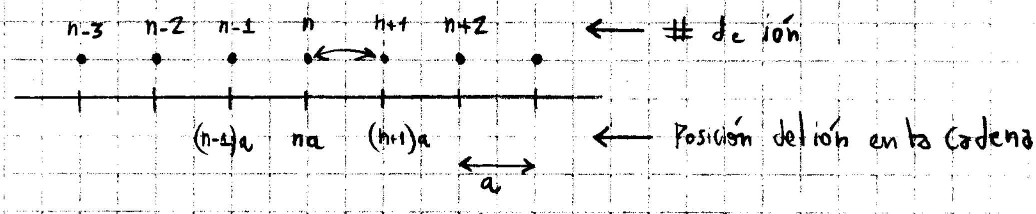
GUÍA 7: Dinámica de Redes

1.

Cadena 1D



Consideramos interacción a primeros vecinos (de intensidad K)

$$U^{harm} = \frac{1}{2} K \sum_n \left[u(na) - u([n+1]a) \right]^2 \quad ; \quad K = \frac{d^2 \phi(x)}{dx^2}$$

Ahora aplicamos las ecuaciones de Newton en el sitio 'n' situado en (na)

$$M \ddot{u}(na) = -\frac{\partial U^{harm}}{\partial u(na)} = -\frac{K}{2} \cdot 2 \left(u[na] - u[(n+1)a] \right)$$

masa de los iones

$$M \ddot{u} = -K \left\{ u(na) - u([n+1]a) + u(na) - u([n-1]a) \right\}$$

$$M \ddot{u} = -K \left[2u(na) - u([n-1]a) - u([n+1]a) \right]$$

Planteo las condiciones de contorno

$$\begin{cases} u([N+1]a) = u(a) & \text{(Cuando se termina vuelve a empezar)} \\ u(0) = u(Na) & \text{(EL 1ero es igual al último)} \end{cases}$$

Pensamos en ondas planas:

$$u(na,t) \propto e^{i(kna - \omega t)}$$

$$\dot{u}(na,t) \propto e^{i(kna-\omega t)}(-i\omega) = -i\omega \, e^{i(kna-\omega t)}$$

$$\ddot{u}(na,t) \propto e^{i(kna-\omega t)}(-i\omega)^2 = -\omega^2 \, e^{i(kna-\omega t)}$$

$$-M\omega^2 e^{i(kna-\omega t)} = -K\left[2e^{i(kna-\omega t)} - e^{i(k[n-1]a-\omega t)} - e^{i(k[n+1]a-\omega t)}\right]$$

$$-M\omega^2 = -K\left[2 - e^{-ika} - e^{ika}\right]$$

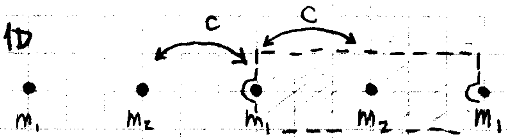
$$-M\omega^2 = -K\left[2 - 2\cos(ka)\right]$$

$$\omega^2 = \frac{2K}{M}\left[1 - \cos(ka)\right]$$

$$\boxed{\omega = \sqrt{\frac{2K}{M}\left(1 - \cos[ka]\right)}}$$

Esta es la relación de dispersión pedida. Aquí k es discreto de las C.C.

2. Cadena 1D



Interacción a primeros vecinos C

# ion		n-1	n	n+1	n+2
Posición (celda)	(n-1)a		(na)		(n+1)a

$\longleftarrow a \longrightarrow$

base $\begin{cases} \vec{0} \\ \vec{d} = \frac{a}{2}\hat{x} \end{cases}$

a) $$U^{harm} = \frac{C}{2}\sum_n \left[U_1(na) - U_2(na) \right]^2 + \frac{C}{2}\sum_n \left[U_1(na) - U_2([n-1]a)\right]^2$$

Hemos contado una sola vez cada interacción.

$$\begin{cases} U_1 = U_1^0 \cdot e^{i(kR(n) - \omega t)} \\ U_2 = U_2^0 \cdot e^{i(kR(n) - \omega t)} \end{cases}$$

Cada ión oscila torno a su posición de equilibrio

$$m_1 \cdot \ddot{U}_1 = -m_1 U_1^0 \omega^2 e^{i(kna - \omega t)} = -\frac{C}{2} \cdot 2\left(U_1(na) - U_2(na)\right) - \frac{C}{2} \cdot 2\left(U_1(na) - U_2([n-1]a)\right)$$

$$= -C\left[2U_1(na) - U_2(na) - U_2([n-1]a)\right]$$

$$= -C\left[2U_1^0 e^{i(kna-\omega t)} - U_2^0 \cdot e^{i(kna-\omega t)} - U_2^0 \cdot e^{i(k[n-1]a - \omega t)}\right]$$

$$-m_1 U_1^0 \omega^2 = -C\left[2U_1^0 - U_2^0 - U_2^0 e^{-i(ka)}\right]$$

$$m_2 \cdot \ddot{U}_2 = -m_2 U_2^0 \omega^2 e^{i(kna - \omega t)} = +\frac{C}{2} \cdot 2\left[U_1(na) - U_2(na)\right] + \frac{C}{2} \cdot 2\left[U_1([n+1]a) - U_2(na)\right]$$

$$= C\left[-2U_2(na) + U_1(na) + U_1([n+1]a)\right]$$

$$-m_2 U_2^0 \omega^2 = C\left[-2U_2^0 + U_1^0 + U_1^0 \cdot e^{ika}\right]$$

$$\begin{cases} m_1 U_1^0 \omega^2 = C2U_1^0 - CU_2^0 - CU_2^0 e^{-ika} \\ m_2 U_2^0 \omega^2 = C2U_2^0 - CU_1^0 - CU_1^0 e^{ika} \end{cases}$$

$$\begin{cases} U_1^0(m_1\omega^2 - 2C) + U_2^0(C + Ce^{-ika}) = 0 \\ U_2^0(m_2\omega^2 - 2C) + U_1^0(C + Ce^{ika}) = 0 \end{cases}$$

$$\begin{pmatrix} m_1\omega^2 - 2C & C(1+e^{-ika}) \\ C(1+e^{ika}) & m_2\omega^2 - 2C \end{pmatrix}\begin{pmatrix} U_1^0 \\ U_2^0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$(m_1\omega^2 + 2C)(m_2\omega^2 + 2C) - C^2\underbrace{(1+e^{-ika})(1+e^{ika})}_{1 + e^{-ika} + e^{ika} + 1} = 0$$

$$m_1 m_2 \omega^4 + 4C^2 - m_1\omega^2 2C - m_2\omega^2 2C - 2(1 + \cos ka)C^2 = 0$$

HOJA:

FECHA

$$m_1 m_2 \omega^4 - (m_1+m_2) 2C\omega^2 - 2(1+\cos ka)C^2 + 4C^2 = 0$$

define $z = \omega^2$

$$z^2 - \underbrace{\left(\frac{m_1+m_2}{m_1 \cdot m_2}\right) 2Cz}_{b = \frac{2C}{\mu} z} - \frac{2(1+\cos(ka))C^2}{m_1 \cdot m_2} + \frac{4C^2}{m_1 m_2} = 0$$

$$\mu \equiv \frac{m_1 \cdot m_2}{m_1+m_2}$$

$$z = \left(\frac{+2C}{\mu} \pm \sqrt{\frac{4C^2}{\mu^2} - 4\cdot\left(\frac{-2(1+\cos[ka])C^2}{m_1 \cdot m_2} + \frac{4C^2}{m_1 m_2}\right)}\right)\frac{1}{2}$$

$$\omega^2 = +\frac{C}{\mu} \pm \sqrt{\frac{C^2}{\mu^2} - \frac{C^2}{\mu^2}\left(\frac{-2(1+\cos(ka))\mu^2}{(m_1 \cdot m_2)} + \frac{4\mu^2}{m_1 \cdot m_2}\right)}$$

$$\omega^2 = \frac{C}{\mu}\left(1 \pm \sqrt{1 - \frac{\mu^2}{(m_1 \cdot m_2)}[2 - 2\cos(ka) + 4]}\right)$$

$$\boxed{\omega^2 = \frac{C}{\mu}\left(1 \pm \sqrt{1 - \frac{2\mu^2}{m_1 \cdot m_2}(1-\cos[ka])}\right)}$$

+ óptica
− acústica

b)

En $k = \pi/a$ es $\begin{cases} \cos(\pi) = -1 \\ e^{-ika} = e^{-i\pi} = -1 \end{cases} \Rightarrow \omega^2 = \frac{C}{\mu}\left[1 \pm \sqrt{1 - \frac{4\mu^2}{m_1 \cdot m_2}}\right]$

$$\omega_A = \sqrt{\frac{C}{\mu}}\left(1 + \sqrt{1 - \frac{4\mu^2}{m_1 \cdot m_2}}\right)^{1/2}$$

$$\omega_B = \sqrt{\frac{C}{\mu}}\left(1 - \sqrt{1 - \frac{4\mu^2}{m_1 \cdot m_2}}\right)^{1/2}$$

$$1 - \frac{4\mu^2}{m_1 m_2} = 1 - \frac{4 m_1 \cdot m_2}{(m_1+m_2)^2} = \frac{m_1^2 + m_2^2 + 2m_1 m_2 - 4m_1 m_2}{(m_1+m_2)^2} = \left(\frac{m_1 - m_2}{m_1+m_2}\right)^2$$

NOTA

$$\sqrt{\left(\frac{m_1 - m_2}{m_1+m_2}\right)^2} = \left|\frac{m_1 - m_2}{m_1+m_2}\right|$$

Suponemos $m_1 > m_2$ pero es similar con $m_2 > m_1$

$$\omega_+^2 = \frac{C}{\mu}\left(1 + \frac{m_1 - m_2}{m_1+m_2}\right) = \frac{C}{\mu}\left(\frac{2m_1}{m_1+m_2}\right) = \frac{C\,2\mu}{\mu\, m_2}$$

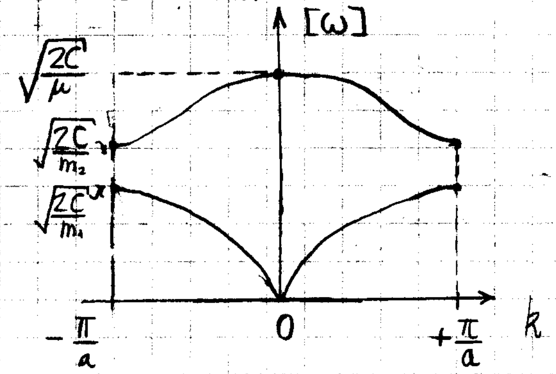
$$\omega_-^2 = \frac{C}{\mu}\left(1 - \frac{m_1 - m_2}{m_1+m_2}\right) = \frac{C}{\mu}\left(\frac{2m_2}{m_1+m_2}\right) = \frac{C}{\mu}\frac{2\mu}{m_1}$$

$\Rightarrow$ * $\omega_+ = \omega \rightarrow \omega = \sqrt{\frac{2C}{m_2}}$ $\quad U_1^0\left(\frac{m_1}{m_2}2C - 2C\right) = 0 \rightarrow \begin{cases} U_2^0 \propto cte \\ U_1^0 = 0 \end{cases}$

* $\omega_- = \omega \rightarrow \omega = \sqrt{\frac{2C}{m_1}}$ $\quad U_2^0\left(\frac{m_2}{m_1}2C - 2C\right) = 0 \rightarrow \begin{cases} U_2^0 = 0 \\ U_1^0 \propto cte. \end{cases}$

NOTA

En el borde de zona tenemos a un tipo de ion oscilando, y el otro inmóvil. EL cociente $U_1^{(0)}/U_2^{(0)}$ será nulo o infinito



NOTA

Al haber supuesto $m_1 > m_2$ resulta $\frac{1}{m_1} < \frac{1}{m_2}$ lo cual define que curva se halla por encima

▲ viene de la nota anterior

c)

$m_1 \gg m_2 \rightarrow 1 \gg \frac{m_2}{m_1}$ $\qquad 1 \gg \frac{m_2}{m_1} \rightarrow$

$$\mu = \frac{m_1 \cdot m_2}{m_1+m_2} = \frac{m_2}{\left(1+\frac{m_2}{m_1}\right)}$$

$$\frac{\mu^2}{m_1 . m_2} = \frac{m_1 \cdot m_2}{(m_1+m_2)^2} = \frac{m_2}{m_1\left(1+\frac{m_2}{m_1}\right)^2}$$

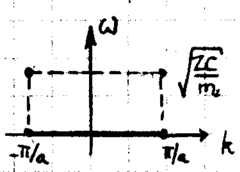
$$= \frac{m_2}{m_1}\left(1 - 2\frac{m_2}{m_1}\right) \simeq \frac{m_2}{m_1}$$

$$\mu \approx m_2 \cdot \left(1 - \frac{m_2}{m_1}\right)$$

$$\mu \approx m_2 - \frac{m_2^2}{m_1} \rightarrow \mu \simeq m_2$$

$$\omega^2 = \frac{C}{m_2}\left(1+\frac{m_2}{m_1}\right)\left(1 \pm \sqrt{1 - 2\frac{m_2}{m_1}\left[1-\cos(ka)\right]}\right)$$ ← No veo expresión fácil de analizar

Si $m_1 \to \infty \Rightarrow$ $\quad \omega^2 \approx \frac{C}{m_2}(1 \pm 1) =$ $\nearrow \omega_+^2 = \frac{2C}{m_2}$ (óptica) $\searrow \omega_-^2 = 0$ (acústica)



→ $U_1^0 = 0$, $U_2^0 = 0$ salvo en $k = \pi/a$ donde $U_2^0 = $ cte.

→ se mueven todos en bloque

d) En el caso de igualdad valdrán:

$$\mu = \frac{m_1 . m_2}{m_1+m_2} = \frac{m^2}{2m} = \frac{m}{2} \rightarrow \qquad \frac{\mu^2}{m^2} = \frac{m^2}{4m^2}$$

$$\omega^2(k) = \frac{2C}{m}\left(1 \pm \sqrt{1 - \frac{2}{4}\left[1-\cos(ka)\right]}\right)$$

$$\omega^2(k=0) = \frac{2C}{m}(1 \pm 1)$$ $\nearrow \sqrt{\frac{4C}{m}} = \omega$ óptica $\searrow 0 = \omega$ acústica

$$\left[1 \pm \sqrt{\underbrace{1 - \frac{1}{2}}_{1/2} + \frac{1}{2}\cos(ka)}\right]$$

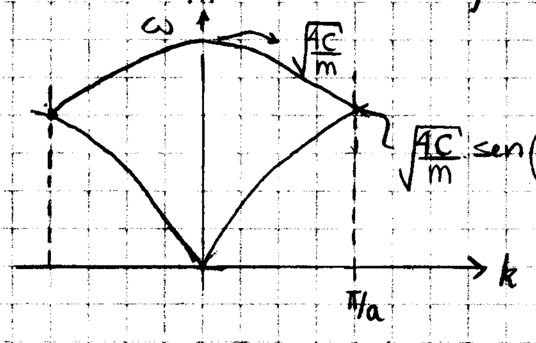
$$\boxed{\omega^2(k) = \frac{2C}{m}\left[1 \pm \cos\left(\frac{ka}{2}\right)\right]}$$

donde $\begin{cases} - & \text{corresp. acústica} \\ + & \text{corresp. óptica} \end{cases}$

Vemos que coincide con lo obtenido para la cadena monoatómica en el caso en que tomamos el signo (−); es decir la rama acústica.

$$\omega_{AC}^2 = \frac{2C}{m}\left(1 - \cos\left[\frac{ka}{2}\right]\right) = \frac{2C}{m} \cdot 2\,\mathrm{sen}^2\left(\frac{ka}{4}\right)$$

$$\omega_{OP}^2 = \frac{2C}{m}\left(1 + \cos\left[\frac{ka}{2}\right]\right) = \frac{2C}{m} \cdot 2\cos^2\left(\frac{ka}{4}\right)$$

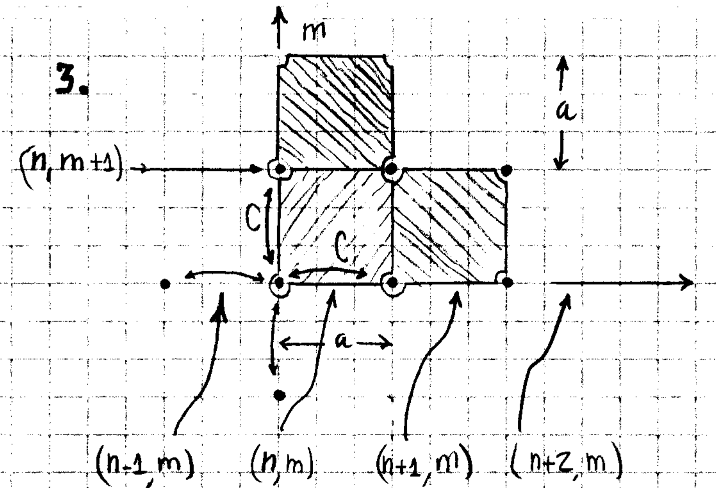


$$\sqrt{\frac{4C}{m}} \to \text{(máximo)}$$

$$\sqrt{\frac{4C}{m}}\,\mathrm{sen}\left(\frac{\pi}{4}\right) = \sqrt{\frac{4C}{m}} \cdot \cos\left(\frac{\pi}{4}\right)$$

Con $m_1 = m_2$ las ramas ópticas y acústicas se hallan degeneradas en el borde de zona.

3.



Red cuadrada plana (2D) monoatómica de parámetro a.

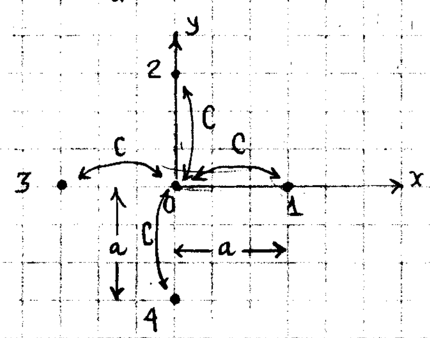
Interacción C a primeros vecinos

$$U = \vec{U}^{(0)} e^{i(\vec{k}\cdot\vec{R} - \omega t)}$$

$$U^{harm} = \frac{1}{2}\left(\frac{1}{2}\right)\sum_n \sum_m C\Big\{\left[U(na, ma) - U([n+1]a, ma)\right]^2 + \left[U(na, ma) - U([n-1]a, ma)\right]^2$$
$$+ \left[U(na, ma) - U(na, [m+1]a)\right]^2 + \left[U(na, ma) - U(na, [m-1]a)\right]^2$$

$$U^{harm} = \frac{1}{2}\sum_n \sum_m C\Big\{\left[u(na, ma) - u([n+1]a, ma)\right]^2 + \left[u(na, ma) - u(na, [m+1]a)\right]^2$$

a) Para el armado de la matriz se reetiquetarán los primeros vecinos:



$$\phi = -\frac{e^2}{r} = -\frac{e^2}{\sqrt{x^2+y^2}}$$

$$C \equiv \left.\frac{\partial^2 \phi}{\partial r^2}\right|_{r_0} = e^2 \frac{\partial^2}{\partial r^2}(-r^{-1}) = -\left.\frac{e^2 2}{r^3}\right|_{r_0}$$

$$-\frac{2e^2}{r_0^3} = C, \quad r_0^3 = a^3$$

$$\overleftrightarrow{D}^{01}_{[-1,0]} = \begin{pmatrix} -C & 0 \\ 0 & 0 \end{pmatrix}$$

$$\overleftrightarrow{D}^{02}_{[0,-1]} = \begin{pmatrix} 0 & 0 \\ 0 & -C \end{pmatrix}$$

$$\overleftrightarrow{D}^{03}_{[1,0]} = \begin{pmatrix} -C & 0 \\ 0 & 0 \end{pmatrix}$$

$$\overleftrightarrow{D}^{04}_{[0,1]} = \begin{pmatrix} 0 & 0 \\ 0 & -C \end{pmatrix}$$

$$\overleftrightarrow{D}^{00} = \begin{pmatrix} 2C & 0 \\ 0 & 2C \end{pmatrix}$$

$$\epsilon_1^{01} = \hat{x}$$

$$D^{0n}_{ij} = -\mathcal{X}^{0n} \epsilon_i^{0n} \epsilon_j^{0n}$$

$$\frac{\partial^2 \phi}{\partial y \partial x} = -e^2 \frac{\partial}{\partial y}\left(-\frac{1}{2}(x^2+y^2)^{-3/2} 2x\right) = -e^2(-1)\left(-\frac{3}{2}\right) x (x^2+y^2)^{-5/2} 2y$$

$$\frac{\partial^2 \phi}{\partial y \partial x} = -\frac{e^2 3xy}{(x^2+y^2)^{5/2}}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{\partial}{\partial x}\left[x (x^2+y^2)^{-3/2}\right](e^2)$$
$$= -e^2\left[(x^2+y^2)^{-3/2} - x(x^2+y^2)^{-5/2} \frac{3}{2} 2x\right]$$

$$\frac{\partial^2 \phi}{\partial x^2} = -e^2\left(\frac{y^2 - 2x^2}{(x^2+y^2)^{+5/2}}\right)$$

$$\frac{\partial^2 \phi}{\partial y^2} = -e^2\left(\frac{x^2 - 2y^2}{(x^2+y^2)^{+5/2}}\right)$$

$$\left.\frac{\partial^2 \phi}{\partial x^2}\right|_{x=a} = -e^2\left(\frac{-2a^2}{a^5}\right) = \frac{2e^2}{a^3}$$

$$\overleftrightarrow{D}(\vec{k}) = \sum_{\vec{R}_n} \overleftrightarrow{D}(\vec{R}_n)\, e^{i\vec{k}\cdot\vec{R}_n} \quad ; \text{ con } \quad \vec{R}_n = \vec{0};\ (a,0);\ (-a,0);\ (0,a);\ (0,-a)$$

$$\Rightarrow \quad \overleftrightarrow{D}(\vec{k}) = \begin{pmatrix} -C & 0 \\ 0 & 0 \end{pmatrix} e^{ik_x a} + \begin{pmatrix} -C & 0 \\ 0 & 0 \end{pmatrix} e^{-ik_x a} + \begin{pmatrix} 2C & 0 \\ 0 & 2C \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & 0 \\ 0 & -C \end{pmatrix} e^{ik_y a} + \begin{pmatrix} 0 & 0 \\ 0 & -C \end{pmatrix} e^{-ik_y a}$$

$$\boxed{\overleftrightarrow{D}(\vec{k}) = \begin{pmatrix} -C & 0 \\ 0 & 0 \end{pmatrix} 2\cos(k_x a) + \begin{pmatrix} 0 & 0 \\ 0 & -C \end{pmatrix} 2\cos(k_y a) + \begin{pmatrix} 2C & 0 \\ 0 & 2C \end{pmatrix}}$$

b)

$$M\omega^2 \vec{u} = \overleftrightarrow{D}(k)\vec{u} \rightarrow$$

$$\det\left[M\omega^2 - \overleftrightarrow{D}(k)\right] = 0 \rightarrow$$

$$\begin{bmatrix} M\omega^2 + 2C[\cos(k_x a) - 1] & 0 \\ 0 & M\omega^2 + 2C[\cos k_y a - 1] \end{bmatrix}$$

$$\begin{bmatrix} M\omega^2 - 2C\, 2\,\text{sen}^2\left(\frac{k_x a}{2}\right) & 0 \\ 0 & M\omega^2 - 2C\, 2\,\text{sen}^2\left(\frac{k_y a}{2}\right) \end{bmatrix}$$

$$\left[M\omega^2 - 4C\,\text{sen}^2\left(\frac{k_x a}{2}\right)\right]\left[M\omega^2 - 4C\,\text{sen}^2\left(\frac{k_y a}{2}\right)\right]$$

$$M^2\omega^4 - M\omega^2 4C\,\text{sen}^2\left(\frac{k_x a}{2}\right) - M\omega^2 4C\,\text{sen}^2\left(\frac{k_y a}{2}\right) + 16C^2\,\text{sen}^2\left(\frac{k_x a}{2}\right)\text{sen}^2\left(\frac{k_y a}{2}\right) = 0$$

$$M^2\omega^4 - \omega^2 M 4C\left[\text{sen}^2\left(\frac{k_x a}{2}\right) + \text{sen}^2\left(\frac{k_y a}{2}\right)\right] + 16C^2\,\text{sen}^2\left(\frac{k_x a}{2}\right)\text{sen}^2\left(\frac{k_y a}{2}\right) = 0$$

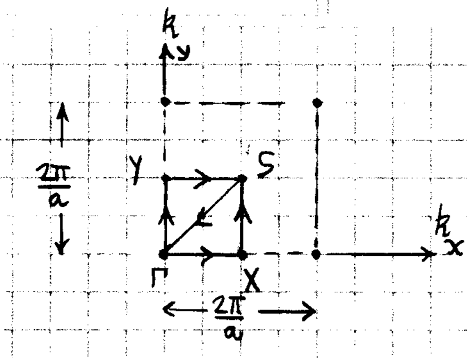
$$\omega^2 = \frac{4CM[\text{sen}^2(X) + \text{sen}^2(Y)]}{2M^2} \mp \frac{\sqrt{4^2C^2M^2[\text{sen}^2(X) + \text{sen}^2(Y)]^2 - 4M^2 16C^2\,\text{sen}^2(X).\,\text{sen}^2(Y)}}{2M^2}$$

$$\frac{2C}{M}(\text{sen}^2 X + \text{sen}^2 Y) \mp \frac{1}{2M^2}\sqrt{16C^2M^2\left[\text{sen}^4 X + \text{sen}^4 Y + 2\,\text{sen}^2 X\,\text{sen}^2 Y - 4\,\text{sen}^2 X\,\text{sen}^2 Y\right]}$$

$$\omega^2(k) = \frac{2C}{M}(\text{sen}^2 X + \text{sen}^2 Y) \mp \frac{\overset{2}{\cancel{4}}CM}{\cancel{2}M^2}(\text{sen}^2 X - \text{sen}^2 Y) \qquad \text{con } \begin{array}{l} X \equiv k_x a/2 \\ Y \equiv k_y a/2 \end{array}$$

$$\omega^2(k) = \frac{2C}{M}\left[\text{sen}^2 X + \text{sen}^2 Y \mp (\text{sen}^2 X - \text{sen}^2 Y)\right]$$

$$\omega_-^2(k) = \frac{2C}{M}\, 2\,\text{sen}^2\left(\frac{k_y a}{2}\right) \quad ; \quad \omega_+^2(k) = \frac{2C}{M}\, 2\,\text{sen}^2\left(\frac{k_x a}{2}\right)$$



$\Gamma \to X \to S \to \Gamma \to Y \to S$: recorrido

$\left(\frac{\pi}{a}, 0\right)$ $\left(\frac{\pi}{a}, \frac{\pi}{a}\right)$ $\left(0, \frac{\pi}{a}\right)$

* $\Gamma \to X$: $\omega_+^2(k) = \frac{2C}{M}\left[\operatorname{sen}^2\left(\frac{k_x a}{2}\right) 2\right]$

* $X \to S$: $\omega_+^2 = \frac{2C}{M}[1+1]$

$\omega_-^2 = \frac{2C}{M}\left[2 \operatorname{sen}^2\left(\frac{k_y a}{2}\right)\right]$

* $S \to \Gamma$: $\omega_+^2 = \frac{2C}{M}\left[2 \operatorname{sen}^2\left(\frac{k a}{\sqrt{2}\cdot 2}\right)\right]$ $\omega_-^2 = \frac{2C}{M}\left[2 \operatorname{sen}^2\left(\frac{k a}{\sqrt{2}\cdot 2}\right)\right]$

$k_x = \frac{k}{\sqrt{2}}$

$k_y = \frac{k}{\sqrt{2}}$

$2\left(\frac{\pi}{a}\right)^2 = \frac{2\pi^2}{a^2}$

* $\Gamma \to Y$: $\omega_-^2 = \frac{2C}{M}\left[2 \operatorname{sen}^2\left(\frac{k_y a}{2}\right)\right]$

* $Y \to S$: $\omega_+^2 = \frac{2C}{M}\left[2 \operatorname{sen}^2\left(\frac{k_x a}{2}\right)\right]$

$\omega_-^2 = \frac{2C}{M}[2]$

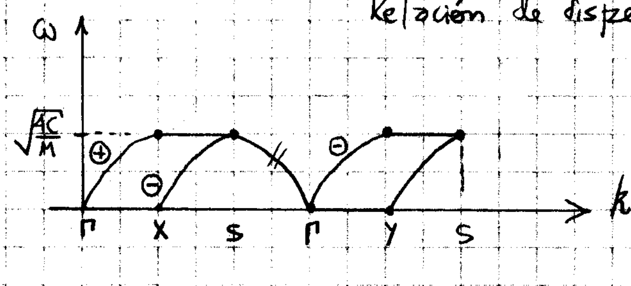
Ahora evaluamos los autovectores correspondientes:

$\left[M\omega_-^2 - 4C \operatorname{sen}^2\left(\frac{k_x a}{2}\right)\right] u_x^{(0)} = 0$ $\omega_-(k) \to \vec{u}_{(-)} \propto \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\left[\operatorname{sen}^2\left(\frac{k_y a}{2}\right) - \operatorname{sen}^2\left(\frac{k_x a}{2}\right)\right] u_x^{(-)} = 0 \to$

$\omega_+(k) \to \vec{u}_{(+)} \propto \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Relación de dispersión



Desde $\Gamma \to X$ el modo de ω_+ es el longitudinal y el de ω_- el transversal