Práctica 3

1. Ecuación indefinida $\longrightarrow F_i + \frac{\partial}{\partial x_j}\overleftrightarrow{\sigma}_{ji} = \rho . a_i$ Continuidad $\frac{d\rho}{dt} + \rho . div(\vec{v}) = 0$

F_i conservativas $\Rightarrow$

$f = -grad(\Phi)$

$F = -\frac{1}{V} . grad\, \Phi$

$-\frac{\partial}{\partial x_j} P . \delta_{ij} = -\frac{\partial P}{\partial x_i}$

a) fluido ideal $\sigma_{ji} = -P . \delta_{ij}$

$F_i - \frac{\partial P}{\partial x_i} = \rho . a_i$

incompresibilidad $div(\vec{v}) = 0 \rightarrow \frac{d\rho}{dt} = 0$

en reposo $\longrightarrow 0 = F_i - \frac{\partial P}{\partial x_i}$

$\vec{F} = grad(P) = \rho . f$

fuerza por unidad de masa

<u>Definiciones</u>

$\vec{F} \equiv \frac{\vec{f}}{V}$

$\vec{f} \equiv \frac{\vec{f}}{m} = -\frac{\vec{\nabla}(\Phi)}{m} = -\vec{\nabla}(\Omega)$

La fuerza tradicional

$\vec{F} = \frac{\vec{f}}{\cancel{V}} . \rho . \frac{\cancel{V}}{m} = -\rho . \vec{\nabla}(\Phi)$

$\rho . f = -\rho . grad(\Omega)$

$0 = -\rho . grad(\Omega) - grad(P)$

$\rho \neq \rho(x, y, z)$

$0 = grad\left(\Omega + \frac{P}{\rho}\right)$

$$\boxed{\Omega + \frac{P}{\rho} = cte}$$

$\Omega \equiv$ potencial por unidad de masa

con $-\vec{\nabla}\Omega = f$ ← fuerza por unidad de masa [aceleración]

b)

entalpía $\underbrace{H = U + P.V}_{\text{definición de entalpía}} \Rightarrow h = \frac{U}{m} + \frac{P.V}{m} = \frac{U}{m} + \frac{P}{\rho}$

$dH = dQ - P.dV + P.dV + V.dP$

$dH = +V.dP$ adiabático

$\frac{dH}{m} = \frac{V}{m} . dP = \frac{dP}{\rho}$

$dh = \frac{1}{\rho} . dP$

$h = \frac{1}{\rho} . P \Rightarrow \Omega + \frac{P}{\rho} = cte.$

$$\boxed{\Omega + h = cte.}$$

c)

$G = U - T.S + P.V$

$dG = dU - T.dS + S.dT + P.dV + V.dP$

$\cancel{dQ} - \cancel{P.dV} - \cancel{dQ} + S.dT + \cancel{P.dV} + V.dP$

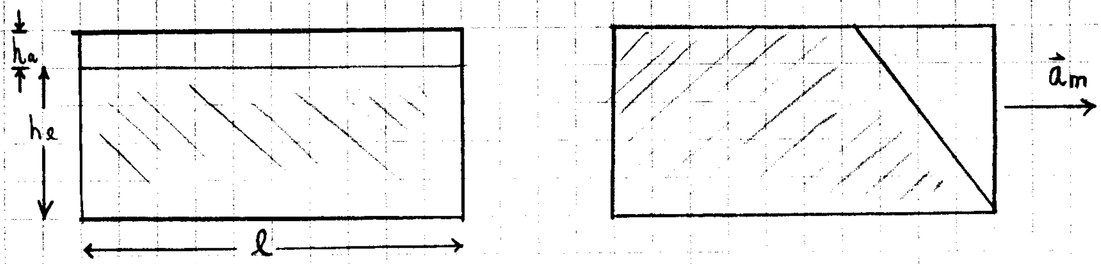
$dG = S.dT + V.dP \rightarrow$ en isotérmicos es $dG = V.dP$

$$\Rightarrow \quad \frac{dG}{m} = \frac{V}{m}.dP = \frac{dP}{\rho} \quad \rightarrow \quad dg = \frac{1}{\rho}.dP \quad \rightarrow \quad g = \frac{P}{\rho}$$

Gibbs específico

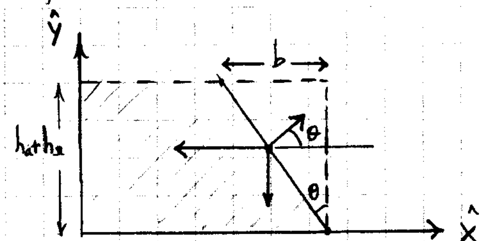
$$\therefore \quad \Omega + \frac{P}{\rho} = cte \quad \Rightarrow \quad \boxed{g + \Omega = cte}$$

2.



Fluido ideal - hidroestática

- Método 1 (sistema No inercial en el tanque)



$$(\vec{F} + \rho\vec{a}) - \vec{\nabla}P = 0$$

$$\rho(\vec{g} + \vec{a}) - \vec{\nabla}P = 0$$

$\hat{x})\quad -\rho a - |\vec{\nabla}P|.\cos\theta = 0$

$\hat{y})\quad -\rho.g - |\vec{\nabla}P|.sen\,\theta = 0$

$$\tan\theta = \frac{sen\,\theta}{\cos\theta} = \frac{\cancel{\rho}.g/|\vec{\nabla}P|}{-\cancel{\rho}.a/|\vec{\nabla}P|} = \frac{-g}{a}$$

; Por la incompresibilidad
área aire inicial = área aire final

$$\tan\theta = \frac{b}{(h_a+h_e)} = \frac{-g}{a}$$

$$h_a.\ell = (h_a+h_e).\frac{b}{2}$$

$$-g\frac{(h_a+h_e)}{a} = \frac{2.h_a.\ell}{(h_a+h_e)}$$

$$\boxed{a_m = \frac{-g(h_a+h_e)^2}{2.h_a.\ell}}$$

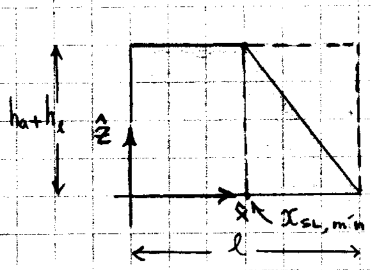
- Método 2

$$\frac{P}{\rho} + \Phi = K \quad \rightarrow \quad \text{meto } f_{NI} = -a\hat{x} \quad \rightarrow \quad \vec{F}_{NI} = -\vec{\nabla}(ax)$$

$\Phi = +g.z$

$-\nabla\Phi = -gz\hat{z} = \vec{f}$ $\xrightarrow{\text{Fuerza x unidad de masa}}$ $\Phi = g.z + a.x$

$$\frac{P}{\rho} + g.z + a.x = K$$



sobre la sup. libre es $P = P_0$ $\rightarrow$

$$\frac{P_0}{\rho} + g.z_{SL} + a.x_{SL} = K$$

$$g z_{SL} + a x_{SL} = K_2 = K - \frac{P_0}{\rho}$$

$x_{SL} = \ell,\ z_{SL} = 0 \rightarrow \quad a.\ell = K_2$

$x_{SL}^{m} = \quad ,\ z_{SL} = h_e + h_a \quad g(h_a+h_e) + a x_{SL}^{m} = K_2$

vínculo

$$h_a . \ell = \frac{(\ell - x_{sL}^m)(h_\ell + h_a)}{2}$$

$$\frac{1}{(x_{sL}^m - \ell)} = -\frac{1}{2}\frac{(h_\ell + h_a)}{h_a . \ell}$$

$$a\,(x_{sL}^m - \ell) = -g\,(h_\ell + h_a)$$

$$a_m = \frac{-g(h_\ell + h_a)}{(x_{sL} - \ell)}$$

$$\boxed{a_m = \frac{g . (h_\ell + h_a)^2}{2 . h_a . \ell}}$$

$$a . \ell = K - \frac{P_0}{\rho} \Rightarrow K = \frac{P_0}{\rho} + a . \ell$$

$$\frac{P}{\rho} + g z + a x = \frac{P_0}{\rho} + a . \ell$$

$$P = \rho\left(\frac{P_0}{\rho} + a(\ell - x) - g . z\right) = P_0 + a . \rho\,(\ell - x) - \rho . g . z$$

$$\boxed{P = P_0 + \rho . g\,\frac{(h_\ell + h_a)^2}{2 \ell\, h_a}(\ell - x) - \rho g z}$$

Distribución de presiones

Pared posterior del tanque

$$P = P_0 + \rho . g . \frac{(h_\ell + h_a)^2}{2 \ell\, h_a} . \ell - \rho . g . z$$

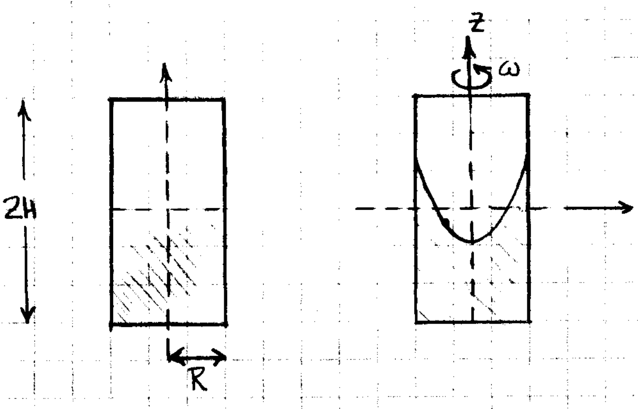
en el origen

$$P(0,0) = P_0 - \rho^2 \ell . a_m$$

$$P(z = h_a + h_\ell, 0) = P_0 + \rho^2 g\,(h_a + h_\ell) - \rho^2 a_m \ell$$

$$\rho^2 \frac{g\,(h_\ell + h_a)^2}{2 h_a \ell}$$

3.



$$\frac{P}{\rho} + \Phi = K$$

$$\frac{P}{\rho} + g . z - \frac{\omega^2 r^2}{2} = K$$

$g(z + H)$ en $z = -H$ es $\Phi_{grav.} = 0$

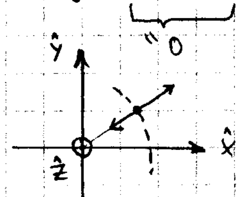
Me paso a un sistema NO inercial en el recipiente en donde vale hidroestática → all r:

Si la $|\vec{a}|$ es $\omega^2 r \Rightarrow -\frac{1}{2}\omega^2 r^2$

$$-\frac{\partial}{\partial r}\left(\quad\right) = \omega^2 r$$

$$-\vec{\nabla}\left(-\frac{\omega^2 r^2}{2}\right) =$$

$$\vec{a}_{NI} = \vec{a}_i - \underbrace{\dot{\vec{\Omega}} \times \vec{r}'}_{=0} \underbrace{- 2 \vec{\Omega} \times \underbrace{\dot{\vec{r}}'\big|_{No\ IN}}_{=0}}_{=0} - \vec{\Omega} \times (\vec{\Omega} \times \vec{r}')$$

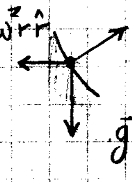


$$-\omega \hat{z} \times (\omega \hat{z} \times r \hat{r})$$

$$-\omega \hat{z} \times (\omega r \hat{\theta})$$

$$+\omega^2 r\,\hat{r}$$

$\vec{a}_{NI} = \omega^2 r \hat{r}$

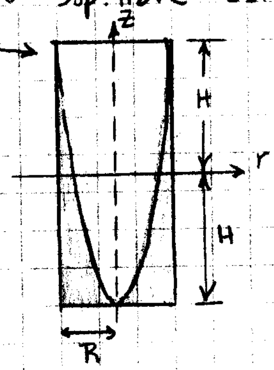


a) En la sup. Libre es $\frac{P_0}{\rho} + g(z_{SL}+H) - \frac{\omega^2 r_{SL}^2}{2} = K$

$$z_{SL} = \frac{1}{g}\left[K + \frac{\omega^2 r_{SL}^2}{2} - \frac{P_0}{\rho}\right] - H$$

$\boxed{\text{En cualquier plano } zr \text{ que cortemos tenemos: } z = z(r^2) \rightarrow \text{ es una parábola}}$

b) Necesito sup. libre así:



considero régimen en el que no desborda por arriba

Sea $z = -H$; $r = 0 \Rightarrow$

Para este régimen tendré

$$0 = \frac{1}{g}\left(K - \frac{P_0}{\rho}\right)$$

$$K = \frac{P_0}{\rho}$$ ← Tengo la sup. libre tocando fondo

$$\frac{P_0}{\rho} + g(z_{SL}+H) - \frac{\omega^2 r^2}{2} = \frac{P_0}{\rho}$$

$$g(z_{SL}+H) - \frac{\omega^2 r_{SL}^2}{2} = 0$$

$$g z_{SL} = \frac{\omega^2 r_{SL}^2}{2} - H.g$$

$$z_{SL} = \frac{\omega^2 r_{SL}^2}{2g} - H$$

$$V_\ell = H.\pi R^2 \quad \text{(inicial)}$$

$$V_\ell = \int_0^{2\pi}\int_0^{R}\int_{-H}^{\frac{\omega^2 r^2}{2g}-H} 1 r\, dz.dr.d\theta$$

$$H\pi R^2 = \int_0^{2\pi}\int_0^{R} \frac{\omega^2 r^3}{2g}\, dr.d\theta =$$

$$H.\pi \cancel{R^2} = \cancel{2}\pi . \frac{\omega^2 R^{\cancel{4}\,2}}{4\cancel{8}g}$$

$$\omega^2 = 4g\frac{H}{R^2}$$

El límite refleja que no se a que altura llegará el líquido

$$\boxed{\omega_F = \frac{2}{R}\sqrt{gH}}$$

único ω que conserva el volumen con fondo seco

si $r_{SL} = R \rightarrow$

$$z_{SL} = \frac{\omega^2 R^2}{2g} - H$$

ahora será:

(ω permitidos) que no producen desborde y tienen fondo seco

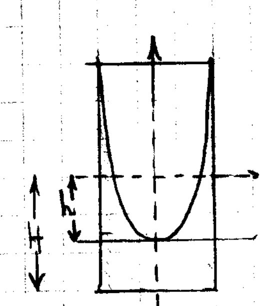
$$\underbrace{\frac{\omega^2 R^2}{2g}}_{>0} - H \leq H$$

$$\omega^2 \leq \frac{2g\,2H}{R^2}$$

$$\omega \leq \sqrt{\frac{4gH}{R^2}} = \frac{2\sqrt{gH}}{R}$$

$\Rightarrow$ si $\omega > \frac{2}{R}\sqrt{gH}$ hay desborde

c)



$r_{SL} = 0 \rightarrow z_{SL} = -h$

$$\frac{P_0}{\rho} + g(-h+H) = K$$

$$\frac{P_0}{\rho} + g(z_{SL}+H) - \frac{\omega^2 r_{SL}^2}{2} = K$$

$$\frac{\omega_F^2 R^2}{4g} = H$$

$$z = \frac{K}{g} - \frac{P_0}{\rho g} + \frac{\omega^2 r^2}{2g} - H$$

$$\cancel{\frac{P_0}{\rho}} + gz + \cancel{gH} - \frac{\omega^2 r^2}{2} = \cancel{\frac{P_0}{\rho}} - gh + \cancel{gH}$$

$$z = \frac{\omega^2 r^2}{2g} - h$$

$$\sqrt{\frac{2g}{R^2}(H+h)} = \omega_D$$

$$\boxed{\omega_D = \frac{\sqrt{2g(H+h)}}{R}}$$

condición de desborde $\rightarrow z(r=R) = H$

$$\frac{\omega^2 R^2}{2g} - h = H$$

$$\omega_F = \frac{\sqrt{4gH}}{R}$$

$$\cancel{2}g\frac{(H+h)}{\cancel{R^2}} < \frac{\overset{2}{\cancel{4}}gH}{\cancel{R^2}}$$

$$H+h < 2H$$

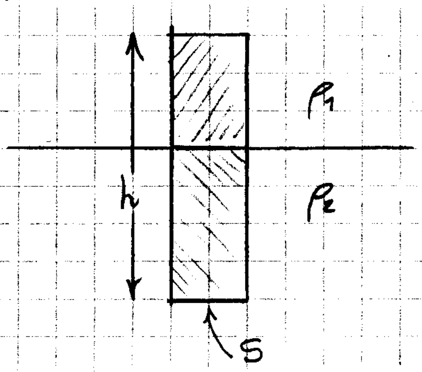
$h < H \Rightarrow \omega_F > \omega_D \Rightarrow$ Al ir subiendo ω desde cero desbordará primero

$$\omega_D = \frac{\sqrt{4gH}}{R}\left(\frac{1}{2}+\frac{h}{2H}\right)^{1/2} \qquad \text{si } \frac{h}{H} = \xi$$

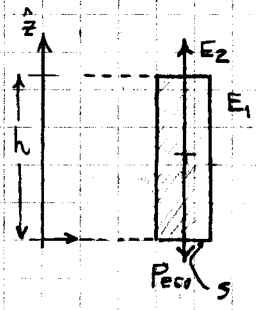
$$\omega_F = \frac{\sqrt{4gH}}{R}$$

El líquido comienza a desbordar con $\omega_D = \sqrt{\dfrac{2 \cdot 9{,}8\,\frac{m}{s^2}\,(0{,}075m + h)}{0{,}05m}}$

4.



$V = S.h$



$V_2 = S.h_2$
$V_1 = S.h_1$

$V = V_2 + V_1 = S(h_2 + h_1)$

$h = h_1 + h_2$

* Método A

$E_2 = \rho_2 g h_2 . S$ con $\vec{E}_2 = E_2 \hat{z}$

$E_1 = \rho_1 g h_1 . S$ con $\vec{E}_1 = +E_1 \hat{z}$

equilibrio $\rightsquigarrow$ $\rho . \cancel{g} . h . \cancel{S} = (\rho_2 h_2 + \rho_1 h_1) \cancel{S} \cancel{g}$

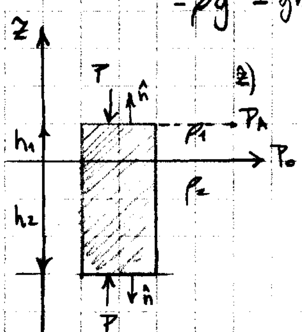
$(\rho \; h_1 + h_2 \; \rho) = \rho_2 h_2 + \rho_1 h_1$

$-\rho_1 h_1 + \rho \; h_1 = \rho_2 h_2 - h_2 \rho$

$$\boxed{\frac{h_1}{h_2} = \left(\frac{\rho_2 - \rho}{\rho - \rho_1}\right)}$$

* Método: B

$-\rho g - \text{grad}_z(P) =$



z) $\frac{\partial P}{\partial z} = -\rho g$

$P - P_0 = -\rho . g (z - z_0)$

$P = P_0 - \rho . g z - \rho . g \underset{\shortparallel 0}{z_0}$

$P = P_0 - \rho . g . z$

$\vec{E} = -\int p \, \hat{n} \, dS$

$\vec{E} = -(\hat{z}) \left(P_0 - \rho_1 g h_1 \right) . S$

$\vec{E} = -(-\hat{z}) \left(P_0 + \rho_2 g h_2 \right) . S$

$\vec{E} + \vec{P}_{eso} = 0$

$\hat{z}$) $-\rho_{cilindro} \, g \, S . (h_1 + h_2) = (P_0 - \rho_1 g h_1) S - (p_0 + \rho_2 g h_2) S$

$-\rho_c \cancel{g} (h_1 + h_2) = \cancel{P_0} - \rho_1 \cancel{g} h_1 - \cancel{P_0} - \rho_2 \cancel{g} h_2$

$h_1 (\rho_1 - \rho_c) = h_2 (\rho_c - \rho_2)$

$\frac{h_1}{h_2} = \frac{\rho_c - \rho_2}{\rho_1 - \rho_c}$

$$\boxed{\frac{h_1}{h_2} = \frac{(\rho_2 - \rho)}{(\rho - \rho_1)}}$$

El cuerpo se halla en equilibrio $\Rightarrow$

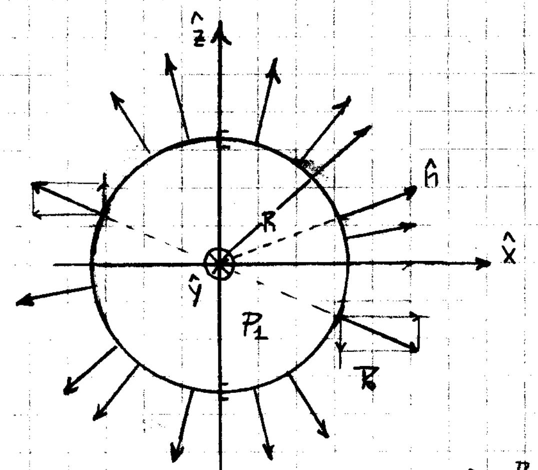
$-\rho g - \text{grad}(P) = 0$

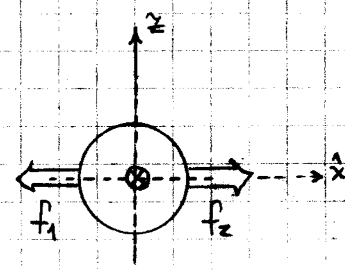
empuje por unidad de volumen

z) $-\rho g = \text{grad}(P) = e$

$-\rho g = -\oint_S p \, \hat{z} \, dS = e$

6.





$$\vec{F} = -\oint p\,\hat{n}\,dS = 0$$

→ Por simetría. Dados que las fuerzas f_1, f_2 son iguales y opuestas (pues tiran de cada lado 8 caballos) no habrá desplazamiento del centro de masa. Tampoco se producirán torques.

$$|\vec{f}_1| = |\vec{f}_2| \equiv f$$

La fuerza necesaria para separar los hemisferios es $f = -F$ que hace la atmósfera sobre un hemisferio solo.

$$f_x = -\int_{-\pi/2}^{\pi/2}\int_0^{\pi}(P_1-P_0)\,R^2\,\text{sen}\,\theta\,.\,d\theta\,.\,d\varphi\,.\,\cos\varphi\,.\,\text{sen}\,\theta\;\hat{x}$$

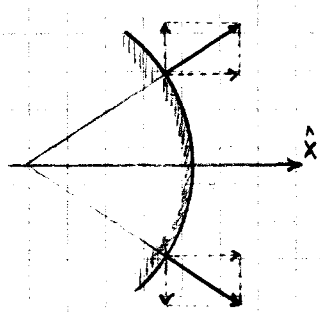
$$-\int_0^{\pi}(P_1-P_0)R^2\,\text{sen}^2\theta\,d\theta\,.\,\text{sen}\,\varphi\,\Big|_{-\pi/2}^{\pi/2} = -(P_1-P_0)\,R^2\int_0^{\pi}\text{sen}^2\theta\,.\,d\theta\,.\,(1+1)$$

$$= -(P_1-P_0)\frac{\pi}{\cancel{2}}\,R^2\,\cancel{2}$$

Fuerza sobre un hemisferio

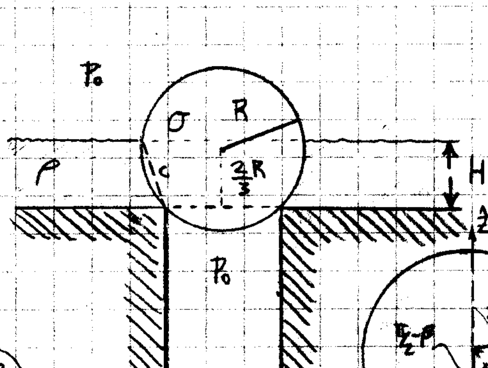
$$\boxed{f = \Delta p\,.\,\pi R^2}$$

Fuerza que debió haber hecho cada tiro de caballos para separar los hemisferios



Las componentes en $\hat{y}, \hat{z}$ se cancelan por simetría → solo tengo una componente en $\hat{x}$

7.



Equilibrio hidrostático

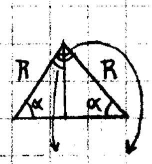
a) El empuje será igual al peso del líquido desalojado.

$$E = \sigma\,.\int_0^{2\pi}\int_{\pi/2+\beta}\frac{R^3}{3}\,\text{sen}\,\theta\,.\,d\theta\,.\,d\varphi$$

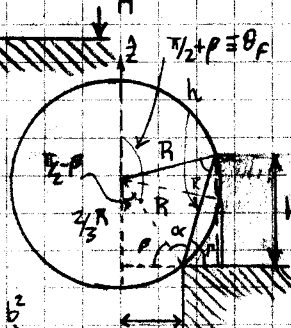
$$H = \text{sen}\,\Gamma\,.\,h \quad (h = \cos\alpha\,.\,R\,.\,2) \quad = \text{sen}\,\Gamma\,.\cos\alpha\,.\,R$$

$$\alpha + \beta + \Gamma = \pi$$

$$\tan\beta = \frac{2/3\,R}{\sqrt{5}/3\,R} = \frac{2}{\sqrt{5}} \rightarrow \beta = \text{atan}\left(\frac{2}{\sqrt{5}}\right)$$



$\frac{\pi}{2} - \alpha$ $\quad\pi - 2\alpha$

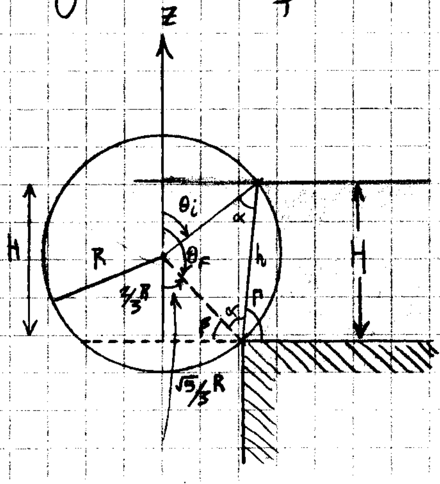


$$R^2 = \left(\frac{2}{3}\right)^2 R^2 + b^2$$

$$R^2\left(1 - \frac{4}{9}\right) = \frac{5}{9}R^2 = b^2$$

$$\Rightarrow \theta_i = \pi - \left(\frac{\pi}{2}+\beta\right) - [\pi - 2\alpha]$$
$$= - \quad + 2\alpha = -\frac{\pi}{2} - \beta + 2\alpha$$

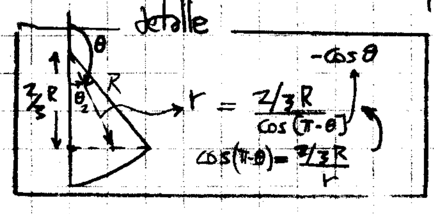
Hagamos un esquema más claro ahora ▼. Caso (A) $H \le \frac{5}{3}R$ *



$$H = R\cos\theta_i + \frac{2}{3}R \rightarrow \frac{H}{R} - \frac{2}{3} = \cos\theta_i$$

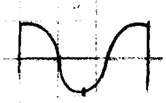
$$R\cos(\theta_f - \pi) = \frac{2}{3}R$$

$$-\cos(\theta_f) = \frac{2}{3} \qquad -\frac{2}{3} = \cos\theta_f$$



$$-\rho g 2\pi \int_{\theta_f}^{\pi/2} \frac{4}{9}\left[\left(\frac{2}{3}R\right)^3 - R^3\right] \frac{\operatorname{sen}\theta}{\cos^2\theta}\, d\theta$$

$$-\frac{19}{27}R^3 \int_{\theta_f}^{\pi/2} \frac{\operatorname{sen}\theta}{\cos^2\theta}\, d\theta$$



$$\underbrace{\left(V_{\substack{\text{sector}\\\text{esférico}\\\text{superior}}} - V_{\substack{\text{calota}\\\text{esférica}\\\text{superior}}}\right.}_{(1)} + V_{\substack{\text{sector}\\\text{esférico}\\\text{inferior}}} - \left.V_{\substack{\text{calota}\\\text{esférica}\\\text{inferior}}}\right)$$

$$E = \rho g \int_0^{2\pi}\int_{\theta_i}^{\theta_f}\int_0^R r^2 \operatorname{sen}\theta\, dr\, d\theta\, d\varphi$$

$$= \rho g \frac{R^3}{3} 2\pi \left(-\cos\theta \Big|_{\theta_i}^{\theta_f}\right)$$

$$\cos\theta_i - \cos\theta_f$$

$$= \rho g \frac{\cancel{R^3}^{\,2}}{3} 2\pi \cdot \frac{H}{\cancel{R}}$$

$$(1) = \frac{2}{3}\pi R^2\left(2R - H - \frac{1}{3}R\right) - \frac{1}{3}\pi\left(2R - H - \frac{1}{3}R\right)^2 \left(3R - \left(2R - H - \frac{1}{3}R\right)\right)$$

$$(1) = \frac{2}{3}\pi R^2\left(\frac{5R}{3} - H\right) - \frac{\pi}{3}\left(\frac{5R}{3} - H\right)^2\left(\frac{4R}{3} + H\right)$$

$$(2) = \frac{2}{3}\pi R^2\left(\frac{1}{3}R\right) - \frac{1}{3}\pi\left(\frac{1}{3}R\right)^2\left(3R - \frac{1}{3}R\right)$$

CA

$$\boxed{\begin{aligned} &\frac{2\pi R^2}{3} - \frac{20R^2\pi}{9\cdot 3} + \frac{4RH\pi}{3\cdot 3} - \frac{5RH\pi}{3\cdot 3} + \frac{H^2\pi}{3} \\ &-\frac{2R^2\pi}{27} - \frac{RH\pi}{9} + \frac{H^2\pi}{3} \\ &\frac{\pi}{3}\left(-\frac{2R^2}{9} - \frac{RH}{3} + H^2\right)\end{aligned}}$$

$$\frac{2}{3}\pi R^2\left(\frac{5}{3}R - H\right) - \frac{\pi}{3}\left(\frac{5}{3}R - H\right)^2\left(\frac{4}{3}R + H\right) + \frac{2}{3}\pi R^2\left(\frac{R}{3}\right) - \frac{\pi}{3}\left(\frac{R}{3}\right)^2\left(\frac{8}{3}R\right)$$

$$\left(\frac{5}{3}R - H\right)\underbrace{\left[\frac{2\pi R^2}{3} - \frac{\pi}{3}\left(\frac{5R}{3} - H\right)\left(\frac{4R}{3} + H\right)\right]}_{} + \frac{2}{9}\pi R^3 - \frac{8}{81}\pi R^3$$

Ver CA

$$E = \rho g \pi R^2 \frac{2}{3} H + \rho g\left(\frac{5}{3}R - H\right)\left[\qquad\qquad\right] + \rho g \frac{2}{9}\pi R^3 - \rho g \frac{8}{81}\pi R^3$$

$$E = \rho g \pi R^2 \frac{2}{3}H + \rho g \frac{\pi}{3}\left(-\frac{10R^3}{27} - \frac{5R^2H}{9} + \frac{5RH^2}{3} + \frac{2R^2H}{9} + \frac{RH^2}{3} - H^3\right)$$

$$+ \rho g \frac{\pi}{3}\left(-\frac{10R^3}{27} - \frac{R^2H}{3} + 2RH^2 - H^3\right)$$

$$\rho g\frac{\pi}{3}\cdot\frac{5}{3}R^2H \underbrace{- \rho g \frac{\pi}{3}\frac{10R^3}{27}} + \rho g \frac{\pi}{3} 2RH^2 - \rho g\frac{\pi}{3}H^3 \underbrace{+ \rho g\frac{\pi}{3}\cdot\frac{2R^3}{3}} \underbrace{- \rho g \frac{\pi}{3}\cdot\frac{8R^3}{27}}$$

$$E = \rho g \frac{\pi}{3}(0) + \rho g\frac{\pi}{3}\left(\frac{5}{3}R^2H + 2RH^2 - H^3\right) - \rho g \frac{\pi}{3}\left(\frac{\sqrt{5}R}{3}\right)^2 \frac{2}{3}R + \sigma g \frac{\pi}{3}\left(\frac{\sqrt{5}R}{3}\right)^2\frac{2}{3}R$$

$$E = \rho g \frac{\pi}{3}\left(\frac{5}{3}R^2H + 2RH^2 - H^3 \cdot \frac{2\sqrt{5}}{9}R^3\right) \qquad \rho g \frac{\pi}{3}\frac{10R^3}{9}$$

$$\frac{dE}{dH} = \rho g \frac{\pi}{3}\left(\frac{5}{3}R^2 + 4RH - 3H^2\right) \Rightarrow \frac{dE}{dH} = 0$$

$$H^2 - \frac{4}{3}RH - \frac{5}{9}R^2 = 0$$

Sea $\text{Peso} = \sigma g \frac{4}{3}\pi R^3 \rightarrow$ Habrá obturación siempre que:

$$\sigma g \frac{4}{3}\pi R^3 \geq E \quad \rightarrow \text{sea} \quad \sigma = \alpha\rho \quad y \quad \begin{array}{l} E = E_{MAX}(H_1) \; y \\ E = E_{MAX}(H_2) \end{array}$$

$$-\alpha\rho g \, \pi R^3 + \alpha\rho g \frac{4}{3}\pi R^3 \geq \rho g \frac{\pi}{3}\left(-\frac{5}{3}\frac{R^3}{3} + \frac{2R^3}{9} + \frac{R^3}{27} - \frac{5}{9}R^3\right) = \rho g \frac{\pi}{3}\left(-\frac{23}{27}\right)$$

$$\alpha \cancel{\rho g} \frac{\cancel{4}}{\cancel{3}}\cancel{\pi R^3} \geq \cancel{\rho g}\frac{\cancel{\pi}}{\cancel{3}}\left(-\frac{8}{21}\right)R^3$$

$$-\alpha\rho g \frac{10}{27}\pi R^3 + \alpha\rho g \frac{4}{3}\pi R^3 \geq \rho g \frac{\pi}{3}\left(\frac{5}{3}R^2\frac{5}{3}R + 2R\frac{5^2}{3^2}R^2 - \left(\frac{5}{3}\right)^3 R^3 - \frac{10}{9}R^3\right) =$$

$$\left(\frac{26}{27}\right) \geq \quad \frac{70}{27}$$

8.

$$p = K\left(\frac{\rho-\rho_0}{\rho_0}\right)$$

a) $\vec{f} = (0,0,g)$

$$\rho\vec{f} + \vec{\nabla}p = 0$$

$$*(\hat{z}) \quad -\rho g + K\frac{1}{\rho_0}\frac{\partial\rho}{\partial z} = 0$$

$$-\frac{\rho_0 g}{K}\rho + \frac{\partial\rho}{\partial z} = 0$$

$$\frac{\partial\rho}{\partial z} = \rho\frac{\rho_0 g}{K}$$

$$\int\frac{1}{\rho}d\rho = \int\frac{\rho_0 g}{K}dz$$

$$\ln(\rho/\rho_0) = \frac{\rho_0 g}{K}z$$

$$\boxed{p = \frac{K}{\cancel{\rho_0}}\cancel{\rho_0}\left(e^{\frac{\rho_0 g}{K}z} - 1\right)} \leftarrow \boxed{\rho = \rho_0 e^{\frac{\rho_0 g}{K}z}}$$

b) si $\rho = \rho_0 \Rightarrow$ agua incompresible $\Rightarrow$

$$-\rho_0 g + \frac{\partial P}{\partial z} = 0$$

$$\int dP = \int \rho_0 g z$$

$$\boxed{P = P_0 + \rho_0 g z}$$

$$\boxed{\rho = \rho_0}$$

c) $z = -1000\ m \rightarrow$

incompresible $\quad p = K\left(e^{-\rho_0 g\frac{1000m}{K}} - 1\right)$

Compresible $\quad P = P_0 + (-\rho_0 . g . 1000\ m)$

con $K = 2 10^9\ N/m^2$

$\rho_0 = 1.10^3\ Kg/m^3 \qquad P_0 =$

$$P = 2{,}10^9.(-0{,}004888) = -9{,}78 . 10^6\ \frac{N}{m^2}$$

$$P = P_0 - 9{,}8.10^6\ \frac{N}{m^2} = 1{,}01\ 10^5\frac{N}{m^2} - 9{,}8\ 10^6\frac{N}{m^2} = -9{,}7.10^6\frac{N}{m^2}$$

presión al nivel del mar

$$\Delta = \frac{P_\rho}{P_{\rho_0}} = \frac{-9{,}78 . 10^6\ N/m^2}{-9{,}70\ 10^6\ N/m^2} = 1{,}008$$

$$\Delta = 0{,}8\ \%$$

$\boxed{\text{La diferencia es menor al } 1\%}$

9.

$$P = \frac{\rho}{\mu} R.T \longleftarrow \text{gas perfecto} \ , \ \text{con} \ T, \rho = T, \rho(z)$$

reposo en un campo gravitatorio $\Rightarrow$ $\hat{z})$ $\quad -\rho . g - \frac{\partial}{\partial z}\left(\frac{\rho}{\mu} R.T\right) = 0$

Ⓐ $\rho . g + \frac{RT}{\mu}\frac{\partial \rho}{\partial z} + \frac{\rho R}{\mu}\frac{\partial T}{\partial z} = 0$

o equivalentemente

Ⓑ $+\frac{\mu g}{RT} P + \frac{\partial}{\partial z} P = 0 \quad \Rightarrow$ resolvemos Ⓑ aquí:

$$\frac{\partial P}{\partial z} = -\frac{\mu g}{RT(z)} P$$

$$\int \frac{dP}{P} = -\frac{\mu g}{R}\int \frac{dz}{T(z)}$$

$$\ln\left(\frac{P}{P_0}\right) = -\frac{\mu g}{R}\int_0^z \frac{d\xi}{T(\xi)}$$

$$\boxed{P = P_0 . e^{-\frac{\mu g}{R}\int_0^z d\xi / T(\xi)}}$$

$$\frac{\mu P_0}{R} . e^{-\frac{\mu g}{R}\int_0^z T^{-1}(\xi) . d\xi} = \rho . T$$

$$-\frac{\mu g}{R} \cdot \frac{\mu P_0}{R} . e^{-\frac{\mu g}{R}\int_0^z d\xi / T(\xi)} \cdot \frac{d}{dz}\int_0^z \frac{d\xi}{T(\xi)} = \frac{\partial \rho}{\partial z} . T + \frac{\partial T}{\partial z} . \rho$$

$$\underbrace{-\frac{g P_0 \mu}{R} . e^{-\frac{\mu g}{R}\int_0^z d\xi / T(\xi)} \frac{\partial}{\partial z}\left(\int_0^z \frac{d\xi}{T(\xi)}\right)}_{} = \frac{R}{\mu} T \frac{\partial \rho}{\partial z} + \rho . \frac{R}{\mu}\frac{\partial T}{\partial z}$$

$$\longrightarrow \quad -\frac{\mu g}{R} P \cdot \frac{\partial}{\partial z}\int_0^z \frac{d\xi}{T(\xi)} = -\frac{\mu g P}{R . T(z)}$$

Pero $\rho g = \frac{\mu . g}{R . T} P_0 . e^{-\frac{\mu g}{R}\int_0^z d\xi / T(\xi)} \quad \Rightarrow$

Si $T = T(x, y, z) \Rightarrow$

$$\hat{x}) \quad \frac{\partial P}{\partial x} = P_0 . e^{-\frac{\mu g}{R}\int_0^z d\xi / T(x, y, \xi)} \cdot -\frac{\mu g}{R} \underbrace{\frac{\partial}{\partial x}\int_0^z d\xi / T(x, y, \xi)}_{} = 0$$

$$\int_0^z -\frac{1}{T^2(x, y, \xi)} \cdot \frac{\partial T}{\partial x}(x, y, \xi) . d\xi$$

$\Rightarrow$ solo es nulo si $\frac{\partial T}{\partial x} = 0$

NOTA

$$\boxed{\frac{\partial}{\partial z}\left(\int_0^z \frac{1}{T(\xi)} . d\xi\right) = \frac{1}{T(z)}}$$

TFC

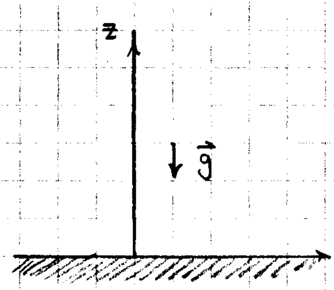
$\frac{\partial P}{\partial y} = 0 \rightarrow$ lleva idénticamente a:

$$\int_0^z -\frac{1}{T^2(x, y, \xi)} \frac{\partial T}{\partial y}(x, y, \xi) . d\xi = 0$$

el cual es nulo solo si $\frac{\partial T}{\partial y} = 0$

$\Rightarrow$ $\boxed{\text{con } T = T(x, y, z) \ \nexists \ \text{solución hidrostática}}$

10.



$$\vec{f} = (0,0,-g)$$

$$p \cdot \rho^{-\gamma} = C$$

$$p = C \cdot \rho^{\gamma}$$

$$\rho \cdot \vec{f} + \operatorname{grad}(p) = 0$$ ⟵ Atmósfera en reposo

$\Rightarrow$ en $\hat{z}$

$$-\rho \cdot g + \frac{\partial}{\partial z} C \cdot \rho^{\gamma} = 0$$

$$\frac{\partial}{\partial z}\rho^{\gamma} = \frac{\rho}{C} g$$

$$\gamma \cdot \rho^{\gamma-1} \frac{\partial \rho}{\partial z} = \rho \cdot g \cdot \frac{1}{C}$$

$$\rho^{\gamma-2} \frac{\partial \rho}{\partial z} = \frac{g}{C \cdot \gamma}$$

$$\int \rho^{\gamma-2} d\rho = \int \frac{1}{C \cdot \gamma} \cdot g \cdot dz$$

$$\frac{1}{(\gamma-1)}\left[\rho^{\gamma-1} - \rho_0^{\gamma-1}\right] = \frac{g}{C\,\gamma} z$$

$$\boxed{\rho^{\gamma-1} = \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{g}{C}\right) z + \rho_0^{\gamma-1}}$$

Usando ecuación del gas perfecto $\Leftarrow$

$$\boxed{p = C \cdot \left(\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{g}{C}\right) z + \rho_0^{\gamma-1}\right)^{\frac{1}{\gamma-1}\gamma}}$$

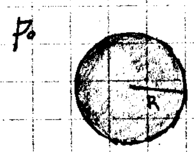
$$T = \frac{p \cdot \mu}{\rho \cdot R} \Rightarrow$$

$$T = \frac{\mu}{R} \frac{C\left[\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{g}{C}\right) z + \rho_0^{\gamma-1}\right]^{\frac{\gamma}{\gamma-1}}}{\left[\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{g}{C}\right) z + \rho_0^{\gamma-1}\right]} = \frac{\mu C}{R}\left(\left[\frac{\gamma-1}{\gamma}\right]\left(\frac{g}{C}\right) z + \rho_0^{\gamma-1}\right)^{\frac{\cancel{\gamma-\gamma}+1}{\gamma-1}}$$

$$\boxed{T = \frac{\mu C}{R}\left(\left[\frac{\gamma-1}{\gamma}\right]\frac{1}{C} g\, z + \rho_0^{\gamma-1}\right)^{\frac{1}{\gamma-1}}}$$

11.

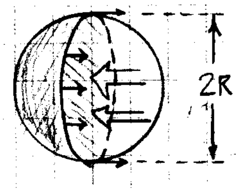
$$R = \frac{10}{2}\overbrace{m\mu m}^{\text{micrómetro?}} = 5.10^{-6}\,m$$



↑gota

tensión superficial $0{,}5\,\frac{N}{m} = \sigma = \left.\frac{f}{\ell}\right]$ → tensión por unidad de longitud

Suponemos que al estar las gotas en equilibrio la σ está actuando de modo de contener la expansión del líquido contra la presión externa



Con la gota en equilibrio en cada lonja circular vale:

$$\sigma . 2\pi \cancel{R} = -\Delta p . \pi . R^{\cancel{2}} \rightarrow \boxed{-\Delta p = \frac{2\sigma}{R}}$$

↑gota

↑burbuja

$$f . d\ell = -\Delta p \; dV$$

$$\sigma . \cancel{2\pi r} . \cancel{dr} = -\Delta p . \cancel{4\pi} r^{\cancel{2}} . \cancel{dr}$$

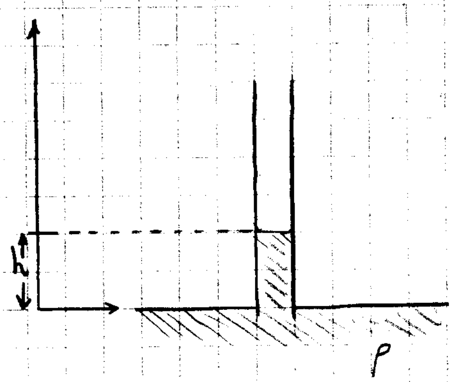
$$\boxed{-\Delta p = \frac{\sigma}{2R}}$$

↑burbuja

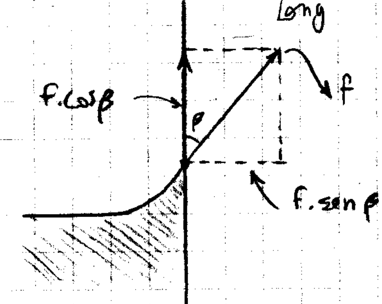
$$V = \frac{4}{3}\pi r^3$$

$$dV = 4\pi r^2 dr$$

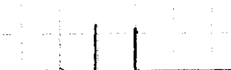
12.



Tensión superficial $\sigma = \frac{f}{Long} \Rightarrow \sigma\,Long = f$



Si $\nexists$ tensión superficial $\Rightarrow$



misma Patm.

La altura del líquido dentro del tubo abierto y fuera es la misma. Pero, como asciende hay una fuerza que lo sostiene cuya componente vertical tiene el mismo valor absoluto que el peso de la columna de líquido ascendido. Entonces:

$$f . \cos\beta = \rho . h . \pi . \left(\frac{d}{2}\right)^2 . g = \sigma . \overbrace{\pi . d}^{\text{Longitud del contacto}} . \cos\beta$$

(circunferencia del contacto líquido-pared tubo)

$$\boxed{h = \frac{\sigma . \cos\beta . 4}{\rho . g . d}}$$

13. hidrostática $\rightarrow$ $\vec{F} - \vec{\nabla} p = 0$; si nos informan $\vec{F}_v = \dfrac{\vec{F}}{V} = \vec{J} \times \vec{B}$

$$\Rightarrow \int \vec{J} \times \vec{B} \cdot dV - \int \vec{\nabla} p \cdot dV = 0$$

$$\Rightarrow \boxed{\vec{J} \times \vec{B} = \text{grad}(p)}$$

; pero supongamos que partimos de la fuerza de Lorentz

$$\vec{F} = q \cdot \left(\frac{\vec{V}}{c} \times \vec{B}\right) \qquad \text{pero} \qquad \vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

hidrostática $\int \vec{F} dV = \int \rho \left(\frac{\vec{V}}{c} \times \vec{B}\right) dV$

$$\int \vec{\nabla} p \, dV = \int_{i} \vec{J} \times \vec{B} \cdot dV \longrightarrow \boxed{\vec{\nabla} p = \vec{J} \times \vec{B}}$$

en estáticos $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

$$\text{grad}(p) = \frac{1}{\mu_0} (\vec{\nabla} \times \vec{B}) \times \vec{B}$$

$$\text{grad}(p) = \frac{1}{\mu_0} \left[(\vec{B} \cdot \text{grad}) \vec{B} - \frac{1}{2} \vec{\nabla}(B^2) \right]$$

de la práctica 1 ej. 3

$$\boxed{\begin{aligned} &\text{XVIII)} \quad \vec{\nabla}(\vec{U} \cdot \vec{U}) = 2(\vec{U} \cdot \vec{\nabla})\vec{U} + 2\, \vec{U} \times (\vec{\nabla} \times \vec{U}) \\ &(\vec{\nabla} \times \vec{U}) \times \vec{U} = -\frac{\vec{\nabla}}{2}(U^2) + (\vec{U} \cdot \vec{\nabla})\vec{U} \end{aligned}}$$

$$\boxed{\text{grad}\left(p - \frac{B^2}{2\mu_0}\right) = \frac{1}{\mu_0} (\vec{B} \cdot \text{grad}) \vec{B}}$$