda^0}=\mu_s} = \frac{-1/3\,.\,e}{2\,.\,m_s} = \frac{-1\,.\,e}{3.2.540\,\text{MeV}} = \boxed{-3{,}08.10^{-4}e}$$

El experimento da $(-0{,}61).\frac{e\hbar}{2m_pc} = -0{,}61.\frac{e}{2.938\,\text{MeV}} = -3{,}25.10^{-4}e$

Se ve que difieren en un 5% con respecto al experimento.