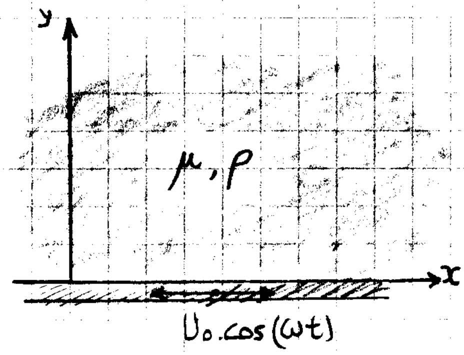
Practica 8

1.



c) La velocidad será una función:

$\vec{v} = v(y)\,\hat{x}$

No consideramos gravedad

Dado que hay simetría de plano ∞ en x, z

$\rightarrow \vec{v} \neq \vec{v}(x,z)$

ρ uniforme $\rightarrow$ $\operatorname{div}(\vec{v}) = 0$ $\quad \frac{\partial v_x}{\partial x} = 0$

$$\frac{\partial v}{\partial t} + \underbrace{v_x \frac{\partial}{\partial x} v(y)_x}_{=0} = -\frac{1}{\rho}\cdot\frac{\partial P}{\partial x} + \nu \frac{\partial^2 v}{\partial y^2}$$

$$\left.\begin{aligned} 0 &= -\frac{1}{\rho}\cdot\frac{\partial P}{\partial y} \\ 0 &= -\frac{1}{\rho}\cdot\frac{\partial P}{\partial z} \end{aligned}\right\} \Rightarrow P = P(x)$$

Pero en realidad $P =$ constante pues $\frac{\partial P}{\partial x} = f(y)$

$\rightarrow \frac{\partial P}{\partial x} = 0$

Nos queda la ecuación: $\longrightarrow$

ecuación de difusión: $\quad \frac{\partial v}{\partial t} = \nu \frac{\partial^2 v}{\partial y^2}$, CC $\begin{cases} v(y=0,t) = U_0 \cos(\omega t) \\ v(y\to\infty, t) = 0 \end{cases}$

Podemos pensar en una variable $\mathbb{C}$ con dependencia temporal armónica:

$$V(y,t) = U(y)\, e^{i\omega t}$$

$$i\omega U e^{i\omega t} = \nu e^{i\omega t} \frac{\partial^2 U}{\partial y^2} \qquad \begin{aligned} U(y=0) &= U_0 \\ U(y\to\infty) &= 0 \end{aligned}$$

$$i\omega U = \nu \cdot \frac{\partial^2 U}{\partial y^2}$$

$* \ \frac{\partial^2 U}{\partial y^2} - \frac{i\omega}{\nu} U = 0 \qquad U = A e^{\lambda y}$

$$A e^{\lambda y}\left(\lambda^2 - \frac{i\omega}{\nu}\right) = 0 \rightarrow \qquad \lambda = \pm\sqrt{\frac{i\omega}{\nu}} = \begin{cases} +\sqrt{\frac{\omega}{2\nu}} + i\sqrt{\frac{\omega}{2\nu}} \\ -\sqrt{\frac{\omega}{2\nu}} - i\sqrt{\frac{\omega}{2\nu}} \end{cases}$$

$$U = A_1 e^{\eta y} e^{i\eta y} + A_2 e^{-\eta y} e^{-i\eta y}$$

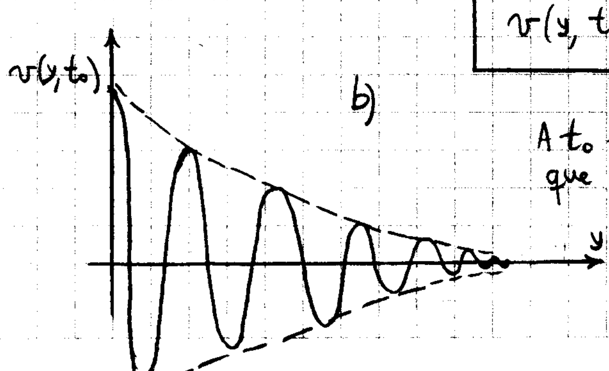
con $y \to \infty$ $\quad U = 0 \rightarrow A_1 = 0$

$$U(y=0) = U_0 = A_2 \quad \rightarrow \quad U = U_0 e^{-\eta y} e^{-i\eta y}$$

$$V(y,t) = U_0 \cdot e^{-\eta y} \cdot e^{-i(\eta y - \omega t)}$$

$$\boxed{v(y,t) = U_0\, e^{-\eta y} \cdot \cos[\eta y - \omega t]}$$

b)



A t_0 fijo el perfil de velocidades es una sinusoidal que decae exponencialmente con la distancia y.

A y_0 fijo es simplemente una oscilación sinusoidal en el tiempo.

a) La dependencia de la δ (distancia de penetración) la intuimos mirando la ecuación diferencial $\frac{\partial^2 U}{\partial y^2} - i\frac{\omega}{\nu} U = 0 \Rightarrow$

cc = U

$$\delta = f(\nu, \omega)$$, con lo cual:

	δ	ν	ω
L	1	2	0
M	0	0	0
T	0	-1	-1

$$3-2=1 \ \#\pi$$

$$\pi_1 = \delta\, \nu^a \omega^b$$

$$L^0 T^0 = L^1 L^{2a} T^{-a} T^{-b}$$

$$1+2a=0 \rightarrow a = -1/2$$
$$-a-b=0 \qquad b=-a=1/2$$

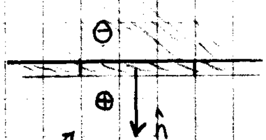
$$\pi = \delta \cdot \nu^{-1/2} \omega^{1/2} \rightarrow$$

$$\boxed{\delta = K\sqrt{\frac{\nu}{\omega}}}$$

Usando $\delta = f(\mu, \rho, \omega)$ se llega a la misma conclusión.

* Para la potencia media tendremos:

$$\text{Potencia} = \frac{dE}{dt} = \vec{F}\cdot\vec{v}$$



F del plano sobre el fluido usará esta normal

esfuerzos cortantes

$$dF = \overleftrightarrow{\sigma}\cdot\hat{n}\, dS = \tau_{\hat{n}}\, dS = -\mu \cdot \frac{dv}{dy}\Big|_{y=0} \cdot dS$$

$$P = \mu \cdot \frac{dv}{dy}\Big|_{y=0} \cdot \text{Area} \cdot v \quad , \text{ sea } \overline{P} = \frac{P}{\text{Area}}$$

$$\overline{P} = \mu \cdot U_0 \cdot \left(e^{-\eta y}(-\eta)\cos(\eta y - \omega t) - e^{-\eta y}\,\text{sen}(\eta y - \omega t)\,\eta\right)\Big|_{y=0}$$
$$U_0 e^{-\eta y} \cos(\eta y - \omega t)$$

$$= \mu \cdot U_0 \cdot \eta \cdot [\cos(-\omega t) - \text{sen}(-\omega t)] \cdot U_0 \cos(-\omega t)$$

Potencia instantánea por unidad de área $\longrightarrow$

$$P(t) = \mu U_0^2 \sqrt{\frac{\omega}{2\nu}}\left(\cos(\omega t) + \text{sen}(\omega t)\right)\cos(\omega t)$$

$$P(t) = \mu U_0^2 \sqrt{\frac{\omega}{2\nu}}\cos^2(\omega t) + \mu U_0^2 \sqrt{\frac{\omega}{2\nu}}\cos(\omega t)\cdot\text{sen}(\omega t)$$

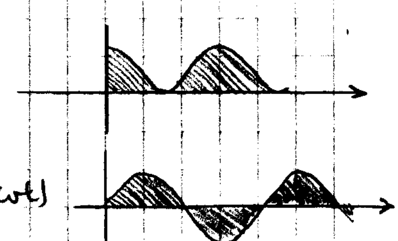
$$\langle P(t)\rangle = \mu U_0^2 \sqrt{\frac{\omega}{2\nu}}\langle\cos^2(\omega t)\rangle + \mu U_0^2\sqrt{\frac{\omega}{2\nu}}\underbrace{\langle\cos(\omega t)\cdot\text{sen}(\omega t)\rangle}_{=0}$$

$$\langle P(t)\rangle = \frac{1}{2}\mu U_0^2 \sqrt{\frac{\omega}{2\nu}}$$ Potencia promedio por unidad de área

Es la potencia media que el plano le entrega al fluido

$\cos^2(\omega t)$

$\cos(\omega t)\cdot\text{sen}(\omega t)$



* Utilizando análisis dimensional será

$$\overline{P} = f(\mu, \rho, \omega, U_0) \quad , \overline{P} \text{ potencia por unidad de área}$$

	P	μ	ρ	ω	U_0
L	0	-1	-3	0	1
M	1	1	1	0	0
T	-2	-1	0	-1	-1

① $5-3=2\ \#\pi$

	P	ν	ω	U_0	μ
L	0	2	0	1	-1
M	1	0	0	0	1
T	-2	-1	-1	-1	-1

② $5-3=2\ \#\pi$

Desde ②

Como en la ecuación de la velocidad figura agrupado $\sqrt{\frac{\omega}{2\nu}}$ preveemos que aparecerá en esa forma y pedimos

$$\pi_1 = P\ U_0^a \mu^b$$

$$L^0M^0T^0 = M^1T^{-2}\ L^aT^{-a}\ L^{-b}M^bT^{-b} \Rightarrow$$

$+a-b=0 \to a=b$

$1+b=0 \qquad b=-1$

$-2-a-b=0$

$a=-2-b$

$a=-2+1=-1$

$$\pi_1 = P\ U_0^{-1}\mu^{-1}$$

$$\pi_2 = \nu\ \omega^a U_0^b$$

$$L^0M^0T^0 = L^2T^{-1}\ T^{-a}\ L^bT^{-b} \rightarrow$$

$2+b=0 \to b=-2$

$-1-a-b=0 \qquad a=-1-b=-1+2$

$a=1$

$$\pi_2 = \nu\omega U_0^{-2}$$

$$\frac{P}{U_0\mu} = f\left(\frac{\nu\omega}{U_0^2}\right)$$

$$\boxed{P = U_0\mu \cdot f\left(\frac{\nu\omega}{U_0^2}\right)}$$

Desde ①

$$\pi_1 = \mu\rho^a\omega^bU_0^c \rightarrow L^0M^0T^0 = L^{-1}M^1T^{-1}\ L^{-3a}M^a\ T^{-b}\ L^cT^{-c}$$

$$\pi_1 = \mu\rho^{-1}\omega U_0^{-2}$$

$c-1-3a=0 \qquad c=-3+1=-2$

$1+a=0 \to a=-1$

$-c-1-b=0 \qquad 2-1=b=1$

$$\pi_2 = P\ U_0^a\mu^b \rightarrow L^0M^0T^0 = M^1T^{-2}\ L^aT^{-a}\ L^{-b}M^bT^{-b}$$

$$\pi_2 = P\ U_0^{-1}\mu^{-1}$$

$1+b=0 \to b=-1$

$-2-a-b=0 \qquad -2+1+1=0$

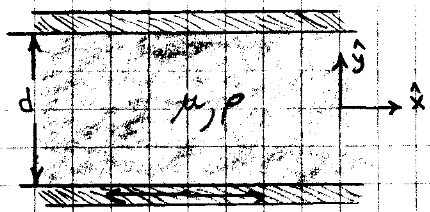
$a-b=0 \qquad a=b=-1$

$$\frac{P}{U_0\mu} = f\left(\frac{\mu\omega}{\rho U_0^2}\right)$$

$$\boxed{P = U_0\mu\ f\left(\frac{\mu\omega}{\rho U_0^2}\right)}$$

Como se ve, han sido obtenidos los mismos resultados con el subgrupo 1 que con el subgrupo 2.

2.



Fluido muy viscoso $\quad \dfrac{|(\vec{v}\cdot \text{grad})\vec{v}|}{|\nu \nabla^2 \vec{v}|} \ll 1$

→ $\# R_e \sim 0$

→ tiro el término convectivo

Desprecio efectos gravitatorios

$$\vec{v} = v(y)\hat{x}$$

$$\frac{\partial v}{\partial t} = \nu \frac{\partial^2 v}{\partial y^2} \qquad CC = \begin{cases} v(y=0,t) = U_0 \cos(\omega t) \\ v(y=d,t) = 0 \end{cases}$$

La CC en $y=d$ cambia viéndose forzada a ser $v=0$ por la condición de No deslizamiento.
Hay dos casos:

① $\delta \propto \sqrt{\frac{\nu}{\omega}} \gg d$ ② $\delta \propto \sqrt{\frac{\nu}{\omega}} \ll d$

ω baja / penetración alta — ω alta / penetración pequeña

Planteamos la ecuación $\dfrac{\partial^2 U}{\partial y^2} - \dfrac{i\omega}{\nu} U = 0$ con $V = U(y)\, e^{i\omega t}$ $\quad CC = \begin{cases} U(y=0) = U_0 \\ U(y=d) = 0 \end{cases}$

$$U = A_1 e^{\eta y} e^{i\eta y} + A_2 e^{-\eta y} e^{-i\eta y}$$

$$\text{con} \quad \eta = \sqrt{\frac{\omega}{2\nu}} = \frac{1}{\sqrt{2}\,\delta}$$

$$U(y=d) = A_1 e^{\eta d} e^{i\eta d} + A_2 e^{-\eta d} e^{-i\eta d} = 0$$

$$\eta d = \frac{d}{\sqrt{2}\,\delta}$$

→ caso ① $\quad 1 \gg d/\delta \quad \rightarrow \begin{cases} e^{\eta d} \rightarrow 1 & A_1 \\ e^{-\eta d} \rightarrow 1 & A_2 \end{cases}$

→ caso ② $\quad 1 \ll d/\delta \quad \rightarrow \begin{cases} e^{\eta d} \rightarrow \infty \rightarrow A_1 = 0 \\ e^{-\eta d} \rightarrow 0 \rightarrow A_2 \neq 0 \end{cases}$

Para el caso ② la solución es igual que la del ejercicio 1, porque el fluido no llega a enterarse del plano cerrando en $y=d$. (Al menos el fluido que se moverá). En ese caso:

$$\boxed{v_{caso\,2} = U_0\, e^{\frac{1}{\sqrt{2}\delta} y} \cdot \cos\left(\frac{y}{\sqrt{2}\,\delta} - \omega t\right)}$$

Para el caso ①

$$e^{\eta d} A_1 e^{i\eta d} + A_2 e^{-i\eta d} e^{-\eta d} = 0$$

$$A_1 + A_2 = U_0$$

$A_1 \cos(\eta d + \phi_1) + A_2 \cos(\eta d + \phi_2) = 0$

$A_1 \operatorname{sen}(\eta d + \phi_1) - A_2 \operatorname{sen}(\eta d + \phi_2) = 0$

$A_1 \cos\phi_1 + A_2 \cos\phi_2 = U_0$

$A_1 \operatorname{sen}\phi_1 + A_2 \operatorname{sen}\phi_2 = 0$

$$|A_1| e^{\eta d} e^{i\eta d} e^{i\phi_1} + |A_2| e^{-\eta d} e^{-i\eta d} e^{i\phi_2} = 0$$

$$|A_1| e^{i\phi_1} + |A_2| e^{i\phi_2} = U_0$$

$$\Rightarrow \quad |A_1| = U_0 e^{-i\phi_1} - |A_2| e^{i\phi_2} e^{-i\phi_1}$$

$$U_0 e^{\eta d} e^{i\eta d} - |A_2| e^{i\phi_2} e^{\eta d} e^{i\eta d} + |A_2| e^{i\phi_2} e^{-\eta d} e^{-i\eta d} = 0$$

$$|A_2| = \frac{-U_0 . e^{i\eta d} e^{\eta d}}{e^{i\phi_2}\left[e^{i\eta d} e^{\eta d} + e^{-i\eta d} e^{-\eta d}\right]}$$

$$|A_1| = U_0 . e^{-i\phi_1} - \frac{-U_0 e^{i\eta d}\, \cancel{e^{i\phi_2}} . e^{-i\phi_1} e^{\eta d}}{\cancel{e^{i\phi_2}}\left[e^{i\eta d} e^{\eta d} + e^{-i\eta d} e^{-\eta d}\right]}$$

$$|A_1| = U_0 . e^{-i\phi_1}\left[1 + \frac{e^{i\eta d} e^{\eta d}}{[\quad]}\right]$$

$$|A_1| = \frac{U_0\, e^{-i\eta d} . e^{-\eta d} . e^{-i\phi_1}}{\left[e^{i\eta d} . e^{\eta d} + e^{-i\eta d} . e^{-\eta d}\right]}$$

$$\vec{U}_{CASO\,1} = \frac{U_0\, e^{-\eta d - i\eta d}}{\left[e^{i\eta d} e^{\eta d} + e^{-i\eta d} . e^{-\eta d}\right]} . e^{\eta y + i\eta y} - \frac{U_0 . e^{+\eta d + i\eta d}}{\left[e^{i\eta d} . e^{\eta d} + e^{-i\eta d} . e^{-\eta d}\right]} . e^{-\eta y - i\eta y}$$

$$\boxed{\vec{v}_{CASO\,1} = Re\left\{\frac{U_0}{\left[e^{i\eta d} . e^{\eta d} + e^{-i\eta d} . e^{-\eta d}\right]}\left(e^{-\eta d - i\eta d} . e^{\eta y + i\eta y} - e^{\eta d + i\eta d} . e^{-\eta y - i\eta y}\right) e^{i\omega t}\right\}}$$

Podemos probar algunos límites

Si $\eta d \ll 1 \rightarrow e^{\eta d} \rightarrow 1$ $\quad e^{\pm i\eta d} \rightarrow 1$ $\qquad \eta d \rightarrow 0$ $\qquad \frac{\cancel{e^{-i\eta d}}\, \cancel{e^{-\eta d}}}{-e^{2i\eta d} e^{2\eta d} + \cancel{e^{-i\eta d}} . \cancel{e^{-\eta d}}}$ (1, 1)

$e^{-\eta d} \rightarrow 1$

$$\left[\left(\frac{U_0}{-e^{2i\eta d + 2\eta d} + 1}\right) e^{\eta y + i\eta y} - \left(\frac{U_0}{-1 + e^{-2i\eta d - 2\eta d}}\right) e^{-\eta y - i\eta y}\right] e^{i\omega t}$$

Si $\eta d \gg 1 \rightarrow e^{\eta d} \rightarrow \infty$

$e^{-\eta d} \rightarrow 0$

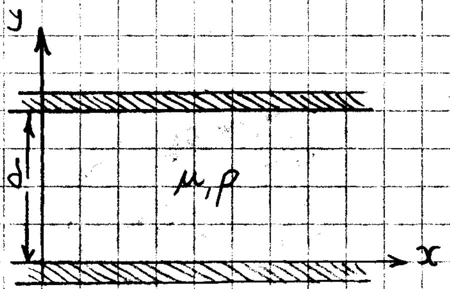
$$Re\left\{U_0\left(-e^{-\eta y - i\eta y}\right) . \left(\cos(\omega t) + i . sen(\omega t)\right)\right\}$$

$$Re\left\{U_0 e^{-\eta y} . \left[-\cos(\eta y) + i . sen(\eta y)\right]\left(\cos(\omega t) + i . sen(\omega t)\right)\right\}$$

$$U_0 . e^{-\eta y}\left[-\cos(\eta y) . \cos(\omega t) - sen(\eta y) . sen(\omega t)\right]$$

$$v_{CASO_2} \cong -U_0 . e^{-\eta y} . \cos(\eta y - \omega t)$$

3.



Los planos son infinitos $\rightarrow \vec{v} \neq \vec{v}(x)$
$\rightarrow \vec{v} \neq \vec{v}(z)$

$\mathrm{grad}(p) = b\cos(\omega t)$, b constante

Tendremos incompresibilidad $\Rightarrow \mathrm{div}(\vec{v}) = 0$
$\frac{\partial v_y}{\partial y} = 0 \rightarrow$

$$\vec{v} = v(y,t)\,\hat{x}$$

$\hat{x})$ $$\frac{\partial v(y,t)}{\partial t} + \underbrace{(\vec{v}(y)\cdot \mathrm{grad})\,v(y,t)}_{= v_x \frac{\partial}{\partial x} v(y,t) = 0} = -\frac{1}{\rho} b\cos(\omega t) + \nu \frac{\partial^2 v(y,t)}{\partial y^2}$$

La ecuación a resolver $\Big\} \longrightarrow$ $$\boxed{\frac{\partial v}{\partial t} = -\frac{b}{\rho}\cos(\omega t) + \nu \frac{\partial^2 v}{\partial y^2}}$$

$$CC = \begin{cases} v(0,t) = 0 \\ v(d,t) = 0 \end{cases}$$ $\rightarrow$ Tengo condiciones de contorno homogéneas pero la ecuación es No homogénea.

Ensayemos $V(y,t) = U(y)\, e^{i\omega t} \rightarrow$ $$i\omega U e^{i\omega t} = -\frac{b}{\rho}\cos(\omega t) + \nu \frac{\partial^2 U}{\partial y^2} e^{i\omega t}$$

tal que $v = \mathrm{Re}\{V(y,t)\}$ $$i\omega U e^{i\omega t} = -\frac{b}{\rho} 2 (e^{i\omega t} + e^{-i\omega t}) + \nu \frac{\partial^2 U}{\partial y^2} e^{i\omega t}$$

Esto no ayuda, pero da idea de que $U = U_1 e^{i\omega t} + U_2 e^{-i\omega t}$ podría funcionar $\Rightarrow$

$$i\omega U_1 e^{i\omega t} - i\omega U_2 e^{-i\omega t} = -\frac{b2}{\rho} e^{i\omega t} - \frac{b2}{\rho} e^{-i\omega t} + \nu \frac{\partial^2 U_1}{\partial y^2} e^{i\omega t} + \nu \frac{\partial^2 U_2}{\partial y^2} e^{-i\omega t}$$

$$i\omega U_1 \cancel{e^{i\omega t}} = -\frac{2b}{\rho} \cancel{e^{i\omega t}} + \nu \frac{\partial^2 U_1}{\partial y^2} \cancel{e^{i\omega t}}$$

CC: $U(y=0) = 0$, $U(y=d) = 0$

① $$i\omega U_1 = -\frac{2b}{\rho} + \nu \frac{\partial^2 U_1}{\partial y^2}$$

② $$-i\omega U_2 = -\frac{2b}{\rho} + \nu \frac{\partial^2 U_2}{\partial y^2}$$

$$CC \begin{cases} U_1(y=0) = 0 = U_2(y=0) \\ U_1(y=d) = 0 = U_2(y=d) \end{cases}$$

① $$\frac{\partial^2 U_1}{\partial y^2} - \frac{i\omega}{\nu} U_1 = \frac{2b}{\rho\nu}$$ $\lambda^2 - \frac{i\omega}{\nu} = 0$ $\lambda = \pm(1+i)\sqrt{\frac{\omega}{2\nu}}$ $\rightarrow$ $\eta \equiv \sqrt{\frac{\omega}{2\nu}}$

$$U_1 = A_1 e^{\eta y} e^{i\eta y} + A_2 e^{-\eta y} e^{-i\eta y} + \frac{+2bi}{\rho\omega}$$

$$\left.\begin{aligned} U_1(y=d) &= 0 = |A_1| e^{\eta d} e^{i\eta d} e^{i\phi_1} + |A_2| e^{-\eta d} e^{-i\eta d} e^{i\phi_2} + \frac{2bi}{\rho\omega} \\ U_1(y=0) &= 0 = |A_1| e^{i\phi_1} + |A_2| e^{i\phi_2} + \frac{2bi}{\rho\omega} \end{aligned}\right\} \Rightarrow$$

$$|A_1| e^{i\phi_1} = -|A_2| e^{i\phi_2} - \frac{2bi}{\rho\omega}$$

$$-\frac{2bi}{\rho\omega} e^{\eta d} e^{i\eta d} - |A_2| e^{i\phi_2} e^{\eta d} e^{i\eta d} + |A_2| e^{i\phi_2} e^{-\eta d} e^{-i\eta d} = 0$$

$$|A_2| e^{i\phi_2}(-e^{\eta d} e^{i\eta d} + e^{-\eta d} e^{-i\eta d}) = \frac{2bi}{\rho\omega} e^{\eta d} e^{i\eta d}$$

$\Rightarrow$

$$|A_2|e^{i\phi_2} = \frac{2bi}{\rho\omega}\,\frac{e^{\eta d}e^{i\eta d}}{[e^{-\eta d}e^{-i\eta d}-e^{\eta d}e^{i\eta d}]}$$

$$|A_1|e^{i\phi_1} = -\frac{2bi}{\rho\omega}\,\frac{e^{\eta d}e^{i\eta d}}{[\qquad\qquad]} - \frac{2bi}{\rho\omega}$$

$$|A_1|e^{i\phi_1} = -\frac{2bi}{\rho\omega}\cdot\frac{e^{-\eta d}e^{-i\eta d}}{[e^{-\eta d}e^{-i\eta d}-e^{\eta d}e^{i\eta d}]}$$

$$U_1 = \frac{2bi}{\rho\omega}\cdot\frac{1}{[e^{-\eta d}e^{-i\eta d}-e^{\eta d}e^{i\eta d}]}\left(-e^{\eta y}e^{i\eta y}e^{-\eta d}e^{-i\eta d} + e^{\eta d}e^{i\eta d}e^{-\eta y}e^{-i\eta y}\right) + \frac{2bi}{\rho\omega}$$

$$v_1 = \mathrm{Re}\left\{U_1\, e^{i\omega t}\right\}$$

$$\lambda^2 = -\frac{i\omega}{D} \rightarrow \quad \lambda = \sqrt{-\frac{i\omega}{D}} = \pm i\,(1+i)\sqrt{\frac{\omega}{2D}} = \pm(1-i)\sqrt{\frac{\omega}{2D}} = \pm(1-i)\,\eta$$

$$U_2 = A_1 e^{\eta y}e^{-i\eta y} + A_2 e^{-\eta y}e^{i\eta y} - \frac{2b}{\rho\omega}i$$

$$U_2(y=d) = A_1 e^{\eta d}e^{-i\eta d} + A_2 e^{-\eta d}e^{+i\eta d} - \frac{2bi}{\rho\omega} = 0$$

$$U_2(y=0) = |A_1|e^{i\phi_1} + |A_2|e^{i\phi_2} - \frac{2bi}{\rho\omega} = 0$$

$$|A_1|e^{i\phi_1} = -|A_2|e^{i\phi_2} + \frac{2bi}{\rho\omega}$$

$$\frac{2bi}{\rho\omega}e^{\eta d}e^{-i\eta d} - A_2 e^{\eta d}e^{-i\eta d} + A_2 e^{-\eta d}e^{+i\eta d} - \frac{2bi}{\rho\omega} = 0$$

$$A_2 = \frac{2bi}{\rho\omega}\,\frac{[1-e^{\eta d}e^{-i\eta d}]}{[e^{-\eta d}e^{i\eta d}-e^{\eta d}e^{-i\eta d}]}$$

$$A_1 = \frac{2bi}{\rho\omega}\,\frac{[e^{\eta d}e^{-i\eta d}-1]}{[\qquad - \qquad]} + \frac{2bi}{\rho\omega}$$

$$A_1 = \frac{2bi}{\rho\omega}\left[\frac{[\qquad\qquad]}{\qquad} + 1\right]$$

$$\left[\cancel{e^{\eta d}e^{-i\eta d}} - 1 + e^{-\eta d}e^{i\eta d} - \cancel{e^{\eta d}e^{-i\eta d}}\right]$$

$$A_1 = \frac{2bi}{\rho\omega}\cdot\frac{(e^{-\eta d}e^{i\eta d}-1)}{[e^{-\eta d}e^{i\eta d}-e^{\eta d}e^{-i\eta d}]}$$

$$U_2 = \frac{2bi}{\rho\omega}\cdot\frac{1}{[e^{-\eta d}e^{i\eta d}-e^{\eta d}e^{-i\eta d}]}\left[(e^{-\eta d}e^{i\eta d}-1)\,e^{\eta y}e^{-i\eta y} + (1-e^{\eta d}e^{-i\eta d})\,e^{-\eta y}e^{i\eta y}\right] - \frac{2bi}{\rho\omega}$$

$$v_2 = \mathrm{Re}\left\{U_2\, e^{-i\omega t}\right\}$$

$$\frac{\partial v}{\partial t} = -\frac{b}{\rho}\cdot\cos(\omega t) + \nu\cdot\frac{\partial^2 v}{\partial y^2}$$

si tomamos $v = \mathcal{Re}\left\{ \mathbb{V} = U.e^{i\omega t} \right\} \rightarrow$ resulta :

$$i\omega\, U e^{i\omega t} = -\frac{b}{\rho}.e^{i\omega t} + \nu.e^{i\omega t}\frac{\partial^2 U}{\partial y^2} \rightarrow i\omega U = -\frac{b}{\rho} + \nu.\frac{\partial^2 U}{\partial y^2}$$

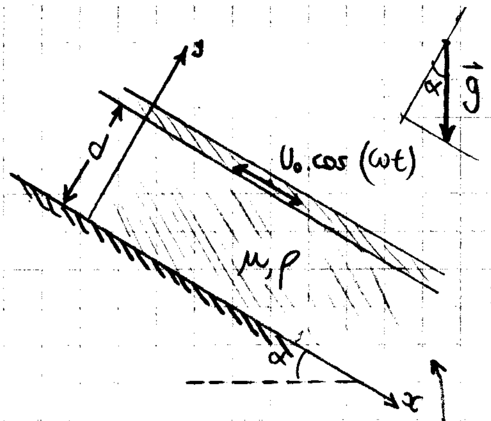
si le tomamos parte real es

$$-\operatorname{sen}(\omega t).\,U\omega = -\frac{b}{\rho}\cos(\omega t) + \nu.\cos(\omega t)\frac{\partial^2 U}{\partial y^2}$$

$$-\operatorname{sen}(\omega t)(U+iV)\omega = -\frac{b}{\rho}\cos(\omega t) + \nu.\cos(\omega t).\left[\frac{\partial^2 U}{\partial y^2} + i\frac{\partial^2 V}{\partial y^2}\right]$$

$$-\operatorname{sen}(\omega t)\,U\omega = -\frac{b}{\rho}\cos(\omega t) + \nu.\cos(\omega t)\frac{\partial^2 U}{\partial y^2}$$

4.



La velocidad del fluido será:

$$\vec{v} = v(y)\,\hat{x}$$

$$\hat{x})\quad \frac{\partial v}{\partial t} = -\frac{1}{\rho}\cdot\frac{\partial P}{\partial x} + \nu\cdot\frac{\partial^2 v}{\partial y^2} + g\,\text{sen}\,\alpha\;\hat{x}$$

$$0 = -\frac{1}{\rho}\cdot\frac{\partial P}{\partial y} - g.\cos\alpha \;,\; 0 = -\frac{1}{\rho}\cdot\frac{\partial P}{\partial z}$$

Al estar abierto el fondo del plano consideramos que no hay gradientes de presión externos (en $\hat{x}$)

$\Rightarrow \dfrac{\partial P}{\partial x} = 0$

$$\begin{cases} P = P(x,y) \text{ pero} \\ \dfrac{\partial P}{\partial y} = -\rho.g.\cos\alpha \\ P = -\rho.g.\cos\alpha.y + f(x) \end{cases}$$

a) Usando todo esto llegamos al siguiente sistema:

$$\text{CC}\quad \begin{cases} v(y=d,t) = U_0.\cos(\omega t) \\ v(y=0,t) = 0 \end{cases} \qquad \boxed{\frac{\partial v}{\partial t} = \nu.\frac{\partial^2 v}{\partial y^2} + g.\text{sen}\,\alpha}$$

$$\text{CI}\quad \{\, v(y=d,0) = U_0$$

Supongo que la longitud de penetración δ es tal que $\delta \ll d \rightarrow$
$v(y=0,t) = 0$

$\boxed{1}$ Probamos una variable compleja $\mathbb{V}(y,t) = U(y)\,e^{i\omega t}$ con $v = \text{Re}\{\mathbb{V}(y,t)\}$

$$i\omega U\,e^{i\omega t} = \nu\,e^{i\omega t}\frac{\partial^2 U}{\partial y^2} + g.\text{sen}\,\alpha \qquad \begin{matrix} U(y=0) = 0 \\ U(y=d) = U_0 \end{matrix}$$

ahora consideramos $U = U_1(y)\,e^{i\omega t} + U_2(y) \Rightarrow$

$$i\omega U_1 e^{i\omega t} = \nu.e^{i\omega t}\frac{\partial^2 U_1}{\partial y^2} + \nu.\frac{\partial^2 U_2}{\partial y^2} + g.\text{sen}\,\alpha$$

se parte en:

$$① \quad i\omega U_1 = \nu.\frac{\partial^2 U_1}{\partial y^2} \qquad \text{CC}\begin{cases} U_1(y=0) = 0 \\ U_1(y=d) = U_0 \end{cases}$$

$$② \quad -g.\text{sen}\,\alpha = \nu.\frac{\partial^2 U_2}{\partial y^2} \qquad \text{CC}\begin{cases} U_2(y=0) = 0 \\ U_2(y=d) = 0 \end{cases}$$

②

$$\frac{\partial U_2}{\partial y} = -g/_{\nu}.\text{sen}\,\alpha.y + C$$

$$U_2 = -g/_{\nu}.\text{sen}\,\alpha\,\frac{y^2}{2} + Cy + D$$

$$D = 0 \qquad 0 = -\frac{g}{2\nu}\,\text{sen}\,\alpha\,d^2 + Cd$$

$$C = \frac{g}{2\nu}\,\text{sen}\,\alpha\,d$$

$$U_2 = -g/_{2\nu}\,\text{sen}\,\alpha\,(y^2 - dy)$$

①

$$\frac{\partial^2 U_1}{\partial y^2} = \frac{i\omega}{\nu}U_1 \qquad \begin{matrix} U_1 = A.e^{\lambda y} \\ U_1' = A.e^{\lambda y}.\lambda \\ U_1'' = A.\lambda^2 e^{\lambda y} \end{matrix}$$

$$Ae^{\lambda y}\left(\lambda^2 - \frac{i\omega}{\nu}\right) = 0$$

$$\lambda = \pm\sqrt{\frac{i\omega}{\nu}} = \pm\,\square = \pm(1+i)\sqrt{\frac{\omega}{2\nu}} = \pm\sqrt{\ }\pm i\sqrt{\ }$$

$$U_1 = A_1 e^{+\square y} + A_2 e^{-\square y}$$

$$U_1 = |A_1|\,e^{+\sqrt{\ }y}e^{+i\sqrt{\ }y} + |A_2|\,e^{-\sqrt{\ }y}e^{-i\sqrt{\ }y}$$

$$A_1 = -A_2 \rightarrow$$

$$U_1 = A.\left[e^{+\sqrt{\ }y}e^{+i\sqrt{\ }y} - e^{-\sqrt{\ }y}e^{-i\sqrt{\ }y}\right]$$

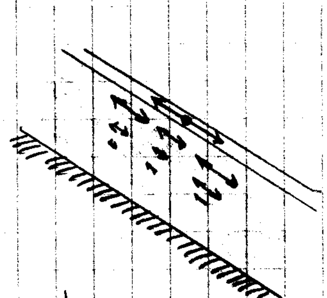
$$U_0 = A.\left(e^{(1+i)\sqrt{\frac{\omega}{2\nu}}\,d} - e^{-(1+i)\sqrt{\frac{\omega}{2\nu}}\,d} \right)$$

$$\boxed{U = \frac{U_0\, e^{i\omega t}}{e^{(1+i)\sqrt{\frac{\omega}{2\nu}}\,d} - e^{-(1+i)\sqrt{\frac{\omega}{2\nu}}\,d}} \cdot \left(e^{(1+i)\sqrt{\frac{\omega}{2\nu}}\,y} - e^{-(1+i)\sqrt{\frac{\omega}{2\nu}}\,y} \right) - \frac{g}{2\nu}\cdot \operatorname{sen}\alpha \left[y^2 - y.d \right]}$$

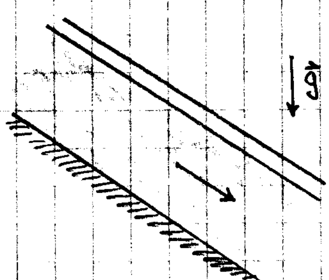
$$U = U_1.e^{i\omega t} + U_2$$

$$\boxed{v(y,t) = \mathcal{Re}\{U\}}$$

AL separar la ecuación en dos ecuaciones hemos realizado la superposición mostrada abajo.



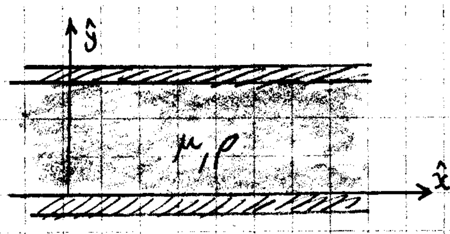
no estacionario con dependencia temporal armónica (el fluido se sacude)



estacionario (el fluido cae por gravedad)

La razón que permite hacer esto es la linealidad de las ecuaciones y de las condiciones de contorno.

5.



Ahora la ecuación a resolver es

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho} a - \frac{1}{\rho} b \cos(\omega t) + \nu \cdot \frac{\partial^2 v}{\partial y^2}$$

$$CC \begin{cases} v(y=0, t) = 0 \\ v(y=d, t) = 0 \end{cases} \qquad \text{con} \quad \vec{v} = v(y,t)\hat{x}$$

Ensayaremos $\quad \mathbb{V}(y,t) = U(y) . e^{i\omega t}$

$$i\omega\, U . e^{i\omega t} = -\frac{1}{\rho} a - \frac{1}{\rho} b\, 2 \left(e^{i\omega t} + e^{-i\omega t}\right) + \nu\, e^{i\omega t} \frac{\partial^2 U}{\partial y^2}$$

Esto no parece funcionar, entonces probamos:

$$U = U_1(y) . e^{i\omega t} + U_2(y) . e^{-i\omega t} + U_3(y)$$

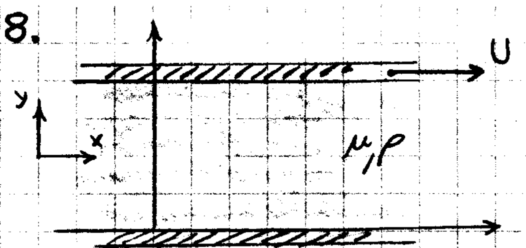
$$i\omega\, U_1 e^{i\omega t} - i\omega\, U_2 e^{-i\omega t} = -\frac{a}{\rho} - \frac{2b}{\rho} e^{i\omega t} - \frac{2b}{\rho} e^{-i\omega t} + \nu e^{i\omega t} \frac{\partial^2 U_1}{\partial y^2} + \nu e^{-i\omega t} \frac{\partial^2 U_2}{\partial y^2} + \nu . \frac{\partial^2 U_3}{\partial y^2}$$

① $\quad i\omega\, U_1 e^{i\omega t} = -\frac{2b}{\rho} e^{i\omega t} + \nu . e^{i\omega t} \frac{\partial^2 U_1}{\partial y^2} \qquad \begin{cases} U_1(y=0) = 0 \\ U_1(y=d) = 0 \end{cases}$

② $\quad -i\omega\, U_2 e^{-i\omega t} = -\frac{2b}{\rho} e^{-i\omega t} + \nu . e^{-i\omega t} \frac{\partial^2 U_2}{\partial y^2} \qquad \begin{cases} U_2(y=0) = 0 \\ U_2(y=d) = 0 \end{cases}$

③ $\quad \frac{a}{\rho} = \nu . \frac{\partial^2 U_3}{\partial y^2} \qquad \begin{cases} U_3(y=0) = 0 \\ U_3(y=0) = 0 \end{cases}$

8.



fluido muy viscoso $\Rightarrow$ #Re chico $\rightarrow$ Flujo laminar

$$\frac{|(v\cdot\text{grad})v|}{|\nu\,\nabla^2 v|} \ll 1 \rightarrow Re \ll 1$$

No considero gravedad.

$$\vec{v} = v(y)\,\hat{x}$$

$$\hat{x})\quad \frac{\partial v}{\partial t} = -\frac{1}{\rho}\cdot\frac{\partial P}{\partial x} + \nu\frac{\partial^2 v}{\partial y^2}$$

$$\frac{\partial P}{\partial y} = 0$$

$$\frac{\partial P}{\partial z} = 0$$

CC

$v(y=d, t) = U \qquad t>0$

$v(y=0, t) = 0 \qquad t\geq 0$

Pero $P \neq P(x)$ por la infinitud del plano, y porque no hay bombas en la dirección $\hat{x}$

$$\frac{\partial v}{\partial t} = \nu\frac{\partial^2 v}{\partial y^2}$$

No podemos usar $V - U.e^{i\omega t}$ porque no hay dependencia armónica de las CC, o en la ecuación.

Pensamos $\quad v(y,t) = v_1(y).\,v_2(t)$

$$v_1.\frac{\partial v_2}{\partial t} = \nu\, v_2.\frac{\partial^2 v_1}{\partial y^2}$$

Sin embargo esta elección no me permite separar las condiciones de contorno.

Entonces postulamos

$$v(y,t) = v_1(y) + v_2(y,t)$$

$$\frac{\partial v_2}{\partial t} = \nu\frac{\partial^2 v_1}{\partial y^2} + \nu.\frac{\partial^2 v_2}{\partial y^2}$$

* CC

estac.	transitoria
$v_1(0)=0$	$v_2(0,t)=0$
$v_1(d)=U$	$v_2(d,t)=0$

busco que sean CC homogéneas en la que depende del t.

$\Rightarrow$ Le puedo aplicar separación de variables

* estacionaria

$$0 = \nu.\frac{\partial^2 v_1}{\partial y^2}$$

$$v_1 = A.y + B$$

$$\left.\begin{array}{l} B=0 \\ \frac{U}{d}=A \end{array}\right\} \rightarrow v_1 = \frac{U}{d}.y$$

* transitoria

$$\frac{\partial v_2}{\partial t} = \nu.\frac{\partial^2 v_2}{\partial y^2}$$

$$v_2 = f(y).\,g(t)$$

$$f.g' = \nu.f''g$$

$$g'/g = \nu.\,f''/f = -\lambda^2$$

$$g' = -\lambda^2 g$$

$$\frac{1}{g}dg = -\lambda^2.dt$$

$$\ln g = -\lambda^2.t$$

$$g = e^{-\lambda^2 t}$$

$$f'' = -\frac{\lambda^2}{\nu}f$$

$$f = A.e^{i\sqrt{\lambda^2/\nu}.y} + B.e^{-i\sqrt{\lambda^2/\nu}.y}$$

$$0 = A + B \rightarrow A = -B$$

$$0 = A.e^{i\sqrt{\ }d} + B.e^{-i\sqrt{\ }d}$$

$$0 = A.(e^{i\sqrt{\ }d} - e^{-i\sqrt{\ }d})$$

$$0 = A.\,\text{sen}\left(\sqrt{\lambda^2/\nu}.\,d\right)$$

$$\sqrt{\lambda^2/\nu}.\,d = n\pi$$

$$\lambda = \sqrt{\lambda^2} = \frac{n\pi}{d}.\sqrt{\nu} \quad \text{con } n\in\mathbb{N}$$

$$v_2(y,t) = \sum_{n=0}^{+\infty} A_n.\,\text{sen}\left(\frac{n\pi}{d}y\right) e^{-\frac{n^2\pi^2}{d^2}.\nu.t}$$

$$v(y,t) = \sum_{n=0}^{+\infty} A_n \cdot \operatorname{sen}\left(\frac{n\pi}{d} y\right) \cdot e^{-\frac{n^2\pi^2\nu}{d^2} t} + \frac{U}{d} y$$

Faltaría determinar los A_n con las condiciones iniciales

$$v(y,0) = 0 = \sum_{n=0}^{\infty} A_n \operatorname{sen}\left(\frac{n\pi y}{d}\right) + \frac{U}{d} y$$