GUÍA 2: Electrones Libres

HOJA
FECHA

1. i) K (# atom = 19) sea Z = # de electrones de conducción
$M_A = 39$
$Z = 1$

$N_A = 6{,}022 \cdot 10^{23}\ \frac{\text{átomos}}{\text{mol}}$, 1 mol ___ 39g

moléculas por volumen $\rightarrow n = \frac{N}{V} = \frac{nN_a}{V} = \rho_m N_a$ $\quad \rho = 0{,}86\ g/cm^3 \rightarrow \rho_M = 0{,}022\ \frac{mol}{cm^3} = \frac{n}{V}$
(densidad molar)

$$n = 6{,}022 \cdot 10^{23}\ \frac{\text{átomos}}{\text{mol}}\ 0{,}022\ \frac{\text{mol}}{\text{cm}^3} \cong 1{,}33 \cdot 10^{22}\ \frac{\text{átomos}}{\text{cm}^3}$$

$$n(\text{electrones conducción}) = \rho_m N_a Z_1 \Rightarrow n = 1{,}33 \cdot 10^{22}\ \frac{\text{átomos}}{\text{cm}^3}\ 1\ \frac{\text{electrón}}{\text{átomos}}$$

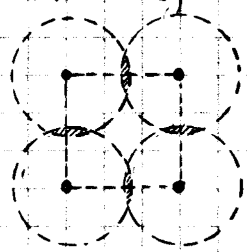
$$\boxed{1{,}33 \cdot 10^{22}\ \text{electrones por cm}^3}$$

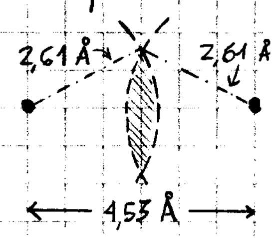
Se define r_s como el radio de la esfera cuyo volumen es igual al volumen por electrón de conducción

$$\frac{4}{3}\pi r_s^3 = \frac{1}{n} \quad \rightarrow \quad r_s = \left(\frac{3}{4\pi}\frac{1}{n}\right)^{1/3} \quad r_s = 2{,}61 \cdot 10^{-8}\ cm$$

$$\boxed{r_s = 2{,}61\ Å}$$

Comparando con la distancia a primeros vecinos 4,53 Å se ve que aproximadamente hay un electrón de conducción por ión con una leve superposición





ii)

$$\rho(77°K) = 1{,}38\ m\Omega.cm$$
$$\rho(273°K) = 6{,}1\ m\Omega.cm$$

$$\vec{E} = \rho\vec{J}$$

$$\vec{J} = -n\vec{v}e$$
$$\vec{J} = ne\frac{e\vec{E}\tau}{m}$$
$$\vec{J} = \frac{ne^2\tau}{m}\vec{E}$$

$$\frac{m\,kg}{s^2.C} = \frac{C}{s.m^2}$$

$$\frac{m^3\,kg}{s\,C^2} = [\rho]$$

$$\boxed{R = \frac{\rho.L}{A}} \leftarrow \text{conductor "lineal"}$$

$$\frac{m^{\cancel{2}}}{\cancel{m}}\frac{m^2\,kg}{s.C^2} = [\rho]$$

$$\boxed{\rho = \frac{m}{ne^2\tau}} \rightarrow \tau = \frac{m}{ne^2\rho}$$

$$\tau = \frac{9{,}1\ 10^{-31}\ kg}{1{,}33.10^{22}\frac{1}{cm^3}\ 2{,}56\ 10^{-38}\ C^2\ \frac{1{,}38}{1000.000}\ \Omega.\frac{1}{100}\ m} = \frac{9{,}1.10^{-31}\ kg}{1{,}33.10^{22}\ 10^{16}\frac{1}{m^3}\ 2{,}56.10^{-38}\ C^2\ \frac{138}{1000}\ \Omega.\ \frac{1}{100}\ m} = 1{,}9.10^{-13}\ s$$

$$\frac{m^3\,kg}{s\,C^2}$$

$$\tau(77°K) \cong 2.10^{-13}\ seg$$

NOTA

$$T = \frac{9{,}1.10^{-31}\,kg}{1{,}33.10^{22}\,10^{6}\,\frac{1}{m^3}\;256.10^{-38}\,C^2\;\frac{6{,}1}{1.10^{6}}\,\frac{m^2 kg}{s.C^2}\,.\,\frac{1}{100}} = 4{,}38.10^{-14}\,s$$

$$\boxed{T(273°K) \cong 4{,}1.10^{-14}\ seg}$$

$$\boxed{T(77°K) \cong 20.10^{-14}\ seg}$$

→ A mayor T el e^- viaja en promedio menor tiempo antes de sufrir una colisión. (Colisiones más frecuentes).

iii) $$\frac{1}{\cancel{2}} m v_0^2 = \frac{3}{\cancel{2}} k_b T$$

$$V_0 = \sqrt{\frac{3 k_b T}{m}} \qquad \left(\frac{3.\,1{,}38.10^{-23}\,\frac{J}{\cancel{K}}\,.\,77\cancel{°K}}{9{,}1.10^{-31}\,kg}\right) \qquad \frac{m^2}{s^2}\frac{\cancel{kg}}{\cancel{kg}}$$

$$\underset{\text{libre medio}}{\Lambda} = V_0 . T = \left(\frac{3 k_b T}{m}\right)^{1/2} . T \longrightarrow \Lambda(77°K) = 59.200\,\frac{m}{\cancel{s}}\,20.10^{-14}\,\cancel{s}$$

$$\boxed{\Lambda(77°K) = 1{,}2.10^{-8}\,m} \sim 1\,Å$$

$$\Lambda(273°K) = 111\,500\,\frac{m}{\cancel{s}}\,4{,}4.10^{-14}\,\cancel{s}$$

$$\boxed{\Lambda(273°K) = 4{,}9.10^{-9}\,m} \sim 0{,}5\,Å$$

Las distancias que viaja el electrón sin colisionar son del orden del Å y la velocidad de $\sim 10^8\,\frac{cm}{s}$ (0,03 % de la velocidad de la luz)

iv) $$R_H = -\frac{1}{nec} = -\frac{1}{1{,}33.10^{22}\,\frac{1}{cm^3}\;4{,}8.10^{-10}\,stC\;3.10^{10}\,\frac{cm}{s}} = \boxed{-5{,}22\,10^{-24}\,\frac{cm^2 s}{C}}$$

La constante Hall se halla solo 5% por sobre el valor experimental lo cual da una excelente aproximación

2.

i) $$\frac{1}{2} m v^2 = \frac{P^2}{2m} = \frac{\hbar^2 k^2}{2m} \rightarrow E = \sum_{k<k_F} \frac{\hbar^2 k^2}{2m}$$

$$1 = \frac{V}{8\pi^3}\,\delta k \rightarrow \qquad E = \sum_{k<k_F} \frac{\hbar^2 k^2}{2m}\,\delta k\,\frac{V}{8\pi^3}$$

$$N = 2.\frac{k_F^3 V}{6\pi^2} = \frac{k_F^3 V}{3\pi^2}$$

/ esféricas

$$E = \frac{V.2}{8\pi^3}\iiint^{k_F} \frac{\hbar^2 k^2}{2m}\,k^2 \operatorname{sen}\theta . dk\,d\theta . d\varphi$$

$$n = \frac{k_F^3}{3\pi^2}$$

$$\left(3\pi^2 \frac{N}{V}\right)^{1/3} = k_F$$

$$\frac{E}{V} = \frac{\cancel{2}\,.\,\cancel{4\pi}}{\cancel{8}\pi^{3\,2}}\int_0^{k_F} \frac{\hbar^2 k_F^4}{2m}\,dk$$

$$\frac{E}{V} = \frac{1}{\pi^2}\left(\frac{\hbar^2 k_F^2}{2m}\right)\frac{k_F^3}{5}$$

$$\frac{E}{V} = \frac{N}{V}\frac{3}{5}\epsilon_F \rightarrow \boxed{\frac{E_0}{N} = \frac{3}{5}\epsilon_F}$$

ii) $$dE = dQ - P.dV + \mu.dN$$

$$dE = T.dS - P.dV + \mu.dN \rightarrow \qquad P = -\left.\frac{\partial E}{\partial V}\right|_{N,S} \leftarrow \text{Termodinámica}$$

$$E = \frac{V}{\pi^2}.\frac{\hbar^2 k_F^5}{10\,m} = \frac{V}{\pi^2}\frac{\hbar^2}{10m}\left(3\pi^2\frac{N}{V}\right)^{5/3} = \frac{\hbar^2(3\pi^2 N)^{5/3}}{\pi^2 . 10m\;V^{2/3}}$$

$$\left.\frac{\partial E}{\partial V}\right|_N = \frac{\hbar^2(3\pi^2N)^{5/3}}{\pi^2\, 10m}\cdot\left(-\frac{2}{3}\right)\cdot\frac{1}{V^{5/3}} = -\frac{2}{3}\frac{\hbar^2}{\pi^2}\left(\frac{3\pi^2N}{V}\right)^{5/3}\frac{1}{10m} = -P$$

$$\frac{2}{3\pi^2}\left(\frac{\hbar^2 k_F^5}{2m}\right)\cdot\frac{1}{5} = P$$

$$\frac{2}{5}\epsilon_F\frac{k_F^3}{3\pi^2} = P$$

$$\frac{2}{5}\epsilon_F\frac{N}{V} = P$$

$$\boxed{P = \frac{2}{3}\left(\frac{E_0}{V}\right)} \leftarrow \frac{2}{3}\left(\frac{3}{5}\epsilon_F N\right)\frac{1}{V} = P$$

iii) Bulk modulus es $1/\varkappa$ donde $\varkappa$ = compresibilidad

$$B = \frac{1}{\varkappa} = -V\frac{\partial P}{\partial V}$$

$$B = -V\cdot\frac{\partial}{\partial V}\left(\frac{2}{3}\frac{\hbar^2}{\pi^2}(3\pi^2N)^{5/3}\frac{1}{10m}V^{-5/3}\right)$$

$$B = -V\left(-\frac{2}{3}\cdot\frac{5}{3}\frac{\hbar^2}{\pi^2}(3\pi^2N)^{5/3}\frac{1}{10m}\frac{1}{V^{8/3}}\right)$$

$$B = \frac{10}{9}\frac{\hbar^2}{\pi^2}\left(\frac{3\pi^2N}{V}\right)^{5/3}\frac{1}{10m}$$

$$B = \frac{2}{9}\frac{1}{\pi^2}\left(\frac{\hbar^2k_F^2}{2m}\right)\cdot k_F^3 = \frac{2}{9}\epsilon_F\frac{3N}{V}$$

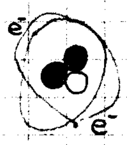
$$B = \frac{2}{9}\frac{5E_0}{V}$$

$$\boxed{B = \frac{5}{3}P} \leftarrow B = \frac{10}{9}\left(\frac{3}{2}P\right) \xleftarrow{\text{usando ii}} \boxed{B = \frac{10}{9}\left(\frac{E_0}{V}\right)}$$

3.

4.

i) ^3He



2p
1n

$81 \ kg/m^3$

$m_p, m_n \sim 1{,}6.10^{-27} \ kg$

$m_e \sim 9{,}1.10^{-31} \ kg$

1 átomo $^3He \rightarrow$ $4{,}8.10^{-27} \ kg$ (Núcleo)
$1{,}8.10^{-30} \ kg$ (nube)

$\rho(\text{electrones}) \cong 0{,}031 \ \frac{kg}{m^3}$

$n = \frac{\#\text{electrones}}{\text{Volumen}} \leftarrow \left(\text{masa electrones}\right) = 0{,}00038 \left(\text{masa núcleo}\right)$

$n = \frac{\text{masa electrones}}{\text{volumen}} \cdot \frac{1}{\text{masa 1 electrón}} = \frac{\rho}{m_e} \rightarrow n = \frac{N}{V} \cong 3{,}38 . 10^{28} \ \frac{1}{m^3}$

$n = \frac{k_F^3}{3\pi^2} \rightarrow k_F = (3\pi^2 n)^{1/3} \rightarrow T_F = \frac{\varepsilon_F}{k_B} = \frac{\hbar^2 k_F^2}{2m \, k_B}$

$T_F = \frac{\hbar^2}{2m_e} \frac{1}{k_B} (3\pi^2 n)^{2/3} \approx \boxed{11000 \ °K}$

ii) Neutron-STAR $\qquad m_n \sim 1{,}6.10^{-27} \ kg$

$\rho = 10^{17} \ \frac{kg}{m^3} \rightarrow n \cong 6{,}25.10^{43} \ \frac{1}{m^3}$

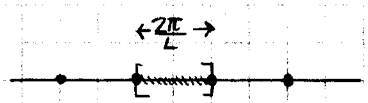
$T_F = \frac{\hbar^2 k_F^2}{2m \, k_B}$

$T_F = \frac{\hbar^2}{2m_p} \frac{1}{k_B} (3\pi^2 n)^{2/3} \approx \boxed{3{,}8.10^{11} \ °K}$

5.

Gas de electrones libres

i) * 1D: $\qquad k_x = \frac{2\pi}{L} n$



$\#$ puntos en un volumen Ω $= \frac{\Omega}{\frac{2\pi}{L}} = \frac{\Omega L}{2\pi}$ $\qquad$ Volumen total del espacio k $\quad \Omega = k_F$

$\#$ densidad de puntos en el espacio k $= \boxed{\frac{L}{2\pi}}$ ← niveles en el espacio k $\Rightarrow$ $\#$ electrones $= 2 . \Omega . \frac{L}{2\pi} = \frac{k_F L}{\pi} = N \rightarrow n = \frac{k_F}{\pi}$

(2 electrones por k)

$\varepsilon_F = \frac{\hbar^2 k_F^2}{2m}$ $\qquad$ $\varepsilon_F = \frac{\hbar^2 (n\pi)^2}{2m} = \frac{\hbar^2 \pi^2}{2m} n^2 \rightarrow n = \frac{\sqrt{2m\varepsilon_F}}{\hbar \pi}$

$\Delta \vec{k} = \Delta k = \frac{2\pi}{L}$

$$E = 2\sum_{k<k_F} \frac{\hbar^2 k_F^2}{2m} \longrightarrow \quad \Delta k = \frac{2\pi}{L} \quad \rightarrow \quad E = \frac{L \cdot 2}{2\pi} \sum_{k<k_F} \frac{\hbar^2 k^2}{2m} \Delta k \cdot f_{FD}$$

$$E = \frac{L \cdot \cancel{2}}{\cancel{2}\pi} \int_0^{k_F} \frac{\hbar^2 k^2}{2m}\, dk$$

→ **1D**: no hace falta pasar a esféricas

$$E = \frac{\cancel{2}L}{6\pi} \frac{\hbar^2 k_F^3}{\cancel{2}m} \rightarrow \frac{E}{L} = \epsilon = \frac{\hbar^2 k_F^3}{6\pi m}$$

Fundamental ($T=0$)

$$E = \cancel{2}\int \frac{\cancel{2}L}{\cancel{2}\pi} \frac{\hbar^2 k^2}{\cancel{2}m} f_{FD}\, dk$$

$$\frac{E}{L} = \epsilon = \int dk\, f_{FD} \cdot \frac{\hbar^2 k^2}{\pi m} \rightarrow \epsilon = \int \frac{\sqrt{2m}}{\cancel{2}\hbar} \frac{d\varepsilon}{\sqrt{\varepsilon}} \frac{\cancel{2m}\varepsilon}{\pi \cancel{m}} f_{FD}$$

$$\Rightarrow \boxed{g(\varepsilon) = \frac{\sqrt{2}\sqrt{m}}{\pi\hbar\sqrt{\varepsilon}}}$$

Por otro camino

$$N = 2\sum_k f_{FD}$$

$$N = 2\sum \frac{\Delta k}{2\pi} L$$

$$n = 2\int \frac{1}{2\pi} dk \quad \text{sphe.} \quad \frac{2}{\pi}\int_0^k dk$$

$$n = \frac{k_F 2}{\pi}$$

$$n = \frac{\sqrt{2m\varepsilon}}{\hbar\pi} 2$$

$$\frac{\hbar^2 k^2}{2m} = \varepsilon$$

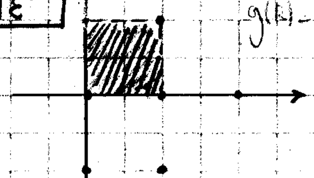
$$k = \frac{\sqrt{2m\varepsilon}}{\hbar}$$

$$dk = \frac{\sqrt{2m}}{2\hbar} \frac{1}{\sqrt{\varepsilon}} d\varepsilon$$

$$\frac{dn}{d\varepsilon} = \frac{\sqrt{2m}}{\hbar\pi} \frac{\cancel{2}}{\cancel{2}} \frac{1}{\sqrt{\varepsilon}} = \frac{\sqrt{2}\sqrt{m}}{\hbar\pi\sqrt{\varepsilon}}$$

ii. * 2D:

$$k_x = \frac{2\pi}{L} n_x \qquad k_y = \frac{2\pi}{L} n_y$$



$$g(k) = \frac{m}{\hbar^2 k \pi}$$

$$\#_{\text{puntos}} = \frac{\Omega}{\left(\frac{2\pi}{L}\right)^2} = \frac{\Omega L^2}{4\pi^2} \rightarrow$$

$$\boxed{\text{densidad de niveles en el k-spa} = \frac{L^2}{4\pi^2}}$$

$$\Omega = \pi k_F^2$$

$$N = 2\,\Omega \cdot \frac{L^2}{4\pi^2} = \frac{\cancel{2}\pi k_F^2 L^2}{\cancel{4}\pi^2} = \frac{k_F^2 L^2}{2\pi}$$

$$\frac{N}{L^2} = \frac{k_F^2}{2\pi} = n$$ ↘ densidad electrónica

$$\varepsilon_F = \frac{\hbar^2 n 2\pi}{2m} \rightarrow \varepsilon_F = \frac{\hbar^2 \pi}{m} n$$

$$N = 2\sum_k f_{FD}$$

$$N = 2\sum \frac{\Delta k}{(2\pi)} L^2 f_{FD}$$

$$\frac{N}{L^2} = 2\frac{1}{4\pi^2} \int d\vec{k}\, f_{FD}$$

$$n = \frac{2}{4\pi^2} \int\int k\,dk\, d\varphi\, f_{FD}$$

$$n = \frac{\cancel{2}\,\cancel{2\pi}}{\cancel{4}\pi^2} \int k\, dk\, f_{FD} = \int \frac{m}{\pi\hbar^2} d\varepsilon\, f_{FD}$$

$$n = \int d\varepsilon\, f_{FD} \left(\frac{m}{\pi\hbar^2}\right)$$

$$\frac{\hbar^2}{\cancel{2}m} \cancel{2}\, dk\, k = d\varepsilon$$

$$\frac{\hbar^2 k^2}{2m} = \varepsilon$$

$$\frac{\cancel{2}k\,dk\,\hbar^2}{\cancel{2}m} = d\varepsilon$$

$$dk = \frac{d\varepsilon\, m\, \cancel{\hbar}}{\hbar^{\cancel{2}} \sqrt{2m\varepsilon}} = \sqrt{\frac{m}{2\varepsilon}} \frac{d\varepsilon}{\hbar}$$

$$E = 2 \cdot \frac{L^2}{4\pi^2} \int_0^{\vec{k}_F} \frac{\hbar^2 k^2}{2m}\, d\vec{k}$$

Pasa a Polares (2D)

$$= 2 \cdot \frac{L^2}{4\pi^2} \int_0^{2\pi}\int_0^{k_F} \frac{\hbar^2 k^2}{2m} k \cdot dk\, d\varphi$$

$$E = \frac{\cancel{2}L^2 \cancel{2\pi}}{4\pi^{\cancel{2}}} \frac{\hbar^2}{\cancel{2}m} \frac{k_F^4}{4} = \frac{L^2 \hbar^2 k_F^4}{8\pi m}$$

$$\frac{E}{L^2} = \epsilon = \frac{\hbar^2 k_F^4}{8\pi m}$$

Fundamental ($T=0$)

$$E = \frac{\cancel{2}L^2}{\cancel{4}\pi^2} \cancel{2\pi} \int_0^{k'} \frac{\hbar^2 k^3}{2m}\, dk \longleftarrow \text{Con } T \neq 0$$

$$\frac{E}{L^2} = \int_0^\infty \frac{1}{\pi}\, \varepsilon \frac{\sqrt{\cancel{2}m\cancel{\varepsilon}}}{\hbar} \sqrt{\frac{m}{\cancel{2\varepsilon}}} \frac{1}{\hbar} f_{FD}\, d\varepsilon$$

$$\epsilon = \int_0^\infty \varepsilon \frac{m}{\hbar^2 \pi} f_{FD}\, d\varepsilon$$

$$\boxed{g(\varepsilon) = \frac{m}{\hbar^2 \pi}}$$

$$E = 2\sum_k \varepsilon(k) f(\varepsilon[k])$$ ↗ # de estados con energía ε

$$E = \frac{2L^d}{(2\pi)^d} \sum_k \varepsilon \cdot f_{FD}(\varepsilon)\, \delta^d k$$

$$\epsilon = \int \varepsilon \cdot f_{FD}(\varepsilon) \cdot g(\varepsilon)\, d\varepsilon$$

General

Hay que pasar $\delta^d k$ a δk (1D en esféricas) y luego a $\delta\varepsilon$

iii.

* $\underline{3D}$

$$\underset{\substack{\text{en el spa}\\ \text{k de Bloch}}}{\#\text{puntos}} = \frac{\Omega}{\left(\frac{2\pi}{L}\right)^3} = \frac{\Omega L^3}{8\pi^3}$$

$$\boxed{\text{densidad de estados en el espacio } k = \frac{L^3}{8\pi^3}}$$

$$\Omega = \frac{4}{3}\pi k_F^3$$

$$N = 2\left(\Omega \frac{L^3}{8\pi^3}\right) = \cancel{2}\,\frac{\cancel{4}}{3}\cancel{\pi}\, k_F^3 \frac{L^3}{\cancel{8}\pi^{\cancel{3}\,2}}$$

$$N = \frac{k_F^3 L^3}{3\pi^2}$$

$$n = \frac{k_F^3}{3\pi^2}$$

$$\varepsilon_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$$

$$\varepsilon = \frac{\hbar^2 k^2}{2m}$$

$$\frac{\sqrt{2m\varepsilon}}{\hbar} = k$$

$$\frac{\sqrt{2m}}{\hbar}\frac{1}{2\sqrt{\varepsilon}}\,d\varepsilon = dk$$

$$d\varepsilon = \frac{\hbar^2}{\cancel{2}m}\cancel{2}k\,dk$$

$$E = 2.\frac{L^3}{8\pi^3}\int_0^k \frac{\hbar^2 k^2}{2m}\,d\vec{k}$$

$$E = \frac{2\,L^3}{8\pi^3}\int_0^{2\pi}\int_0^{\pi}\int_0^{k_F} \frac{\hbar^2 k^4}{2m}\,\text{sen}\,\theta\, dk\, d\theta\, d\varphi$$

$$E = \frac{\cancel{2}.L^3.4\cancel{\pi}}{\cancel{8}\pi^{\cancel{3}\,2}}\;\frac{\hbar^2 k_F^5}{10\,m}$$

$$\frac{E}{L^3} = \frac{\hbar^2 k_F^5}{10\,\pi^2 m} \quad \text{fundamental } (T=0)$$

* Con el N

$$N = 2\sum_k f_{FD}.\frac{\Delta k\, L^3}{(2\pi)^3}$$

$$\frac{N}{L^3} = n = \frac{2}{8\pi^3}\int d\vec{k}\, f_{FD}$$

$$n = \frac{1}{4\pi^3}.4\pi\int k^2\, dk\, f_{FD}$$

$$n = \int \frac{1}{\pi^2} f_{FD}\,\frac{\cancel{2}m\cancel{\varepsilon}^{\sqrt{\varepsilon}}}{\hbar^2}\,\frac{\sqrt{2m}}{\hbar}\,\frac{1}{\cancel{2}}\frac{d\varepsilon}{\cancel{\sqrt{\varepsilon}}}$$

$$\boxed{g(\varepsilon) = \frac{m^{3/2}\, 2^{1/2}\,\varepsilon^{1/2}}{\pi^2\hbar^3}}$$

Para $T \neq 0 \longrightarrow$

$$E = 2.\frac{L^3}{8\pi^3}\,4\pi\int_0^k \frac{\hbar^2 k^4}{2m}\,dk\, f_{FD}$$

$$E = \frac{L^3.\cancel{8}\pi}{\cancel{8}\pi^{\cancel{3}\,2}}\int_0^\infty \varepsilon\left(\frac{\cancel{2}m\varepsilon}{\hbar^2}\right)\frac{\sqrt{2m}\,d\varepsilon}{\hbar\,\cancel{2}\sqrt{\varepsilon}}\, f_{FD}$$

$$E = \int_0^\infty \varepsilon.\frac{L^3}{\pi^2}\,\varepsilon^{1/2}\,\frac{2^{1/2}m^{3/2}}{\hbar^3}\, f_{FD}.\,d\varepsilon$$

$$\frac{E}{L^3} = \epsilon = \int_0^\infty \varepsilon.\frac{\sqrt{2m\varepsilon}.\,m}{\hbar^3\pi^2}\, f_{FD}.\,d\varepsilon$$

$$\boxed{g(\varepsilon) = \frac{m\sqrt{2m\varepsilon}}{\pi^2\hbar^3}}$$

6.

i) $N = \sum_\varepsilon \langle n_\varepsilon\rangle \longrightarrow N = \frac{L^d}{(2\pi)^d}\int_0^\infty 2.\,f_{FD}.\,d^dk$

a $T=0$ es $\frac{N}{L^d} = \frac{1}{(2\pi)^d}\,2\int_0^{k_F} d^dk$

Sea $d=2 \rightarrow$

$L^2 = A$

$$\frac{N}{A} = n = \frac{1}{2\pi^2}\int_0^{2\pi}\int_0^{k_F} k.\,dk.\,d\varphi$$

$$\frac{\hbar^2 k^2}{2m} = \varepsilon \rightarrow, \quad \frac{\hbar^2}{m}k\,dk = d\varepsilon$$

$$n = \frac{\cancel{2}\pi}{\cancel{2}\pi^2}\int_0^{k_F} k.\,dk$$

$$n = \frac{1}{\pi}\int_0^{\varepsilon_F} \frac{m}{\hbar^2}\, d\varepsilon = \frac{m\,\varepsilon_F}{\pi\hbar^2} \rightarrow \boxed{n = \frac{m.\varepsilon_F}{\pi\hbar^2}}$$

$$n = \frac{\cancel{m}}{2\pi\cancel{\hbar^2}} \cdot \frac{\cancel{\hbar^2} k_F^2}{2\cancel{m}} \rightarrow \boxed{n = \frac{k_F^2}{2\pi}}$$

ii)

$$E_F = \mu + k_B T\, \ln(1+e^{-\mu/k_B T})$$

$$n = \frac{1}{2\pi}\int_0^{\infty} \frac{m}{\hbar^2} f_{FD}\, d\varepsilon$$

$$n = \frac{m}{2\pi\hbar^2}\int_0^{\infty} \frac{d\varepsilon}{z^{-1}e^{\beta\varepsilon}+1}$$

$$\frac{d}{d(\beta\varepsilon)}\left[\ln(z\,e^{-\beta\varepsilon}+1)\right] = \frac{1}{z\,e^{-\beta\varepsilon}+1} \cdot z\,e^{-\beta\varepsilon} \cdot \beta =$$

$$= \frac{-z^{-1}e^{+\beta\varepsilon}(-z^{-1}e^{\beta\varepsilon}.\beta)}{1+z^{-1}e^{+\beta\varepsilon}} = \frac{-\beta}{z^{-1}e^{\beta\varepsilon}+1}$$

$$n = \frac{m}{2\hbar^2\pi}\int_0^{\infty} \frac{d}{d\varepsilon}\left(\ln\left[z\,e^{-\beta\varepsilon}+1\right]\right)\frac{1}{-\beta}\, d\varepsilon$$

$$n = -\frac{m}{\hbar^2\pi\beta}\ln(z\,e^{-\beta\varepsilon}+1)\Big|_0^{\infty}$$

$$n = -\frac{m}{\hbar^2\pi\beta}\left(-\ln(z+1)\right) = \frac{m}{\hbar^2\pi\beta}\ln(e^{\beta\mu}+1)$$

$$\frac{\cancel{m}\,\varepsilon_F}{\cancel{\pi\hbar^2}} = \frac{\cancel{m}}{\cancel{\hbar^2\pi}\beta}\ln(e^{\beta\mu}+1)$$

$$e^{\beta\varepsilon_F} = e^{\beta\mu}+1$$

$$e^{\beta\varepsilon_F}e^{-\beta\mu} = 1+e^{-\beta\mu}$$

$$e^{\beta(\varepsilon_F-\mu)} = 1+e^{-\beta\mu}$$

$$\frac{\varepsilon_F-\mu}{kT} = \ln(1+e^{-\beta\mu})$$

$$\boxed{\varepsilon_F = \mu + kT.\ln(1+e^{-\mu/kT})}$$

iii)